Revised ERL-101

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A UNIFIED THEORY OF SYNTHESIS WITH UNSYMMETRIC FOUR-ELEMENT LOSSLESS LATTICES*

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ABSTRACT

A complete theory is given for the use of unsymmetric lossless lattice with four elements in the cascade synthesis of transfer functions and driving-point functions. The major results are a new lattice equivalent for a Darlington-C section under specified constraints on the driving-point impedance and a new, simple condition of applicability for the Miyata Lattice. For both lattices explicit formulas for the element values are developed in terms of the given impedance.

* The research herein was supported by the Joint Services Electronics Program (Air Force Office of Scientific Research, Army Research Office, Office of Naval Research) under Grant AF-AFOSR-139-64.

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SUMMARY

1. Introduction

It has recently been shown that the Foster and Cauer realizations of lossless positive real functions can be replaced by cascades of unsymmetric four-element lattices[1] and Miyata[2] has shown that under certain conditions a cascade of a Brune section and a Darlington section can be converted to an unsymmetric four-element lossless lattice. These results naturally lead us to ask two questions:

1. Are there any other unsymmetric four-element lossless lattices which can be used to replace sections of more complicated driving point or transfer-function realizations?

2. Is there a direct method of obtaining the Miyata lattice without first realizing the equivalent Brune and Darlington sections?

The limitation to lattices of only four elements in question 1 is a practically meaningful one, since it guarantees that the new realizations are simple and hence economical. The second question is important since it deals with the practical applicability of the lattices.

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In this paper complete answers are given to both questions 1 and 2. In answering the questions the two major results of the paper are:

1. It is proved that the lattice in Fig. 1(c) is equivalent to the network in the dotted box of Fig. 5 under certain given restrictions. A direct realization method is given in terms of the lattice.

2. A direct realization scheme is given for the Miyata lattice and a condition of applicability is given in terms of the driving-point impedance.

The approach to the problem, a detailed presentation of the significant results and two examples are given below. The derivations of the results are to be found in the paper.

Except for duality and the trivial situation in which the lattice contains only one type of element, the four-element lattices can be divided into type I used by H. B. Lee, type II used by F. Miyata, and type III consisting of three elements of one kind and one of the other. In the paper a new method is given for the networks in type II and the representative network of type III in Fig. 1(c) is considered. The other networks in type III can be treated in exactly the same fashion.

2. Synthesis Using a Type III Lattice

For the lattice in Fig. 1(c) to be used as a section in a driving-point synthesis, conditions I(a) - III(a) must be satisfied.

I(a) $\mathrm{Re}Z(s) = 0$ at $s = \sigma_o$, σ_o real.

II(a) $Z(s)$ can be represented as in Fig 2(a) or 2(b) with $Z_2(s)$ positive real.

III(a) $$\frac{L_i}{L_c'} > \frac{\dfrac{Z_2(\sigma_o)}{\sigma_o} + Z_2'(\sigma_o)}{\dfrac{Z_2(\sigma_o)}{\sigma_o} - Z_2'(\sigma_o)} \quad \text{where } Z_2'(\sigma_o) = \left.\frac{dZ_2(s)}{ds}\right|_{s=\sigma_o}$$

First split L_c' into $L_c + L_c''$ where L_c is defined by:

$$L_c = L_i \frac{\dfrac{Z_2(\sigma_o)}{\sigma_o} - Z_2'(\sigma_o)}{\dfrac{Z_2(\sigma_o)}{\sigma_o} + Z_2'(\sigma_o)} \tag{1}$$

We may then proceed to find the lattice by following equations

$$C_1 = C \qquad (2) \qquad\qquad L_1 = \frac{L_c(L_c - M)}{L_b + L_c} \qquad (3)$$

$$L_3 = M\frac{M - L_c}{L_b + M} \qquad (4) \qquad\qquad L_4 = \frac{L_c(L_b + M)}{L_b + L_c} \qquad (5)$$

where

$$M = \frac{-2}{\dfrac{Z_2(\sigma_o)}{\sigma_o} - Z_2'(\sigma_o)} \qquad (6) \qquad\qquad C = \frac{\dfrac{2}{\sigma_o^2}}{\dfrac{Z_2(\sigma_o)}{\sigma_o} - Z_2'(\sigma_o)} \qquad (7)$$

$$L_b = \frac{\dfrac{Z_2(\sigma_o)}{\sigma_o}}{\left(\dfrac{Z_2(\sigma_o)}{\sigma_o}\right)^2 - (Z_2'(\sigma_o))^2} \,. \qquad (8)$$

We then calculate the remaining function and complete the synthesis.

<u>Example 1</u>: Let $Z(s) = \dfrac{2s^4 + 3s^3 + 7s^2 + 6s}{5s^3 + 6s^2 + 4s + 3}$ having a transmission zero $\sigma_o = 1$. Removing two inductors as in Fig. 3 we have

$$Z_2(s) = \frac{\frac{3}{4}s^2 + \frac{27}{4}s + 6}{4s^2 + 4.5s + \frac{1}{2}} \,.$$

$$Z_2(\sigma_o) = Z_2(1) = \frac{13.5}{9} \qquad\qquad Z_2'(\sigma_o) = -\frac{7}{6},$$

and

$$\frac{\dfrac{Z_2(\sigma_o)}{\sigma_o} + Z_2'(\sigma_o)}{\dfrac{Z_2(\sigma_o)}{\sigma_o} - Z_2'(\sigma_o)} = \frac{1}{8} \,.$$

Since $\dfrac{L_i}{L_c'} = \dfrac{1}{4} > \dfrac{1}{8}$ the method applies. L_c' is separated into two inductors such that $L_c = 4$.

Using (1) - (8) we have the realization in Fig. 4. The equivalent Darlington realization is in Fig. 5.

3. Synthesis Using a Type II Lattice

A type II latice can be used as a section in a driving-point synthesis if and only if:

I(b) $\operatorname{Re} Z(s) = 0$ at $s = j\omega_o$, ω_o real.

$$\text{II(b)}\quad \frac{\dfrac{Z(\omega_o)}{\omega_o} - Z'(\omega_o)}{\dfrac{Z(\omega_o)}{\omega_o} + Z'(\omega_o)} = \frac{X'(\omega_o) - \dfrac{X(\omega_o)}{\omega_o}}{X'(\omega_o) + \dfrac{X(\omega_o)}{\omega_o}}$$

where

$$Z(\omega_o) = Z(s)\Big|_{s=\omega_o} \qquad X(\omega_o) = \frac{1}{j} Z(j\omega_o)$$

$$Z'(\omega_o) = \frac{dZ(s)}{ds}\Big|_{s=\omega_o} \qquad X'(\omega_o) = \frac{dX(\omega)}{d\omega}\Big|_{\omega=\omega_o}$$

If condition II(b) is not satisfied, removal theorems for singularities at zero or infinite frequency can be used to force the equalities to be satisfied. That is for example for a given impedance $Z_o(s)$ an inductor L can partially be removed. Where

$$L = \frac{Z_o'(\omega_o)\, X_o'(\omega_o) - \dfrac{Z_o(\omega_o)\, X_o(\omega_o)}{\omega_o^{\,2}}}{X_o'(\omega_o) - \dfrac{X_o(\omega_o)}{\omega_o} - \dfrac{Z_o(\omega_o)}{\omega_o} + Z_o'(\omega_o)} \tag{9}$$

For the method to work, L should be smaller than the residue of $Z_o(s)$. (Similar cases can be applied to shunt inductor or capacitors.) We can thus pull out the lattice from $Z(s) = Z_o(s) - Ls$. The element values of the lattice can directly be calculated from the following equations where L_1, L_2, C_1 and C_2 are defined by Fig. 1(b).

$$L_1 = \frac{r_1 - r_2}{b - dr_2} \qquad (10) \qquad\qquad L_2 = \frac{r_1 - r_2}{dr_1 - b} \qquad (11)$$

$$C_1 = \frac{r_2(dr_1 - b)}{r_1 - r_2} \qquad (12) \qquad\qquad C_2 = \frac{r_1(b - dr_2)}{r_1 - r_2} \qquad (13)$$

where

$$b = \frac{2[Z(\omega_o) + \omega_o Z'(\omega_o)]}{\omega_o[\, Z^2(\omega_o) - \omega_o Z'(\omega_o)\, X(\omega_o)]} \qquad (14)$$

$$d = \frac{2[\, Z(\omega_o) - \omega_o Z'(\omega_o)]}{\omega_o^{\,3}[Z^2(\omega_o) - \omega_o Z'(\omega_o)\, X(\omega_o)]} \qquad (15)$$

$$r_1, r_2 = \frac{[Z(\omega_o) \pm \sqrt{Z'(\omega_o)\omega_o\, X(\omega_o)}]^{\,2}}{\omega_o^{\,2}[Z^2(\omega_o) - Z'(\omega_o)\omega_o\, X(\omega_o)]} \qquad (16)$$

The remaining function can be found easily once the lattice is obtained.

Example 2: Let

$$Z_o(s) = \frac{8s^3 + 5s^2 + 5s + 2}{2s^2 + s + 1}$$

$$\mathrm{Re}Z_o(s) = 0 \quad \text{at} \quad s = j1.$$

Then, $Z_o(1) = 5$, $Z'(1) = 7/2$, $X_o(1) = 3$, $X_o'(1) = 7$. By Eq. (9) $L = 19/5$ is smaller than the residue = 4 and the method is applicable. With $Z(s) \quad Z_o(s) - 19/5\ s$,

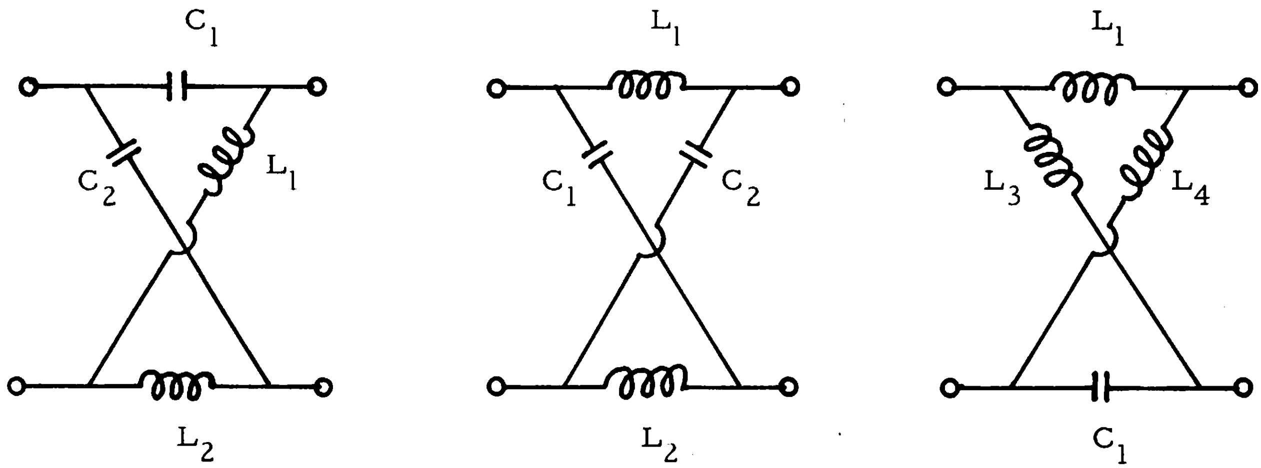
$$Z(1) = \frac{6}{5}, \quad Z'(1) = -\frac{3}{10}, \quad X(1) = -\frac{4}{5}, \quad X'(1) = \frac{16}{5}.$$

From Eqs. (10) - (17) we have the realization in Fig. 6.

REFERENCES

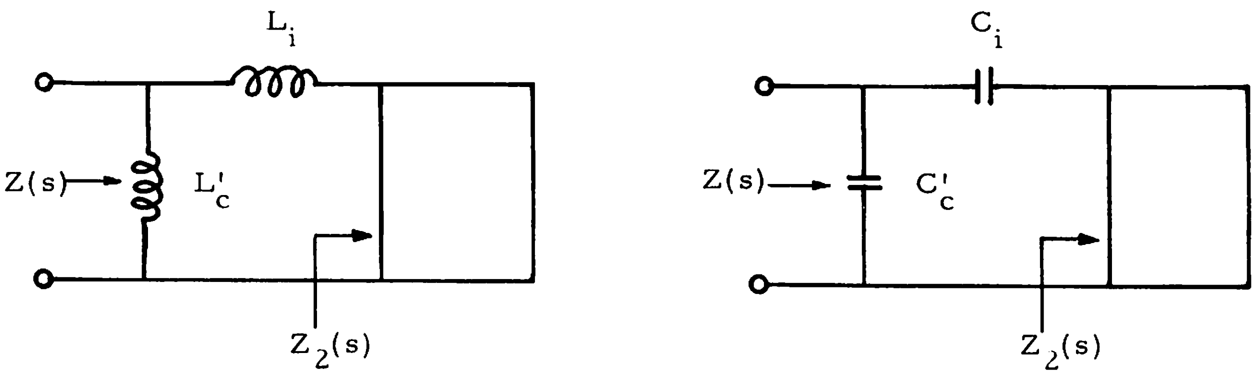
1. H. B. Lee, "A New Canonical Realization Procedure," IEEE Trans., Vol. CT-10, pp. 81-85, March 1963.

2. F. Miyata, "Synthesis of a One-Port Utilizing a Lattice Construction," IEEE Trans., Vol. CT-10, pp. 227-234, June 1963.



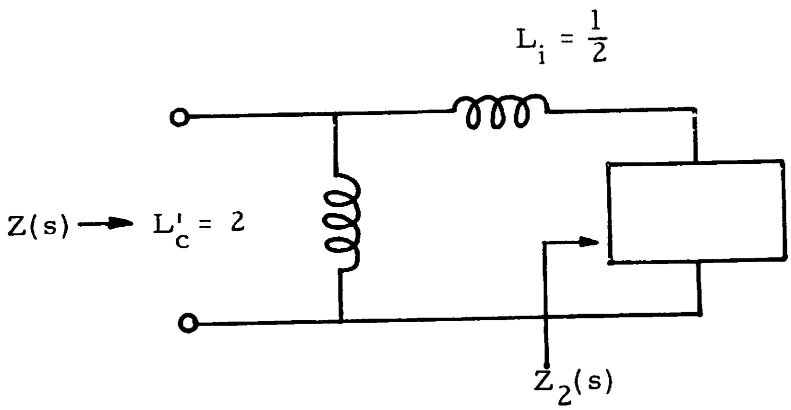
Type I (a) Type II (b) Type III (c)

Fig. 1



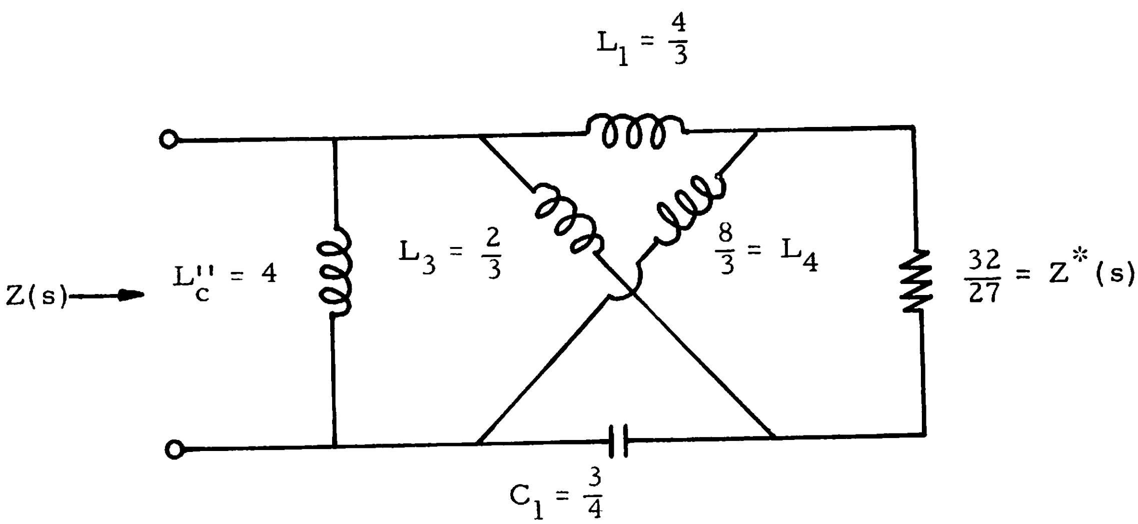
(a) (b)

Fig. 2



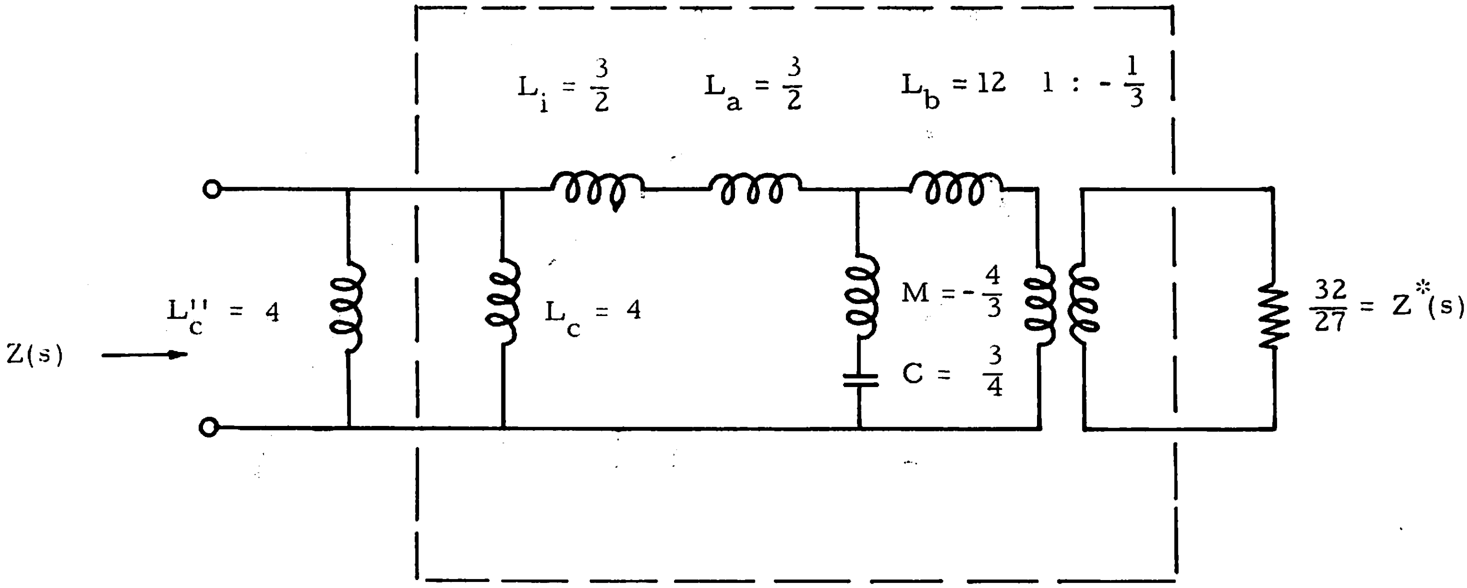
$L_i = \frac{1}{2}$
$Z(s) \longrightarrow$
$L_c' = 2$
$Z_2(s)$

Fig. 3



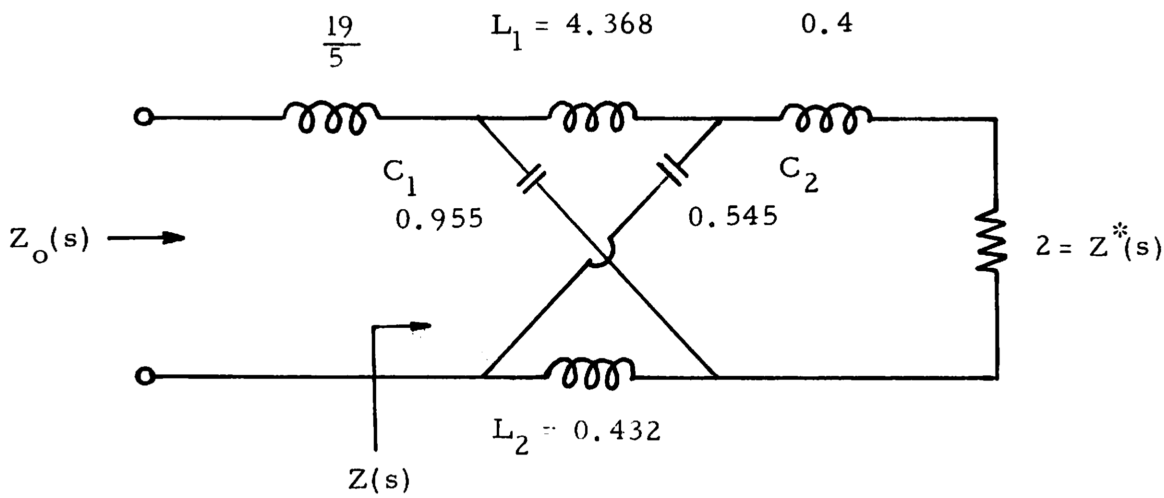
$L_1 = \frac{4}{3}$
$Z(s) \longrightarrow$
$L_c'' = 4$
$L_3 = \frac{2}{3}$
$\frac{8}{3} = L_4$
$\frac{32}{27} = Z^*(s)$
$C_1 = \frac{3}{4}$

Fig. 4



L_i = 3/2
L_a = 3/2
L_b = 12
1 : - 1/3
L''_c = 4
L_c = 4
M = - 4/3
C = 3/4
32/27 = Z*(s)
Z(s)

Fig. 5



19/5
L_1 = 4.368
0.4
C_1
0.955
C_2
0.545
Z_o(s)
2 = Z*(s)
L_2 = 0.432
Z(s)

Fig. 6