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A NEW MODEL FOR SIMPLE NEURAL
NETS AND ITS APPLICATION IN THE
DESIGN OF A NEURAL OSCILLATOR

by

Theodosios Pavlidis

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ABSTRACT

The results of a previous theoretical study of a class of systems are applied for the design of neural nets which try to simulate biological behavior.

Besides the models for single aperiodic and periodic neurons, a "neural oscillator" is developed which consists of two cross-excited neurons. Its response is similar to the firing pattern of certain biological neural oscillators and, by proper change of its parameters, can be made highly irregular, providing a deterministic model for the spontaneous neural activity.

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A NEW MODEL FOR SIMPLE NEURAL NETS
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OSCILLATOR

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I. Introduction. In the recent years a large number of models for neurons and neural nets have been proposed (for review see Harmon, 1962 and Jury, 1963). Most of them however, were concerned rather with analog hardware simulation than with the mathematical analysis of the model (Harmon, 1961; Hiltz, 1963; Lewis, 1963, etc.). Landahl (1961) and Nelson (1962) can be mentioned among the recent contributors who dealt with the mathematical analysis of models while for the previous analytical work one is referred to Rashevsky (1960). The present study is concerned with the use of the results of a theoretical study of a class of systems for the design of models for neural nets. The emphasis will be rather in the study of nets containing a relatively small number of neurons because of the complexity of the proposed model. However, in this way, a more accurate description of the behavior of a certain net may be obtained. The following basic simplifying assumptions were used as starting point for the definition of the system:

1. The wave shape of the neural pulse is not of importance, due to its very short duration (about 1 msec) compared to the usual firing frequencies (Ruch, 1960) as well as to the usual "low pass" properties of the synaptic transmission.

2. The variable threshold refractoriness can be substituted by a negative feedback. Indeed, let the time of the emission of the pulse be determined by a relation of the form

$$p(t) - r(t) = r_0 \quad (1)$$

where $p(t)$ is the membrane potential and $r_0 + r(t)$ the value of the threshold. Then if $q(t) = r(t)$ around the solution of Eq. (1) this can be substituted by

$$p(t) - q(t) = r_0 \quad (2)$$

$q(t)$ will be called the refractory feedback.

3. No effort should be made to simulate the absolute refractoriness because it is of importance only in very high firing frequencies (1000 per sec), well above the usual range.

4. The phenomenon of accommodation can be neglected at first approximation because it is of importance only in very particular cases.

5. The phenomenon of inhibition can be represented by inverting the sign of the pulses.

Of course these assumptions are not enough to specify a simple model but we want to emphasize that no effort will be made to simulate the properties of the neural membrane in detail. The main interest will be the study of aggregates of neurons.

Among the original contributions of this note is a model for a neural net consisting basically from two neurons which are cross-excited. This has an oscillatory behavior and moreover its output can be made highly irregular, so as to possibly represent a sample from a random process. This is not obtained by adding any random input but simply because of the peculiar but completely deterministic structure of the system. In this way a deterministic model for the spontaneous neural activity can be obtained. This problem will be discussed in detail in Section V.

II. Σ - Pulse Frequency Modulation Systems. We define as a neural trigger a device with scalar input $p(t)$ which emits an impulse of area μ whenever its input reaches a value r , which is called the

threshold; immediately after, $p(t)$ is reset to the zero value. If its $(k - 1)$ first derivatives are also reset to the zero value then the trigger will be referred as a k -th order trigger. Consider now an abstract system containing m first order triggers. Let their inputs be p_1, p_2, \dots, p_m and their outputs $\Delta_1, \Delta_2, \dots, \Delta_m$ where

$$\Delta_i(t) = \mu_i \sum_{j=1}^n \epsilon_j \cdot \delta(t - t_j) \text{ for } t_n \leq t < t_{n+1} \quad (3)$$

where $\epsilon_j = \pm 1$, depending on the sign of the pulses and μ_i their areas. All the other state variables of the system are denoted by x_1, x_2, \dots, x_n and the inputs by u_1, u_2, \dots, u_ℓ . Then the following vectors are defined as

$$\underline{p}(t) = \text{col } (p_1, p_2, \dots, p_m) \quad (4)$$

$$\underline{\Delta}(t) = \text{col } (\Delta_1, \Delta_2, \dots, \Delta_m) \quad (5)$$

$$\underline{x}(t) = \text{col } (x_1, x_2, \dots, x_n) \quad (6)$$

$$\underline{u}(t) = \text{col } (u_1, u_2, \dots, u_\ell) \quad (7)$$

Then the following two state equations (Zadeh, 1963) give a complete description of the system

$$\frac{d\underline{x}}{dt} = \underline{F}_1(\underline{x}, \underline{\Delta}, \underline{p}, \underline{u}) \quad (8)$$

$$\frac{d\underline{p}}{dt} = \underline{F}_2(\underline{x}, \underline{\Delta}, \underline{p}, \underline{u}) \quad (9)$$

The above form is very general but too complicated to deal with. Some simplification can be readily made if we have in mind that our purpose is to simulate neural nets. \underline{p} corresponds to the membrane potentials and \underline{x} to the various agents between input and membrane potential. Hence there is no direct connection between \underline{u} or $\underline{\Delta}$ and \underline{p} . A second assumption is that \underline{p} does not act directly on \underline{x} but only through $\underline{\Delta}$. This is debatable from the physiological point of view but there are many cases where it can be true. Then Eq. (8) and (9) are simplified to

$$\frac{d\underline{x}}{dt} = \underline{F}_1(\underline{x}, \underline{\Delta}, \underline{u}) \quad (10)$$

$$\frac{d\underline{p}}{dt} = \underline{F}_2(\underline{x}, \underline{p}) \quad (11)$$

The Δ_i 's have constant amplitude and hence can be expressed in a linear form. Moreover it can be seen that the following change of notation can be made (Pavlidis, 1964b)

$$\Delta_i(t) = \mu_i \operatorname{sgn} [p_i(t)] \cdot \delta [r_i(t) - p_i(t)] \quad (12)$$

Now we can use a mathematical trick to express analytically the resetting of the p_i to zero. This can be done by adding a term

$$-r_i \operatorname{sgn} (p_i) \cdot \delta (r - p_i) \quad (13)$$

to each one of the Eqs. (11) (Pavlidis, 1964b).

Then by defining

$$\underline{r} = \text{col } (r_1, \dots, r_m) \quad (14)$$

$$\underline{R} = \text{diagonal matrix } (r_1, r_2, \dots, r_m) \quad (15)$$

$$\underline{\delta}(\underline{p} - \underline{r}) = \text{col } [\delta(r_1 - p_1) \dots \delta(r_m - p_m)] \quad (16)$$

$$\underline{S}(\underline{p}) = \text{diagonal matrix } [\text{sgn}(p_1) \dots \text{sgn}(p_m)] \quad (17)$$

$$\underline{M} = \text{diagonal matrix } [\mu_1, \mu_2, \dots, \mu_m] \quad (18)$$

Eq. (10) and (11) are written

$$\frac{d}{dt} \underline{x}(t) = \underline{F}_1 [\underline{x}(t), u(t)] + \underline{M} \cdot \underline{S} [p(t)] \cdot \underline{\delta} [p(t) - \underline{r}(t)] \quad (19)$$

$$\frac{d}{dt} \underline{p}(t) = \underline{F}_2 [\underline{x}(t), \underline{p}(t)] - \underline{R} \cdot \underline{S} [p(t)] \cdot \underline{\delta} [p(t) - \underline{r}(t)] \quad (20)$$

This class of systems will be called Σ - Pulse Frequency Modulation Systems.

They are a generalization of Integral Pulse Frequency Modulation Systems [Li, 1961] and they fit better with some of the basic properties of neural nets, as for example the presence of input threshold.

Eqs. (19) and (20) can be said to represent a "neural net," containing m neurons. Obviously the behavior of such a net is not going to be the same with one of a biological neural net but nevertheless they will have many common features. It is expected that these features are the essential ones for some of the functions of the nervous system.

If \underline{F}_1 and \underline{F}_2 are linear functions with respect to \underline{x} and \underline{u} Eqs. (19) and (20) are simplified to

$$\frac{d}{dt} \underline{x}(t) = \underline{A}_1 \underline{x}(t) + \underline{A}_2 \underline{u}(t) + \underline{MS} [p(t)] \cdot \delta [p(t) - \underline{r}(t)] \quad (21)$$

$$\frac{d}{dt} p(t) = \underline{B}_1 \underline{x}(t) + \underline{G} [p(t)] + \underline{RS} [p(t)] \cdot \delta [p(t) - \underline{r}(t)] \quad (22)$$

This will be called an almost-linear Σ -P. F. M. System. There A_1 is $n \times n$, A_2 is $n \times l$ and B_1 is $m \times n$ matrices with constant elements.

If $\underline{G}(\cdot)$ is also linear, i. e.,

$$\underline{G} [p(t)] = \underline{B}_2 \cdot p(t)$$

where \underline{B}_2 is a $m \times m$ matrix with constant elements then the system will be called a linear Σ -P. F. M. System.

The analytical study of such systems has been presented elsewhere. (Pavlidis, 1963a, 1963b, 1964a, 1964b) It was proven that a modification of the Liapunov Method can be applied for the investigation of their stability.

Moreover, if the system contains an antagonistic pair of "neurons," i. e., two triggers with opposite signs of pulse weights then a quasi-describing function can be used for their study (Pavlidis, 1963b).

Here we will present the application of these results in the design of "neural nets."

III. Models of Single Neurons. The system of Fig. 1 can be considered as model of a single neuron. Its state equations are

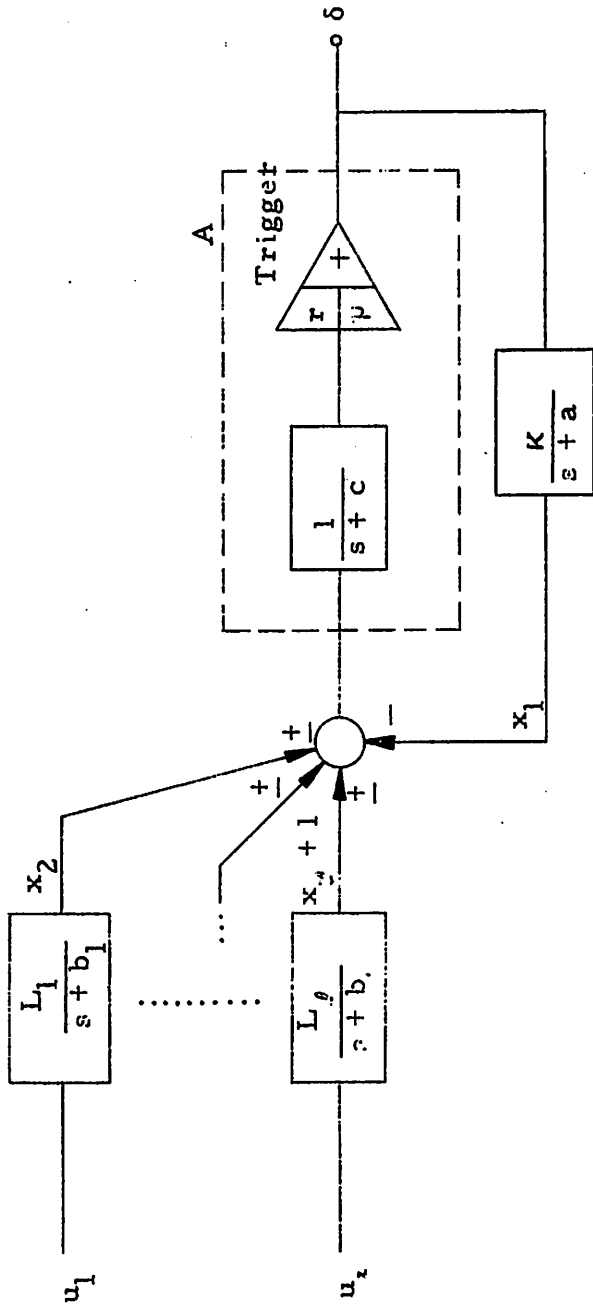


Fig. 1.
Model of an "aperiodic neuron."

$$\dot{x}_1 = -ax_1 + K\mu \delta(r - p) \quad (24)$$

$$\dot{x}_{j+1} = -b_j x_{j+1} + L_j u_j \quad \text{for } j = 1, 2, \dots, l \quad (25)$$

$$\dot{p} = -x_1 + \sum_{j=2}^{r+1} x_j - cp - r \cdot \delta(r - p) \quad (26)$$

We do not proceed in detailed analysis of this model because it is similar to other proposed models, and it has been also analyzed in detail elsewhere (Bertaux, 1963, Pavlidis, 1964a). Here we note that it presents refractoriness, temporal summation, spatial summation, inhibition (by inverting the sign of some of the L's) and the strength-duration curve (Pavlidis, 1963a).

The system 'A': inside the dotted lines will be called a Σ - Pulse Frequency Modulator. * From this unit we can start building various neural nets. A simple variation can account for a rate-sensitive neuro-receptor. This can be obtained by adding a second feedback loop with transfer function R/s . Then the corresponding state equation will be

$$\dot{x}_{l+2} = R\mu \cdot \delta(r - p). \quad (27)$$

The response of such a unit is shown in Fig. 2. Note the delay introduced in all cases by the time needed for $p(t)$ to reach the level of the threshold.

* The name Neural P. F. Modulator has been used in the past, but this was considered as implying a greater similarity to neural behavior than the system actually presents. The letter ' Σ ' is used to indicate the "summing up" property of this modulator.

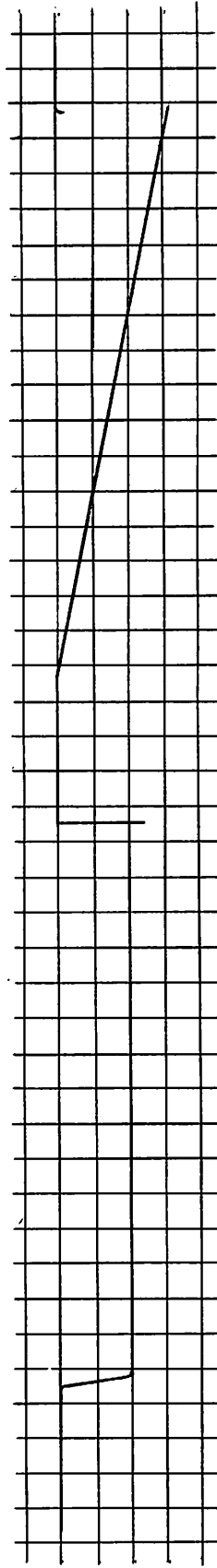
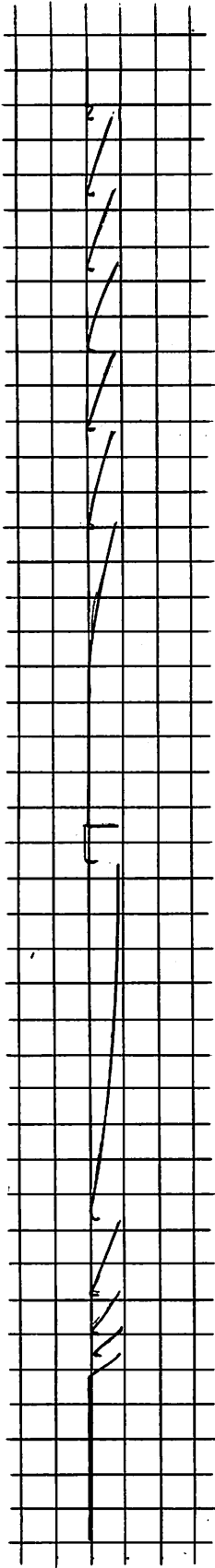


Fig. 2. Response of a rate-sensitive receptor ($c = 0.5$, $r = 5$, $\mu = 1$, $K = 1$, $a = 0.5$, $R = 2$, $L_j/b_j \rightarrow \infty$).
 Lower trace input, upper trace output.

In many cases a neuron may keep on firing periodically with no input. This is the case of a pacemaker or generally a "periodic" neuron (Nelson, 1962). This can be achieved by adding a pair of imaginary poles to the input or the previously mentioned model (Fig. 3). Then the membrane potential will be given by a function whose Laplace Transform is

$$P(s) = \frac{Ld}{(s+c)(s^2+b^2)} + \frac{Bo}{s(s+c)} \quad (28)$$

or by using the appropriate tables (Gardner, 1942, Table C, item 1.308)

$$p(t) = \frac{Lde^{-ct}}{c^2+b^2} + \frac{Ld}{b\sqrt{c^2+b^2}} \cdot \sin(bt - \tan^{-1} \frac{b}{c}) + \frac{Bo}{c}(1 - e^{-ct}) \quad (29)$$

All that is needed for the excitation of this model is a pulse of area d and then firing will go on indefinitely provided that

$$\frac{Bo}{c} + \frac{Ld}{b\sqrt{c^2+b^2}} > r \quad (30)$$

The state equations governing the behavior of such a model are (if we neglect the refractoriness, which may be justified if $a \ll b$).

$$\dot{x}_1 = x_2 \quad (31)$$

$$\dot{x}_2 = -b^2 x_1 \quad (32)$$

$$\dot{p} = -cp + x_1 + Bo \quad (33)$$

where $x_1 = x$ and $x_2 = \dot{x}$. The state space is now 3-dimensional (x_1, x_2, p). A previous model proposed by Nelson (1962) was only

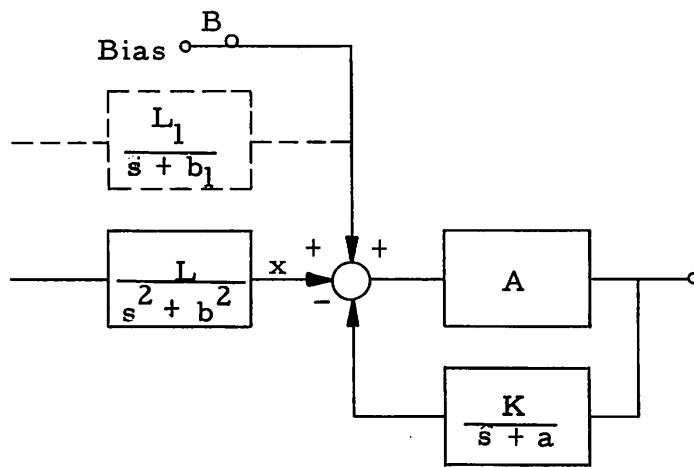


Fig. 3.
Model of a "periodic" neuron.

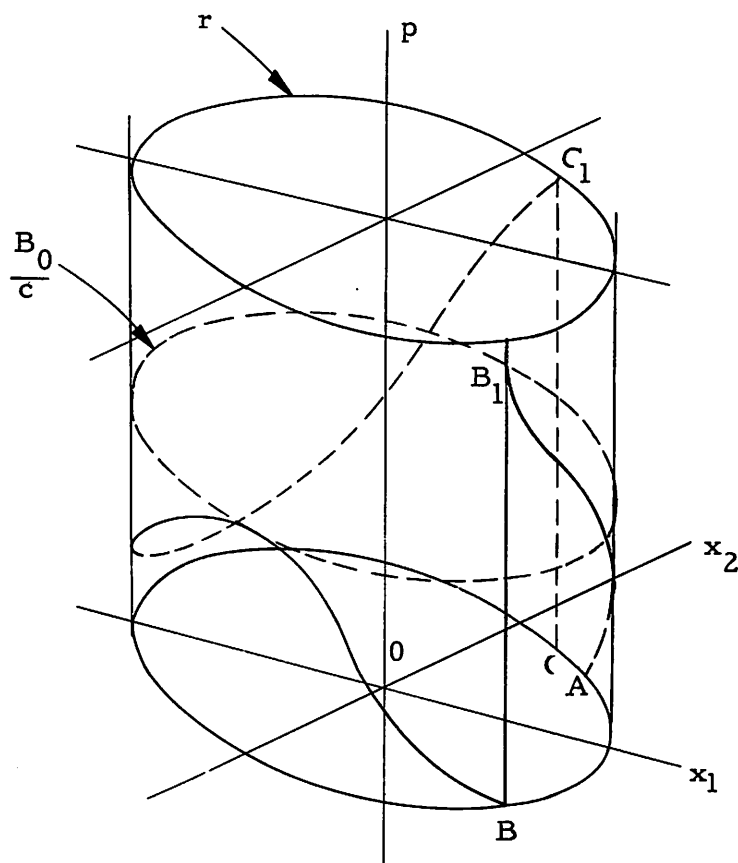


Fig. 4.
State-space trajectory of a "periodic" neuron
(two firings: BB_1 and CC_1).

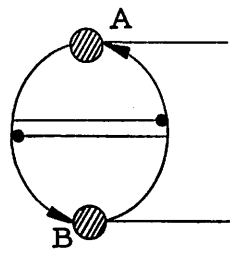
two-dimensional but it required resetting of two initial conditions after the firing of a pulse while the present model requires only one. This is an advantage because the existing physiological data do not give any indication about the resetting of a second parameter.

The projection of the trajectories of the system on the $x_1 - x_2$ plane are ellipses as one can readily see by integrating equations (31) and (32). Then the trajectory is a helix on an orthogonal cylinder with axis parallel to \overline{op} (Fig. 4). The size of the axes of the ellipse depend on Ld . Each time the trajectory reaches the plane $p = r$, the firing of a pulse occurs and $p(t)$ returns to the zero value. Typical trajectories are shown in Fig. 4.

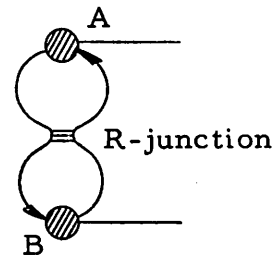
If the model is modified as in Fig. 3 (dotted lines) then input pulses will have the effect of facilitating the firing. This is also in agreement with experimental observation and the behavior of Nelson's model (ibid). The addition of refractoriness can be illustrated by adding a fourth dimension (if the arrangement of Fig. 3 is to be kept) or by substituting the plane $p = r$ by a time-varying surface.

A more detailed study of this model and the choice of specific values for its parameters is beyond the scope of this note.

IV. Design of an Oscillating Neural Net. Now we proceed in a more interesting case, the one of a "neural oscillator." This has never been studied analytically before. It consists of two units of the form of Fig. 1 which are cross-excited. At the same time provision has been made not to allow simultaneous cross-excitation of the system. The physical neural net which will operate on this principle is shown in Fig. 5a and the corresponding $\Sigma - F.F.M.$ system in Fig. 6. Note the way that the mutual inhibition of the two "cross-excitors" is obtained in the model. The impulse sign of one unit is inverted and is "mixed" with the other by passing through the block $G(s)$ and then they are again separated by the diodes. No response should occur when both units fire almost simultaneously.



(a)



(b)

Fig. 5.

Possible connections of two neurons forming a natural neural oscillator.

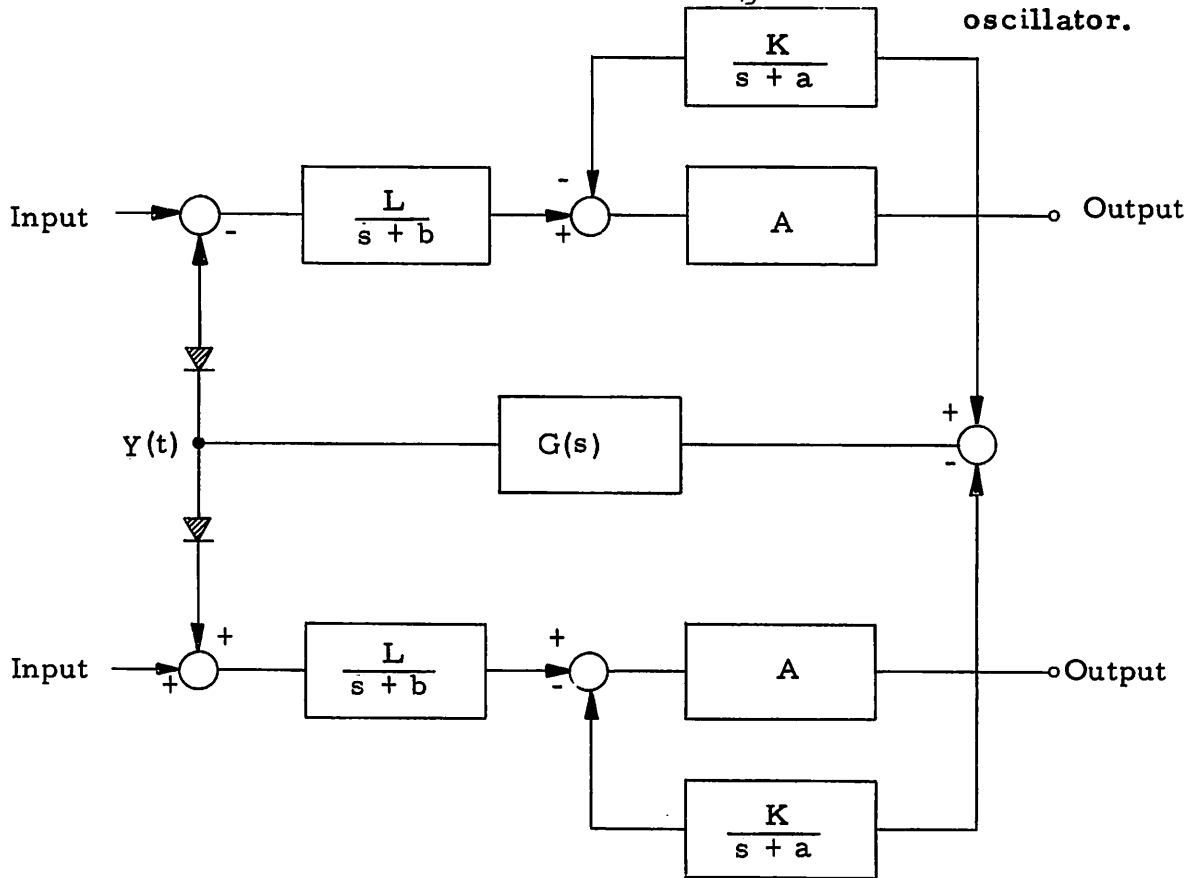


Fig. 6.

Model of an oscillating neural net.

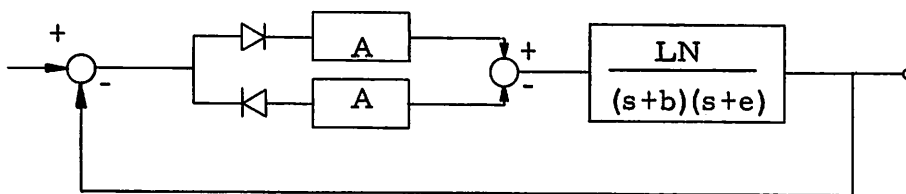


Fig. 7.

Simplification of the model of Fig. 6.

For this purpose $G(s)$ should be of the form $N/(s + e)$. A slightly different interpretation of this mode of operation is the one shown in Fig. 5b. There two pulses travelling in opposite directions cancel each other. This is comparable to the R-junction described by H. D. Crane (1962).

One could write the state equations of the system but for this case we will rather try to apply the concept of the quasi-describing function.

To this end the system of Fig. 6 should be simplified. First, we note that if the frequency of oscillations is much smaller than "a", then the refractory feedback can be omitted. Second the "mixing" can be carried also through the second block $L/s + b$. Then the system of Fig. 7 results which looks like a conventional control system. In this case the quasi-describing function can be used to predict the nature of the occurring oscillations.

In Fig. 8 the plot of a normalized quasi-describing function is shown (Pavlidis, 1963b, 1964a). If S is the amplitude of the output and ω its angular velocity then $x = S/rc$ and $\theta = \omega/\pi c$. If $G(s)$ is the transfer function of the linear part [in this case $LN/(s + b)(s + e)$] than a series of curves $G(j\theta):(\mu/\dot{r}) \cdot \sigma$ are plotted where σ varies between $1/2$ and 1 . The points of intersection give a series of modes of oscillations at the output of the system. The sectors where these points lie give the number of pulses per half period. It is easy to see from the graph that for increasing frequency of oscillations the amplitude also increases, which in turn means that if the firing frequency of the neurons is increasing, each one of them will start firing more than one pulse before the other fires. In this way if we want to obtain a particular firing pattern we can fix the points on the complex plane and then choose a function $G(s)$ such as to give the desired intersections. Actually the general form of $G(s)$ can be chosen a priori and only an adjustment of parameters (e. g. gain)

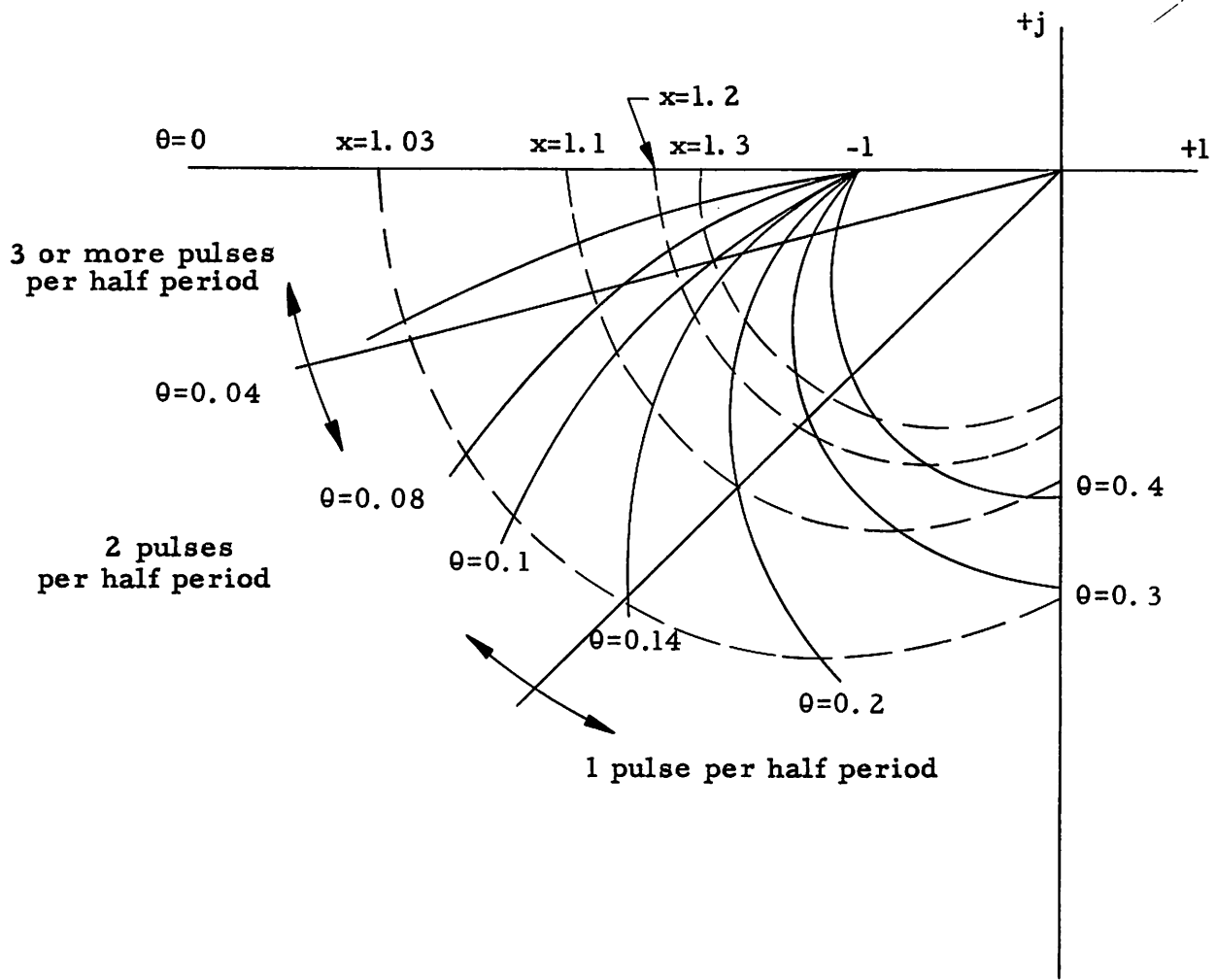


Fig. 8. Complex plane plot of the normalized quasi-describing function of an N. P. F. M. System.

will be sufficient. Recordings of such responses are shown in Figs. 9 and 10. The model used was of the form of Fig. 6 with the indicated values of parameters.

It should be noted that no direct external input is needed for the elicitation of the oscillations. Only a high value of the gain LN is required, which can be computed by the method indicated above. Slight adjustments are needed in going from the approximate system of Fig. 7 to the original model of Fig. 6. The physiological interpretation of the gain increase is facilitation or threshold decrease of the synaptic cross-excitation at the points A and B of Fig. 5. In this way no direct correlation between input and output pattern can be found while the input will still be necessary for the elicitation of the oscillations. After-effect is also possible. This kind of behavior is in agreement with observations of the response of the neurons innervating the elevator and depressor muscles of the wings of certain insects (Wilson, 1961; 1962, 1963).

V. A Model for Spontaneous Neural Activity. Another interesting feature of these oscillations is their irregularity. This is not due to any noise or random variation of parameters but the very nature of the mechanism which generates them. This was represented in the application of the quasi-describing function by the factor σ . A mathematical discussion about this fact can be found elsewhere (Pavlidis, 1963b).

Here we may indicate that the system of Fig. 7 (or 6) does not present a limit cycle but rather a "limit annulus," i. e., a multiplicity of trajectories, all lying in a certain region of the state space. The application of the quasi-describing function is equivalent to the description of the system in terms of an equivalent complex gain. Then this approximate system will have true limit cycles. We have to use a non-countable number of them to cover the original "limit annulus" and this is the significance of the coefficient σ .

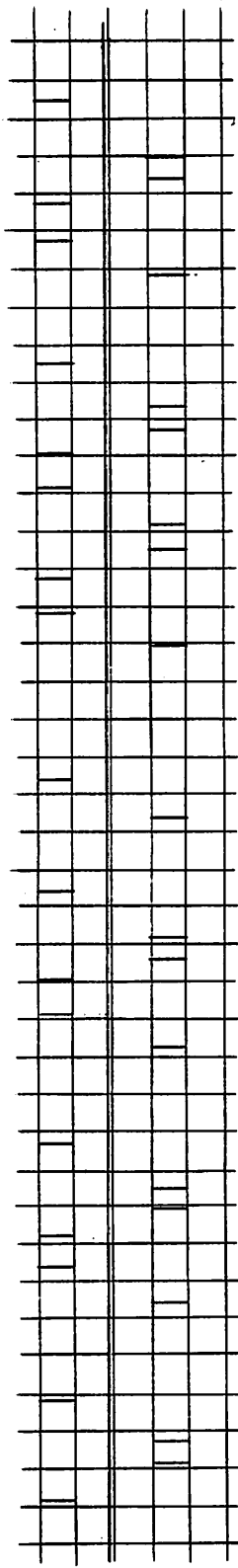
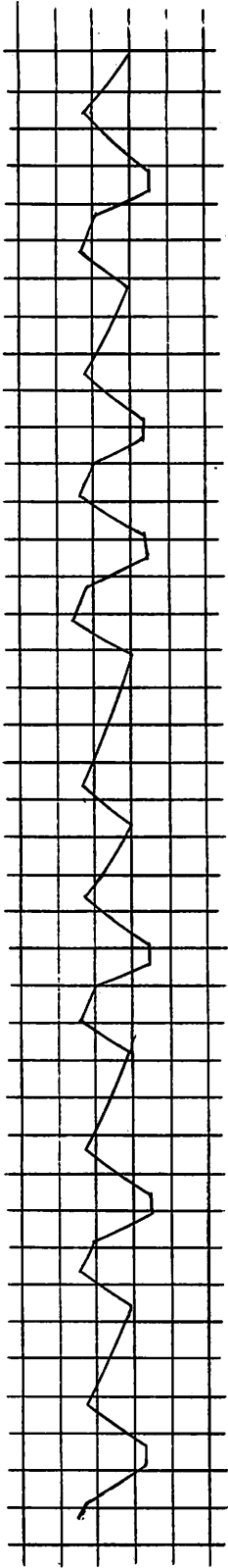


Fig. 9. Analog computer response of system of Fig. 6 with $L = 2$, $a = b = c = 0.5$, $G(s) = (50/s)$, $K = 10$. Upper trace $Y(t)$, lower trace "neuron" firing.

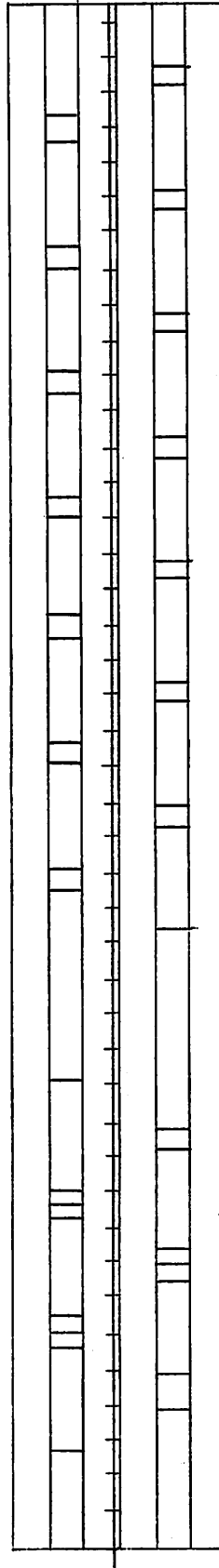
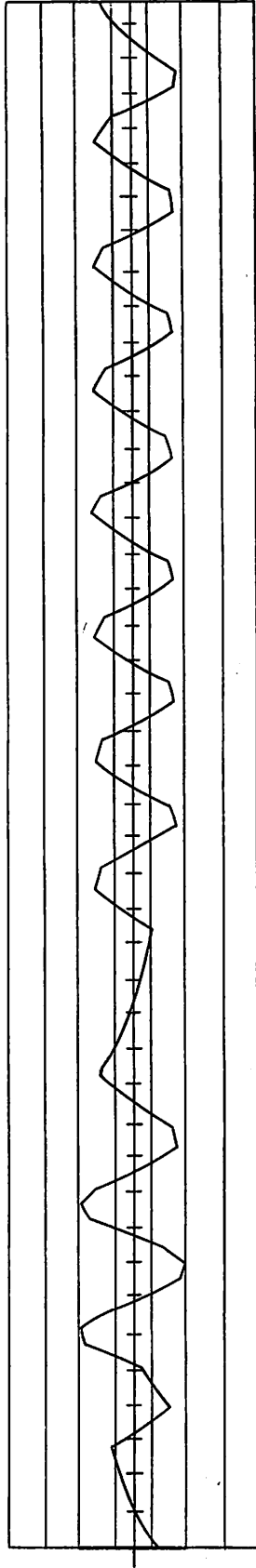


Fig. 10. Analog computer response of system of Fig. 6 with $K = 4$ (smaller refractoriness).

By appropriate variations of the parameters of the system this irregularity can become smaller or greater. In this way the "neural oscillator" can be used as model for the spontaneous neural activity, and the response of two "neurons" is shown in Fig. 11. In a ganglion it is possible to have cross-excitation and this gives a biological justification of the model. The pattern may become more complex if a larger number of neurons is used. This model is basically different than the ones proposed by other investigators (Gernstein, 1964; Fetz, 1963). What seems to be a decisive experiment for the value of the one proposed here is whether the histogram of its response will compare with histograms from actual neurons which show spontaneous activity. The fact that a deterministic model is used does not exclude the possibility that it may represent a random phenomenon. Actually it has been already proven (Kalman, 1956) that a first order Markov process can be represented by the quantized output of a first order nonlinear difference equation. A more general result was derived by Ormsby (1962). In the present case one is faced with a more complicated problem but an analytical study is still possible.

A distinguishing experiment between the two kinds of models will be the simultaneous recording of more than one fiber in the same nerve trunk. The existence of a correlation factor between the histograms of these fibers will be supporting the proposed new model.

Both these experiments and the analytical study will be the subject of future research.

Note: The analog computer used for the simulation of the above described models was a Donner Model 3400.

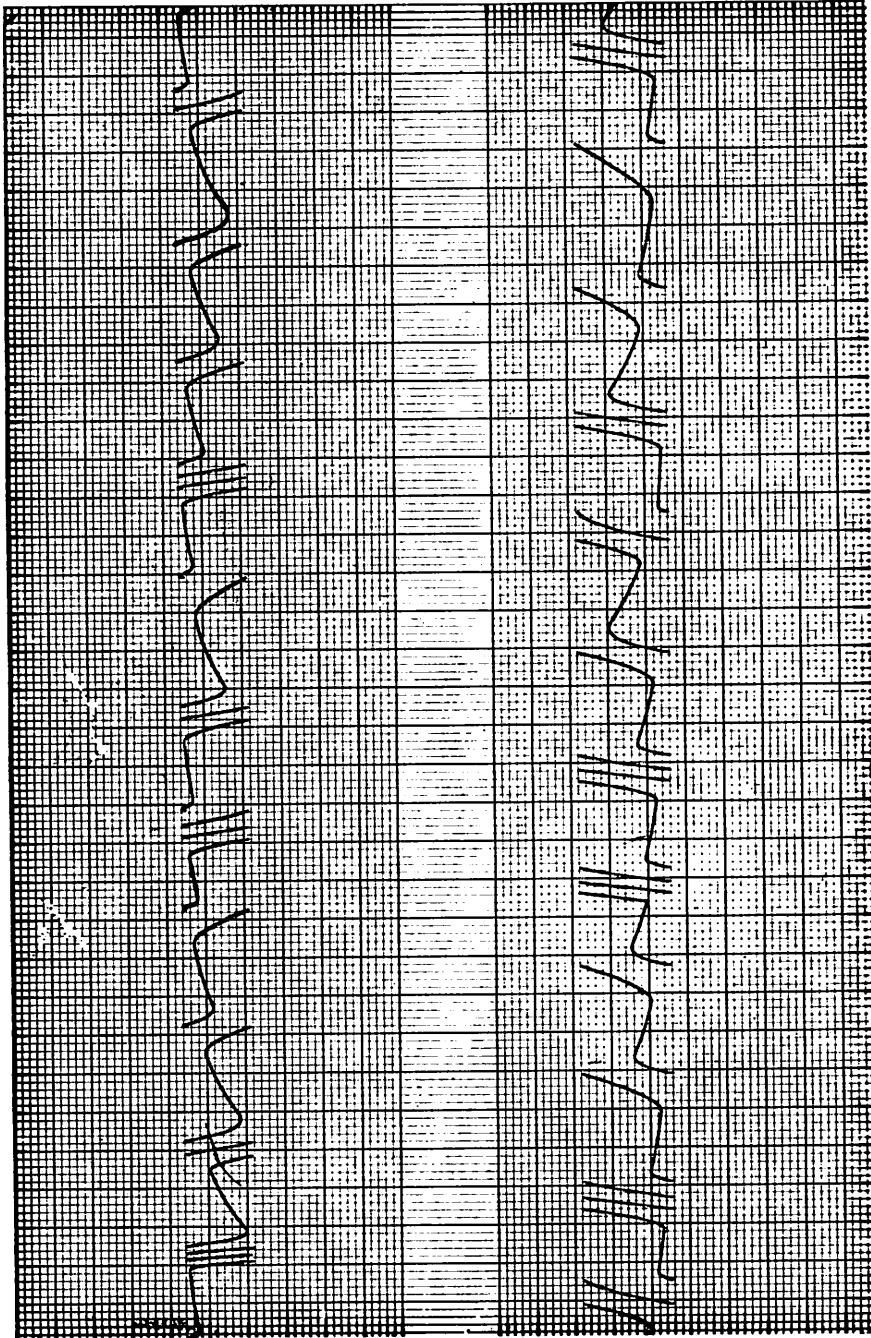


Fig. 11. Behavior of the model of Fig. 5 in an analog computer simulation.

BIBLIOGRAPHY

- Bertaux, D., 1963. Mathematical Model of a Synapse, M. S. Research Project (Plan II), Dept. Electr. Engr., University of California, Berkeley; Sept.
- Crane, H. D., 1962. Neuristor—"A Novel Device and System Concept." IRE Proc. 50, No: 10, 2048-2060.
- Fetz, E. E. and G. L. Gerstein, 1963. "An RC model for spontaneous activity of single neurons." MIT Quarterly Report of the Res. Lab. of Electronics, 71, 249-256; Oct. 15.
- Gardner, M. F. and J. L. Barnes, 1942. Transients in Linear Systems, J. Wiley, New York.
- Gerstein, G. L. and B. Mandelbrot, 1964. "Random Walk Models for the Spike Activity of a Single Neuron." Biophysical Journal, 4, No: 1, pt. 1, 41-68, Jan.
- Harmon, L. D., 1961. "Studies with Artificial Neurons, I: Properties and Functions of an Artificial Neuron." Kybernetik, 1, No: 3, 89-101.
- Harmon, L. D., 1962. "Neural Analogs." Proc. of the Spring Joint Computer Conference, San Francisco, 21, 153-158.
- Hiltz, F. F., 1963. "Artificial Neuron." Kybernetik, 1, No. 6, 231-236.
- Jury, E. I. and T. Pavlidis, 1963. "A Literature Survey of Biocontrol Systems." IEEE Trans. on Automatic Control, 8, No. 3, 210-217, July.
- Kalman, R. E., 1956. "Nonlinear Aspects of Sampled-Data Control Systems." Proc. of the Symposium on Nonlinear Circuit Analysis, Brooklyn Pol. Inst., 273-313.
- Landahl, H. D., 1961. "A Note on Mathematical Models for the Interaction of Neural Elements." Bul. Math. Biophysics, 23, 91-97.
- Lewis, E. R., 1963. "An Electronic Analog of the Neuron Based on the Dynamics of Potassium and Sodium Ion Fluxes." Semi-Annual Report No. 6, The Lab. for Automata Research, General Precision, Inc., 57-109, Jan.

- Li, C. C., 1961. Integral Pulse Frequency Modulated Control Systems. Ph. D. Dissertation, Northwestern University.
- Nelson, P. P., 1962. "Un modele du neurone." Bul. of Math. Biophysics, 24, 159-181.
- Ormsby, J. F. A., 1962. On the Generation of Markov Chains. Electr. Res. Lab. Report No. 60-480, University of California, Berkeley.
- Pavlidis, T., 1963a. Neural Pulse Frequency Modulation Control Systems. Int. Tech. Memo, M-15, Electronics Research Lab., University of California, Berkeley.
- Pavlidis, T., 1963b. "A Quasidescribing Function and its application to P. F. M. Systems." Notes on System Theory. Vol. IV. Report No. 63-17, Electronics Research Lab, University of California, Berkeley, 47-57.
- Pavlidis, T., 1964a. Neural Pulse Frequency Modulation Systems II. Int. Tech. Memo, M-46, Electronics Research Lab., University of California, Berkeley.
- Pavlidis, T. 1964b. "Equilibrium, Controllability and Stability of N. P. F. M. Control Systems." Notes on System Theory, Vol. V. Report No. 64-8, Electronics Research Lab., University of California, Berkeley.
- Rashevsky, N., 1960. Mathematical Biophysics, 3rd Edition, Vol. 1 and 2, Dover Public, New York.
- Ruch, T. C. and J. F. Fulton, 1960. Medical Physiology and Biophysics, W. B. Saunders, Philadelphia.
- Wilson, D. M., 1961. "The Central Nervous Control of Flight in a Locust." J. Exper. Biology, 38, 888-908.
- Wilson, D. M. and T. Weis-Fogh, 1962. "Patterned Activity of Coordinated Motor Units, Studied in Flying Locusts." J. Exper. Physiology, 39, 643-667.
- Wilson, D. M. and E. Gettrup, 1963. "A Stretch Reflex Controlling Wingbeat Frequency in Grasshoppers." J. Exper. Physiology, 40, 171-185.
- Zadeh, L. A. and C. A. Desoer, 1963. Linear System Theory-The State Space Approach, McGraw-Hill, New York.