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DISCUSSION:

ON THE ABSOLUTE STABILITY OF SAMPLED-DATA SYSTEMS 'THE INDIRECT CONTROL CASE' BY G. P. SZEGÖ AND J. B. PEARSON, JR.

by

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DISCUSSION

On the Absolute Stability of Sampled-Data Systems,*

'The Indirect Control Case' by G. P. Szegö and J. B. Pearson, Jr.

The Lure's problem associated with the class of nonlinear sampled data control systems containing a memoryless gain element and direct control linear has been treated, as mentioned in this paper, by Kalman and Szegö[1] and Szegö.[2] Further, the sufficient conditions for stability of these systems with both direct and indirect control linear parts, were recently presented by Jury and Lee[3, 4] using Popov's method.[5]

The main contribution of this paper lies in the inclusion of at most one integral in the plant, i.e., indirect control case, and the proof of Theorem 3. Both of these two extensions are of much use and in particular the latter theorem completes the necessary part of the Lure's problems for this class of non-linear discrete systems which was not discussed in Ref. 2.

In the introductory part of the paper, the authors had implied that the subclasses A_2 and A_3 of their systems are the same as the systems of Γ_1 considered by Jury and Lee.[3] This implication is not quite accurate, since the conditions which specify the subclasses A_2 and A_3 are given by Eq. (2) of the paper as follows:

$$\begin{aligned} &(a)\quad \phi(0) = 0, \\ &(b)\quad 0 < \phi(\sigma) \le k\sigma^2, \\ &(c)\quad 0 < \frac{d\phi}{d\sigma} < \mu, \\ &(d)\quad \frac{d\phi}{d\sigma} \le k; \end{aligned} \qquad (2)$$

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whereas the corresponding specifications for the subclass Γ_1 considered by Jury and Lee are given by

$$
\begin{aligned}
&(a) \quad \phi(0) = 0, \\
&(b) \quad 0 < \sigma\phi(\sigma) \leqq k\sigma^2, \\
&(c) \quad \left|\frac{d\phi}{d\sigma}\right| < \mu, \qquad (2') \\
&(d) \quad k \leq \mu.
\end{aligned}
$$

First it may benoted that condition (2d) appears to be inconsistent with conditions (2b) and (2c). Secondly, the difference between conditions (2c) and (2'd) is clearly significant. For example, a system with a nonlinear gain function which has both positive and negative values of slopes is included in the subclass Γ_1 but would be excluded by the conditions specified for subclasses A_2 and A_3. This would therefore imply that subclass Γ_1 is in fact more general than A_2 and A_3. Inasmuch as the same frequency domain criterion obtained by Jury and Lee in Ref. 3 for the absolute stability of systems in Γ_1 also appears as the main premise of both Theorems 1 and 2 of the present paper, it appears that these theorems are unnecessarily restricitve.

Upon closer inspection of steps leading to these theorems, the crucial step which requires application of conditions (2a) to (2c) is inequality (5) of the present paper. However, as was pointed out in Refs. 3 and 6, a combined application of the well-known Theorem of the Mean and condition (2'b) would yield the result that inequality (5) is valid not only for functions specified by conditions (2a) to (2d) but also for functions specified by conditions (2'a) to (2'd). This conclusion furthermore can be established for positive or negative real values of δ^*. (See Ref. 6.) Theorems 1 and 2 therefore could have been established for the more general subclass Γ_1 of systems.

* The symbol δ in present paper is equivalent to the parameter q in Refs. 3 and 6.

Another point which requires clarification is that inequality (15) appears to be missing from premises of both Theorems 1 and 2. Yet in the steps leading to these theorems inequality (15) appears to play an important role. Unless this inequality can be shown to be redundant in the sense that it is implied by inequality (14), inequality (15) should have been included in the premises of these theorems.

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