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# NEURAL PULSE FREQUENCY MODULATION SYSTEMS - II

by

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# FORWARD

About a year ago a first report was published under the title "Neural pulse frequency modulated control systems." This report contains certain points of the research done on the subject since that time but its main purpose is to emphasize the relation of this class of systems to biological models.

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# I. INTRODUCTION

Although a detailed mathematic description of single neurons has been offered by the Hodgkin-Huxley equations, very little has been done in the direction of studying neural nets in detail. One reason is the high degree of complexity of the Hodgkin-Huxley equations which makes impossible the simultaneous solution of a system of equations representing more than one neuron. Another reason is that little is known about the way neurons are interconnected.

One way to circumvent the first difficulty is to substitute the Hodgkin-Huxley equations by a simpler model which simulates only those properties of the neuron which are essential in the operation of a group of them (neurons). That is of course a vague statement because we do not really know exactly which are these essential properties. However some assumptions seem reasonable:

1. The waveshape of the neural pulse is not of importance. On one hand the pulse duration (about 1 msec)[1] is much smaller than the interval between two successive pulses under most circumstances (at least 10 msec corresponding to frequency 100 c/s).[1] On the other hand it is established that the synaptic transmission has low pass properties.[1]

2. The variable threshold negative refractoriness can be substituted by a negative feedback. Indeed if the time of the emission of a pulse is determined by a relation of the form:

$$p(t) = r(t) + r_o \tag{1}$$

where p is the membrane potential and $r(t) + r_o$ the value of the threshold this can be easily rewritten as

$$p(t) - q(t) = r_o \tag{2}$$

where q(t) is due to the negative feedback. We will call this <u>refractory feedback.</u>

3. No effort should be made to simulate the absolute refractoriness as this is of importance only in very high firing frequencies (1000 cps), well above the usual range.

4. The phenomenon of accommodation can be neglected at first because it is of importance only in very particular cases. For the problem of interconnection, however, one can make only some arbitrary assumptions.

5. The phenomenon of inhibition can be represented by inverting the sign of the pulses.

6. The equations describing the transmission of the pulses across the synapse are linear.

Based on these assumptions we are going to indtroduce a system simulating some of the features of neural nets.

## II. AN ATTEMPT FOR A GENERAL MODEL FOR NEURAL NETS

We define first as a neural trigger a device with scalar input $p(t)$ which emits an impulse of area $\delta$ whenever its input reaches a value $r$, which is called the threshold. Immediately after, $p(t)$ is reset to the zero value. Consider now a system containing $m$ triggers. Let their inputs be $p_1, p_2, \ldots p_m$ and their outputs $d_1, d_2, \ldots d_m$ where

$$d_i = \sum_{j=1}^{n} \delta_i(t - t_j) \quad \text{for } t_n \leq t < t_{n+1}. \tag{3}$$

$\delta_i(t - t_j)$ represents an impulse of area $\delta_i$ emitted at time $t_j$. We define furthermore the following vectors.

$$\underline{p} = \text{col}(p_1, p_2, \ldots p_m) \tag{4}$$

$$\underline{d} = \text{col}(d_1, d_2, \ldots d_m) \tag{5}$$

$$\underline{u} = \text{col}(u_1, u_2, \ldots u_r) \tag{6}$$

$$\underline{x} = \text{col}(x_1, x_2, \ldots x_n) \tag{7}$$

where $u_i$ are various inputs to the system and $x_i$ auxiliary variables necessary also for the description of the synaptic transmission and the neural receptors. Now we can write the following equations:

$$\overset{o}{\underline{X}} = \underline{A}_1 \underline{x} + \underline{A}_2 \underline{d} + \underline{A}_3 \underline{u} \tag{8}$$

$$\overset{o}{p} = \underline{B}(\underline{p}, \underline{x}, \underline{u}) \tag{9}$$

where $\underline{A}_1$ is an n x n matrix, $\underline{A}_2$ n x m and $\underline{A}_3$ n x r and $\underline{B}$ a vector function.

It is easy to check that the above equations represent a system which resembles a neural net under the assumptions of Sec. I. An input $u_k$ can cause either directly [Eq. (9)] or indirectly [Eq. (8)] a change in $p_i(t)$ which may result in the firing of a pulse. An equation of the form of Eq. (8) can represent the refractory feedback while a group of them can be used to describe the synaptic transmission which eventually will affect another trigger with input $p_j(t)$, etc.

Actually Eq(9) need not be so complicated. It is reasonable to assume that there exists no terms of the form $p_j k_i$, $p_j u_k$, $k_i u_k$ and also that the effect of the $x_i$'s on the "generation" of the $p_j$'s is linear. Then Eq. (9) is written

$$\overset{o}{\underline{p}} = \underline{B}_1(p) + \underline{B}_2 \underline{x} + \underline{B}_3(\underline{u}) \tag{10}$$

where $\underline{B}_2$ is an m x n matrix.

In Sec. VI it will be shown that by using equations of the form of Eq. (8) and (10) one can construct a model of a single neuron which has a large number of properties of the biological original.

Now the following question arises: Given that equations of the form of Eq. (8) and (10) account for a good model of a single neuron, it is possible to consider Eq. (8) and (10) as giving an equally good general model for neural nets?

The answer is obviously negative. However they are expected to present a behavior resembling the behavior of actual neural nets and the study of such a class of systems may be helpful in understanding some processes occurring in the nervous system of animals.

## III. NEURAL PULSE FREQUENCY MODULATION SYSTEMS

The class of systems represented by Eqs. (8) and (10) will be called Neural Pulse Frequency Modulation (N. P. F. M.) Systems. We think that by using this name no implication is made that the above systems are an exact model of neural nets while the existence of some common basic features between the two is also brought to our attention.

The same name has been used by the writer[2] before to describe only a special case of the above class of systems, which now for distinction will be referred as special N. P. F. M. The class of Integral P. F. M. systems[3, 4] is also a special case of the N. P. F. M. systems. This can be seen by putting $\underline{B}_1 = \underline{0}$ in Eq. (10). However although it is much easier to study Eq. (8) and (10) under this assumption, at the same time the connection with neural nets is dropped completely. This is because I. P. F. M. systems present no input threshold[3] which is a very essential property of neurons, while N. P. F. M. have an input strength duration curve very similar to the one of the neurons.[2]

The study of N. P. F. M. systems can be interesting not only from the biological point of view but also for technological reasons. Pulse frequency modulation is widely used in many applications and the modulator described by Eq. (8) can be implemented with very simple components. The results will be higher reliability as well as smaller weight and occupied space. I. P. F. Modulators have been already used in space vehicles.[5] In this respect N. P. F. M. systems present also certain advantages over I. P. F. M. systems in terms of performance criteria.

A special case of Eq. (10) results when the i-th component of $\underline{B}_1(\underline{p})$ depends only on $p_i$. If $\underline{B}_1$ is a mareix this amounts to say that it is diagonal. This simplification is particularly justified when one seeks to design models for neural nets.

Another simplification may be done when the system is such that to every trigger emitting negative pulses corresponds one emitting positive and the sum of the inputs to the two of them is zero. This is necessarily the case in a control system[2, 3] as well as in a neural net containing pairs of antagonistic neurons. Then one can approximate the pair by one trigger emitting pulses of both signs depending on the sign of the input. The number of scalar equations included in Eq. (10) is then reduced to the half. Thus the class of double signed (D. S.) N. P. F. M. systems is defined.

Each one of the scalar equations

$$\overset{o}{p}_i = b_{ii}^{(1)} p_i + \sum_{j=i}^{n} b_{ij}^{(2)} x_j + \sum_{k=i}^{r} b_{ik}^{(3)} u_k \tag{11}$$

represents one <u>Modulator.</u>

Eq. (11) represents in particular a linear modulator. A nonlinear modulator will result if $b_{ii}^{(1)}$ is looked not an constant but as a nonlinear operator.

## IV. ANALYTICAL STUDY OF N. P. F. M. SYSTEMS

One can readily see that the study of N. P. F. M. systems is not possible by using either differential or difference equations. One could work recursively if the solution of the equation

$$p_i(t) = r_i \tag{12}$$

with respect to $t$ was possible but except when $b_{ii}^{(1)} = 0$ this is always a transcendental equation.

If only one modulator is included in the system one may try the use of a difference equation with independent variable the number of the pulses emitted. However the difficulty of the solution of Eq. (12) still remains.

An equivalent gain approximative approach has been described elsewhere.[2, 6] According to this approach, a modulator is substituted by a nonlinear, frequency dependent continuous element. In this case if

$$\overset{o}{x}_j = \sum_{i=1}^{n} a_{ji}^{(1)} \; x_i + \sum_{k=1}^{m} a_{jk}^{(2)} K_k \sum_{j=1}^{n} b_{ks}^{(2)} \; x_s \tag{14}$$

or

$$\overset{o}{x}_j = \sum_{i=1}^{n} \left[ a_{ji}^{(1)} + \sum_{k=1}^{m} a_{jk}^{(2)} \; K_k \; b_{ki}^{(2)} \right] \; x_i . \tag{15}$$

This is then written as

$$\underline{\overset{o}{x}} = \left( \underline{A}_1 + \underline{A}_2 \; \underline{K} \; \underline{B}_2 \right) \underline{x} \tag{16}$$

where $\underline{K}$ is a diagonal matrix m x m with elements $K_k$.

A further simplification can be made if we assume that the $K_k$'s do not depend on the frequency. Then Eq. (16) can be written as

$$\underline{\overset{o}{x}} = \left[ \underline{A}_1 + \underline{A}_2 \, \underline{K} \, (\underline{x}) \, \underline{B}_2 \right] \; \underline{x}. \tag{17}$$

Any further simplification results in too large derivations from the actual behavior of the system. (Such examples will appear in a future report.)

## V. STABILITY OF N. P. F. M. SYSTEMS

The problem of stability when the system contains only one D. S. modulator has been investigated by the use of a quasi-describing function.[6] For a more complex system a possible approach is to use Liapunov's method on the system presented by Eq. (17). This however is not very accurate as one can see by considering the above mentioned special case. Fig. 1 shows the graph of the normalized quasi-describing function. If we applied the simplification of Sec. IV, and we reject all the points with nonzero imaginary parts, then certain systems will seem to be asymptotically stable while they actually present sustained oscillations.

Let us consider however the initial system represented by Eq. (8) and (10) (for zero input).

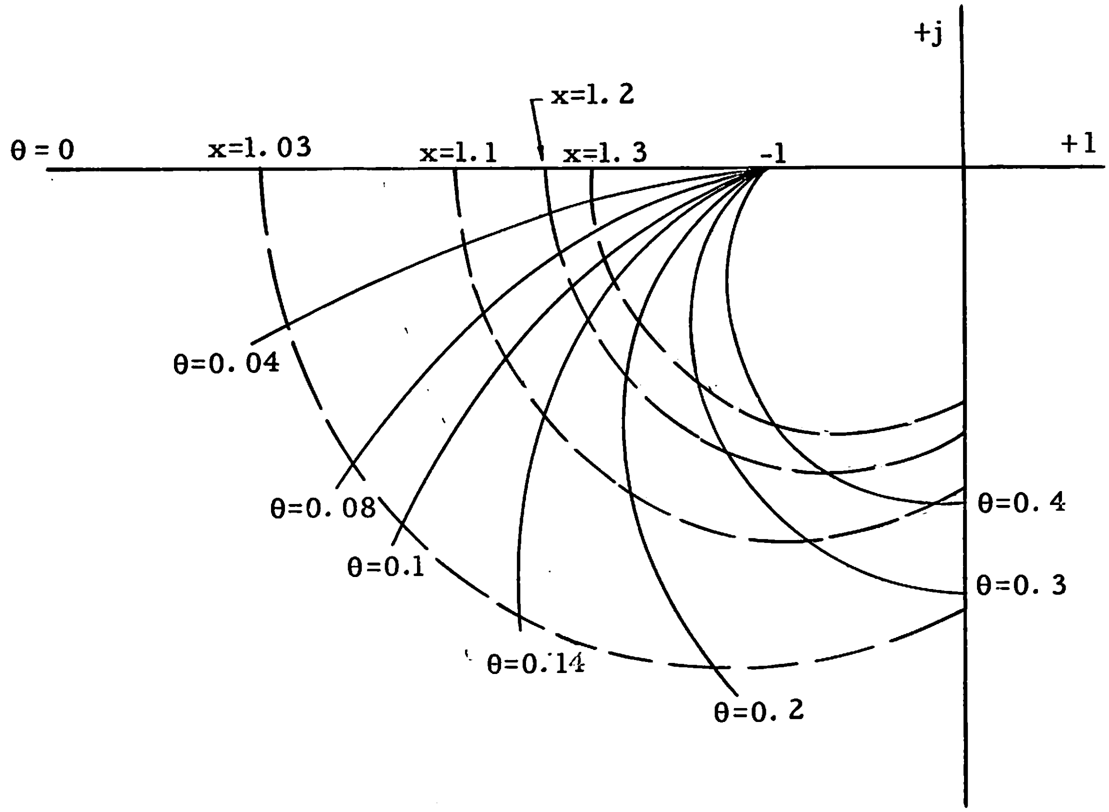


Fig. 1. Complex plane plot of the normalized quasi-describing function of a N. P. F. M. System[6]

$$S \begin{cases} \overset{o}{\underline{x}} = \underline{A}_1 \underline{x} + \underline{A}_2 \underline{d} & (8') \\ \overset{o}{\underline{p}} = \underline{B}_1 \underline{p} + \underline{B}_2 \underline{x} & (10') \end{cases}$$

A necessary condition for the structural stability of the system S is that $\underline{A}_1$ and $\underline{B}_1$ are stable matrices. S will be stable if the element of $\underline{A}_2$ and $\underline{B}_2$ are small enough in absolute value. This compares with the earlier results of Ref. 6, where certain conditions necessary for the stability of an N. P. F. M. feedback system were also sufficient when the gain of the loop was made small enough.

The above result is of course of little practical value by itself, however it justifies the use of approximate methods in order to find the proper values of the elements of $\underline{A}_2$ and $\underline{B}_2$.

# VI. MODELS OF A NEURON

We will try now to suggest a system that simulates the properties of a single neuron except the ones mentioned in Sec. I. We need only one modulator, hence one part will be described by the equation:

$$\mathring{p} = f(p) + \underline{b}' \cdot \underline{x} + g(u). \tag{21}$$

The refractory feedback must enter through the term $\underline{b}' \cdot \underline{x}$. A fatigue feedback can be possibly added Hence we define

$$\mathring{x}_1 = a_1 \cdot x_1 + k_1 \cdot d \tag{22}$$

$$\mathring{x}_2 = -a_2 x_2 + k_2 d \tag{23}$$

with $a_2 << a_1$,

and choose $b_1 < 0$, $b_2 < 0$.

As far as the input is concerned we have to consider the fact of the limited temporal summation.[1] This can be described by defining

$$\mathring{x}_3 = - a_3 \cdot x_3 + k_3 \cdot u. \tag{24}$$

Then $b_3$ 0 and $g = 0$.

In Ref. 2 it is shown that the strength duration curve is obtained for $f(p) = - cp$. Hence the final model is described by:

$$\underline{\mathring{x}} = \underline{A} \cdot \underline{x} + \underline{K} \cdot d + \underline{L} \cdot u \tag{24a}$$

$$\mathring{p} = -cp + \underline{b}' \underline{x} \tag{25}$$

where

$$\underline{A} = \begin{pmatrix} -a_1 & 0 & 0 \\ & -a_2 & 0 \\ 0 & 0 & -a_3 \end{pmatrix} \qquad (a_i > 0)$$

$$\underline{K} = \begin{pmatrix} K_1 \\ K_2 \\ 0 \end{pmatrix} \quad \underline{b} = \begin{pmatrix} -1 \\ -1 \\ 1 \end{pmatrix} \quad \underline{L} = \begin{pmatrix} 0 \\ 0 \\ L_3 \end{pmatrix}.$$

The arbitrary choice of $b_1$, $b_2$, $b_3$ is due to the fact that only the product $K_i\,b_i$ is of importance in the system.

Because $\underline{A}$ is obviously a stable matrix and the trigger is single signed the system is always stable. Its block diagram is shown in Fig. 2. When the input of such a model is an impulse then it will emit another impulse with a delay depending on the strength of , or it may not fire at all. A detailed study of such a model has been done elsewhere.[7] Here we will mention only the phenomenon of fractional ratio of firing frequencies. If the input of the model is a train of pulses at fixed time intervals and say two of them are needed to cause the firing of one, then the system will also fire every three pulses sometimes. Such a response is shown in Fig. 3 and it is in agreement with experimental results.

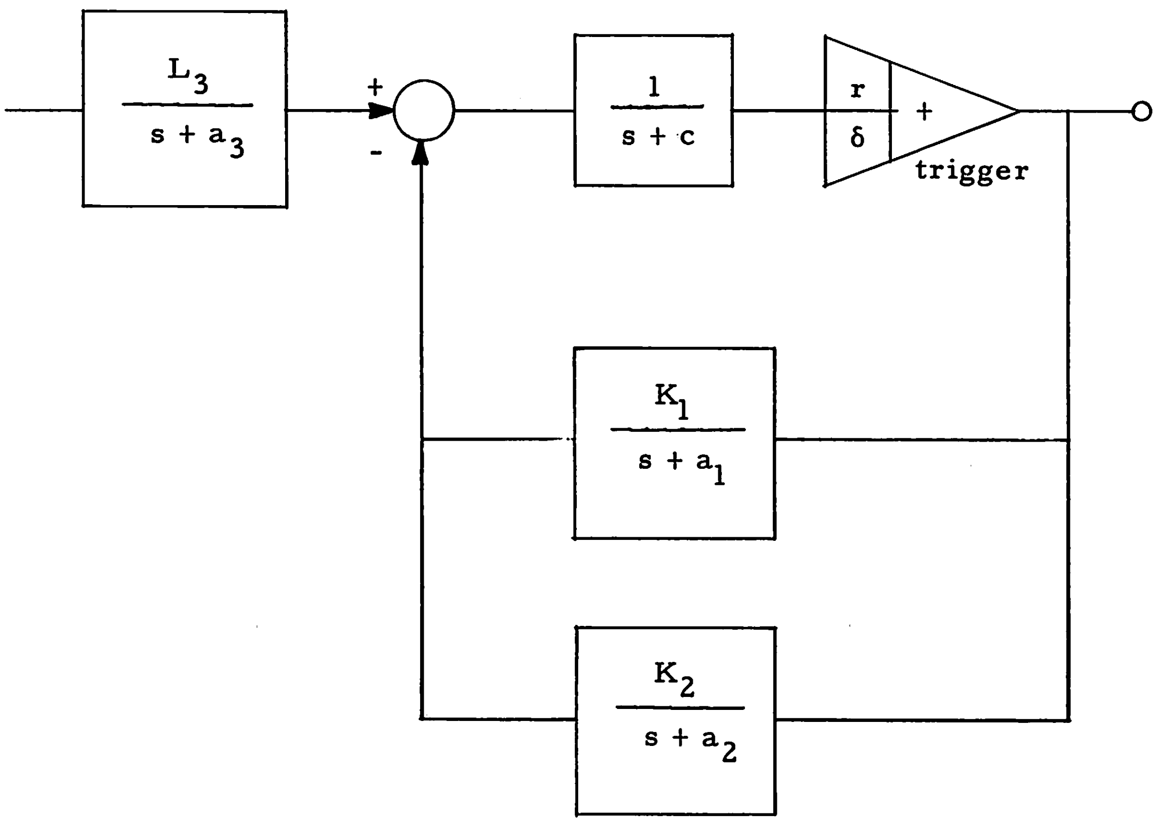


Fig. 2. Block diagram for a model of a neuron.

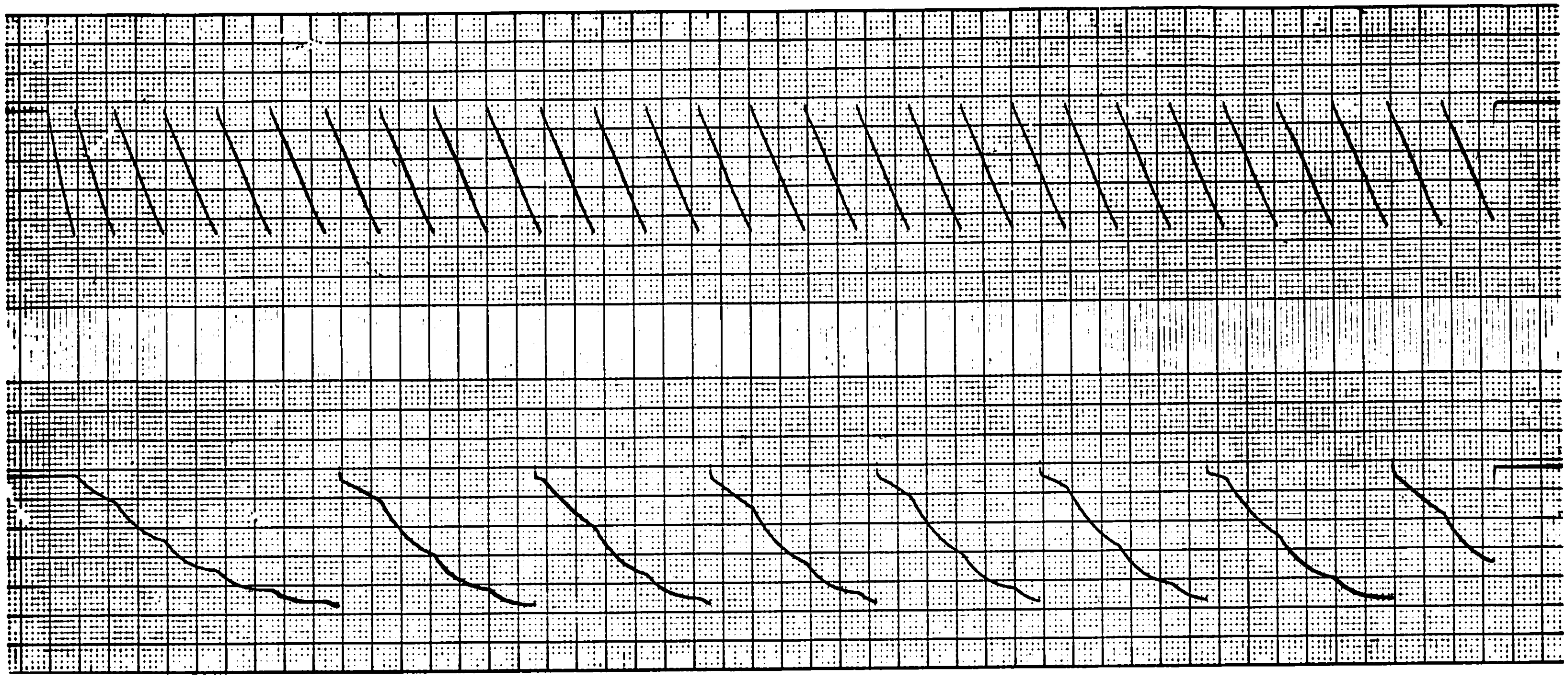

Fig. 3. Analog computer response of the model of Fig. 2 for $C = .5$, $a_1 = .5$ $a_3 = .2$, $K_1 = 2$, $K_2 = 0$, $L_3 = 2$. Upper trace: Input, Lower trace: Output.*

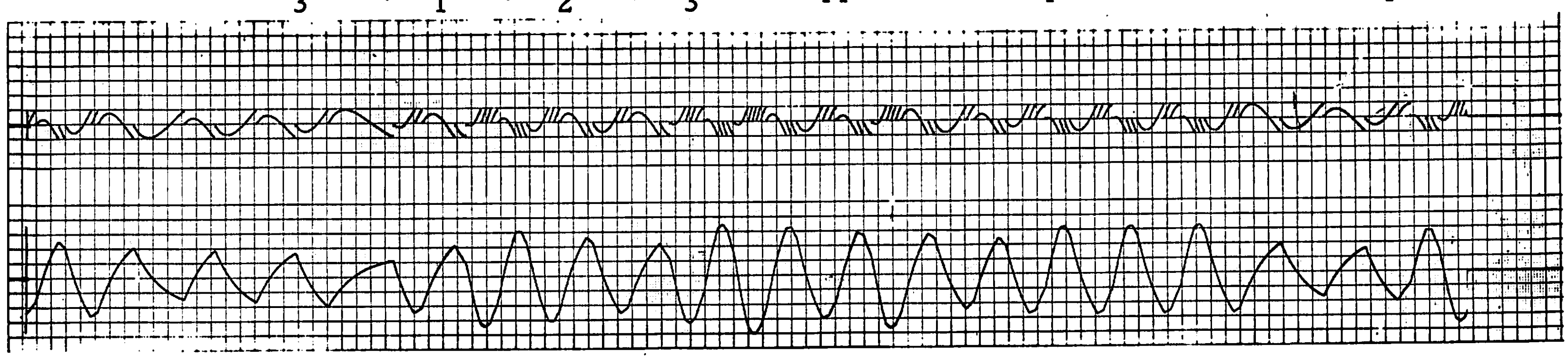

Fig. 4. Sustained oscillations in a feedback N. P. F. M. system.

* The input is a series of pulses generated by an N. P. F. modulator. Recorded in both cases is the quantity p(t). Note firing pattern: 6-3-4-3-3-4.

## VII. SOME ADDITONAL REMARKS

A typical characteristic of N. P. F. M. systems is their <u>pseudo randomness.</u> This is especially prominent in the case of sustained oscillations in a feedback system where a continuum of modes of oscillation exists.[3, 5, 6] Such a response is shown in Fig. 4. The upper trace presents p(t) and the lower trace the output of a unity feedback system using a D. S. N. P. F. modulator and forward path linear plant $[10/s(s + 0.5)]$. The input to the system is zero. The discontinuities in the diagram of p(t) represent emission of pulses. From a physiological point of view this system can be considered as model of two antagonistic neurons which are cross exhibited. (This type of oscillation fits very well the experimental data of various biological oscillators.) A not very careful inspection of the response of any of the "neurons" would lead to the conclusion of a "spontaneous random response." However the system is completely deterministic! With this example as starting point one may ask the more general question--

"Is the so-called random spontaneous neural activity really random or is it deterministic but following a law unknown to us?"

The answer to this question exceeds the purpose of this report. It should be actually the subject of very extensive research.

We can however already suggest a theory for the explanation of the "random" spontaneous activity of neurons. A model for that behavior is shown in Fig. 5 and the output of the two units in an analog computer simulation is shown in Fig. 6. The physiological interpretation of this model is that the firing of the two "neurons" is due to their cross-excitation. This can a fortriori happen in a trunk of many neurons or in a ganglion. In this way the spontaneous activity of neurons is explained in terms of cross excitation without ay need to assume any random input, as it is the case with other proposed models.[8] The model of Fig. 5 is a too simple one to explain all the characteristics of the spontaneous activity of neurons but is incorporates the basic idea of the new approach. A more complicated model is under study.

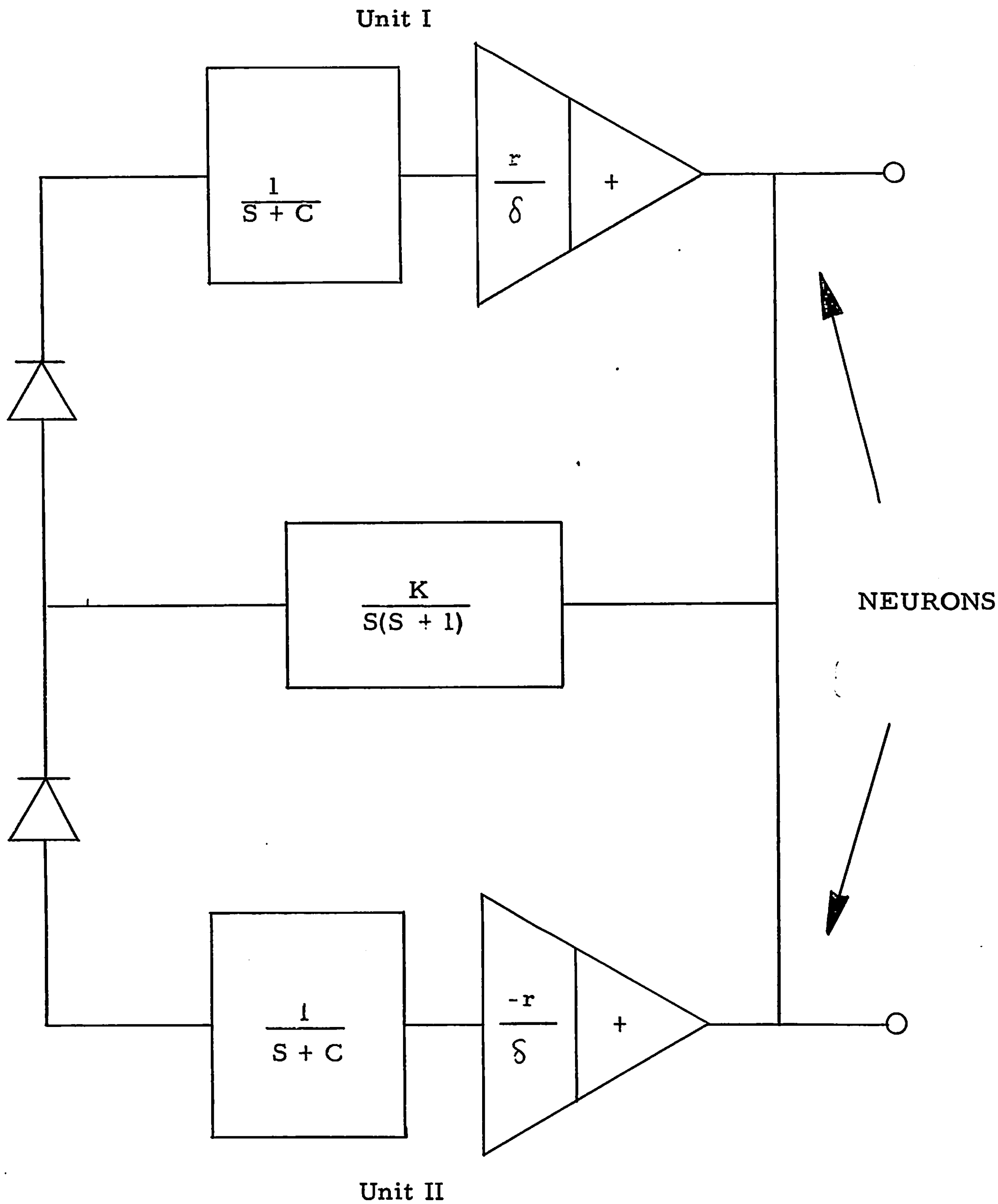


Fig. 5. Model for the spontaneous activity of neurons.

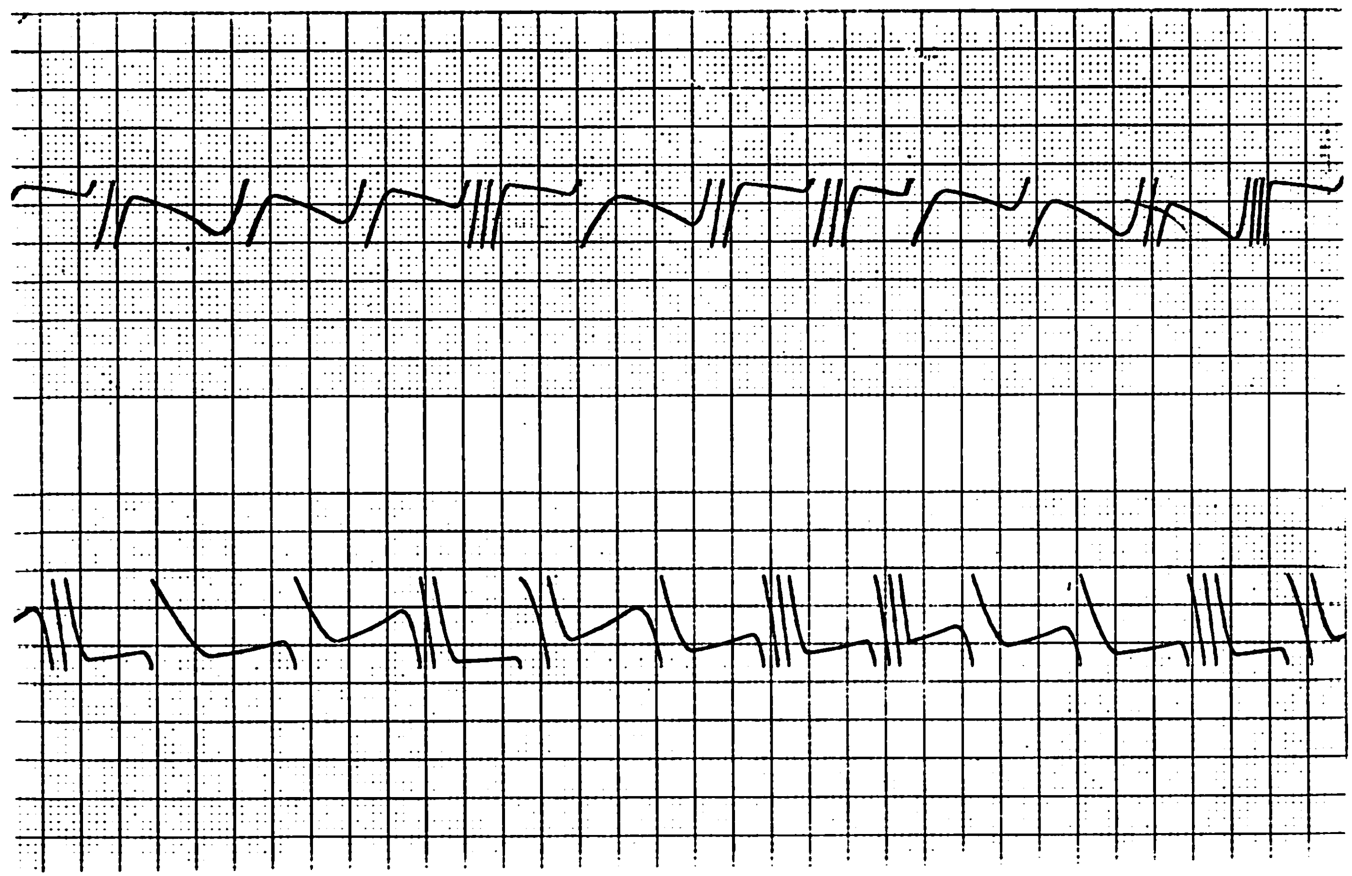

Fig. 6. Behavior of the model of Fig. 5 in an analog computer simulation.

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