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NATURAL FREQUENCIES OF NETWORKS CONTAINING TUNNEL DIODES

by

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I. INTRODUCTION

With the invention of an electrical device a number of questions must be answered by circuit theorists in order that designers can achieve the full potentialities of the new device. For example, a practically important problem is to determine exactly characteristics that can be achieved using the new device imbedded in a passive circuit. In particular, this question might reduce to the following: If the device is imbedded in a passive circuit, what are the necessary and sufficient conditions on the resulting driving point function? If the device is a tunnel diode represented by the complete linear equivalent circuit shown in Figs. 1a and 1b, this question is still unanswered, although a number of significant attempts have been made.

Kinariwala[1] first found the limitations on the natural frequencies of a tunnel diode imbedded in a passive network with the tunnel diode treated as a negative resistance in parallel with a capacitor. For this same equivalent circuit necessary and sufficient conditions on the driving point impedance were then found independently by Kinariwala[2] and Sandberg.[3] Golosman and Newcomb[4] next found the allowable natural frequencies using an equivalent circuit consisting of the circuit in Fig. 1 with either the series resistor or series inductor absent. For the same circuits, Sandberg[5] then found necessary and sufficient conditions on the impedance obtained by imbedding these circuits in a passive network.

In this paper we extend these results to define the regions of the complex frequency plane in which natural frequencies can be achieved for the tunnel diode imbedded in a passive network, using the complete linear equivalent circuit of the tunnel diode. The technique used is the same as that of Golosman and Newcomb, namely the application of the theory developed from energy considerations by Desoer and Kuh[6] for determining "active points" in the right half of the complex frequency plane. However, whereas Golosman and Newcomb only carry through the analysis for the simplified equivalent circuits for the tunnel diode, we are able to define exactly allowable regions for the natural frequencies for the complete equivalent circuit.

II. STATEMENT OF THE PROBLEM AND SUMMARY OF RESULTS

A real, linear, time-invariant, one-port network N is said to be "active" at the complex frequency p_0 with $\mathrm{Re}\ p_0 \geq 0$, if there exists a passive one-port N_P such that the combined network $N - N_P$ formed by imbedding N in N_P as in Fig. 2 can support a mode of the form $I(t) = I e^{p_0 t}$ where I is some fixed current. In other words, N is active at p_0 if there is an N_P such that the network $N - N_P$ has a short circuit natural frequency at p_0. Stating it in still another way, p_0 is an active point if there exists an N_P such that $Z_P(p_0) + Z(p_0) = 0$ where $Z_P(p_0)$ is the impedance of the passive network N_P and $Z(p_0)$ is the impedance of the diode. The point p_0 is called an active point and the region in the complex frequency plane containing all points at which N is active is called the "active region." The problem then becomes to determine the active regions for the tunnel diode using the complete linear equivalent circuit.

Desoer and Kuh have derived necessary conditions (shown to be sufficient for a one port[7]) for a point p_0 to be an active point for an n-port device imbedded in a passive n-port network. When specialized to a one-port device imbedded in a one-port passive network these conditions reduce to 1a, 1b and 1c below, where $p_0 = \sigma_0 + j\omega_0$. Im Z(p) denotes the imaginary part of the impedance at p_0 and $\mathrm{Re}\ Z(p_0)$, the real part.

$$\mathrm{Re}\ Z(p_0) \leq 0 \qquad \text{for all } \omega_0 \text{ and } \sigma_0 \geq 0 \qquad (1a)$$

$$\mathrm{Im}\ Z(p_0) - \omega_0\ \mathrm{Re}\ Z(p_0) \leq 0 \qquad \text{for } \omega_0 > 0 \text{ and } \sigma_0 \geq 0 \qquad (1b)$$

$$\mathrm{Im}\ Z(p_0) + \omega_0\ \mathrm{Re}\ Z(p_0) \geq 0 \qquad \text{for } \omega_0 > 0 \text{ and } \sigma_0 \geq 0. \qquad (1c)$$

Note that for $p_0 = j\omega_0$, (1b) and (1c) reduce to (1a). For $\omega_0 = 0$ (1b) and (1c) become vacuous statements.

The physical meaning of (1a, b, c) for the one port network is readily apparent. If we are to have

$$Z(p_0) + Z_P(p_0) = 0$$

then Re $p_0 \geq 0$ we must have Re $Z(p_0) \leq 0$ since for a positive real function Re $Z_P(p_0) \geq 0$. Similarly since $Z_P(p_0)$ is positive real it satisfies the conditions $|\arg Z_P(p_0)| \leq |\arg(p_0)|$ for Re $p_0 \geq 0$ which is equivalent to (1b) and (1c) for Re $Z(p_0) \leq 0$. (1a), (1b), (1c) are the conditions applied to the tunnel diode equivalent circuit of Fig. 2b for which we can easily show Re Z(p) and Im Z(p) are given by (2a) and (2b) respectively

$$\mathrm{Re}\, Z(p) = r + 1 + \frac{\sigma - 1}{(\sigma - 1)^2 + \omega^2} \tag{2a}$$

$$\mathrm{Im}\, Z(p) = \ell\omega - \frac{\omega}{(\sigma - 1)^2 + \omega^2} \quad . \tag{2b}$$

The remainder of the paper is concerned with the detailed solutions and interpretations of Eqs. (1) and (2). The results of the investigations are displayed in Tables 3 and 4 for all possible relative values of r and ℓ. The curves labeled ① and ② are described by the Eqs. (3a) and (3b) to be derived later.

$$\omega^2 = \frac{1}{r}(1 - 2\sigma) - (\sigma - 1)^2 \tag{3a}$$

$$\omega^2 = \frac{1}{2\ell\sigma + r} - (\sigma - 1)^2 \quad . \tag{3b}$$

For $\sigma_0 = 0$ the maximum frequency of oscillation ω_i is given by

$$\omega_i = \sqrt{\frac{1}{r} - 1} \quad . \tag{4}$$

The largest σ_0 for $p_0 = \sigma_0 + j\omega_0$, with $\omega_0 = 0$ is given by

$$\sigma_i = \frac{1}{2}\left(1 - \frac{r}{\ell}\right) \quad . \tag{5}$$

The other critical points indicated on the curves are given in (6)-(10).

$$\sigma_1 = \frac{1}{2}(1 - r/\ell) + \frac{1}{2}\sqrt{(1 + r/\ell)^2 - 4/\ell} \tag{6}$$

$$\sigma_2 = \frac{1}{2}(1 - r/\ell) - \frac{1}{2}\sqrt{(1 + r/\ell)^2 - 4/\ell} \tag{7}$$

$$\sigma_3 = \alpha_1, \ \sigma_4 = \alpha_2 \tag{8}$$

where α_1 and α_2 are the real roots of the equation

$$\sigma^3 + (\frac{r}{2\ell} - 1)\sigma^2 + (1 - r/\ell)\sigma + (\frac{r - 1}{2\ell}) = 0 \tag{9}$$

given in Appendix B. Finally we have

$$\omega_b = \sqrt{1/\ell - 1/4(1 + r/\ell)^2} . \tag{10}$$

III. TUNNEL DIODE ACTIVE REGION

We now proceed to derive the results described in the previous section. Substituting (2a) and (2b) into (1a, b, c) we have, respectively, (11), (12), and (13) as follows.

$$\ell\sigma^3 + \ell\sigma\omega^2 + (r - 2\ell)\sigma^2 + r\omega^2 + (1 - \ell - 2r)\sigma + r - 1 \leq 0 \tag{11}$$

$$r\sigma^2 + r\omega^2 + 2(1 - r)\sigma + r - 1 \leq 0 \tag{12}$$

$$2\ell\sigma^3 + 2\ell\sigma\omega^2 + (r - 4\ell)\sigma^2 + r\omega^2 + (2\ell - 2r)\sigma + r - 1 \leq 0 . \tag{13}$$

Rearranging, we have:

$$\text{from (11)} \quad \omega^2 \leq \frac{1 - \sigma}{r + \ell\sigma} - (\sigma - 1)^2 \tag{14}$$

from (12) $\omega^2 \leq 1/r(1 - 2\sigma) - (\sigma - 1)^2$ (15)

from (13) $\omega^2 \leq \frac{1}{2\ell\sigma + r} - (\sigma - 1)^2$. (16)

The regions of the complex frequency plane satisfying (14), (15) and (16) then consists of the points in the first quadrant of the complex frequency plane enclosed by the axes and the points satisfying (17), (18) and (19), respectively.

$$\omega^2 = \frac{1 - \sigma}{r + \ell\sigma} - (\sigma - 1)^2 \tag{17}$$

$$\omega^2 = \frac{1}{r}(1 - 2\sigma) - (\sigma - 1)^2 \tag{18}$$

$$\omega^2 = \frac{1}{2\ell\sigma + r} - (\sigma - 1)^2 \ . \tag{19}$$

Values of ω^2 satisfying (17), (18), and (19) will be denoted by ω_I^2, ω_{II}^2, and ω_{III}^2, respectively. Then a point σ_0 on the real axis will be an active point if and only if $\omega_I^2 \geq 0$ for that σ_0. A point ω_0 on the imaginary axis will be an active point if and only if ω_0^2 is not greater than the value of ω_I^2 at $\sigma = 0$. A point $p_0 = \sigma_0 + j\omega_0$ in the first quadrant (not including the axes) is an active point provided that ω_I^2, ω_{II}^2 and ω_{III}^2 are all positive at σ_0, and provided that $\omega_0^2 \leq \text{Min}(\omega_I^2, \omega_{II}^2, \omega_{III}^2)$ at $\sigma = \sigma_0$. For a given value of σ, ω_I^2 or ω_{II}^2 or ω_{III}^2 is said to be "dominant" or to be the "dominant restriction" if it is equal to $\text{Min}(\omega_I^2, \omega_{II}^2, \omega_{III}^2)$. Eq. (17), (18) or (19) is said to be the "dominant curve" or the "dominant equation" if it yields the dominant restriction. It is shown in Appendix A that the relative values of ω_I^2, ω_{II}^2, and ω_{III}^2 and, hence, the dominant restrictions for different ranges of r, ℓ and σ are as given in Table 1.

The problem of finding the active regions in the first quadrant for the tunnel diode has therefore been reduced to examining the dominant restrictions for different ranges of r, ℓ, and σ. The active regions for the fourth quadrant are then found by reflecting the region in the first quadrant about the real axis. In Sec. IV the region is found over which $\omega_I^2 \geq 0$. Since ω_I^2 determines the active region on the real and imaginary axis, these regions are then found and given in Table 2. In Sec. V the region of non-negative ω_{II}^2 is determined. From Table 1 we see that for $r \geq \ell$, ω_{II}^2 is dominant. We can therefore determine the active region (not including the axes) from ω_{II}^2. Combining this region with the active region on the axes found in Sec. IV, the complete active region for $r \geq \ell$ is then given in Table 3. In Sec. VI, the region of non-negative ω_{III}^2 is determined. Combining the results of Secs. IV, V, and VI, the active region for $r < \ell$ is then given in Table 4. In Sec. VII it is shown that the necessary restrictions on the natural frequencies are also sufficient.

IV. REGION OF NON-NEGATIVE ω_I^2 AND ACTIVE REGION FOR $\omega = 0$ OR $\sigma = 0$

We consider (17), which yields the region of activity on the real and imaginary axes. We first identify the regions over which $\omega_I^2 \geq 0$. If $\sigma > 1$, then $\omega_I^2 < 0$. Hence we can limit our investigation to the range $0 < \sigma \leq 1$. If $\sigma = 1$, then $\omega_I^2 \geq 0$ corresponds to a single point, (1, 0) on the σ axis. Next, consider $\sigma < 1$. Rearranging (17) we have

$$\omega_I^2 = \frac{1 - \sigma}{r + \ell \sigma} \left[1 - (1 - \sigma)(r + \ell \sigma)\right] \tag{20}$$

$\frac{1 - \sigma}{r + \ell \sigma}$ is non-negative, for $0 \leq \sigma \leq 1$, since r and ℓ are assumed positive. In order that ω_I^2 be positive, we therefore must have $F(\sigma) \geq 0$ where $F(\sigma)$ is defined by

$$F(\sigma) \equiv 1 - (1 - \sigma)(r + \ell \sigma) \tag{21}$$

or

$$F(\sigma) = \left[\sigma - 1/2(1 - r/\ell)\right]^2 + \left[1/\ell - 1/4(1 + r/\ell\right]^2 \geq 0 . \tag{22}$$

We then have three cases to consider, as follows.

Case I. If $1/\ell - 1/4\,(1 + r/\ell)^2 > 0$ then $F(\sigma)$ is always non-negative for all $\sigma < 1$. This condition is equivalent to $r < 2\sqrt{\ell} - \ell$.

Case II. If $1/\ell - 1/4\,(1 + r/\ell)^2 = 0$, then

(a) for $r \geq \ell$, $\omega_I^2 \geq 0$ for $0 \leq \sigma \leq 1$

(b) for $r < \ell$, $\omega_I^2 \geq 0$ for $1/2\,(1 - r/\ell) < \sigma \leq 1$.

Case III. If $1/\ell - 1/4\,(1 + r/\ell)^2 < 0$, rearrange $F(\sigma)$ in a factored form

$$F(\sigma) = \left[\sigma - 1/2(1 - r/\ell) + 1/2\sqrt{(1 + r/\ell)^2 - 4/\ell}\right]\left[\sigma - 1/2(1 - r/\ell) - 1/2\sqrt{(1 + r/\ell)^2 - 4/\ell}\right] \quad (23)$$

and let

$$\sigma_1 = 1/2(1 - r/\ell) + 1/2\sqrt{(1 + r/\ell)^2 - 4/\ell} \quad (24)$$

$$\sigma_2 = 1/2(1 - r/\ell) = 1/2\sqrt{(1 + r/\ell)^2 - 4/\ell}\,. \quad (25)$$

For $\sigma_1 \leq \sigma \leq \sigma_2$, $F(\sigma)$ is negative. For σ smaller than both σ_1 and σ_2, or greater than both σ_1 and σ_2, $\omega_I^2 \geq 0$. Next consider the locations of σ_1 and σ_2; there are two cases:

(a) if $r \geq \ell$, then $\sigma_2 < 0$ or complex

(i) if $r > 1$, then $1 > \sigma_1 > 0$

(ii) if $r = 1$, then $\sigma_1 = 0$

(iii) if $r < 1$, then $\sigma_1 < 0$ or complex

(b) if $r < \ell$, then $\sigma_2 < 0$ or complex

(i) if $r > 1$, then $\sigma_2 < 0$ or complex

(ii) if $r = 1$ then $\sigma_2 = 0$

(iii) if $r < 1$ then $1 > \sigma_2 > 0$ or complex

In Table 2, the first two columns correspond to different relative values of r and ℓ. The third column contains the regions in the first quadrant of the complex frequency plane for which ω^2 is less than or equal to ω_I^2 and for which ω_I^2 is non-negative. The region in the fourth quadrant is found by symmetry. The fourth column contains the active regions on the axes of the complex frequency plane found by taking the intersection of the regions in the second column with the axes. In the table the maximum real active frequency is denoted by ω_i.

V. REGION OF NON-NEGATIVE ω_{II}^2 AND ACTIVE REGION FOR $r \geq \ell$.

Rewriting (18), we have

$$\left[\sigma - (1 - 1/4)\right]^2 + \omega_{II}^2 = \left[1/r\,(1/r - 1)\right] \tag{26}$$

We see that ω_{II}^2 describes a circle with center at $(1 - 1/r,\ o)$ and radius $R = \sqrt{1/r\,(1/r - 1)}$ as in Fig. 3.

If $r = 1$, the region inside the circle degenerates to a single point $(0, 0)$. σ_j, the maximum value in the region on the real axis, cannot exceed 1/2. For $r \geq \ell$ ω_{II}^2 is dominant. Combining the results for the axis from Table 1, with the dominant region for ω_{II}^2, we then have the active regions in Table 3 for $r \geq \ell$.

VI. REGION OF NON-NEGATIVE ω_{III}^2 AND ACTIVE REGION FOR $r < \ell$

For $r < \ell$ and $1/2(1 - r/\ell) \leq \sigma < 1$, ω_{II}^2 given in Sec. V is dominant. Next consider (19) rewritten as:

$$\omega_{III}^2 = \frac{1}{2\ell\sigma + r}\left[1 - (2\ell\sigma + r)(\sigma - 1)^2\right]. \tag{27}$$

Since ℓ, r and σ are positive, $\frac{1}{2\ell\sigma + r}$ is always positive.

Define $G(\sigma)$ by the equation

$$G(\sigma) = 1 - (2\ell\sigma + r)(\sigma - 1)^2 \quad . \tag{28}$$

Then $\omega_{II}^2 > 0$ if and only if $G(\sigma) \geq 0$. Thus for values of r and ℓ satisfying the constraint $3\ell^{2/3} - 2\ell < r < 2\sqrt{\ell} - \ell$, there are two separate regions on the real axis which degenerate to one region for $r < 3\ell^{2/3} - 2\ell$ or $r \geq 2\sqrt{\ell} - \ell$. In Table 4, σ_3 and σ_4 are given by the real roots of $G(\sigma) = 0$. These values are calculated in Appendix B.

Having now examined ω_I^2 in Sec. IV, ω_{II}^2 in Sec. V, and ω_{III}^2 in this section, we can describe the active regions for $r < \ell$. On the real and imaginary axes the active regions are as given in Table 1. Off the real axis ω_{III}^2 is dominant for $0 \leq \sigma < 1/2(1 - r/\ell)$ and ω_{II}^2 is dominant for $1/2(1 - r/\ell) \leq \sigma \leq 1$. The active regions for $r < \ell$ are plotted in Table 4. In its dominant interval ω_{II}^2 has its maximum value of $\omega_b^2 - 1/\ell - 1/4(1 + r/\ell)^2$ at $\sigma = 1/2(1 - r/\ell)$. ω_b is positive so long as $r > 2\sqrt{\ell} - \ell$. If $r = 2\sqrt{\ell} - \ell$, $\omega_b = 0$.

VII. SYNTHESIS OF PASSIVE IMBEDDING NETWORKS

Simple RLC imbedding networks can be devised to imbed a given active one-port network and obtain any of the allowable natural frequencies. One such imbedding network is included here with its justification as an example. Consider as a general imbedding network a series LC circuit with impedance,

$$Z_P(p_0) = L p_0 = \frac{1}{p_0 C} \tag{30}$$

$$= \sigma_0 L + \frac{\sigma_0}{\sigma_0^2 + \omega_0^2} \; 1/C + j\left[\omega_0 L - \frac{\omega_0}{\sigma_0^2 + \omega_0^2} \; 1/C\right] \tag{31}$$

As stated in the introduction, the sufficient condition for N with impedance $Z(p_0)$ to be active at p_0 is that $Z_P(p_0) + Z(p_0) = 0$. We therefore require that $Z_P(p_0) = -Z(p_0) = -\alpha - j\beta$ where

$$\alpha = \text{Re}\, Z(p_0) \qquad \text{and} \qquad \beta = \text{Im}\, Z(p_0)\,, \tag{32}$$

and

$Z(p_0)$ is the impedance of the tunnel diode. Using (31) and (32), we have

$$\sigma_0 L + \frac{\sigma_0}{\sigma_0^2 + \omega_0^2} \; 1/C = -\alpha \tag{33a}$$

$$\omega_0 L - \frac{\omega_0}{\sigma_0^2 + \omega_0^2} \; 1/C = -\beta \quad . \tag{33b}$$

Solving for L and C, we have

$$L = \frac{-(\alpha \omega_0 + \beta \sigma_0)}{2\sigma_0 \omega_0} \tag{34a}$$

$$C = \frac{2\sigma_0\omega_0}{(\beta\sigma_0 - \alpha\omega_0)(\sigma_0^2 + \omega_0^2)} \quad . \tag{34b}$$

In order that L and C be positive and real (note that ω_0 and σ_0 are assumed positive), it is necessary that

$$\alpha \omega_0 + \beta \sigma_0 \leq 0 \tag{35a}$$

$$\beta \sigma_0 - \alpha\omega_0 \geq 0 \quad . \tag{35b}$$

Equation (35a) corresponds to (lb) and (35b) corresponds to (lc) , i. e., the dominant restrictions for $\omega_0 > 0$. Hence if p_0 as an active point with $\omega_0 > 0$, then L and C are positive and real. The element values are given by

$$L = -\frac{1}{2\sigma_0\omega_0} \left((r + \ell\sigma_0 + \frac{\sigma_0 - 1}{(\sigma_0 - 1)^2 + \omega_0^2}) \omega_0 + \sigma_0(\ell\omega_0 - \frac{\omega_0}{(\sigma_0 - 1)^2 + \omega_0^2}) \right) \tag{36a}$$

$$C = \frac{2\sigma_0\omega_0}{(\sigma_0^2 + \omega_0^2) \left((\ell\omega_0 - \frac{\omega_0}{(\sigma_0 - 1)^2 + \omega_0^2}) \sigma_0 - \omega_0(r + \ell\sigma_0 + \frac{\sigma_0 - 1}{(\sigma_0 - 1)^2 + \omega_0^2}) \right)} . \tag{36b}$$

Thus the series LC circuit covers the whole active range for $\omega_0 > 0$. As an example, consider a tunnel diode with $r = 0.01$, $\ell = 0.085$, and $p_0 = 0.1 + j0.1$. Then $L = 55.0$ and $C = 10.0$. The resulting network is given in Fig. 4.

VIII. CONCLUSION

The exact regions of allowable natural frequencies have been obtained in the right half of the complex frequency plane. Several interesting general conclusions can be drawn from the tables. As r and ℓ increase, the region of activity generally becomes smaller. For $r < \ell$ as r increases from zero, the active region first consists of a single domain. The region off the real axis then splits into two parts. One of these regions then disappears; the region on the real axis then splits, and finally for $r = 1$ all that is left is one part of the real axis. The behavior for $r \geq \ell$ is not quite as interesting but still points out that as r increases the region of activity is severely limited. Two final points should be noted.

First we have considered only natural frequencies in the right half plane since any natural frequencies can be obtained in the left hand plane. Finally, it should be noted that although any active point p_0 can be achieved with a passive imbedding, there is no way of predicting what other active points in addition to the desired active point are obtained with a given imbedding. The realization of isolated active points represents a major problem yet to be solved.

APPENDIX A

DERIVATION OF ENTRIES IN TABLE 1

Define $\Omega_{I\ II}$ as $\omega_I^2 - \omega_{II}^2$. Then

$$\Omega_{I\ II} = \omega_I^2 - \omega_{II}^2 = \frac{1 - \sigma}{r + \ell \sigma} - \left[\frac{1 - 2\sigma}{r}\right]$$

$$= \frac{2 \ell \sigma}{r(r + \ell \sigma)} \left[\sigma - 1/2\ (1 - r/\ell)\right] \qquad \text{(A.1)}$$

Since we have assumed ℓ and r to be positive and $0 \leq \sigma \leq 1$, $\frac{2 \ell \sigma}{r(r + \ell \sigma)}$ is non-negative. However, the sign of $\left[\sigma - 1/2(1 - r/\ell)\right]$ depends on the values of σ, r and ℓ. We consider the following cases:

(a) if $r \geq \ell$, then $\Omega_{I\ II} \geq 0$.

(b) if $r < \ell$, there are three cases:

- (i) if $1 \geq \sigma > 1/2(1 - r/\ell)$, then $\Omega_{I\ II} > 0$
- (ii) if $\sigma = 1/2(1 - r/\ell)$, then $\Omega_{I\ II} = 0$
- (iii) if $1/2(1 - r/\ell) > \sigma \geq 0$, then $\Omega_{I\ II} \leq 0$.

Next, define $\Omega_{I\ III} \equiv \omega_I^2 - \omega_{I\ II}^2$. Then

$$\Omega_{I\ III} = \frac{1 - \sigma}{r + \ell \sigma} - \left[\frac{1}{2\ell \sigma + r}\right]$$

$$= \frac{-2\ell \sigma}{(r + \ell \sigma)(2\ell \sigma + r)} \left[\sigma - 1/2(1 - r/\ell)\right] \qquad \text{(A. 2)}$$

$\frac{-2\ell \sigma}{(r + \ell \sigma)(2 \ell \sigma + r)}$ is nonpositive.

However, the sign of $\sigma - 1/2\,(1 - r/\ell)$ depends on the values of r, ℓ and σ:

(a) if $r \geq \ell$ $\Omega_{I\ III} \leq 0$

(b) if $r < \ell$ there are three cases:

(i) if $1 \geq \sigma > 1/2\,(1 - r/\ell)$ then $\Omega_{I\ III} \leq 0$

(ii) if $\sigma = 1/2(1 - r/\ell)$, then $\Omega_{I\ III} = 0$

(iii) if $1/2(1 - r/\ell) > \sigma \geq 0$, then $\Omega_{I\ III} \geq 0$.

APPENDIX B
CALCULATIONS OF ROOTS OF $G(\sigma) = 0$

Rewriting (28) and setting $G(\sigma)$ equal to zero, we have

$$\sigma^3 + \left(\frac{r}{2\ell} - 1\right) \sigma^2 + (1 - r/\ell)\, \sigma + \left(\frac{r-1}{2\ell}\right) = 0 . \tag{B.1}$$

It can then be shown,[8] that the roots ob (B.1) are given in general by

$$\sigma = \frac{p}{3w} - w - \frac{a_2}{3} \tag{B. 2}$$

where

$$p = a_1 - \frac{a_2^2}{3} \tag{B. 2}$$

$$q = \frac{a_1 a_2}{3} - a_0 - \frac{2a_2^2}{27} \tag{B. 4}$$

and

$$w = \left[\frac{-q}{2} + \sqrt{q^2/4 + p^3/27}\right]^{1/3} \tag{B. 5}$$

and for our case $a_2 = (r/2\ell - 1)$ (B. 6)

$$a_1 = (1 - r/\ell) \tag{B. 7}$$

$$a_0 = (r - 1)/2\ell \quad . \tag{B. 8}$$

The real roots of $G(\sigma) = 0$ are then given by the real roots of (B. 5), substituted into (B. 2).

REFERENCES

1. B. K. Kinariwala, "Theoretical limitations on the Esaki diode as a network element," IRE NEREM Record, 2, pp. 86-87; 1960.

2. B. K. Kinariwala, "The Esaki diode as a network element," IRE Trans., Vol. CT-8, No. 4, pp, 389-397; December 1961.

3. I. W. Sandberg, "The realizability of multiport structures obtained by imbedding a tunnel diode in a loss less reciprocal network," BSTJ, Vol. XLI, No. 3, pp. 857-876; May 1962.

4. B. S. Golosman and R. W. Newcomb, "Natural frequencies of negative resistors with parasitics," IEEE Trans. on Communication and Electronics, No. 68, pp. 527 - 531; September 1963.

5. I. W. Sandberg, "The tunnel diode as a network element, BSTJ, Vol. XLI, No. 5, pp. 1537-1556; September 1962.

6. C. A. Desoer and E. S. Kuh, "Bounds on natural frequencies of linear active networks," Proc. Symp. on Active Networks and Feedback Systems, pp. 415-436, Polytechnic Institute of Brooklyn; 1960.

7. R. W. Newcomb, "Synthesis of passive networks for networks active at p_0 - I," IRE Intl. Conv. Record, Pt. 4, pp. 162-175; 1961.

8. G. Birkhoff and S. MacLane, A Survey of Modern Algebra, The MacMillan Co., New York; 1953.

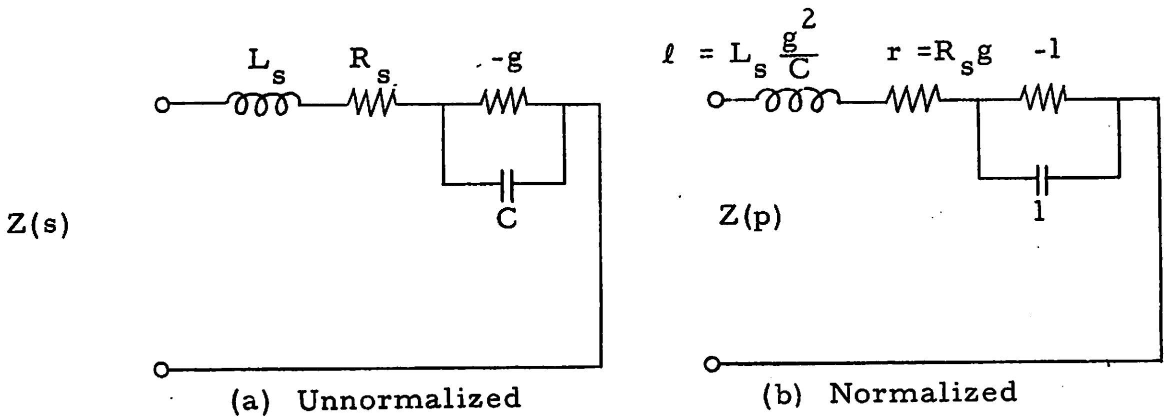


Fig. 1. Equivalent circuit of tunnel diode.

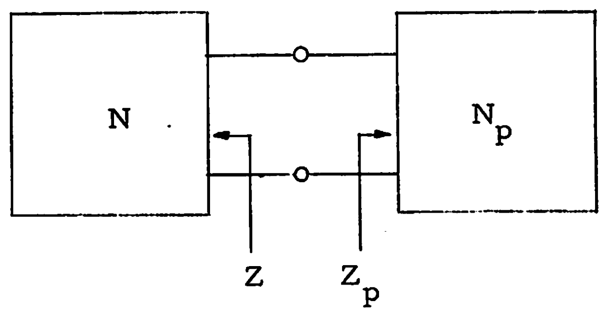


Fig. 2. Active network N imbedded in possitive network N_p.

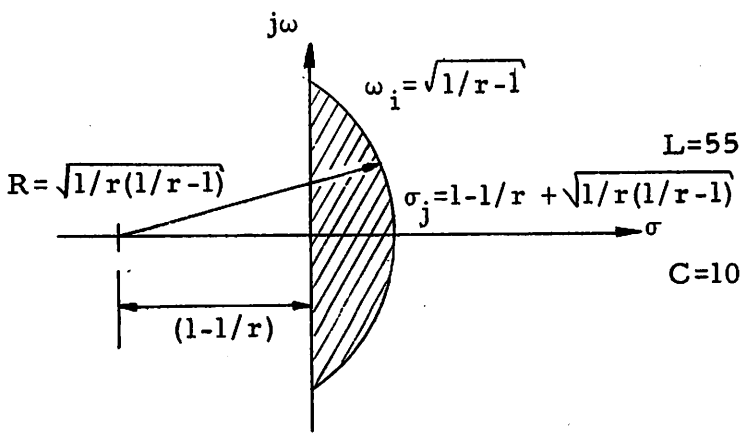


Fig. 3. Region in right half plane with $\omega_{II}^2 \geq 0$ and $\omega^2 \leq \omega_{II}^2$.

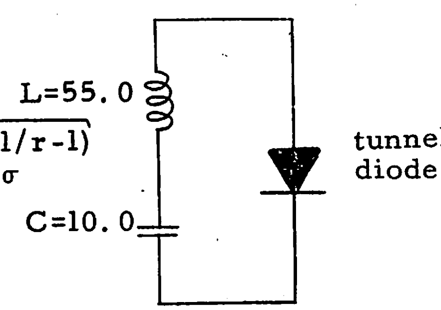


Fig. 4. Tunnel diode imbedded in passive network.

Relative Values of r and ℓ	Range of σ	Relative Values of ω_I^2, ω_{II}^2 and ω_{III}^2	Dominant Restriction
$r \geq \ell$	$0 \leq \sigma \leq 1$	$\omega_{II}^2 \leq \omega_I^2 \leq \omega_{III}^2$	ω_{II}^2
$r < \ell$	$0 \leq \sigma < 1/2(1 - r/\ell)$	$\omega_{III}^2 \leq \omega_I^2 \leq \omega_{II}^2$	ω_{III}^2
	$\sigma = 1/2\ (1 - r/\ell)$	$\omega_I^2 = \omega_{II}^2 = \omega_{III}^2$	$\omega_I^2 = \omega_{II}^2 = \omega_{III}^2$
	$1/2(1 - r/\ell) < \sigma \leq 1$	$\omega_{II}^2 \leq \omega_I^2 \leq \omega_{III}^2$	ω_{II}^2

TABLE 1 Dominant Restrictions for Ranges of r, ℓ and σ

FIGURE AND TABLE CAPTIONS

Figures

Tables

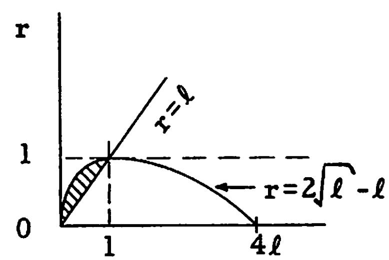
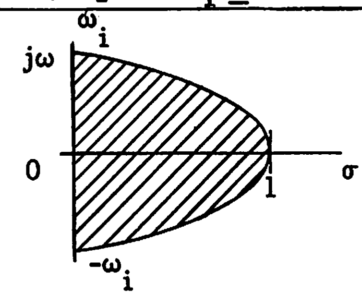
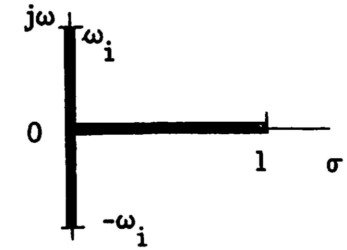
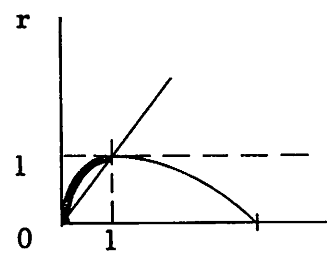
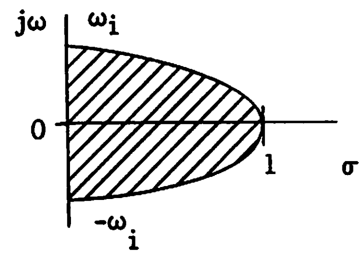
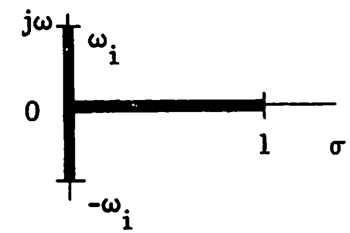
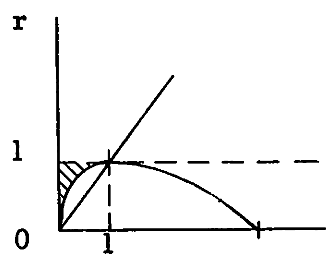
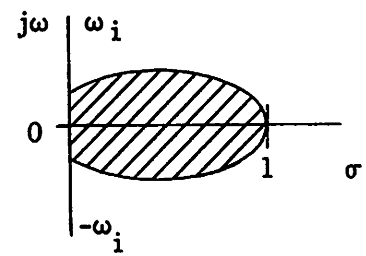
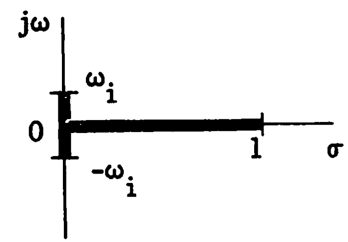
Case	Restrictions on r and ℓ		Region Enclosed by ω_I for $\omega_I^2 \geq 0$	Active Regions on Axes
I	$r < 2\sqrt{\ell} - \ell$			
IIa	$r = 2\sqrt{\ell} - \ell$			
IIIaiii	$1 > r > 2\sqrt{\ell} - \ell$			

TABLE 2 Active Regions for $\omega = 0$ or $\sigma = 0$, $r \geq \ell$

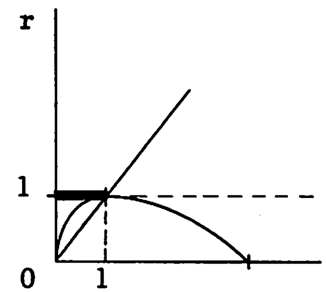
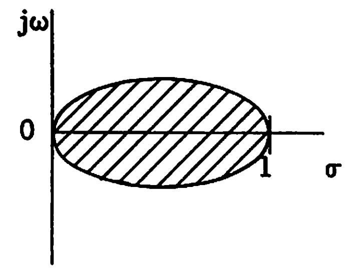
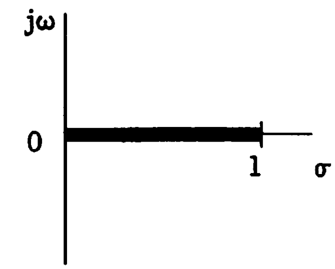
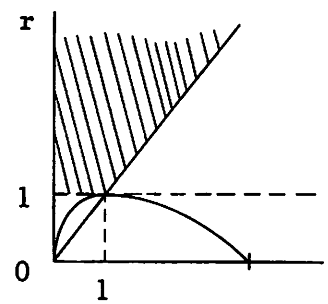
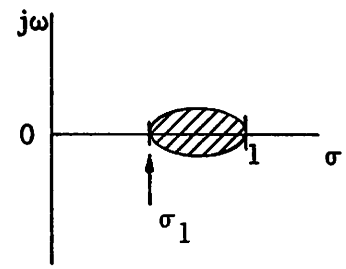
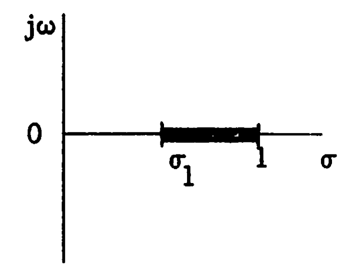
Cases	Restrictions on r and ℓ		Region Enclosed by ω_I for $\omega_I^2 \geq 0$	Active Regions on Axes
IIIaii	$r = 1$			
IIIai	$r > 1$			

TABLE 2 Active Regions for $\omega = 0$ or $\sigma = 0$, $r \geq \ell$ (cont.)

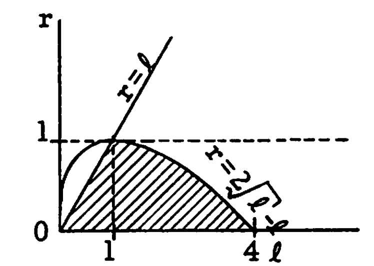
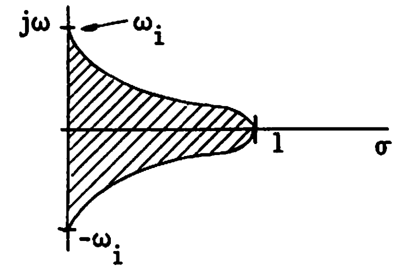
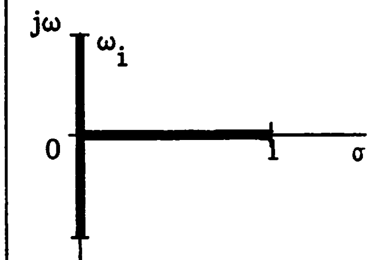
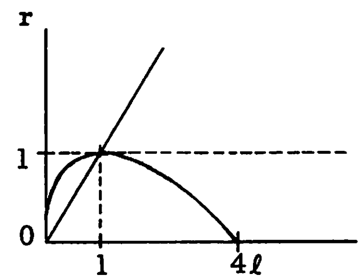
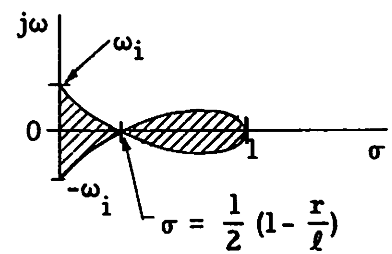
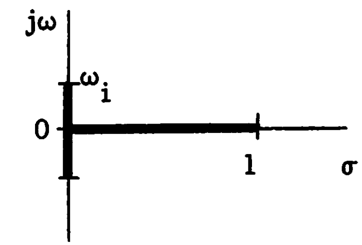
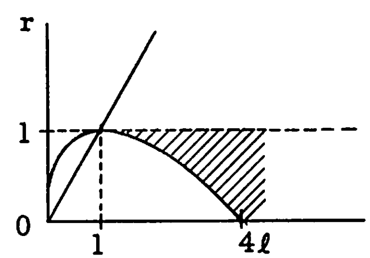
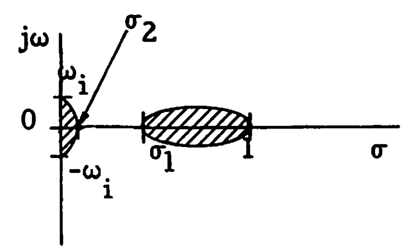
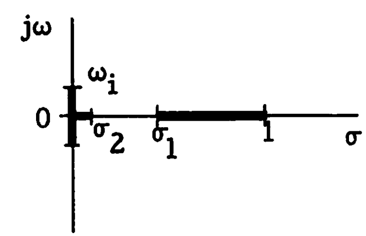
Case	Restrictions on r and ℓ		Region Enclosed by ω_I for $\omega_I^2 \geq 0$	Active Regions on Axes
I	$r < 2\sqrt{\ell} - \ell$			
IIb	$r = 2\sqrt{\ell} - \ell$			
IIIbiii	$1 > r > 2\sqrt{\ell} - \ell$			

TABLE 2 Active Regions for $\omega = 0$ or $\sigma = 0$, $r < \ell$

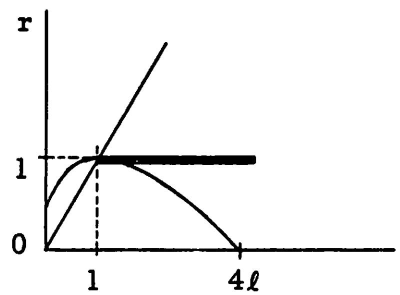
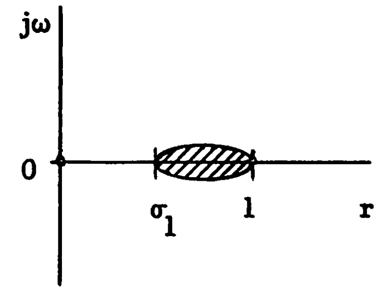
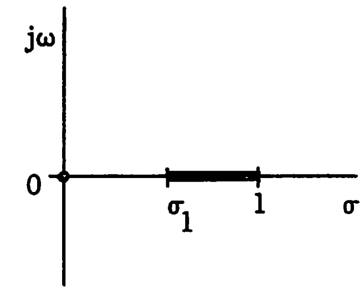
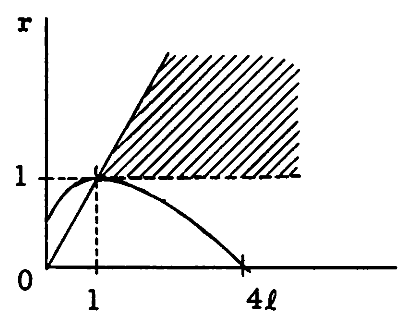
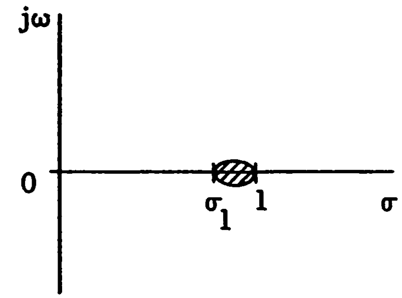
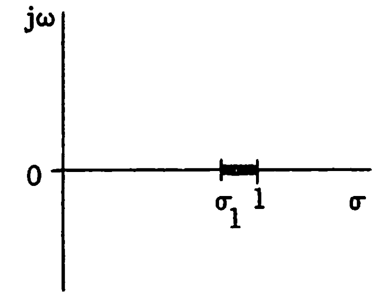
Case	Restrictions on r and ℓ		Region Enclosed by ω_I for $\omega_I^2 \geq 0$	Active Regions on Axes
IIIbii	$r = 1$			
IIIbi	$r > 1$			

TABLE 2 Active Regions for $\omega = 0$ or $\sigma = 0$, $r < \ell$ (cont.)

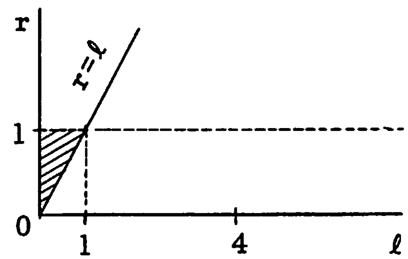
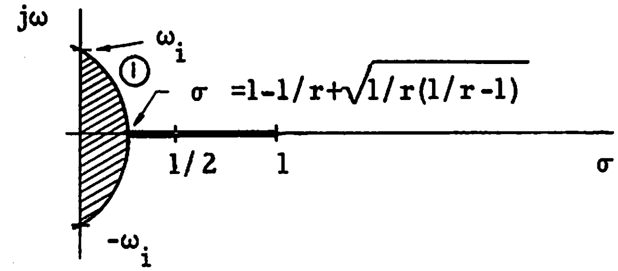
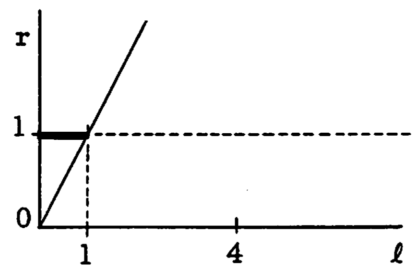
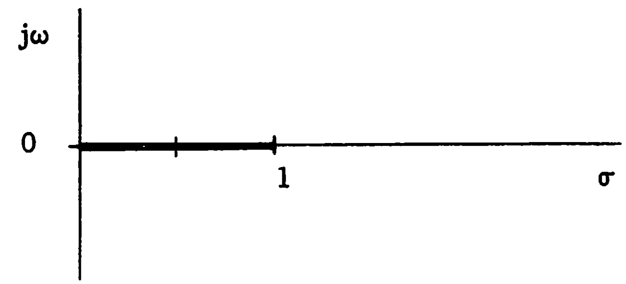
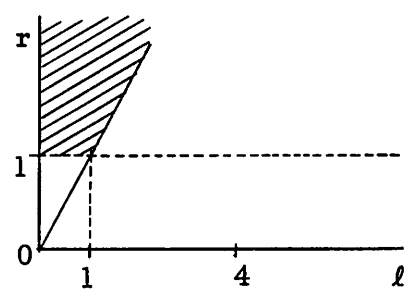
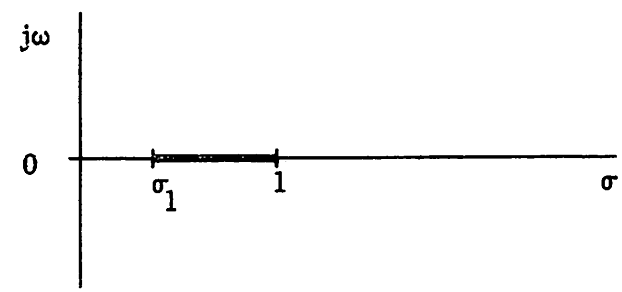
	Constraints on r and ℓ		Active Regions
i	$r < 1$		
ii	$r = 1$		
iii	$r > 1$		

TABLE 3 Active Regions for $r \geq \ell$

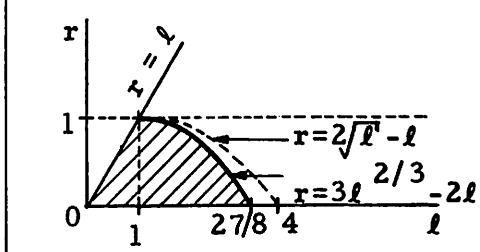
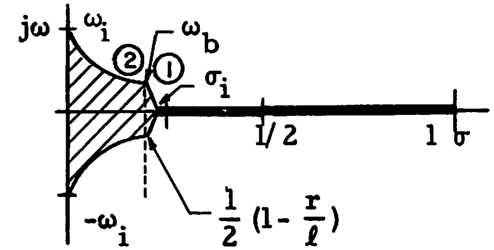
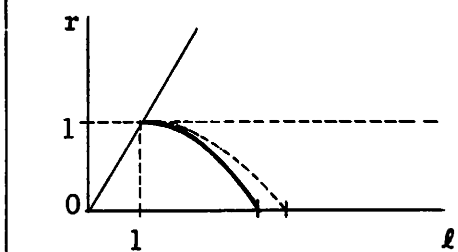
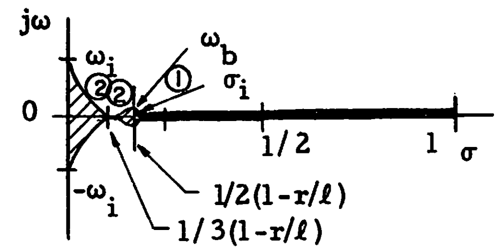
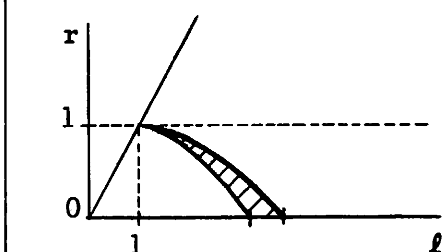
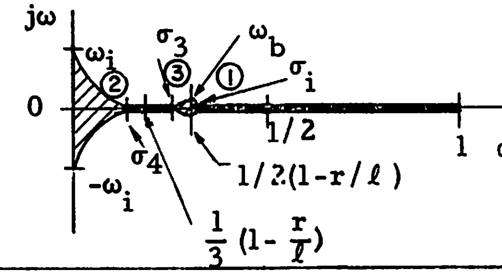
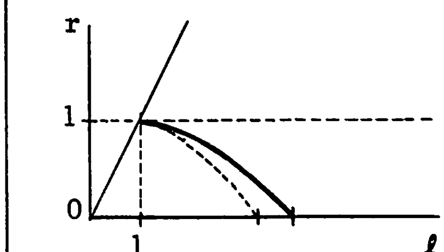
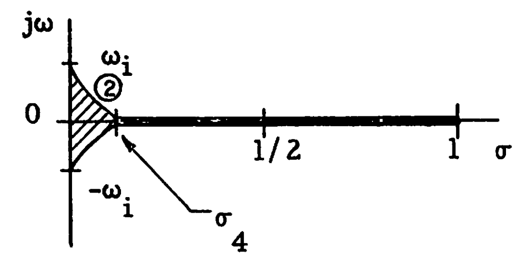
	Constraints on r and ℓ	Active Regions
i $r < 3\ell^{2/3} - 2\ell$		
ii $r = 3\ell^{2/3} - 2\ell$		
iii $3\ell^{2/3} - 2 < r < 2\sqrt{\ell} - \ell$		
iv $r = 2\sqrt{\ell} - \ell$		

TABLE 4 Active Regions for $r < \ell$

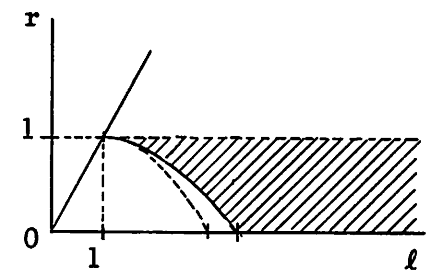
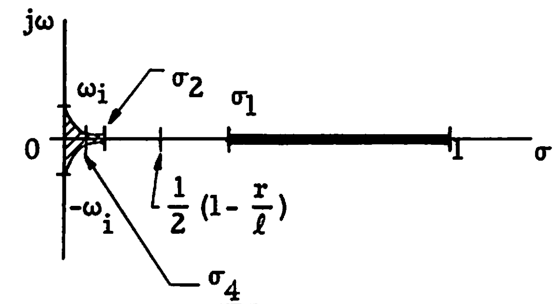
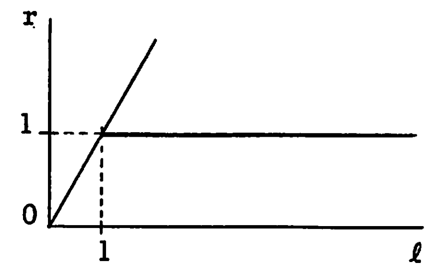
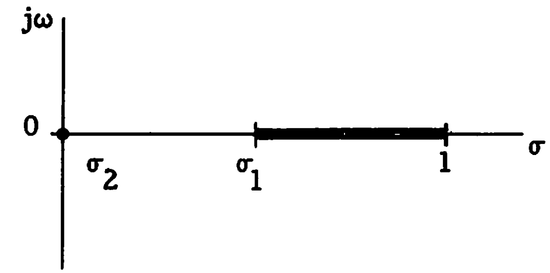
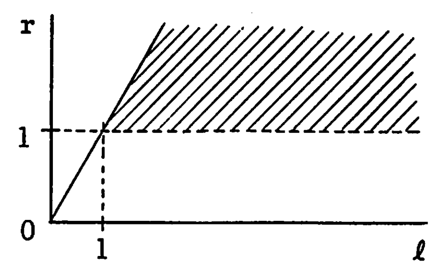
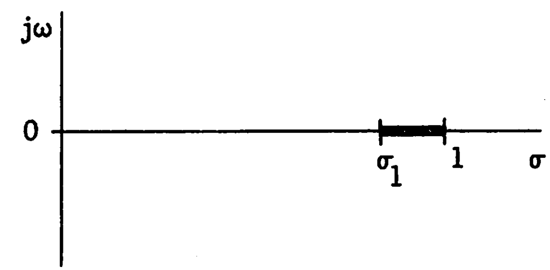
	Constraints on r and ℓ		Active Regions
v	$1 > r > 2\sqrt{\ell} - \ell$		
vi	$r = 1$		
vii	$r > 1$		

TABLE 4 Active Regions for $r < \ell$ (cont.)