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FOCUSING OF AN ELECTRON STREAM WITH RADIO-FREQUENCY FIELDS

by

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SUMMARY

A stream of electrons has been focused solely by the fields of a slow ($v_p < c$) electromagnetic wave.

An analysis was obtained by transforming to the wave frame in which the equations of motion of electrostatic periodic focusing are valid. Using the well known results for the static case, and retransforming to the laboratory frame, focusing behavior and design information (circuit size, frequency, power for a given stream) were obtained. A second but more appropriate analysis was made by transforming to the average electron frame in order to predict both fast and slow wave results; for the axially symmetric modes considered, the slow wave focusing requires orders of magnitude less power than for fast waves and has a more favorable field shape.

Several experiments were performed using a helix as the slow-waveguide. The stream was launched into the helix and was collected both on the helix and by a collector at the opposite end. Typically 10 to 20 per cent of the current would be transmitted with no wave, rising linearly with wave power to a best transmission of over 80 per cent. The best results were obtained with the wave velocity in the same direction and greater than the average velocity of the stream; use of oppositely directed wave and stream velocities gave poorest results. At best, a 1 watt stream (roughly 1 ma at 1000 volts) was focused by about 10 watts of rf wave power at 1000 mc. These results are in rough agreement with the theory. For a given injected stream there appears to be a maximum rf power beyond which focusing decreases; this effect may be due to either a strong localized lens effect at the point of stream injection or to rf voltages large compared with the average stream voltage.

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I. INTRODUCTION AND HISTORY

It is well known that a cylindrical stream of electrons can be focused by static electric or magnetic fields. The fields used may be uniform or periodic in the axial direction. The theory of axially periodic static electric fields for focusing a stream has been developed, for example, by Clogston and Heffner (1954) and by Tien (1954).

Focusing fields with time dependence as well as axial dependence have also been investigated theoretically by several authors dealing with fast waves, $v_p > c$. Weibel and Clark (1958) determined theoretically and verified experimentally that one type of time-dependent field TE_{01} mode in a cylindrical guide at cutoff would focus a stream of electrons. They also showed that a TM_{01} mode in cylindrical guide should focus a stream of electrons traveling in the axial direction. Their approach was that of particle dynamics; the magnetic fields necessitated the use of a velocity dependent potential for the Hamiltonian. The result of this velocity dependence was that a time dependence occurred in the solution and further assumptions were required to yield an approximate solution. Boot, Self and R-Shersby-Harvie (1958) analyzed the plasma problem by treating the plasma as a compressible dielectric fluid and using the methods of hydrodynamics. In a treatment such as this, one loses insight into the actual particle motion caused by the confining fields. The special case of fast waves at cutoff in a cylindrical guide was treated. Because of the configuration of the electromagnetic fields for these fast waves the potential well formed does not extend to the walls of the guide; i.e., all of the power flowing in the guide may not be used for focusing of the stream.

The fields produced by certain structures propagating of electromagnetic waves are periodic in time and space and are quite similar in shape to the fields "seen" by an electron stream in periodic electrostatic focusing. For a wave with phase velocity greater than or equal to the speed of light and a slow stream, the interaction corresponds to electrons moving very rapidly through a series of electrostatic lenses. For a given power in the wave, P, it would appear that the focusing effect of the energy, $W = P/v_g$ (v_g = group velocity), could be increased if it were possible to use small v_g. It would also be desirable to know the behavior if the wave and electron stream were to travel at more nearly the same velocity. One then asks whether slowly traveling waves, $v_g < c$ and $0_p < v_p$, such as produced by any one of numerous delay lines or slow-wave circuits, could not also be used to focus a gas of electrons (the electrons need not drift).

The initial work at the University of California, Berkeley, by Birdsall and Lichtenberg (1959) with a slow wave (on a helix) achieved apparent confinement of a mercury plasma. These results were not conclusive. However, it was felt that the slow-wave focusing had sufficient merit to warrant an unambiguous result as might be obtained with an electron-stream-focusing experiment.

A helix offers several advantages over other structures that might be used. The helix slow-wave circuit is simple in construction and has a high impedance; i.e., large electric fields exist for a given radio frequency power on the helix. The helix also has maximum values of axial and transverse fields at the helix and can easily be made to have quite small phase velocity $v_p \ll c$. This report is concerned with the use of these slow-traveling waves for focusing a cylindrical stream of electrons. The effects of thermal velocities in the electron stream are neglected.

The work presented was performed in 1960 and reported on briefly (Birdsall and Rayfield, (1960); further comment was published in 1961 (Birdsall and Rayfield, 1961).

The analysis is presented in Section II. The known focusing behavior in state-periodic-electrostatic lenses will be employed to obtain the focusing due to a traveling wave, by transforming to the wave frame. A second analysis is made by transforming the electron frame and approximating the fields in order to compare fast- and slow-wave focusing. The experiments are given in Section III and a discussion in Section IV.

II. ANALYSIS

2.01. Fields of a Helix Slow-Wave Circuit

It is assumed that the space-charge of the stream alters the fields of the slow-wave structure very little so that the vacuum fields may be used in the equation of motion. The fields of interest are those inside a helix of radius a, taken to be a sheath helix. The vacuum fields within a sheath helix have been given, for example, by Pierce (1950), as

$$E_z = E_z\,(0) I_0(\gamma r)\, e^{j(\omega t - \beta z)} \tag{1}$$

$$E_r = j\frac{\beta}{\gamma}\, E_z\,(0)\, I_1(\gamma r)\, e^{j(\omega t - \beta z)} \tag{2}$$

$$E_\phi = E_z \frac{I_0(\gamma a)}{I_1(\gamma a)} \tan\psi\, I_1\,(\gamma r)\, e^{j(\omega t - \beta z)} \tag{3}$$

where

$$\gamma^2 = \beta^2 - k^2 = \beta^2 \left(1 - \frac{v_p^2}{c^2}\right) \tag{4}$$

$$k = \frac{\omega}{c} \tag{5}$$

$$\beta = \frac{\omega}{v_p} \tag{6}$$

It is possible to make approximations for $v_p << c$. First, $\gamma \simeq \beta$. Second, to make $v_p << c$, one needs small pitch angle ψ so that $\tan\psi \simeq \psi$; thus we choose to ignore E_ϕ relative to E_z and E_r. This approximation reduces the fields to the so-called forced sinusoidal fields, which are shown in Fig. 1.

The magnetic fields H_z, H_r, H_ϕ have been omitted because their effect on the motion will be slight if the electron velocity is small, $v << c$. This omission is justified by noting

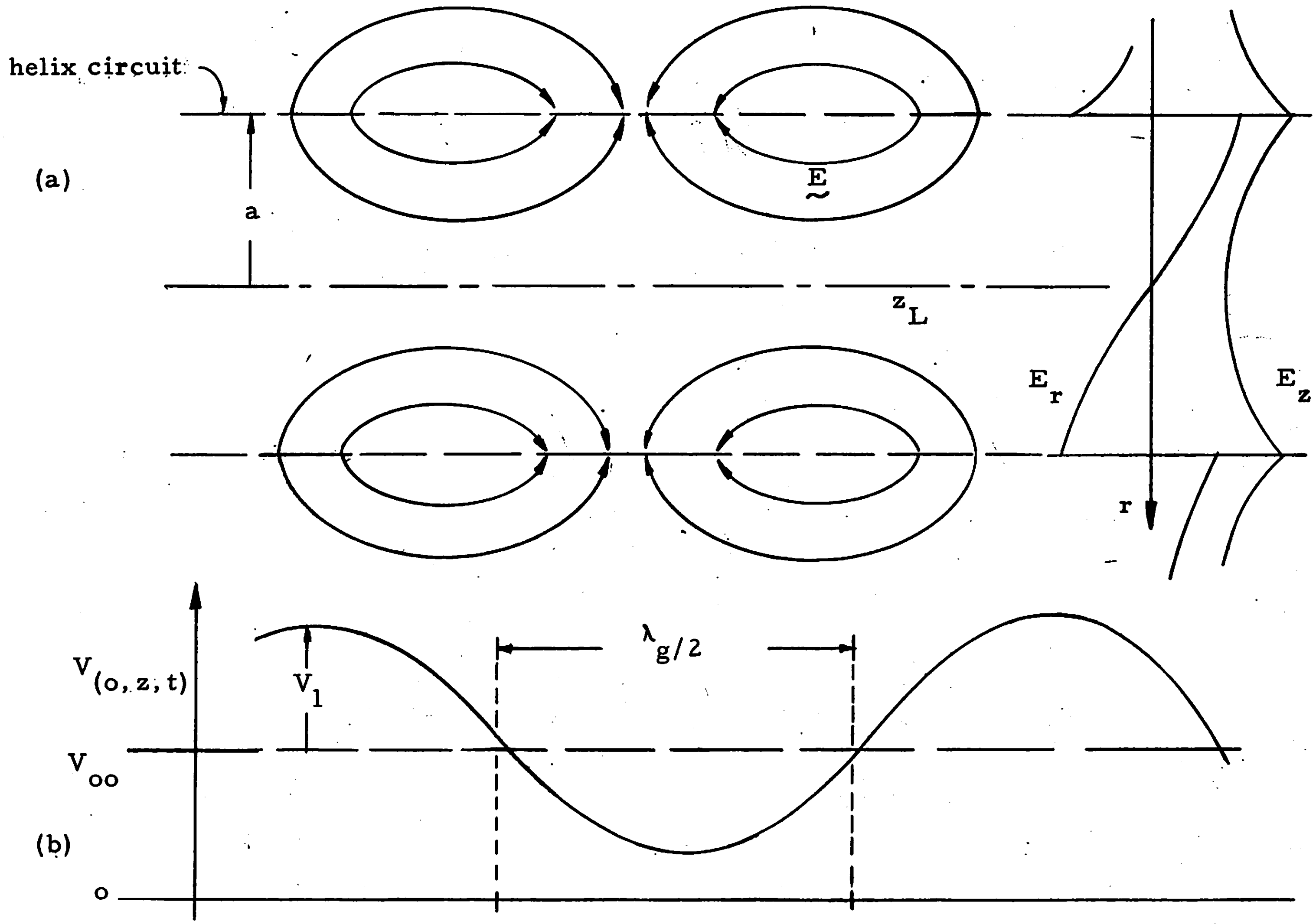


Fig. 1 (a) TM_{01} Electric Fields in a Slow-wave, $v_p < c$, circuit (helix); note that E_z is maximum on the circuit.

(b) Periodic Axial Variation of Potential on the Axes $V(r, z, t) = V(o, z, t)$.

that the ratio of magnetic to electric force is

$$\frac{|e\underset{\sim}{v} \times \underset{\sim}{B}|}{|e\underset{\sim}{E}|} = \frac{|\underset{\sim}{v} \times \mu \underset{\sim}{H}|}{|\underset{\sim}{E}|} \leq v\sqrt{\mu\epsilon \frac{\mu H^2}{\epsilon E^2}} = \frac{v}{c}\sqrt{\frac{W_H}{W_E}} \qquad (7)$$

This ratio is small where $v << c$ and where $W_H \simeq W_E$ as is true throughout most of the circuit.

The E_z and E_r fields may be found from a potential of the following form

$$V(r, z, t) = V_{oo} + V_1 I_0(\gamma r) \cos(\omega t - \beta z_L) \qquad (8)$$

where

V_{oo} = dc potential between cathode and helix in the laboratory

z_L = laboratory coordinate

V_1 = amplitude of rf potential on the axis.

Note that a physically realizable helix is made of a finite number of wires and as such would need an infinite number of spatial harmonics for complete description and that there are higher-order modes as well, each with its spatial harmonics. For our problem, it is sufficient to consider only the zero harmonic of the fundamental mode, as given above.

2.02. Focusing Properties of a Periodic Electrostatic Field

It is well known that a charged particle moving through cylindrical rings of different static potentials will be focused toward the axis. Such motion is shown in Fig. 2, for a simple case. The focusing depends on deflection by the radial fields and acceleration or deceleration by the axial field, and comes about

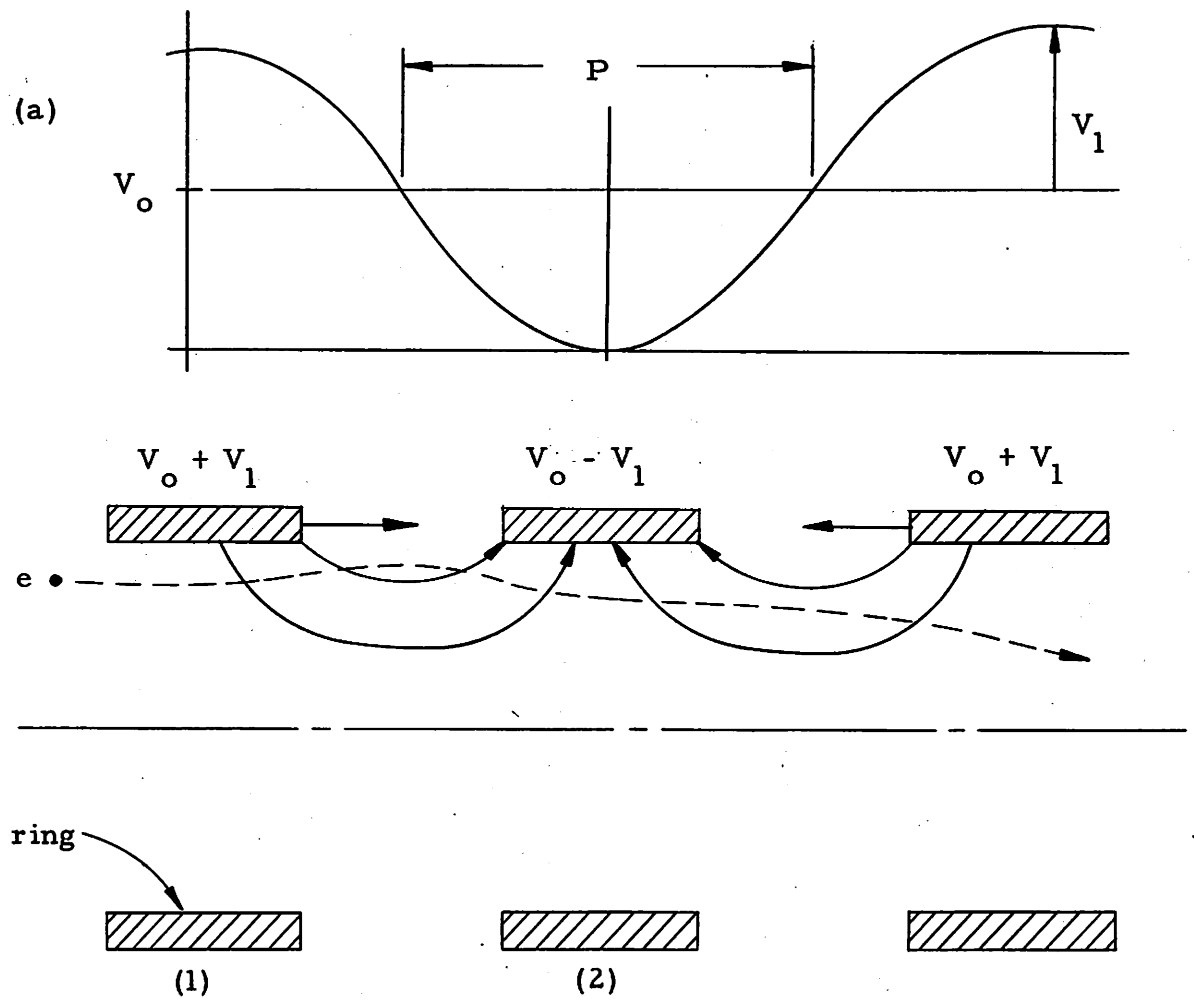


Fig. 2 (a) Time-independent Periodic Potential Formed By a Series of Stationary Rings of Alternating Potential.

(b) Possible Path of Electron Being Focused by a Series of Electrostatic Fields.

due to the difference in time spent in the two radial field regions. The particle may have positive or negative charge and drift in either direction and still be focused.

The potential for the rings is a solution of Laplace's equation, e.g., as given by Tien (1954), for the first harmonic,

$$V(r, z) = V_o + FV_f \frac{I_0(\beta r)}{I_0(\beta a)} \sin \beta z \qquad (1)$$

where

$$\beta = \frac{\pi}{p} \qquad (2)$$

p = pitch (spacing) of the rings

and F is a function of wire size (for typical wire size, F is usually unity).

2.03. Fields of a Cylindrical Pipe, Fast-Wave Circuit.

The TM_{01} waveguide electric fields of a cylindrical pipe are shown in Fig. 3. The points of interest here are that the E_z and E_r components near the axis are similar to those of the helix, but at wall E_z must be zero. Because the type of focusing sought requires both E_z and E_r, focusing should occur in the waveguide near the axis but not near the walls.

The electric fields for the TM_{01} guide are given, for example, [Ramo and Whinnery (1953)] as

$$E_z = AJ_0(k_c r)\, e^{j(\omega t - \beta z)} \qquad (1)$$

$$E_r = j \left[\left(\frac{\omega}{\omega_c}\right)^2 - 1 \right]^{1/2} A J_1 (k_c r)\, e^{j(\omega t - \beta z)} \qquad (2)$$

where $k_c a = 2.405$ is the first root of J_0 and w_c is the cut-off frequency.

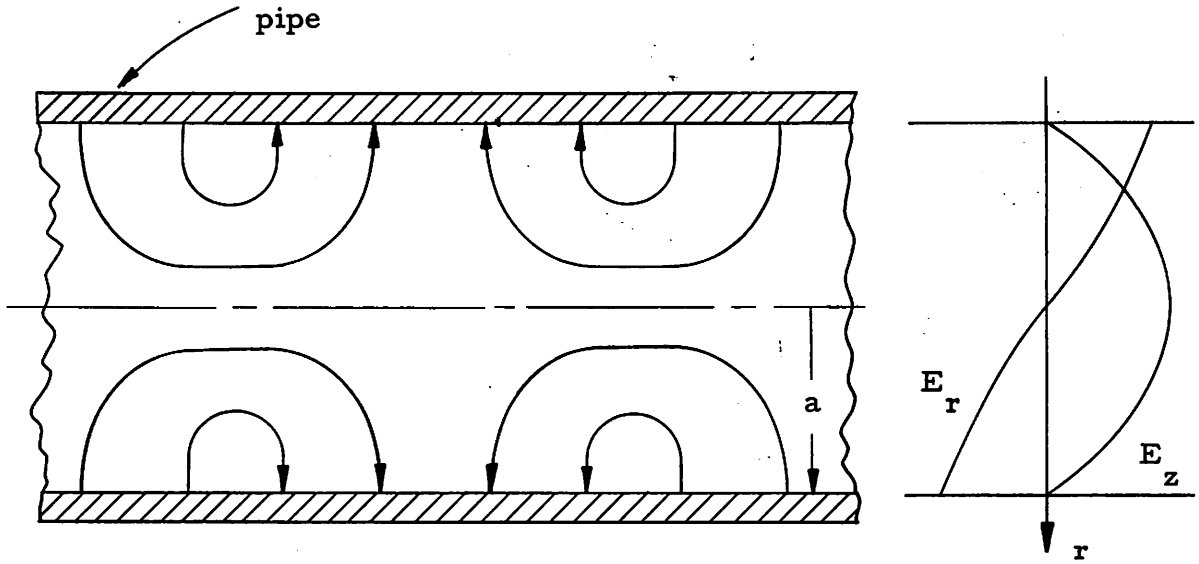


Fig. 3 TM_{ol} Electric Field inside a fast-wave $V_p > c$, cylindrical waveguide. Note that the axial electric field goes to zero on the guide walls.

2.04. Transformation to Wave Frame.

The similarity of fields in the helix and stationary rings leads one to expect focusing in the helix similar to that in the rings. This is evident from the sketches of the fields and from the equations for $V(r, z)$ in the two models. The electrostatic focusing has been solved in the ring case; hence, one may find the focusing in the helix case by transforming to a frame moving at the phase velocity of the wave, where the helix fields are now time-independent and much the same as the ring fields. Only the zero-th spatial harmonic, fundamental mode of the helix and the first harmonic of the rings will be used.

Consider a Galilean velocity transformation from the laboratory frame to a moving frame as shown in Fig. 4; the transformation is allowable because the wave is slow and therefore the induced electric field $(v_p \times B)$ is small and will be ignored. In the laboratory

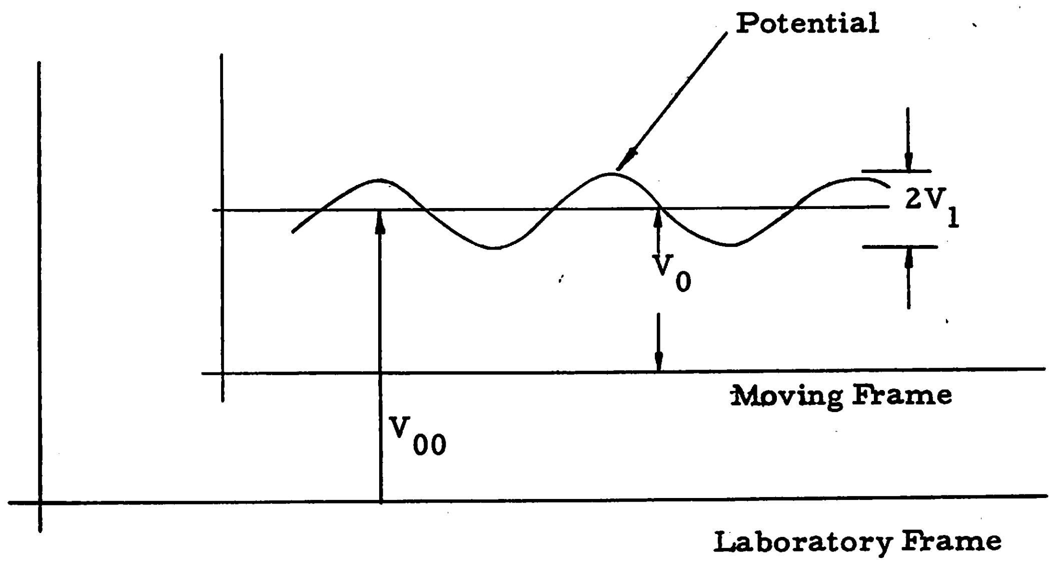


Fig. 4 Transformation from Laboratory to a Moving System So as to Eliminate the Time Dependence in the Potential

frame the cathode-to-helix dc voltage is V_{oo}. Therefore, with no rf, the electron velocity is v_{oo} where

$$\frac{1}{2} m v_{oo}^2 = e V_{oo} \qquad \text{(e is magnitude only)} \tag{1}$$

and with rf, the velocity is v_L. The moving frame has velocity v_p with respect to the laboratory frame; a fictitious laboratory voltage may be assigned to v_p, as $v_p = 2\eta V_p$ with $\eta = e/m$. In the moving frame, the electron velocity, with no rf, is taken to be v_o, positive to the left (because we are mostly interested in the case where $v_p > v_{oo}$), so that

$$v_{oo} = v_p - v_o \tag{2}$$

A fictitious voltage V_o can be written as

$$eV_o = \frac{1}{2} mv_o^2$$

$$= \frac{1}{2} m(v_p - v_{oo})^2 \qquad (3)$$

where V_o is the dc voltage to be used in the ring focusing equations of motion. The average charge density is the total charge Q divided by the total volume, or the average charge density per unit length, $\rho_{\ell o}$, divided by the average cross-section, πa^2. Because $\rho_{\ell o}$ and a do not depend on velocity, the average charge density is taken to be the same in both frames in the analysis. Thus, $\rho_{\ell oo} = \rho_{\ell o}$ or simply ρ_ℓ. Hence, for currents I_{oo} and I_o, one has

$$\rho_\ell = \frac{I_o}{v_o} = \frac{I_o}{\sqrt{2\eta V_o}} \qquad (4)$$

$$= \frac{I_{oo}}{v_{Lo}} = \frac{I_{oo}}{\sqrt{2\eta V_{oo}}} \qquad (5)$$

Then, for transforming currents, one has the relation

$$\frac{I_o}{I_{oo}} = \sqrt{\frac{V_o}{V_{oo}}} \qquad (6)$$

The phase constants are the same in both frames because lengths are invariant (Galilean transformation). The frequency is Doppler-shifted but to zero in the wave frame eliminating dependence on time in the potential (which is the whole object of the transformation). The axial acceleration of the electron in the laboratory is given by:

$$\frac{dv_L}{dt} = \dot{v}_L = \eta E_z = -\eta \frac{\partial}{\partial z_L} \left[V_1 I_o(\gamma r) \cos (\omega t - \beta z_L) \right] \quad (7)$$

To transform to the wave frame the argument of the cosine may be put in terms of the variables of this frame. We already have $\dot{z} = v = v_p - \dot{z}_L$ so that $z = v_p t - z_L$, where the constant of integration is taken to be zero. By definition, $\omega t = \beta v_p t$. Therefore, $\omega t - \beta z_L$ becomes simply βz.

The axial acceleration of the electron in the moving frame is given by

$$\dot{v} = \dot{v}_p - \dot{v}_L = 0 - \eta E_z = \eta \frac{\partial}{\partial z} \left[V_1 I_o(\gamma r) \cos \beta z \right] \quad (8)$$

$$\dot{v} = V \frac{\partial}{\partial z} \left(\frac{\partial z}{\partial t} \right) = v \left(\frac{\partial v}{\partial z} \right) = \frac{1}{2} \frac{\partial}{\partial z} v^2. \quad (9)$$

This equation may be integrated once to yield

$$v^2 = 2\eta V_1 I_o (\gamma r) \cos \beta z + [\text{constant in } z] \quad (10)$$

The added constant is simply $2\eta V_o$ because with no rf on the helix $(V_1 = 0)$, $v^2 = 2\eta V_o$; hence, the velocity of the electron in the moving frame is given by

$$v = \left[2\eta \left(V_o + V_1 I_o (\gamma r) \cos \beta z \right) \right]^{1/2} \quad (11)$$

This velocity corresponds to the laboratory velocity of the electron in periodic electrostatic focusing.

2.05. Electrostatic Focusing Equations

The paraxial-ray equation may be used to determine the equations of motion of an electron in a periodic electrostatic field.

Clogston and Heffner (1954) using the paraxial ray equation have found a focusing relation between the peak value of the periodic field V_1, the average potential of the electron stream, V_o (cathode at zero potential), the plasma frequency of the stream, ω_p, and the frequency of the alternating field seen by the electron, ω_e ($\omega_p^2 = \eta \rho_{oo}/\epsilon_o$). The field was produced by a series of rings having alternately potential $V_o + V_1$ and $V_o - V_1$. Their result is

$$(V_1/V_o) = \frac{16}{3} (\omega_p/\omega_e)^2 \qquad (1)$$

If this relation is satisfied, the electron stream is confined about some mean radius r_o associated with an average charge density, proportional to ω_p^2; the stream will fluctuate in a radius of about v_o. The equation is valid so long as the electron is near the axis of the helix; that is, it is valid in the region in which the paraxial ray equation is valid.

If the outermost boundary is not near the axis so that the paraxial ray equation is not a valid approximation, then the problem may be approached by balancing the forces acting on the outermost electrons. The forces acting on the electrons are taken to be the inertial force due to the acceleration of the electrons, the force due to the electric field produced by the space charges of the electron stream and the force due to the periodic electrostatic field applied in order to confine the electron stream. P. K. Tien (1954) has found a relation between the variables by balancing the forces on the outer boundary of the electron stream; this relation is

$$V'_r \left[V_r + V''_r \left(\frac{1}{\beta}\right)^2\right] = \frac{\sqrt{2}}{\pi\epsilon_o} \frac{I_o V_o^{1/2}}{r_o \sqrt{\eta}} \qquad (2)$$

TABLE 1

Quantity	Lab Frame	Moving Frame at v_p	Interrelations
Average Electron Velocity	v_{oo}	v_o	$v_{oo} = v_p - v_o$
Electron Velocity	v_L	v	$v_L = v_p - v$
Phase Velocity of Wave	v_p	0	
Dc Voltage of Electrons	V_{oo}	V_o	$V_o = (1/2\eta)v^2$ $V_{oo} = (1/2\eta)v^2$ $V_{oo} = (V_H^{1/2} - V_o^{1/2})^2$
Synchronous Dc Voltage of Helix	V_H	0	$V_H = (1/2\eta)v_p^2$
Axial Coordinate	z_L	z	$z_L = v_p t - z$
Radial Coordinate	r	r	
Average Radius of Electron Stream	r_o	r_o	
Dc Injected Current of Electron Stream	I_{oo}	I_o	$I_{oo} = \rho_{oo} v_{oo} \pi r_o^2$ $I_o = \rho_o v_o \pi r_o^2$
Charge per Unit Length	$\rho_{\ell oo}$	$\rho_{\ell o}$	$\rho_{\ell oo} = \rho_{\ell o}$

where $\beta = \frac{\pi}{p}$. p the pitch, is the distance between successive maxima of the electrostatic field. V_r is defined by

$$V_r = V_1 I_0 (\beta r) \tag{3}$$

with V'_r meaning $\partial V_r / \partial r$.

This relation is valid as long as the charge per unit length may be considered approximately constant and the only electromagnetic forces present are those due to electric fields.

2.06. Application of the Electrostatic Focusing Equations to Slow Wave Focusing.

The equations of focusing derived by Clogston and Heffner and P. K. Tien for electrostatic focusing may be directly applied, in the moving frame, to slow-wave focusing. If the paraxial ray treatment of Clogston and Heffner is used the parameters in equation 2.05 (1) may be written in terms of quantities which are considered known or may be inferred,

$$\omega^2 = 2\eta V_o \beta^2 \tag{1}$$

$$\omega_p^2 = \frac{\eta}{\epsilon_o} \rho_{oo} = \frac{\eta}{\epsilon_o} \frac{I_{oo}}{\sqrt{2\eta V_{oo}}} \frac{1}{\pi r_o^2} \tag{2}$$

The value of r_o is not known directly and must be inferred.

If these values are substituted into equation 2.05 (1) the required peak value of the rf field on the axis, V_1, is found from,

$$V_1^2 = \frac{8}{3\pi} \frac{V_o}{\epsilon_o \sqrt{2\eta}} \left(\frac{I_{oo}}{V_{oo}^{1/2}} \right) \frac{1}{(\beta r_o)^2} \tag{3}$$

The value of V_1^2 on the axis of the helix is related to the rf power flowing on the helix and the helix impedance K_H, on the axis, by

$$V_1^2 = (P_{rf})(2K_H) \tag{4}$$

The approximate helix impedance on the axis is given, for example, by Pierce (1950), as

$$K_H \cong \frac{1}{2}\sqrt{\frac{\mu_o}{\epsilon_o}}\ \frac{c}{v_p}\ e^{-2\beta a} \tag{5}$$

which is good as long as $v_p << c$. For idealized field shapes v_g appears in place of v_p in this expression; hence, in general, to increase K, it is important to reduce v_g. The helix expression implies that reduction of v_p increases K_H (certainly true) but $v_g = v_p$ for a helix.

The value of the rf power required for focusing a particular stream using a given helix may now be found by combining (4) and (5) as

$$P_{rf} = \frac{4\sqrt{2}}{3\pi\epsilon\sqrt{\eta}}\ \frac{\epsilon_o}{\mu_o}\ \left(\frac{I_{oo}}{V_{oo}^{1/2}}\right) V_o \left(\frac{v_p}{c}\right) \left(\frac{e^{2\beta a}}{(\beta a)^2}\right) \left(\frac{a}{r_o}\right)^2 \tag{6}$$

The required rf power may also be given in terms of the velocities as

$$P_{rf} = (I_{oo}V_{oo})\left(\frac{8}{3\pi}\right)\left[\left(\frac{v_p - v_{oo}}{v_{oo}}\right)^2 \frac{v_p}{v_{oo}}\right]\left[\frac{e^{2\beta a}}{(\beta a)^2}\right]\left(\frac{a}{r_o}\right)^2 \tag{7}$$

where $I_{oo}V_{oo}$ is recognized as the dc power of the stream.

The value r_o of the stream radius is not easily determined and, in fact, is not even uniquely defined. The rf power required depends on the square of r_o and therefore some arbitrariness exists in the equation.

If most efficient use of rf power is to be obtained, the rf power should be minimized with respect to βa. The minimum of

$e^{2\beta a}/\beta a^2$ occurs when βa is equal to unity, but is reasonably broad. The required power flow in Eq. (6), shows P_{rf} is proportional to the charge density per unit length through the term $(I_{oo}/V_{oo}^{1/2})$. P_{rf} is also proportional to the difference in velocity between the stream and the wave, through V_o. Apparently the smaller the difference in velocity between the phase velocity v_p of the wave and the velocity of the electron V_{oo} the less rf power required to confine a given stream, a result which is physically unacceptable; as the wave and stream become more synchronous, the lenses appear to be further apart so that the excursions of the stream become larger as $v_{oo} \longrightarrow v_p$. However, there is a lower limit on the value of V_o, hence on $v_p - V_{oo}$, and this will be considered in the next section. Going in the other direction, to $v_p < 0$, with wave and stream oppositely directed, the required power is seen to increase rapidly. This is due to the nature of the focusing which depends on the differential time spent in radially inward and radially outward fields; this time decreases rapidly as the relative velocity of stream and wave increases, making oppositely directed wave and stream flow especially poor. The required rf power also depends directly on the ratio (v_p/c). This ratio is introduced in the expression for the helix impedance. The helix impedance increases when the phase velocity of the wave is decreased and hence the power required for focusing a particular stream is reduced proportional to the phase velocity of the wave. The important factor, in general, is the group velocity, as noted earlier under (5).

For streams of larger diameter a similar procedure may be carried through using the electrostatic focusing Eq. 2.05 (2) derived by P.K. Tien. Using V_r from Eq. 2.05 (3) in Eq. 2.05 (2) and evaluating at $r=r_o$, there results.

$$\beta r_o I_1(\beta r_o) \left[I_o(\beta r_o) - \frac{1}{\beta r_o} I_1(\beta r_o) + I_o(\beta r_o)\right] V_1^2 = \frac{\sqrt{2}}{\pi} \frac{I_o V_o^{1/2}}{\epsilon\sqrt{\eta}} \tag{8}$$

For simplicity, let the terms in βr_o be gathered together as

$$A(\beta r_o) \equiv \beta r_o I_1(\beta r_o)\ [I_o(\beta r_o) - \frac{1}{\beta r_o} I_1(\beta r_o) + I_o(\beta r_o)] \qquad (9)$$

Substituting $A(\beta r_o)$ into (8) and solving for V_1^2, the result is:

$$V_1^2 = \frac{\sqrt{2}}{\epsilon_o \pi \sqrt{\eta}} \left(\frac{I_{oo}}{V_{oo}^{1/2}} \right) \frac{V_o}{A(\beta r_o)} \qquad (10)$$

For βr_o small, $A(\beta r_o) \simeq \frac{3}{4} (\beta r_o)^2$ so that Eq. (10) reduces to

$$V_1^2 = \frac{8}{3\pi} \frac{V_o}{\epsilon_o \sqrt{2\eta}} \left(\frac{I_{oo}}{V_{oo}^{1/2}} \right) \frac{1}{(\beta r_o)^2}$$

which is the result found by Clogston and Heffner in solving the paraxial ray equation to first order, Eq. (3).

The equation for power may again be written in terms of the helix impedance on the axis

$$P_{rf} = \sqrt{\frac{\epsilon_o}{\mu_o}} \frac{\sqrt{2}}{\pi \epsilon_o \sqrt{\eta}} \left(\frac{I_{oo}}{V_{oo}^{1/2}} \right) V_o \left(\frac{v_p}{c} \right) \frac{e^{2\beta a}}{A(\beta r_o)} \qquad (11)$$

The field shape factor, $e^{2\beta a}/A(\beta r_o)$, is plotted in Fig. 5. For thin streams the optimum (least power) is seen to be when βa=1 or helix circumference equal to guide wavelength. As the same current fills more of the helix, r_o/a increases, the power needed decreases rapidly and the optimum βa increases slowly.

2.07. Limiting Region of Required Power Equations.

One possible question is immediately presented by the equation for required power flow, Eq. 2.06 (6) and (7). What occurs when $V_o \rightarrow 0$ meaning $V_{oo} \rightarrow V_H$ or $V_{oo} \rightarrow V_p$?

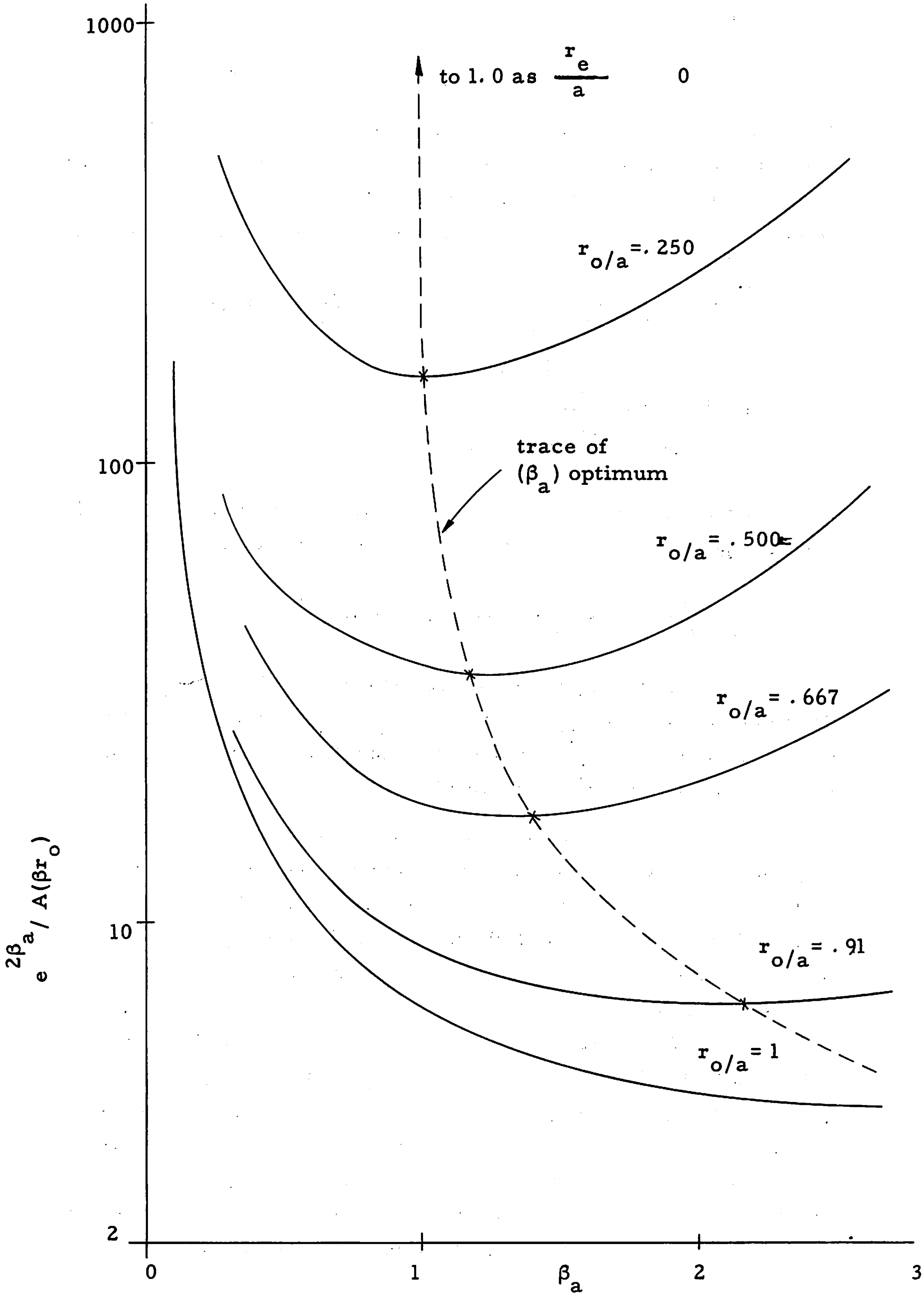


Fig. 5 Variation of $\frac{e^{2\beta_a}}{A(\beta r_o)}$ with β_a for various r_o/a.

Apparently, no rf power is required for focusing. Actually the equation of motion for the electrons has a singular point before this condition is reached. The radial equation of motion for the electrons in the moving frame has been derived by Tien (1954), as

$$\ddot{r}_1 + \frac{1}{v_x}\frac{dv_x}{dx}\dot{r}_1 - \frac{4r_1\eta V''}{\beta^2 v_x^2}\cos 2x = \frac{4\eta V'}{v_x^2\beta^2}\cos 2x + \frac{4\eta}{\beta^2}\frac{1}{v_x^2}\frac{I_o}{v_o \pi r_o^2 \epsilon_o} \quad (1)$$

where $\beta z = 2x$, the prime denotes derivative with respect to r, and the dot denotes derivative with respect to x, also

$$v_x = \sqrt{2\eta}\,[V_o + V_1(r)\cos 2x]^{1/2} = \sqrt{2\eta V}.$$

Equation (1) has a singularity for vanishing v_x, $V_o = - V_1(r)\cos 2x$. This singularity is only possible for $V_o \leq V_1$, hence, if a restriction is made to the region where V_1 is small compared to V_o the solution found earlier will be valid.

The situation may be made a little clearer by referring to Fig. 6. The entering electron has a kinetic energy $\frac{1}{2}mv_o^2$ in the moving frame; this energy level is represented by V_o in Fig. 6. The periodic electrostatic field is seen in the moving frame by the electron as a series of potential wells and barriers. Two values of barrier potential V_1 are shown in Fig. 6; at (a) V_1 is less than V_o and (b) V_1 is greater than V_o. The electron will pass by the barriers and be focused as long as $V_1 < V_o$. When $V_1 > V_o$, as at (b), the electron cannot penetrate the barrier and hence would be trapped in a well of the wave. The case where the electron is trapped will not be discussed (a linear accelerator operates in this region $V_1 > V_o$). It is also seen that when the height of the barrier energy V_1 approaches the kinetic energy

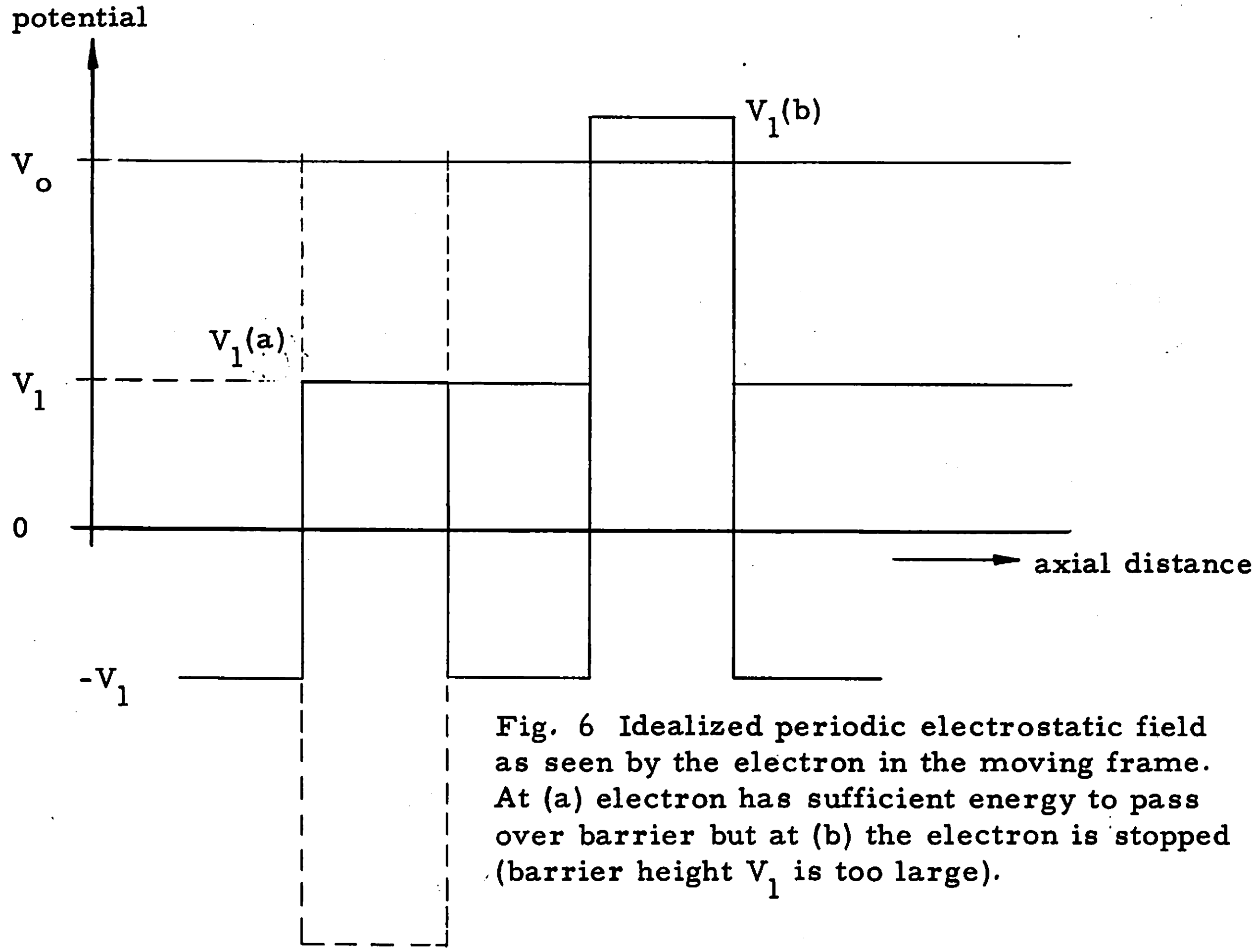


Fig. 6 Idealized periodic electrostatic field as seen by the electron in the moving frame. At (a) electron has sufficient energy to pass over barrier but at (b) the electron is stopped (barrier height V_1 is too large).

of the electron, V_o, the transit time across the barrier becomes large. Large variations of the stream radius about r_o result and the focusing equations lose their validity. Therefore, we treat only the case where $V_o >> V_1$.

2.08. Comparison of Slow and Fast Wave Focusing.

The fields of a slow wave ($v_p << c$) propagating on a helix are, to our approximation, sinusoidal in distance and time. The fields of an electromagnetic wave in a wave guide are also periodic in space and time but are fast waves ($v_p \geq c$). In this section the sinusoidal waves are approximated by a square wave. That is, the wave consists of four equal sections of constant E_r or E_z as illustrated in Fig. 7.

Consider a typical electron on the edge of the beam. Focusing or confinement occurs because this electron spends more time in the radially inward force region than in the outward. By calculating

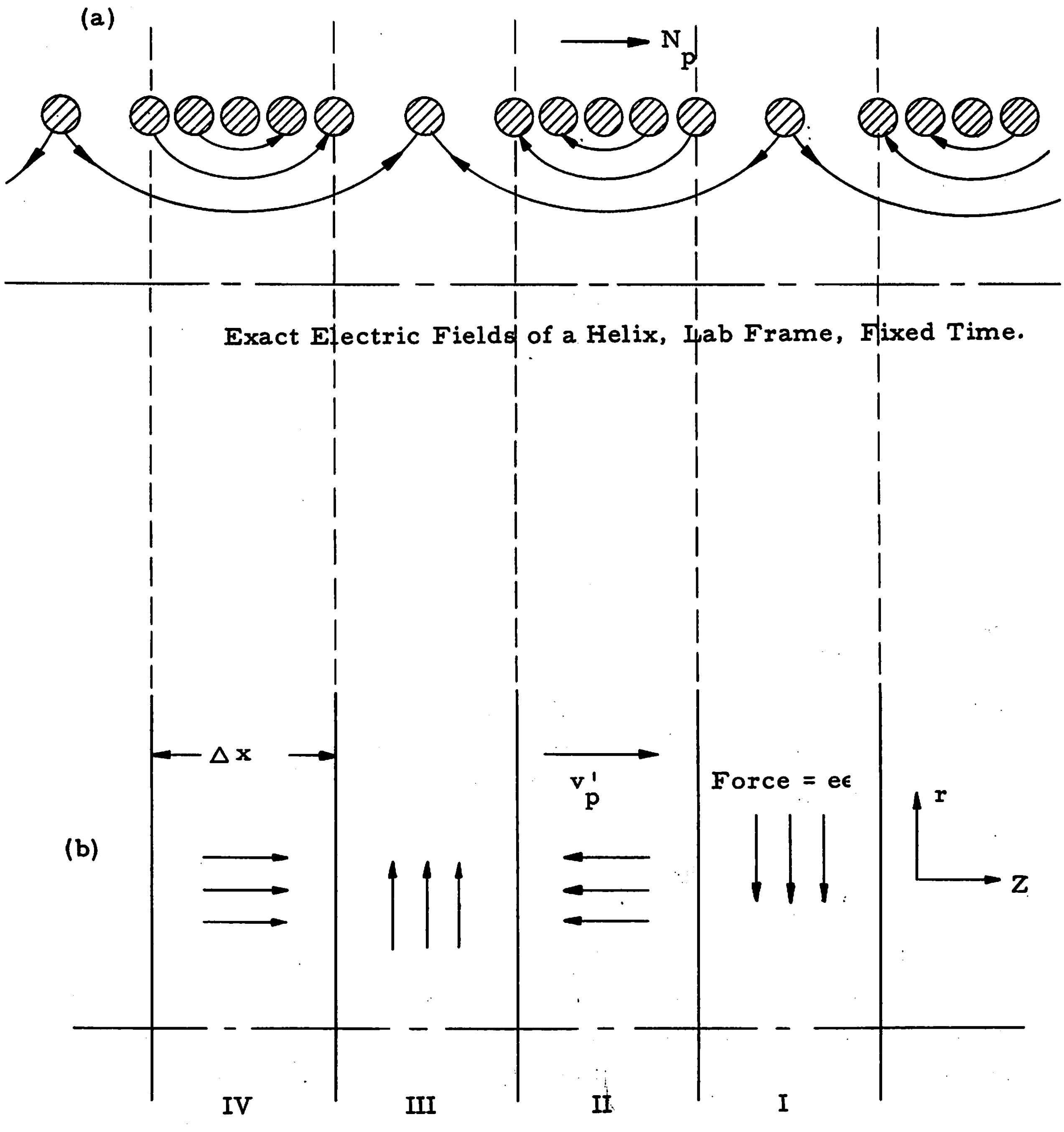


Fig. 7 Approximate Fields of a Helix, Average Electron Frame, for all time; Electron going Slower than Wave: Sees Region I then II, then III, IV.

the difference in time spent by the electron in each region a focusing relation may be found.

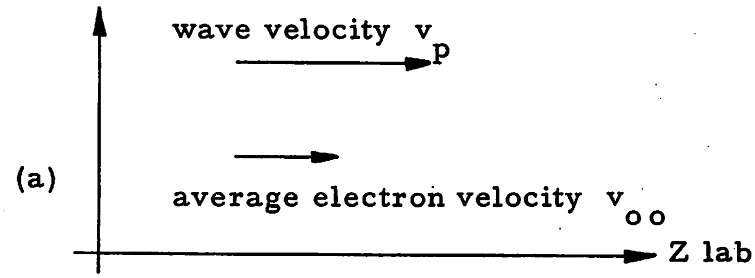
It is desired to have the focusing equations apply even if $v_p \sim c$, and, hence it will not be possible to use the Galilean velocity transformation to the wave frame used in section 2.04. However, a Galilean velocity transformation may be made to the electron frame such that the average velocity of the electron stream is zero (considering slow streams). The regions (1), (2), (3) and (4) then move past the electron and exert an average force on the electron per cycle that just balances the space-charge force. The frame in which the movement of the electron is observed is moving at a velocity v_{oo}. The phase velocity in this frame is $v'_p = v_p - v_{oo}$.

By the choice of frames it is clear that the electron executes some form of periodic motion about a fixed z-coordinate in the moving frame. It is not unreasonable to require this period to be equal to the time taken by the four regions to move past a fixed point on the z_e axis, that is one cycle. Furthermore, because the focusing and space charge forces are to be balanced on the average, one expects a radial motion of this same period as shown in Fig. 8. In a more thorough analysis, one would not require the loop shown to close in one period but would allow also a slower oscillation period to exist. This neglect throws out long wavelength effects which can cause defocusing, well known as the stop bands found in periodic focusings.

The time spent in each region is given by

$$\Delta t_1 = \frac{\Delta x + v_1 \Delta t_1}{v'_p} \qquad (1)$$

$$\Delta t_2 = \frac{\Delta x + v_1 \Delta t_2 - (\frac{e}{m} E_z) \frac{\Delta t_2^2}{2}}{v'_p} \qquad (2)$$



Wave Transformation

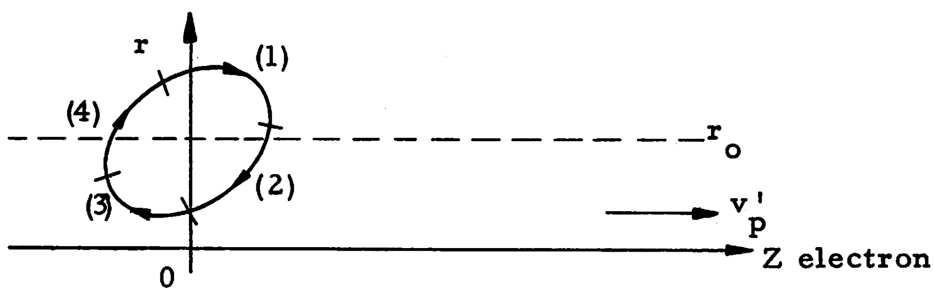


Fig. 8 Focusing Effect of the Fields on an Electron. The electron spends more time in (1) than in (3) because its axial velocity v_{eZ} (Section 1) is in the same direction as v_p'. The times spent in (2) and (4) are equal by the requirement of steady state.

$$\Delta t_3 = \frac{\Delta x + (v_1 - \frac{e}{m} E_z \Delta t_2)\,\Delta t_3}{v'_p} \tag{3}$$

$$\Delta t_4 = \frac{\Delta x + (v_1 - \frac{e}{m} E_z \Delta t_2)\,\Delta t_4 + \frac{e}{2m} E_z \Delta t_4^2}{v'_p} \tag{4}$$

where

Δx = width of one region, $\frac{1}{4}\,\lambda$ guide

Δt_i = time spent in i^{th} region

v_1 = axial velocity when entering region (1)

E_z = magnitude of the axial electric field.

Note that the time spent in each region is determined solely by E_z. This is because for the idealized field chosen here E_z

is not a function of radius. If E_z is strongly dependent on radius ($E_z = E_z(r)$), the Δt_i equations would have to be modified.

Apply the restriction of steady state--that is, when the electron leaves one cycle of fields, the axial exit velocity is the same as the axial velocity when the electron entered the cycle (v_1). Hence

$$v_1 = v_1 - \frac{e}{m} E_z \Delta t_2 + \frac{e}{m} E_z \Delta t_4 \qquad (5)$$

requiring that $\Delta t_2 = \Delta t_4$. Δt_1 and Δt_3 may be solved for directly, and approximated as,

$$\Delta t_1 = \frac{\Delta x}{(v'_p - v_1)} \simeq \frac{\Delta x}{v'_p} \left(1 + \frac{v_1}{v'_p}\right) \qquad (6)$$

$$\Delta t_3 = \frac{\Delta x}{(v'_p - v_1 + \frac{e}{m} E_z \Delta t_1)} \simeq \frac{\Delta x}{v'_p} \left(1 + \frac{v_1}{v'_p} - \frac{e}{m} \frac{E_z \Delta t_2}{v'_p}\right) \qquad (7)$$

where it is assumed that

$$\frac{v_1}{v_p} << 1$$

$$\frac{e}{m} \frac{E_z \Delta t_2}{v'_p} << 1$$

Solving for Δt_2,

$$\Delta t_2 \left(v'_p - v_1 + \frac{e}{m} E_z \frac{\Delta t_2}{2}\right) \simeq \Delta x$$

$$\Delta t_2 \simeq \frac{\Delta x}{v'_p} \left(1 + \frac{v_1}{v_p} - \frac{\Delta x}{v'^2_p} \frac{e}{2m} E_z\right) = \Delta t_4 \qquad (8)$$

Substituting Δt_2 into the expression for Δt_3, (5), and dropping second-order terms,

$$\Delta t_3 = \frac{\Delta x}{v'_p}\left(1 + \frac{v_1}{v'_p} - \frac{e}{m} E_z \frac{\Delta x}{v'^2_p}\right) \qquad (9)$$

By the previous arguments and inspection of Fig. 8, the time required to complete one cycle is

$$\Delta t_1 + \Delta t_2 + \Delta t_3 + \Delta t_4 = \frac{4\Delta x}{v'_p} = 2\Delta t_2 + \Delta t_3 + \Delta t_1 = \tau \qquad (10)$$

Using (5), (7) and (8), (9) becomes

$$2\Delta t_2 + \Delta t_1 + \Delta t_3 = \frac{2\Delta x}{v'_p} + \frac{\Delta x}{v'_p}\left(2\frac{v_1}{v'_p} - \frac{e}{m} E_z \frac{\Delta x}{v'^2_p}\right) + \frac{2\Delta x}{v'_p}\left(1 + \frac{v_1}{v'_p} - \frac{\Delta x}{v'^2_p}\frac{e}{2m}E_z\right)$$

$$= \frac{4\Delta x}{v'_p}$$

Therefore, it is required that the entrance velocity is

$$\frac{v_1}{v'_p} = \frac{e}{2m} E_z \frac{\Delta x}{v'^2_p}$$

Substituting this result into (6), (7) and (8), the result is

$$\Delta t_2 = \Delta t_4 \simeq \frac{\Delta x}{v'_p} \qquad (11)$$

$$\Delta t_1 \simeq \frac{\Delta x}{v'_p}\left(1 + \frac{e}{2m} E_z \frac{\Delta x}{v'^2_p}\right) \qquad (12)$$

$$\Delta t_3 \simeq \frac{\Delta x}{v'_p}\left(1 - \frac{e}{2m} E_z \frac{\Delta x}{v'^2_p}\right) \qquad (13)$$

One other constraint may now be used. Consider one cycle as in deriving (5). It is necessary that the exit velocity in the radial direction equal the entrance velocity in the radial direction.

$$\dot{r}_{exit} = \dot{r}_{entrance} \underbrace{- \frac{e}{m} E_r \left(\Delta t_1 - \Delta t_3\right)}_{\text{net velocity due to applied field}} + \underbrace{\frac{e}{m} E_s \left(\Delta t_1 + \Delta t_2 + \Delta t_3 + \Delta t_4\right)}_{\text{net velocity due to space charge field}} \tag{14}$$

Therefore, this restraint requires that

$$E_{rI} \left(\Delta t_1 - \Delta t_3\right) = E_s \frac{4\Delta x}{v'_p} \tag{15}$$

where the space charge field is

$$E_s = \frac{\rho_\ell}{2\pi\epsilon_o r_o} \tag{16}$$

Substituting Δt_1 and Δt_3 from (12) and (13), into (15), and putting in (16),

$$E_r \frac{\Delta x}{v'_p} \left(1 + \frac{e}{2m} E_z \frac{\Delta x}{{v'_p}^2}\right) - E_r \frac{\Delta x}{v'_p} \left(1 - \frac{e}{2m} E_z \frac{\Delta x}{{v'_p}^2}\right) = \frac{\rho_\ell}{2\pi\epsilon_o r_o} \frac{4\Delta x}{v'_p} \tag{17}$$

which simplifies to

$$\begin{aligned} E_r E_z &= \frac{2}{\pi} \frac{m}{e} \frac{{v'_p}^2}{\Delta x} \frac{\rho_\ell}{\epsilon_o r_o} \\ &= \frac{8}{\pi} \frac{m}{e} \frac{\left(v_p - v_{oo}\right)^2}{\lambda_g} \frac{\rho_\ell}{\epsilon_o r_o} \end{aligned} \tag{18}$$

This relation may be applied to either fast or slow wave circuits and, hence, used for comparison of power required in such circuits.

For a helix, from 2.01 (1), 2.01 (2), at $r = r_o$

$$E_z E_r = E_{zo}^2 \frac{\beta r_o}{2} \qquad (19)$$

Using 2.06 (4),

$$E_{zo}^2 = 2\beta^2 P_{rf} K_H = \beta^2 V_1^2 \qquad (20)$$

then, (18) becomes

$$E_r E_z = \beta^2 V_1^2 \frac{\beta r_o}{2} = \frac{8}{\pi} \frac{m}{e} \frac{(v'_p)^2}{\lambda_g} \frac{I_{oo}}{v_{oo} \epsilon_o r_o} \qquad (21)$$

$$= \frac{16}{\pi \epsilon_o} \frac{V_o}{\lambda_g} \frac{I_o}{v_{oo} r_o} \qquad (22)$$

Hence, the rf voltage required for focusing is,

$$V_1^2 = \frac{16}{\pi^2 \epsilon_o} \frac{1}{(\beta r_o)^2} V_o \left(\frac{I_{oo}}{v_{oo}}\right) \qquad (23)$$

$$= \frac{16}{\epsilon_o \pi^2} \frac{1}{(\beta r_o)^2} \frac{I_{oo}}{v_{oo}}$$

This differs from 2.06 (3) by a factor of $6/\pi$, which is remarkably close.

For a fast-wave guide, with $v_p > c$, and using the same guide wavelength, λ_g (as this leads to about the same field shapes which, if optimum for slow waves, then are also optimum for fast waves), then the required $E_r E_z$ increases at least as fast as $(v_{pfast}/v_{pslow})^2$, which may easily be 10^2 to 10^4. In addition to the larger field required for fast wave focusing, it is necessary to consider the variation of the $E_r E_z$ product with radius for the helix and for a circular pipe with a TM_{01} fast wave, as shown in Fig. 9.

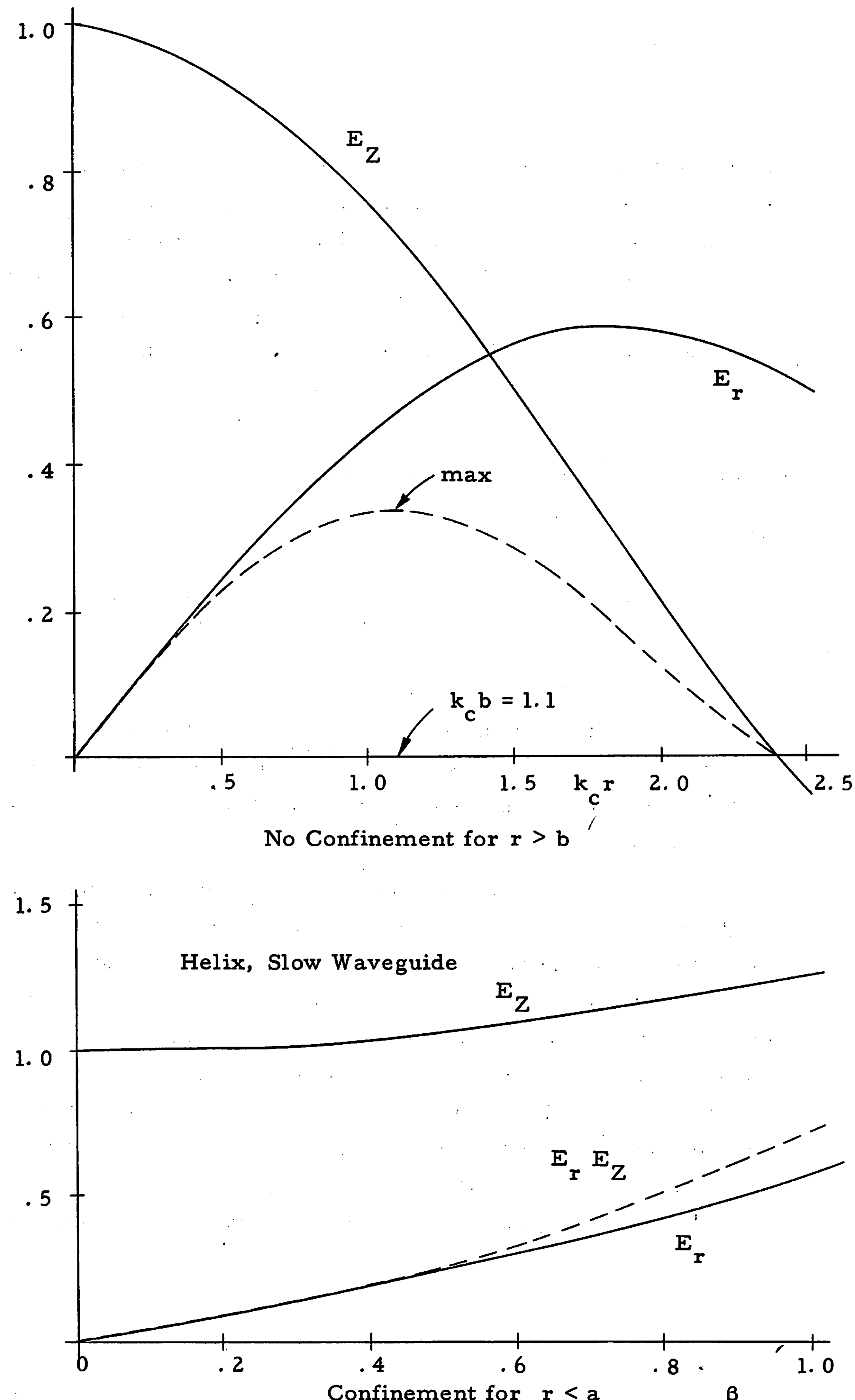


Fig. 9 Focusing field product, $E_r E_Z$, of fast waveguide and helix compared.

In the fast-wave guide, for $r_o > b$, the focusing becomes weaker as r_o increases to a. If the stream initially was at $r_o = b$ and ρ_ℓ was increased, then the stream would expand to $r_o > b$, but into a weaker focusing region and tend to be lost to the wall; only$(1.1/2.4)^2$ or 21 per cent of the cross sectional area of the fast-wave guide is useful. However, with the helix, the stream is stable over the entire range $r < a$.

The next step is to calculate the ratio of powers needed to focus a given stream. This ratio for thin streams, using the same λ_g, is roughly,

$$\frac{P_{rf}(\text{fast wave})}{P_{rf}(\text{slow wave})} \geqq \frac{1}{10} \left(\frac{c}{v_{p\,slow}}\right)^3 \frac{1}{\left(1 - \frac{v_{oo}}{v_{p\,slow}}\right)^2} \tag{24}$$

For $v_{p\,slow}/c$ of 0.1, (2500 volt helix), even ignoring the last factor, this ratio is 100; for $v_{p\,slow}/c$ of 0.04 (400 volt helix) the ratio is 1560. Obviously, slow-wave focusing is immensely better where traveling-wave focusing is used, with no reflections or energy stroage.

Resonant structures could be used where the input power might be much less than the circulating power which was calculated above. If the power required to do work on the stream is much less than that lost to I^2R heating of the fast- or slow-wave circuit (a rather uneconomical arrangement), then fast-wave circuits might appear more competitive because of their relatively lower losses. Calculations of slow-wave circuit losses are difficult because of spiked current distributions on wire or wave surfaces and, to our knowledge, differ appreciably from experimental measurements. Hence, no comparisons are given.

III. EXPERIMENTS

3.01. Introduction to the Experiments

To test the foregoing conclusions regarding slow-wave focusing, an experiment was designed using the electron stream from an electron gun and the slow wave propagated on a helix. In this device the electron stream leaves the gun, passes through the helix and is collected by a suitable electrode at the opposite end. The stream, during its transit from the electron gun to the collector, is confined by the slow wave on the helix. The rf power is fed onto the helix by a coaxial transmission line from the source of rf power and removed in a similar fashion. This is the type of match commonly referred to as a "pin match" in the design of traveling-wave tubes. Fig. 10 shows a schematic fo the assembly. Five tubes were made with the experimental results shown in Table II.

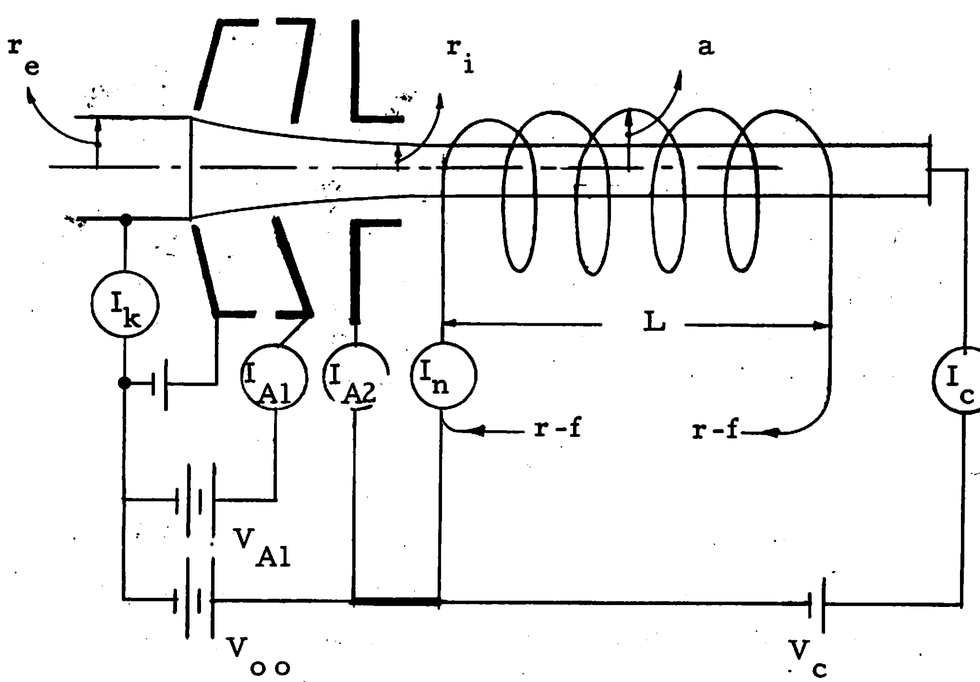


Fig. 10 Schematic of Experiment for Slow Wave Electron Stream Focusing

The electron gun used in the first four experiments was of the parallel-flow type. The gun was composed of a button cathode, 0.090 inch in diameter, a focus plate, to maintain parallel flow, a first anode, to control the emission from the cathode and a second anode, to control the velocity of the stream. Some stream-area convergence was possible with this gun by using the anodes as an electrostatic lens system.

3.02. Tube I

We shall refer to the various test assemblies as tubes because of the similarity to the traveling-wave tube in appearance. The physical dimensions of the tubes are given in Table II and sketched in Fig. 11. In Tube 1, the helix was chosen small in diameter because, in accord with the theory developed, the power required to focus a given beam goes as the square of the ratio of the helix radius to the beam radius.

Due to the difficulties involved in actually building the tube it was necessary to mount the gun relatively far from the beginning of the helix. The beam, because of this spacing, was so disturbed on entering the rf field of the helix that no focusing effect could be observed. This result emphasized that the problem of entrance conditions is quite critical in a scheme of this sort. In particular, the slope of the entering electron stream is assumed to match with the slope of an infinitely long stream traveling in the periodic field at that particular axial position. Note that the infinite stream in the moving frame has a harmonic variation of radial boundary with respect to axial position.

For electrostatic focusing the position of the emerging electrons relative to the periodic field remains constant in space and time so that the trajectories of the emerging electrons may be made to match the steady state condition. That is, the entrance velocity is such that it matches the periodic boundary of an infinitely long stream which is being focused by a periodic electrostatic

TABLE II

	Gun			Helix							
Tube Number	r_c	r_i	$I_{oo}/V_{oo}^{3/2}$ Perveance	a	L		Pitch Angle ψ	Type of Transition	Operating Frequency f	V_h at f	β_a at f
I	.090"	.090"	3.3×10^{-8}	.060"	8"	.030" Wire					
II	.090"	.090"	3.3×10^{-8}	.177"	3"	.020" x .060" tape	.0816 Radians 4.67°	Pin Match	1 kMc	2800 v	.97
III	.090"	← Same →									
IV	As Above			.177"	8"	.020" x .060"	4.67°	Pin Match	1 kMc	2800 v	.97
V	?	.070"	$.136 \times 10^{-6}$	.100"	6"	.015" dia.	.044 rad. 2.52°	Coaxial Transmission Line to Helix	.8 - 1.2 kMc	590 v (Meas.)	1.5

TABLE II

Tube Number	Collector Description	Typical Results					
	βa at f	V_{oo}	I_{oo}	(%) Stream Transmission rf off	(%) Stream Transmission rf on	P_{rf}	Brief Remarks
I							Helix diameter too small for transmission
II	Sheet metal circular cone	900 v	.660ma	88%	92%	5W	Helix too short to show large change in collector current
III	Fluorescent screen						Beam diameter as seen on the collector decreased with application of rf
IV	Fluorescent screen	930 v	.86ma	27%	87%	15W-62W	Region where $V_{oo} \longrightarrow V_H$ could not be investigated because gun injection became poor
V	2 piece collector, probe on axis, plus concentric cone	200 v	.38 ma	Negligible	30%-40%	5W	Overall transmission not as good as IV due to unfavorable entrance conditions.
							All electron guns used were of the triode type essentially producing convergent flow, i. e. $r_i < r_e$ where r_i is the estimated entrance radius.

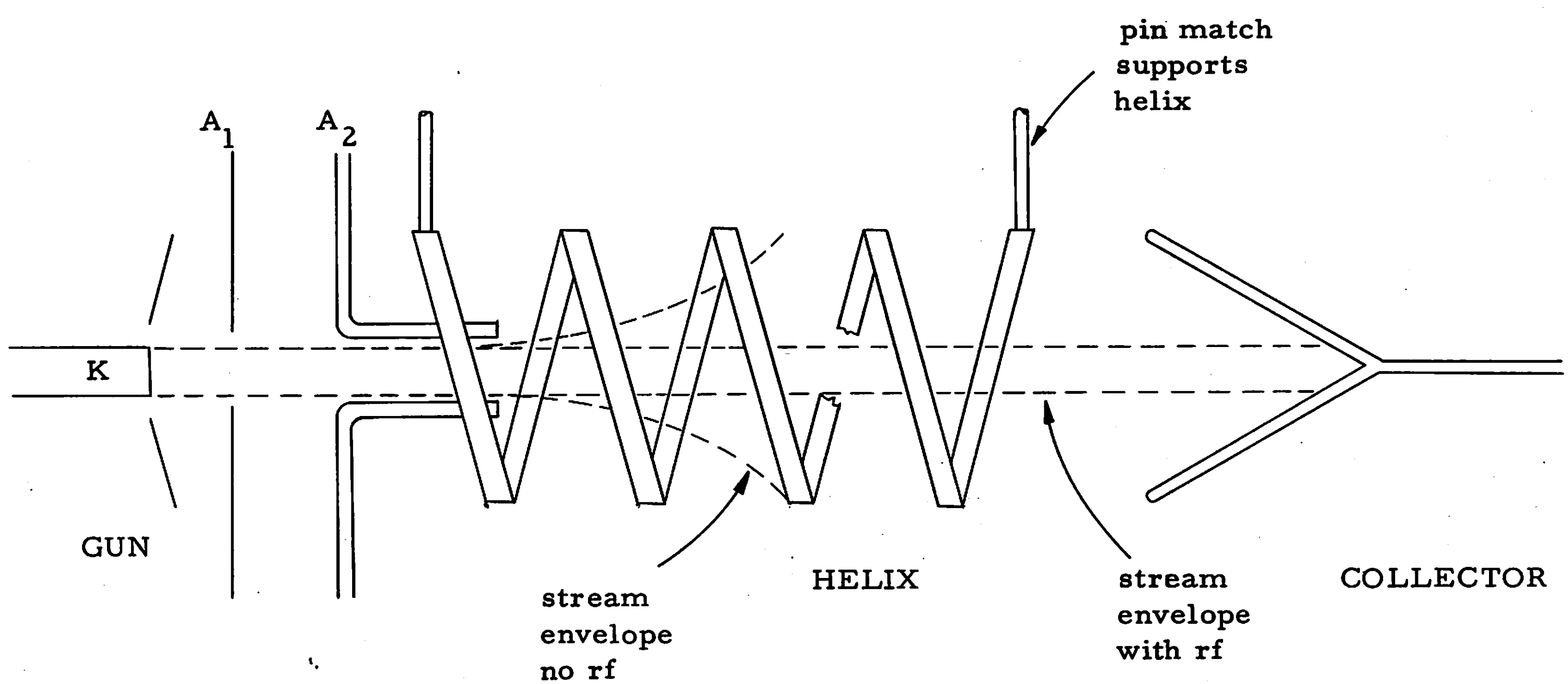


Fig. 11 Schematic of Tube for Testing Slow-Wave Focusing

field. In the case of time-varying slow-wave focusing, however, this is not the case. In the moving system the point where the electrons enter the field varies since the entrance point is moving at v_p relative to the field. At t_1, in Fig 12, the fields match while at a later time t_2 the position of the entering stream has shifted by an amount $(t_2-t_1)v_p$ and they no longer match. Equation 2.06 (3) does not hold until the stream has assumed a harmonic variation of radial coordinate. It is reasonable to expect some irregular motion of the stream until it has traveled far enough for its outer boundary to match the periodic form of the infinite stream. This perturbation of the stream when it enters the periodic focusing field may lead to some interception of the stream at the beginning of the helix. One way to reduce this effect would be to make the diameter of the beam small compared with that of the helix and to allow it to emerge into the field only after the wave is well established on the helix. Unfortunately, the small value of r_o/a will reduce the efficiency of the focusing power.

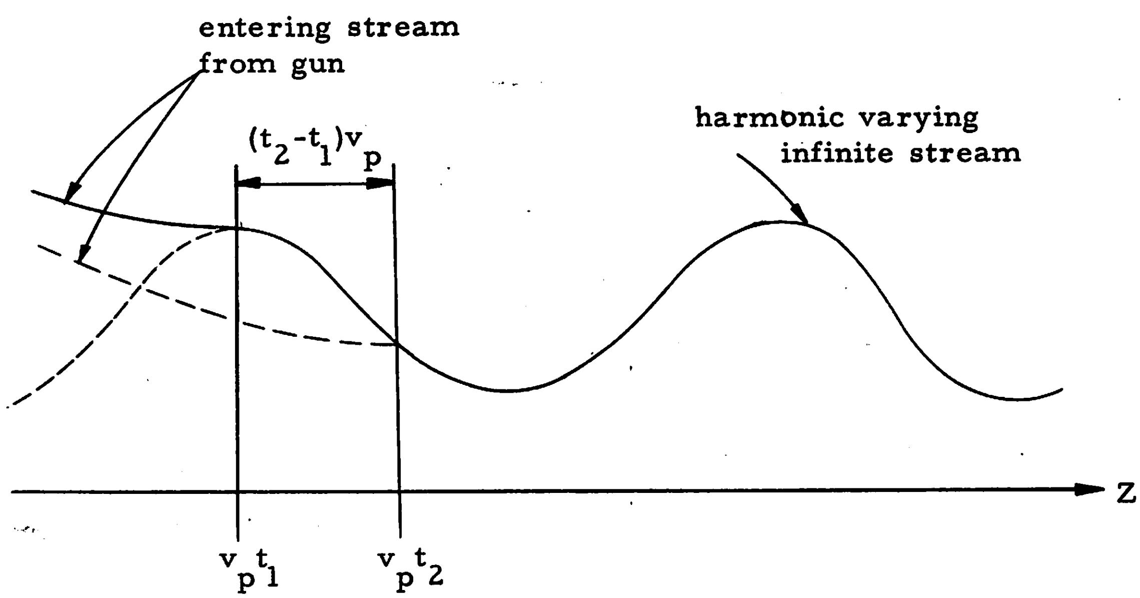


Fig. 12 A Mismatch of Slope Exists between the Entering Electron Stream and a Focused Infinitely Long Electron Stream.

Or, the wave might be built up slowly in amplitude along the structure thereby reducing the initial shock of the wave on the stream. Or, a lens might be put before the helix, driven by the rf in proper phase to provide proper entrance slope and radius.

3.03. Tube II.

The second tube had the same electron gun described previously except that a small tube was attached to the end of the second anode as shown in Fig. 11. This small tube was approximately 0.110" in diameter, just large enough to allow the stream to pass through without an unusual amount of interception to and yet sufficiently small in radius so it does not affect the radio frequency match of coaxial line to the helix. The beam was shielded by this tube from the rf field for the first few turns of the helix. The helix of the second tube was quite stiff because of the large tape used to wind it and its short length; hence it could be supported by the pins used for the radio frequency coupling. The length of the helix was chosen to be about three inches so that some initial current would be transmitted through the helix even with no focusing, thereby giving some transmission with which to begin the focusing studies. Due to the large diameter of the helix compared with that of the electron gun, it was possible to mount the gun very close to the beginning of the helix and still keep the tube simple. The beam diameter was small compared to the helix diameter ($r_o/a=1/4$) so the beam essentially travels on the axis of the helix and, on entering the periodic field, little helix interception of the stream would be expected. The position of the gun relative to the helix is roughly as shown in Fig. 10. The electrode used for collecting the stream on exit from the helix was simply a piece of sheet metal in the shape of a cone.

The tube gave indications that slow-traveling waves would focus the electron beam. Due to the large transmission of current in the absence of a focusing field however little actual change in the transmission was noticed. It was necessary to adjust the voltage of the first and second anodes until the beam entering the periodic field was of the proper shape to be focused.

3.04. Tube III.

In the next modification the collector was changed so that the size of the beam entering the collector could be ascertained. The collector consisted of a wire grid that was coated with a material which would fluoresce when the beam struck it. This tube is shown in Fig. 13. The transfer of radio frequency power from the coaxial feed lines to the helix presented a problem in matching but it was a simple matter to use a tuning stub at the input to insure that all of the available power was being transferred to the tube.

When the radio frequency power was turned on, the fluorescence due to the impinging beam was seen to change from a uniform glow filling the entire screen to a small brilliant spot surrounded by a weaker halo. From the appearance of the screen and observation of the collector current with rf power on and off, it was inferred that slow waves had a focusing effect on the stream.

3.05. Tube IV.

The next step was to determine the order of magnitude of radio frequency power which would be required to focus a stream at some given potential and current. The length of the helix was increased from three inches to eight inches in order to reduce the transmission in the absence of radio frequency on the helix. Two quartz washers, with inside diameter that of the helix and outside diameter that of the glass tubing of the vacuum envelope, were used for supporting the center of the helix. The transmission to the collector with or without radio frequency on the helix varied considerably depending on what potentials were applied to the

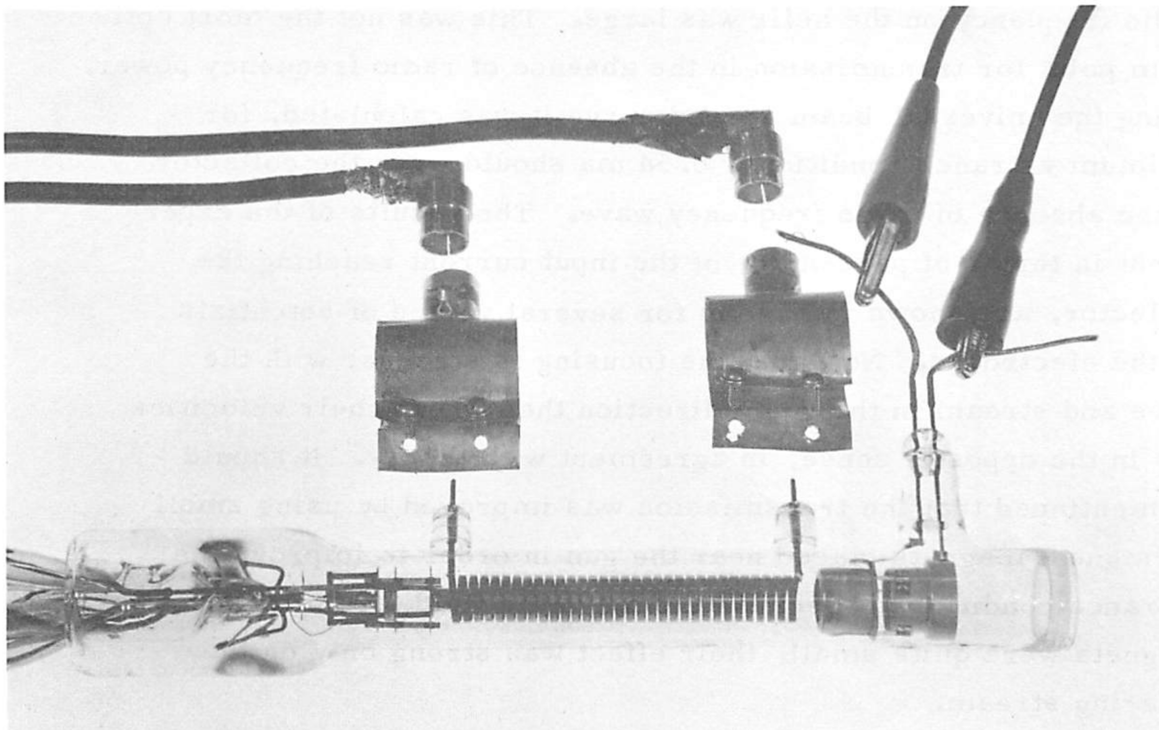


Fig. 13 Photograph of Tube Shown Schematically in Fig. 11.

various electrodes. The potential of the collector had little effect as long as the collector was positive and was therefore set at 150 volts above the helix to eliminate any secondaries that might return to the helix. The potentials of the remianing electrodes were varied until an operating point was found where the transmission with radio frequency on the helix was large. This was not the most optimum point for transmission in the absence of radio frequency power. Using the universal beam spread curve it was calculated, for optimum entrance conditions, 0.54 ma should go to the collector in the absence of radio frequency wave. The results of the experiment in terms of percentage of the input current reaching the collector, are shown in Fig. 14 for several values of potentials on the electrodes. Note that the focusing is stronger with the wave and stream in the same direction than where their velocities are in the opposite sense, in agreement with theory. It should be mentioned that the transmission was improved by using small permanent magnets placed near the gun in order to improve the entrance conditions. Because the magnetic fields from these magnets were quite small, their effect was strong only on the entering stream.

If the beam diameter is chosen to be halfway between that of the cathode and helix the power required experimentally is close to that predicted. The power was difficult to measure exactly due to losses and reflections. If some of the radio frequency wave on the helix was being reflected it would contribute to the focusing (although weakly) but not be measured in the power out.

The fluorescent screen used for the collector gave an indication as to what the stream looked like when it hit the collector. For no wave on the helix the entire screen glowed dimly; as the radio frequency power was increased from zero the glow would shrink to a brilliant spot, corresponding to 10 watts of radio frequency power out of the tube, and then expand as power was increased further, to 15 watts of power, presumably due to

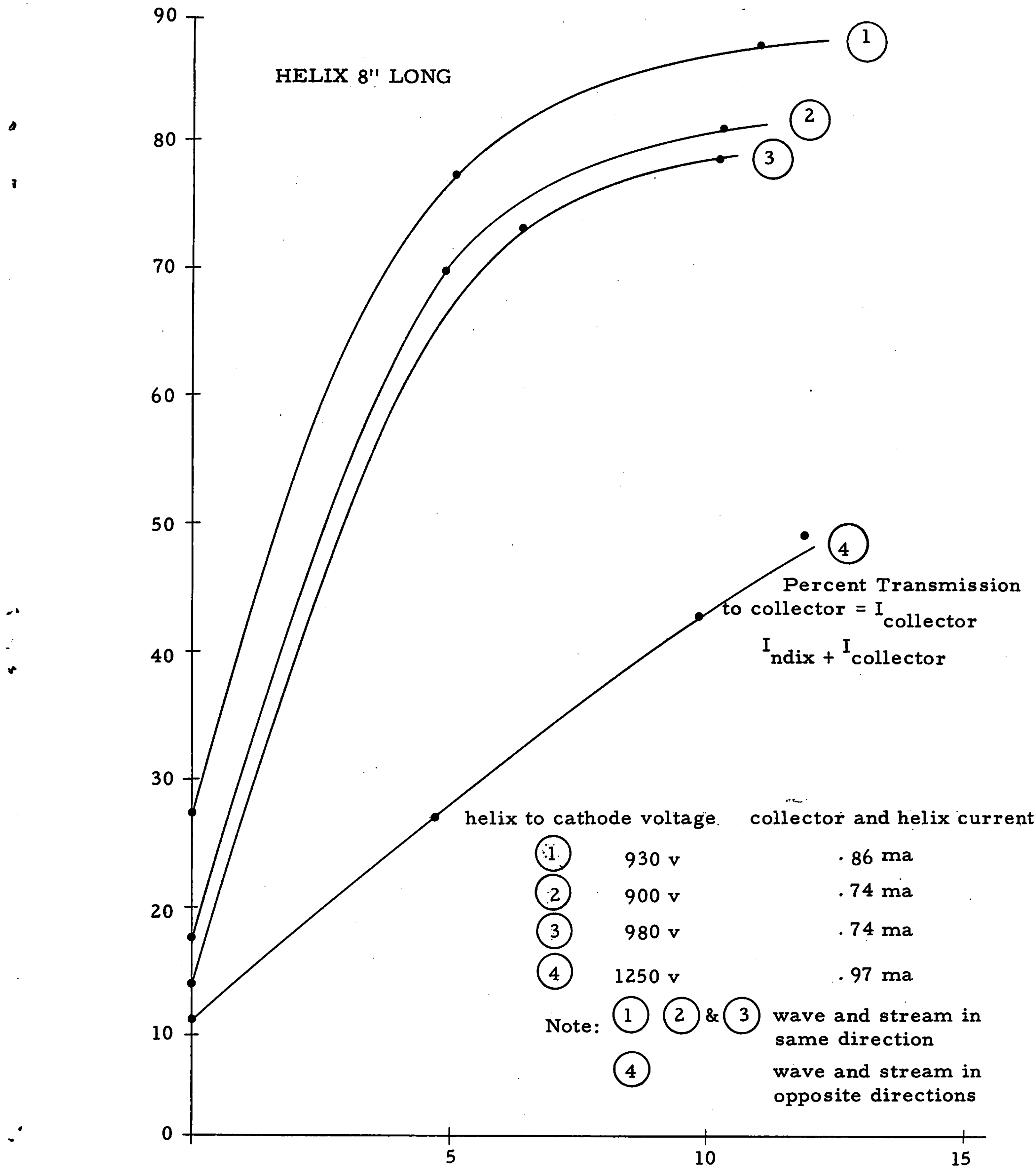


Fig. 14 R-F Power as a Function of Percent Transmission for Tube IV

overfocusing (Section 2.07). Figure 15 shows the fluorescent screen with power on and power off. The diameter of the spot with power on is about 0.080 inches. The diameter of the screen is about 0.300 inches. The gun was somewhat misaligned with respect to the axis of the helix but with the application of rf on the helix the spot seen on the fluorescent screen was seen to move toward the center of the screen.

Due to the configuration of the electron gun in this tube is was impossible to vary V_{oo} over a wide range of values and investigate the focusing effect for V_{oo} near V_H. The electrostatic lens system formed by the electrodes of the gun gave good entrance conditions only for certain potentials thereby restricting the usable range of stream potentials. The theory of section 2.07 indicates no focusing for V_1/V_o near or greater than one; it was important therefore to determine how close to V_H one might make V_{oo} and still have transmission. There exists some optimum point since, according to Eq. s 2.06 (6) and 2.06 (7), the focusing becomes better as V_{oo} approaches V_H. This optimum point is not given theoretically.

3.06. Tube V.

In order that the focusing could be investigated for a large range of V_{oo} above and below V_H, a tube was built with a relatively low phase velocity. A convergent flow gun was used which was supplied by the Sylvania Microwave Tube Laboratory. This gun was designed to operate at 600 volts with a beam current of 6 ma. The gun had a grid for controlling the emission and appeared to perform well for a wide range of beam voltages, V_{oo}. A schematic of the tube is shown in Fig. 16. The collector collects all of the beam that is not intercepted by the probe in the center of the helix. Due to the small diameter of the helix, imposed by the optimum condition, $\beta a = 1$, and for f = 1 kMc and a low phase velocity, it was necessary to make the tube

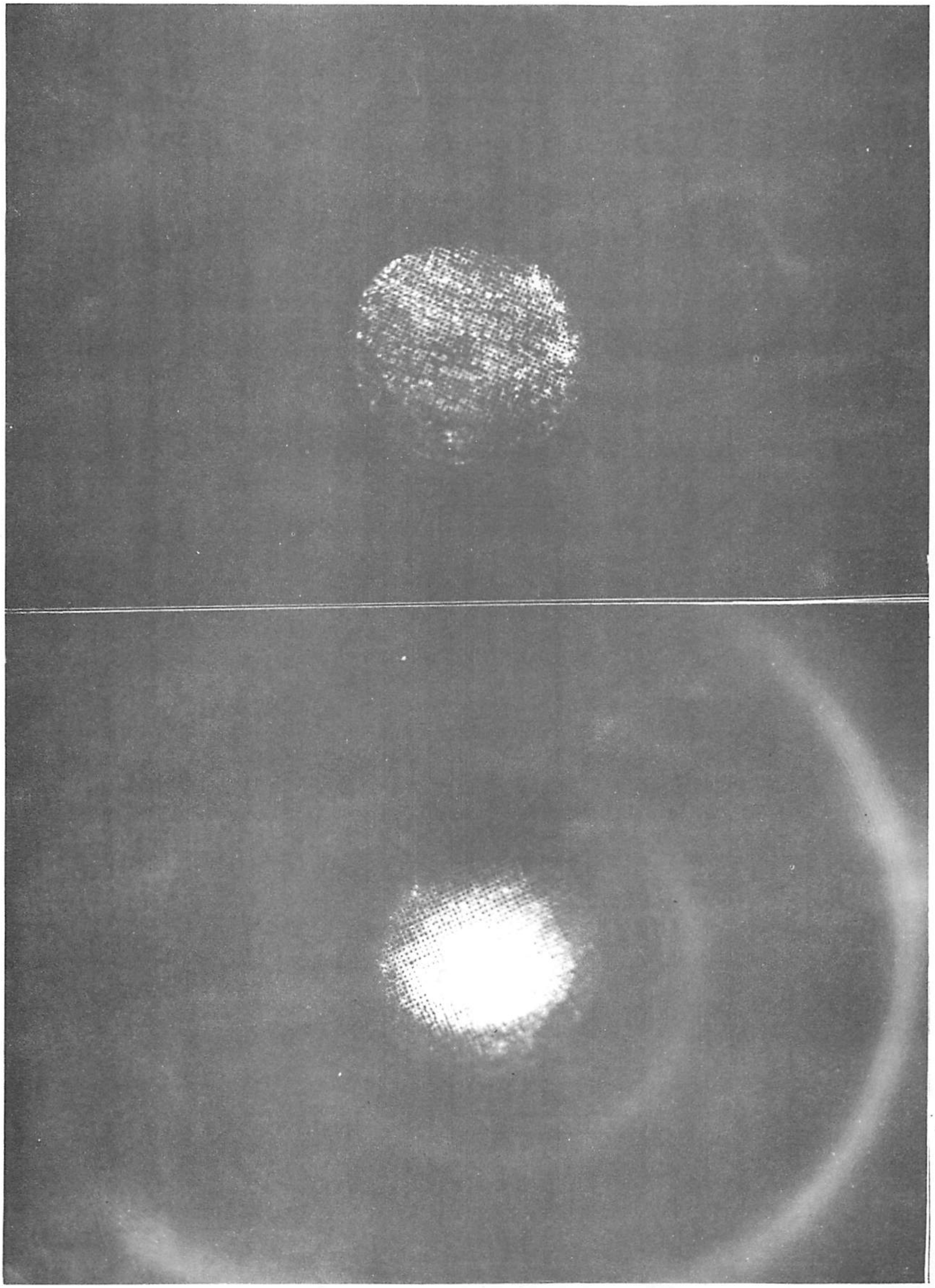


Fig. 15 Appearance of Stream Striking the Collector Power OFF (above) Power ON (below).

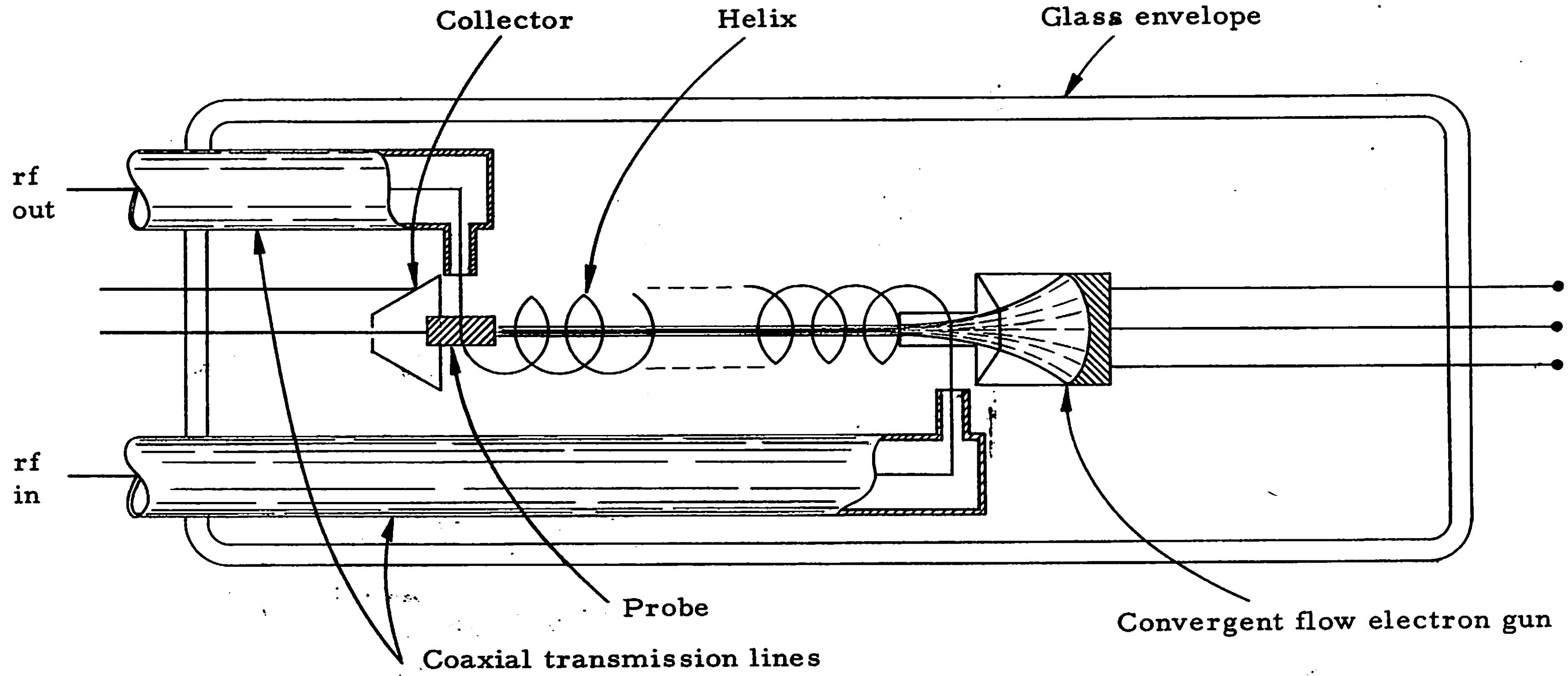


Fig. 16 Schematic of Tube V for Investigating the Region where v_p is near v_e.

considerably more complicated, than the previous ones. In order to put the gun quite near the helix it was necessary to put the coaxial to helix match in the vacuum. Nothing could be done about the entrance conditions once the tube was built. The helix was glass rod supported and some dielectric loading (reducing V_p and K_H) was introduced by this support. The calculated value (sheath model) of V_H was 750 volts and the measured value was 590 volts at 1 kMc and 550 volts at 1.2 kMc.

The match into the tube was adjusted by means of a double stub and a double stub was also inserted at the output of the tube. Although it was possible to get an input VSWR of 1.08, there still was a considerable amount of power lost through the tube and associated radio frequency lines. We have no measurement of VSWR within the helix. There is also some question as to the accuracy of measuring the radio frequency power since a 40 db pad on the output was used to drop the power to a level where a milliwatt meter could be used.

The transmission was poor with regard to total injected current being 25 % with radio frequency on. The rods along the helix showed fluorescence where the beam was intercepted for about the first 1.5 inch of helix and hence it was assumed that the interception took place at the beginning of the helix. The current to the probe and collector is somewhat boldly assumed to comprise the stream current. Without rf on the helix, it was impossible to see any current collected by the probe or the collector under any conditions of operation. For 350 volts $< V_{oo} <$ 800 volts no focusing was observed as was expected since $\frac{V_1}{V_0}$ becomes large. Operation for several values of V_{oo} is shown in Figs. 17 and 18 for f = 1.2 kMc. The data shown in Table I was taken at the maximum value of probe current for the 200 volt curve in Fig. 17.

The agreement with theory is fairly good and V_1/V_o is less than one as expected. As the power is increased a peak value

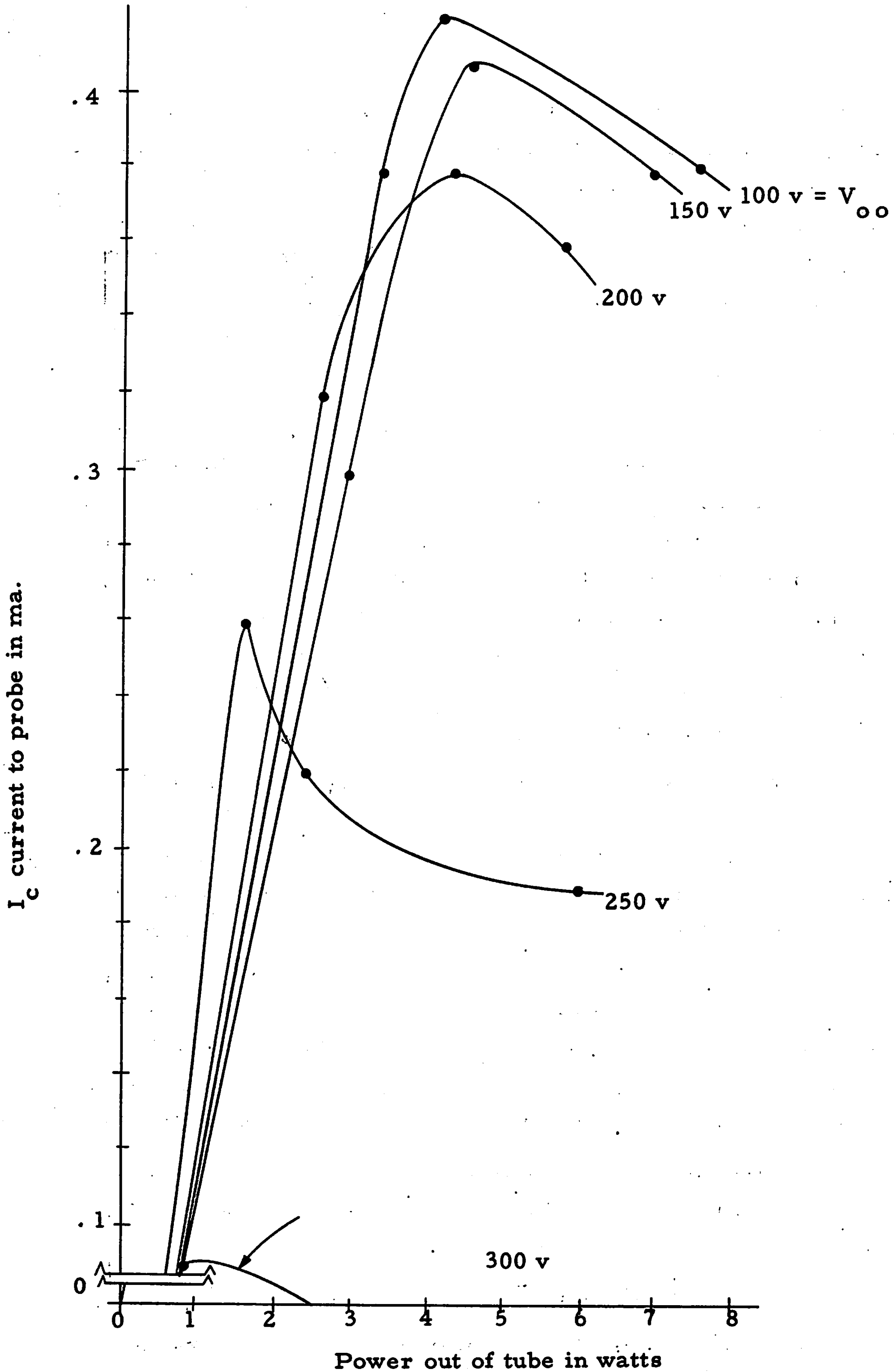


Fig. 17 Probe current vs rf power for tube V with $V_{oo} < V_H$ (V_H=550v).

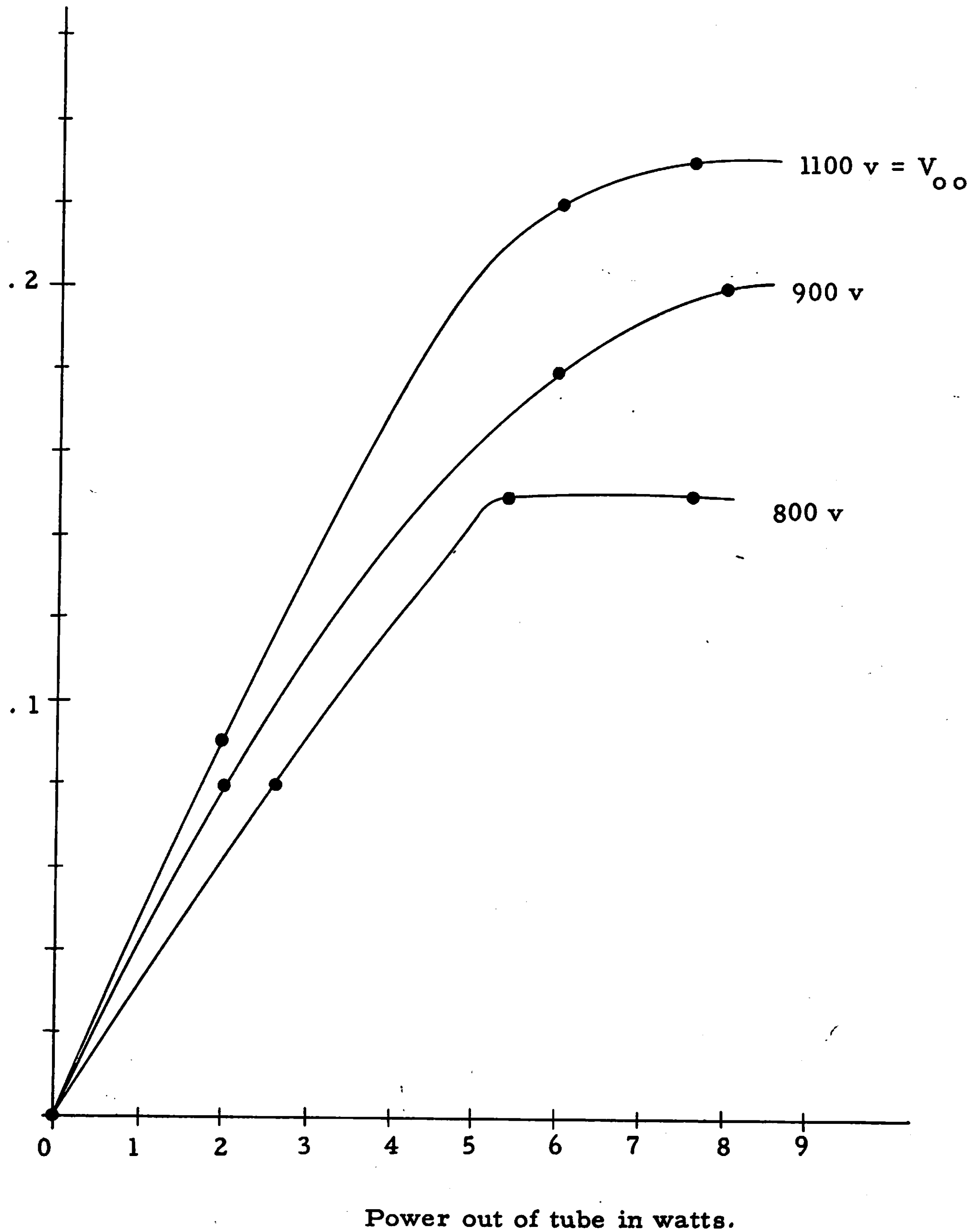


Fig. 18 Probe current vs rf power for Tuve V with $V_{oo} > V_H$ (V_H=550v).

of collector current is observed in Fig. 17. This is a reasonable result since, as the power is increased, the value of V_1/V_o increases and the optimum point mentioned previously is reached and passed with the result that the current drops. The maximum current focused was predicted to vary as V_{oo} for the same value of V_o; thus for $V_{oo} > V_H$, as in Fig. 18, more current should be focused than for $V_{oo} < V_H$ (Fig. 17.), but this does not occur. If, however, the total emission from the cathode is raised, the current focused at the higher voltages is more than the current focused at the lower voltages for the same value of V_o, and, hence, one is led to believe the experiment is at fault and the theory is correct. The electron gun operation was probably at fault in this case, since it was operating at electrode potentials it was not designed for.

When the wave and stream were moving in opposite directions, the focusing with the output mismatched (appreciable reflection) was considerably larger, as one would expect, than M the matched case, since the reflected wave was traveling with the stream and contributed greatly to the focusing. In the region where V_{oo} was near V_H no focusing was noticed due to the reflected wave.

IV. DISCUSSION

The experiments have shown that the slow wave on a helix may be used to confine an electron stream and that Eq. 2. 06 (3) predicts the required radio frequency power to a reasonable degree of accuracy.

Several factors may account for the discrepancy between experiment and theory:

(i) The entrance conditions present a main problem as mentioned in Section 3. 02. It was found experimentally that a small steady magnetic field in the entrance region may help these entrance conditions.

(ii) Although the impedance match was obtained by the use of double stub tuning, there still existed mismatch due to the change in mode from the coaxial line to helix and vice versa; this mismatch will lead to reflections back and forth on the helix. This reflected power was not measured as power out of the tube but nevertheless, contributed to the focusing.

(iii) The magnetic fields due to the wave may also affect the stream focusing and these fields were neglected in the theory. The effect of these fields should be quite small, however, as mentioned in 2. 01.

(iv) In the experiments there existed stray radio frequency fields near the gun and these fields may have affected the space charge in the region between the cathode and anode, thereby influencing the operation of the gun. Some small change was noticed in the anode and grid current with the introduction of radio frequency power on the helix.

(v) The variation of focusing properties of the gun with different values of electrode potential undoubtedly influenced the transmission characteristics.

In the fast-wave focusing experiment of Weibel and Clark (1958) it was found possible to focus an electron stream of the following dimensions: $2r_o = .078''$, $V_{oo} = 400$ volts, $I_{oo} = 3 \times 10^{-4}$ amp, $\omega_p^2 = 1.6 \times 10^{17}$. A TE_{01} mode was used, with $V_o/a \ll 1$. The required radio frequency power was 250,000 watts (pulsed). In the experiment of Section 3.05 an almost identical stream was confined using slow waves with but 15 watts of radio frequency power; roughly 16,000 times smaller!

In a letter, Sugata, Terada, Ura and Ikebucki (1960) stated that waves traveling faster than the beam would defocus, quite contrary to our experiment. Their argument that defocusing occurs if the wave velocity is larger than the stream velocity is as follows: as the wave accelerating (axial) field passes by an electron, the wave accelerates the electron; then, the radially inward (focusing) force of the wave acts on this electron. Similarly, after the wave decelerates an electron, the radially outward (defocusing) force acts on this electron. Thus, it appears that the electron will spend less time in the focusing field and more time in the defocusing field, resulting in net defocusing. However, we wish to point out that the opposite is true and that net focusing occurs.

The accelerated electron tends to catch up with the wave and spends more time in the focusing field than in the defocusing field that follows deceleration, where the electron tends to drop further behind the wave. Or, one may observe the action from the wave coordinate system, as follows: an electron drifting through the decelerating phase is decelerated, to be sure, as viewed by a laboratory observer, but this electron is accelerated away from this region as seen by the wave; similarly, the electron drifting through the accelerating phase is accelerated in the laboratory frame, but is decelerated in the wave frame. Thus, just as with periodic electrostatic focusing, net focusing occurs.

Slow-wave focusing may find applications for focusing the electron stream in a device such as a traveling-wave tube or backward-wave amplifier if the focusing wave can be introduced by reflection or perhaps by some external circuit. The application of slow-wave focusing to ion confinement also shows promise because of the relatively low radio frequency power required in the electron stream experiment. Perhaps in a linear accelerator, where focusing might be desired, use of rf focusing would provide a simple straightforward method of focusing.

However, the primary attempt here was not in showing applications, but was in demonstrating rf containment of electrons and in showing that use of slow-wave guides has very large advantages over use of fast-wave guides.

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