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Internal Technical Memorandum M-57

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PLASMA OSCILLATIONS AND PARTICLE TRAJECTORIES IN A DRIFTING ELECTRON STREAM; PALMER DIAGRAMS (Continuation of IER Report 60-458)

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C. K. Birdsall

February 11, 1964

The report "Plasma Oscillations and Particle Trajectories in a Drifting Electron Stream; Palmer Diagrams," IER Series No. 60, Issue No. 458, June 29, 1962, has been augmented by two sets of trajectories, one for current modulation, and one for traveling-wave interaction. The computer programming was done by Gail Nirdlingler.

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C. K. Birdsall February 11, 1964

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CURRENT MODULATION

In the case of current modulation only, ignoring ϕ the displacement is

$$\frac{\omega}{v_o} x(t, t_a) = \omega_p (t - t_{oa}) + \frac{i_1}{i_o} \frac{\omega}{\omega} \sin(\omega t_a) \cos \omega_p (t - t_a).$$
(1)

This expression appears to be similar to that for velocity modulation but shifted in t_a and $t - t_a$ axes by $\pi/2$, a result expected from the Eulerian analysis. However, the rub comes in that the departure times, t_a , are not uniform for current modulation (by definition!) as given by (5) p. 26; a simple translation is not correct except for vanishingly small excitation. (Thanks to R. Kompfner for reminding me.) The value of t can be inserted here and an expansion made through terms in i_1^{2a} to give,

$$\frac{\omega_{p}}{v_{o}} \mathbf{x}(t, t_{a}) \stackrel{\simeq}{=} \omega_{p}(t - t_{oa}) + \frac{i_{1}}{i_{o}} \frac{\omega_{p}}{\omega} \sin \omega t_{oa} \cos \omega_{p}(t - t_{oa}) - \left(\frac{i_{1}}{i_{o}}\right)^{2} \frac{\omega_{p}}{\omega} \left[\cos \omega t_{oa} \sin \omega t_{oa} \cos \omega_{p}(t - t_{oa}) + \frac{\omega_{p}}{\omega} \sin^{2} \omega t_{a} \sin \omega_{p}(t - t_{oa})\right] + \dots$$

$$(2)$$

Keeping only the i term allows for the translation and use of the previous diagrams for velocity modulation; for values of $(i_1/i_0 \omega_p/\omega) > 0.1$ or so, the higher order terms obviously alter the trajectories appreciably. Plots have been made, using the exact expressions and are given in Figs. 18, 19, and 20 for $\omega/\omega_p = 2$, 1, and 0.5. The contrast with those for velocity modulation is not large, even with $i_1/i_0 \sim 0.3$ as used here.

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INTERACTION WITH A TRAVELING WAVE

The stream may react to its own fields, space-charge fields, and may also be driven by externally applied fields. Such fields, of course, are necessary to provide the initial modulation, v_1 and i_1 , used earlier; these fields presumably are imposed over a short length of stream, a diode or gap region. Particle motion in a gap has been given in great detail by many authors over the past three decades, as found in the classical monograph by Llewellyn (1941) and as outlined by one of the earliest workers with transit time effects, Benham, in Benham and Harris (1957).

Interaction with a traveling wave provides an interesting example of a driven stream, as well as insight into the bunching process in a traveling-wave tube. The problem solved here is very simple but somewhat nonphysical. A slow wave of phase velocity v_p is coupled to a stream of average velocity v_o . The wave produces bunching and may alter the stream kinetic energy; this change in kinetic energy should appear in field energy, some of which may cause the driving wave to increase (as in a traveling-wave tube) or decrease (as in an accelerator), all as a self-consistent process. However, here, the driving field will be taken to be constant. Thus, the model is closely related to the traveling-wave resonant cavity devices, ignoring interaction with the reflected wave, and loosely related to the travelingwave tube amplifier and traveling-wave accelerator.

The electric field of the driving wave is given in terms of laboratory coordinates as,

$$E_{w}(x, t) = E_{w} \sin \left(\omega t - \frac{\omega x}{v_{p}}\right).$$
(1)

The equation of motion for a particle is

$$m \frac{\partial^2 x}{\partial t^2} = e E_{\text{space charge}} + e E_{\text{wave}}$$
(2)

where the fields are to be given at the particle position x. The particle position x appears in E_{wave} , whereas solution would be simpler (an understatement!) if the equilibrium position x_0 appeared. Expand E_w as

$$E_{w}(x, t) = E_{w}(x_{o}, t) + \frac{\partial E_{w}(x_{o}, t)}{\partial x}(x - x_{o}) + \dots$$
(3)

$$= \mathbf{E}_{\mathbf{w}} \left[\sin \left(\omega t - \frac{\omega x_0}{v_p} \right) - \frac{\omega}{v_p} (\mathbf{x} - \mathbf{x}_0) \cos \left(\omega t - \frac{\omega x_0}{v_p} \right) + \ldots \right] \quad . \quad (4)$$

The second term compares with the first as,

$$\frac{\omega}{v_{p}} (x - x_{o}) = 2\pi \frac{x - x_{o}}{\gamma_{w}}$$
(5)

where γ_w is the wavelength of the driving wave. In the limit of small excitation, the deviation $x - x_0$ will be much smaller than γ_w , so that the second term may be ignored. The problem solved is for no initial current moculation and no initial velocity modulation. Thus, $t_{oa} = t_a$. Hence

$$x_{o} = v_{o} (t - t_{a})$$
 (6)

Thus, the phase of E_{w} becomes

$$\omega t - \frac{\omega x_0}{v_p} = \omega t - \frac{\omega v_0}{v_p} (t - t_a)$$
(7)

$$= \omega(1 - \frac{v_o}{v_p}) t + \frac{v_o}{v_p} \omega t_a$$
(8)

$$= \omega' t + \frac{v_0 \omega t_a}{v_p} .$$
 (9)

 ω' is the frequency of the wave as seen by the stream, the usual Doppler shifted value. The response of the stream to the wave is expected to be greatest if ω' is at the resonant frequencies of the stream, $\pm \omega_p$, or

$$\omega \left(1 - \frac{v_o}{v_p}\right) = \pm \omega_p \tag{10}$$

$$\frac{\omega}{v_{p}} = \frac{\omega}{v_{o}} + \frac{\omega_{p}}{v_{o}} \quad . \tag{11}$$

That is, greatest response is expected if the wave velocity is that of the faster or the slower space-charge wave. Physically this requires that the wave should move one wavelength past the stream in one cycle of the plasma frequency; this slip is just that used to obtain maximum growth rate in traveling-wave tubes with moderate-to-large space-charge densities. Choosing this synchronism, one obtains the equation of motion,

$$\frac{\partial^2 x}{\partial t^2} + \omega_p^2 (x - x_o) = \frac{e}{m} E_w \sin\left[\frac{1}{m} \omega_p (t - t_a) + \omega t_a\right]. \quad (12)$$

The solution for $\dot{x} = v_0$ at $t = t_a$ is found to be (e.g., by Laplace transform),

$$\mathbf{x}(t, t_{a}) = \mathbf{v}_{o}(t - t_{a}) - \frac{(e/m)\mathbf{E}_{w}}{2\omega_{p}} \left\{ \pm \omega_{p} (t - t_{a}) \cos\left[\pm \omega_{p} (t - t_{a}) + \omega t_{a}\right] - \cos \omega t_{a} \sin\left[\pm \omega_{p} (t - t_{a})\right] \right\}.$$
(13)

The cost term shows the growth of displacement from equilibrium due to the traveling-wave interaction.

Trajectories are plotted for the six points marked on the ω - β diagram of Fig. 21, for the slower and faster space-charge waves



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Fig. 21. Interaction frequencies for trajectory plots to follow. (a) corresponds to Fig. 22, (b) to Fig. 23, etc.

at $\omega/\omega_p = 2, 1$, and 0.5, and given in Figs. 22-27. It is tacitly assumed that the circuit that provides E_w is well behaved and was sketched as such in Fig. 21, with $v_p = v_g$; in all six interactions, linear, selfconsistent analyses (in a later chapter) show that $E_w \sim \exp(ax)$, or $\cosh(bx)$ or $\cos(cx)$ so that the simplifying assumption of E_w constant is good as $x \rightarrow 0$. Note that the motion is still oscillatory, just as for the waves without a driving field, with electrons sliding through the bunches and not being trapped. Also, note that the growth is in distance and that there is no growth in time t for a fixed position x.

The slower waves (a, b, c) are all very similar in appearance. The bunching of slower electrons, occurring in the retarding phase of E_w , implies a decrease of the stream total kinetic energy, which would lead to an increase in E_w ; had such been allowed, E_w would grow and cause stronger bunching, with further growth, resulting in exponential rather than linear growth.

The faster waves (d, e, f) have faster electrons in the accelerating phase, to be sure, but bunch in special ways depending on $\omega/\omega_{\rm p}$. For $\omega > \omega_n$, the bunches have faster electrons, implying an increase in stream total kinetic energy which would lead to a decrease in E_{y} ; if allowed, E., would decrease and the rate of bunching would decrease, with $E_{w} \sim \cos (kx)$ and density $\sim \sin(kx)$, as known from the coupledmode analysis of Gould (1955) for the Kompfner dip (1950). For $\omega < \omega_{n}$, the wave is reversed ($v_g < 0$) and may be called a backward wave, requiring that it be launched toward x = 0 at some x = L > 0. The bunches form with slower electrons but \overline{W} and $\overline{\overline{P}}_k$ are positive so that power is fed to the stream at the expense of the wave; E_w must decay away from its value at its source, $E_{w} \sim coshKx/coshKL$, as discussed by Gould (1955) and given by Hutter (1960). The behavior at $\omega = \omega_p$ is quite singular; $v \rightarrow \infty$, $v \rightarrow \infty$, and both change sign as ω goes through $\omega_{\rm p}$, altering the interaction from forward-forward to forward-backward. There is no ac charge density (vertical spacings remain constant), no bunching, but there is still $\overline{W}_{k} \approx v_{1}v_{1}^{*} > 0$; where

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 v_1 is large, i_1 is large so that $\overline{P}_k > 0$, implying a stream power increase, requiring a decrease in E_w .

Solutions of the self-consistent problem would be more useful. Also, in either solution, as the deflection grows, particles must cross at some point so that complete solution to maximum growth of the wave or stream energy virtually requires use of a high speed computer; typical machine solutions and trajectories are given by Tien (1955).

ADDITIONAL REFERENCES

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