HORN ANTENNAS WITH UNIFORM POWER PATTERNS AROUND THEIR AXES

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Summary It is shown that a linearly polarized horn which has the same power pattern in all planes through the axis can be made from a synthetic material for which the boundary conditions on $\underset{\sim}{E}$ and $\underset{\sim}{H}$ are the same. An arbitrarily close approximation to this requirement can be realised by using a grooved circular horn.

INTRODUCTION

For many purposes one needs a horn antenna having equal E and H plane patterns. Here we shall be concerned with the more general problem of making the power patterns equal in <u>all</u> planes

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through the axis. The efficient-illumination of a parabolic antenna is one application for this kind of antenna and the accompanying paper by H. C. Minnett and B. M. Thomas describes the development of this approach. Another important application occurs in connection with polarization measurements in radio astronomy. Polarization is measured by measuring the powers $\langle E_x E_x^x \rangle$ $\langle E_y E_y^x \rangle$ and the correlation factor $\langle E_x E_y^x \rangle$, or in some equivalent way. In the case of a paraboloidal radio telescope, the complex signals E_x and E_y represent two orthogonal components received at the focus. Suppose the x and y axes are chosen as the E and H planes of the focal feed horn, so that the horn measures E_x. The measurement of E_y requires that the feed horn effectively or actually be turned through 90°. Now consider the cross section of the antenna beam on the celestial sphere. Ideally we would like it to be circular, but if the E and H patterns of the horn are not equal it is clear that the beam will not have this circular symmetry. Consequently, when the horn is turned through 90° to measure E_y, the telescope sees a different area of the sky. For example in an extreme case it might happen that a randomly polarized source was picked up when the horn was oriented to measure E_x but not when it was oriented to measure E_y. The "measured polarization" of the randomly polarized source would then be 100% linear in the E_x direction. It will be seen from this example that precise uniformity of the power pattern around the axis is essential in polarization measurements of the celestial source distribution.

Thus if E_θ and E_ϕ represent the field transmitted by the horn, θ and ϕ being the usual spherical coordinates for colatitude and longitude, we want to make $|E_\theta|^2 + |E_\phi|^2$ independent of ϕ. Starting from this requirement it will be shown that the walls of the horn have to be such that the boundary conditions on $\underset{\sim}{E}$ are the same as those on $\underset{\sim}{H}$. This condition can be synthesised in practice over a limited band by using a circular waveguide and horn with a ridged surface, as described in the accompanying paper by Minnett and Thomas. To see how polarization measurements would be made with such a horn suppose we measure E_x by placing an electric dipole parallel to the x axis in the waveguide. We can measure E_y by replacing the electric dipole with a magnetic dipole also along the x axis. Since the entire structure affects E and H in the same way it is clear that $\underset{\sim}{E}$ in the first measurement is equal to $\underset{\sim}{H}$ in the second. The power patterns in the two measurements are therefore equal, and so we have a true measure of polarization.

THEORY

At the outset it is clear that if we want the power pattern to be independent of ϕ, we need an antenna which is also independent of ϕ: in other words a circular waveguide and circular horn at the focus of a circular parabolid. In practice the cross section of the waveguide is chosen to cut off all except the modes which vary as $\exp(\pm j\phi)$. Therefore the field radiated by the horn can be expressed as

$$\underset{\sim}{E} = \underset{\sim}{a} \exp(j\phi) + \underset{\sim}{b} \exp(-j\phi) \tag{1}$$

where $\underset{\sim}{a}$ and $\underset{\sim}{b}$ depend on θ but are independent of ϕ. The linearly polarized modes in the waveguide are represented by the functions $\cos\phi$ and $\sin\phi$: they consist of equal amounts of the $\exp(\pm j\phi)$ modes. Thus the linearly polarized patterns of the circular horn are represented by Eq. (1) when $\underset{\sim}{a}$ and $\underset{\sim}{b}$ represent equal powers. The condition for this is

$$\underset{\sim}{a} \cdot \underset{\sim}{a}^{x} = |\underset{\sim}{a}|^{2} = |\underset{\sim}{b}|^{2} = \underset{\sim}{b} \cdot \underset{\sim}{b}^{x} \tag{2}$$

We see from (1) that $\underset{\sim}{a}$ and $\underset{\sim}{b}$ represent the electric fields for the $\exp(\pm j\phi)$ modes. Because the structure is independent of ϕ it is reasonably clear that, apart from a constant, $\underset{\sim}{a}$ and $\underset{\sim}{b}$ will be the same functions of θ except that the phase of b_{ϕ} relative to b_{θ} will differ from that for $\underset{\sim}{a}$ by 180^{o} i.e.

$$\frac{a_{\phi}}{a_{\theta}} = -\frac{b_{\phi}}{b_{\theta}} \tag{3}$$

This can be derived rigorously by expressing $\underset{\sim}{a}$ and $\underset{\sim}{b}$ as general combinations of TE and TM fields. Thus the TE part of $\underset{\sim}{a} \exp j\phi$ is expressed in terms of a scalar $f(\theta) \exp j\phi$ and that of $\underset{\sim}{b} \exp(-j\phi)$ by $C\, f(\theta) \exp(-j\phi)$, C being constant. The standard formulas then show that (3) applies to the TE parts. Similarly for the TM parts. So (3) holds in the general case.

Combining (2) and (3) we see that

$$|a_\theta|^2 = |b_\theta|^2 \tag{4}$$

or

$$a_\theta = b_\theta \exp j\psi' .$$

On referring to the definition (1) of $\underset{\sim}{a}$ and $\underset{\sim}{b}$ we see that the position of the plane $\phi = 0$ can be chosen to make $\psi' = 0$. Thus

$$a_\theta = b_\theta \quad \text{and} \quad a_\phi = -b_\phi \tag{5}$$

Substitution in (1) gives

$$E_\theta = a_\theta \cos \phi \tag{6}$$

$$E_\phi = j\, a_\phi \sin \phi \tag{7}$$

These, then, are the general formulas for the linearly polarized case with a_θ and a_ϕ independent of ϕ.

We now put in the requirement that the power pattern be independent of ϕ: $|E_\theta|^2 + |E_\phi|^2$ must be independent of ϕ. Substitution from (6) and (7) shows that this requires

$$|a_\theta|^2 = |a_\phi|^2 \; . \tag{8}$$

In words: - for the exp $j\phi$ excitation, E_θ and E_ϕ must have the same magnitude for all θ.

Now $\underset{\sim}{a}$ represents a certain state of elliptical polarization. From (8) we have

$$a_\theta = a_\phi \exp j\psi \; .$$

It will be found that this means the polarization ellipse is inclined at 45° to the θ ϕ directions or, in case $\psi = 90^\circ$, the polarization is circular. It will also be found that the polarization on axis $\theta = 0$ is circular in any case, as is well-known for the exp $j\phi$ excitation. It seems reasonably clear that the first case, that of elliptical polarization inclined at 45° to the coordinate directions, can be ruled out as impractical. Actually it could be generated by structures like the "sheath helix" i.e., cylindrical structures which are independent of ϕ but have anisotropically transparent walls. But such structures are not useful as feeds for parabolic antennas and a more general consideration of the focussing problem does indeed rule out the elliptically polarized case. Therefore the far field of the horn must be circularly polarized everywhere when excited in the $\exp j\phi$ mode.

The properties of such fields have already been worked out: they are basic to the theory of frequency independent antennas.[1] They are such that the field vectors $\underline{E}$ and $\underline{H}$ are, apart from a constant, the same everywhere, in the near field as well as at infinity. Specifically

$$\underline{E} = \pm j\, Z_o\, \underline{H} \tag{9}$$

with Z_o the intrinsic impedance of space (377 ohms approximately). Any solution of Maxwell's equations can be expressed as a combination of the two types. The elementary source for such fields is the combination of an electric and a magnetic dipole aligned in the same direction at the same point with moments related by the factor $j\, Z_o$. Any field of type (9) can be expressed in terms of a single scalar function f by the formula

$$\pm j\, Z_o\, \underline{H} = \underline{E} = \nabla\left(\frac{\partial f}{\partial z}\right) + k^2\, \underline{\hat{z}}\, f \pm k\, \nabla \times \underline{\hat{z}}\, f \tag{10}$$

with $$k^2 = \omega^2 \mu\, \epsilon = \left(\frac{2\pi}{\lambda}\right)^2 \quad \text{and} \quad Z_o^2 = \frac{\mu}{\epsilon} \tag{11}$$

$\underline{\hat{z}}$ = unit vector in fixed direction.

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[1] V. H. Rumsey, "A new way of solving Maxwell's equations," IRE Trans. on Antennas and Propagation, AP-9, No. 5, pp. 461-465, Sept. 1961.

The first two terms can be interpreted as the electric dipole component and the last as the magnetic dipole component.

We see from (9) that such fields are supported by any structure which has the same effect on $\underline{E}$ as on $\underline{H}$. An ordinary metal surface does not have this effect: it forces the normal component of $\underline{H}$ to zero but the normal component of $\underline{E}$ is then maximum. Thus a synthetic surface is needed. Consider a grooved metal surface in the z x plane, the width of the grooves being equal to the width of the metal ridge between them. When the width of the groove, denoted by a, is infinitesimal compared to the wavelength λ, the field within the groove must have $\underline{E}$ perpendicular to the sides of the groove. In other words, if the grooves are parallel to the z axis, E_x is the only component of $\underline{E}$, and it is independent of x. Consequently H_y and H_z are the only components of $\underline{H}$ in the groove and they depend only on y and z. If the variation with z is as $e^{jkz \cos \psi''}$ where ψ is a fixed parameter, the variation with y is as $e^{\pm jky \sin \psi''}$. It follows that if the depth of groove, d, fits $4d \sin \psi'' = \lambda$, $H_z = 0$ at the top of the groove. Also $E_z = 0$ at the edge of the groove, so as $a \to 0$, E_z and H_z vanish at the surface formed by the edges of the sheets which form the sides of the grooves. Thus the effect of this surface on $\underline{E}$ is the same as on $\underline{H}$ for all incident fields with the same ψ'', i.e. the same variation along the grooves. This applies to a circular waveguide with the grooves cut perpendicular to the axis for all waveguide modes with a common $e^{j\phi}$ variation, but not if the grooves are parallel to the axis. The formula for d in this case is, however, different. It is expressed in terms of Bessel functions J_1 and N_1: -

$$J_1^1\left(k\left[b - d\right]\right) N_1(kb) = N_1^1\left(k\left[b - d\right]\right) J_1(kb) \tag{12}$$

where b is the radius of the bottom of the grooves. Thus a circular waveguide and horn fabricated of such a synthetic material will support the kind of field required to give a power pattern independent of ϕ with linear polarization.

The tests described by Minnett and Thomas indicate excellent performance from such a model but a considerable amount of work remains to be done to determine the bandwidth and effect of groove width.

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