SELF-DUALITY AND CONSTANT RESISTANCE ONE-PORTS

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Using his concept of system function, Zadeh has shown that every self-dual one-port made of linear time-varying elements is a constant resistance one-port [1]. We have recently given instances of constant resistance one-ports which have nonlinear, time-varying elements, and some of them are self-dual one-ports [2, 3]. In this letter we give precise conditions for the truth of the statement "every self-dual one-port is constant resistance." This proposition has recently acquired more importance since wide classes of self-dual one-ports can easily be generated [4].

When we consider the one-port $\mathfrak{N}$ as a "constant resistance one-port" we only allow $\mathfrak{N}$ to be connected to one-ports $\mathfrak{N}'$ such that, with $\mathfrak{N}$ in the zero-state at time t_o, and $\mathfrak{N}$ connected to $\mathfrak{N}'$ at t_o, the connection $\mathfrak{N}$ - $\mathfrak{N}'$ is determinate, i.e., the port voltage $v(\cdot)$ and the port current $i(\cdot)$ are uniquely determined. Such one-ports $\mathfrak{N}'$ are said to be <u>compatible</u> with $\mathfrak{N}$. If all such connections have the property that, for all t_o, the port voltage $v(\cdot)$ (of $\mathfrak{N}$) is equal to its port current $i(\cdot)$ on $[t_o, \infty)$, we say that $\mathcal{N}$ is constant resistance. Except for a scale factor, this definition includes the case where for all such connections, $v(\cdot) = ki(\cdot)$, where k is a fixed nonzero real number independent of $i(\cdot)$, $v(\cdot)$ and t. We want now to prove the

Theorem: If a one-port $\mathcal{N}$ is self-dual and if, for all $e(\cdot)$, the series connection (shown in Fig. 1a) of $\mathcal{N}$, a one-ohm resistor and the voltage source e is zero-state determinate, then $\mathcal{N}$ is constant resistance. Note that no assumption need be made concerning the nature of the elements of $\mathcal{N}$: they may be linear or nonlinear, lumped or distributed, active or passive, time-invariant or time-varying.

Proof: In the following, we assume that $\mathcal{N}$ is in the zero-state at time t_o and that all connections are soldered at t_o. Fig. 1a shows the series connection of the one-port $\mathcal{N}$, the voltage source e and a one-ohm resistor. By assumption, the waveforms $v(\cdot)$ and $i(\cdot)$ are uniquely determined by the waveform $e(\cdot)$. Figure 1b shows the dual of the circuit of Fig. 1a: since $\mathcal{N}$ is self-dual, Fig. 1b shows the parallel connection of the one-port $\mathcal{N}$, the current source $\hat{j} = e$ and a one-ohm resistor; let $\hat{v}$ and $\hat{i}$ be the port voltage and port current, respectively. By duality

$$v = \hat{i} \quad \text{and} \quad i = \hat{v}. \tag{1}$$

Let us replace the source resistor combination of Fig. 1b by its Thévenin equivalent; the resulting circuit is shown in Fig. 1c. By the Thévenin theorem and by the determinateness assumption, the port voltage and the port current remain the same: $\hat{v}$ and $\hat{i}$, respectively. Noting that the circuit of Fig. 1c is identical with that of Fig. 1a, and noting that both pairs (v, i) and $(\hat{v}, \hat{i})$ are uniquely determined by e, we conclude that

$$v = \hat{v} \quad \text{and} \quad i = \hat{i}. \tag{2}$$

Combining (1) and (2), we get

$$v = \hat{v} = i = \hat{i}.$$

Thus the one-port $\mathfrak{N}$ is zero-state equivalent to a one-ohm resistor when it is driven by any voltage source in series with a one-ohm resistor. By a previous argument, [3], it follows that the one-port $\mathfrak{N}$ is constant resistance in the sense defined above.

Remark: If we examine carefully the proof above, we see that the Thévenin theorem is not necessary; we observe that the zero-state response of $\mathfrak{N}$ to a voltage v is $i = \mathcal{H}(v)$ where $\mathcal{H}(\cdot)$ expresses a relation (rather than a single valued function) in some linear function space of allowed waveforms. Indeed the only assumption needed is that i and v are uniquely defined when $\mathfrak{N}$ is driven by a series connection of a voltage source and a one-ohm resistor. For the circuit of Fig. 1a, $i = \mathcal{H}(v)$ and, by KVL, $e = v + i$, hence

$$e = v + \mathcal{H}(v). \tag{3}$$

The assumption implies that for all allowed waveforms e, this equation has a unique solution v. Now, by duality, for Fig. 1b, $\hat{i} = v$ and $\hat{v} = i$: furthermore, by self-duality $\hat{i} = \mathcal{H}(\hat{v})$. By KCL applied to Fig. 1b, $\hat{j} = e = \hat{v} + \hat{i}$, hence

$$e = \hat{v} + \mathcal{H}(\hat{v}). \tag{4}$$

Since for all allowed e this equation (which is identical with (3)) has a unique solution, we get $v = \hat{v}$. Thus, again $i = v$.

Finally it should be pointed out that the assumption of the theorem has already appeared in the study of linear networks; in particular, postulate P4 by Youla et al. [5] and under the name of solvable networks in Spaulding and Newcomb [6].

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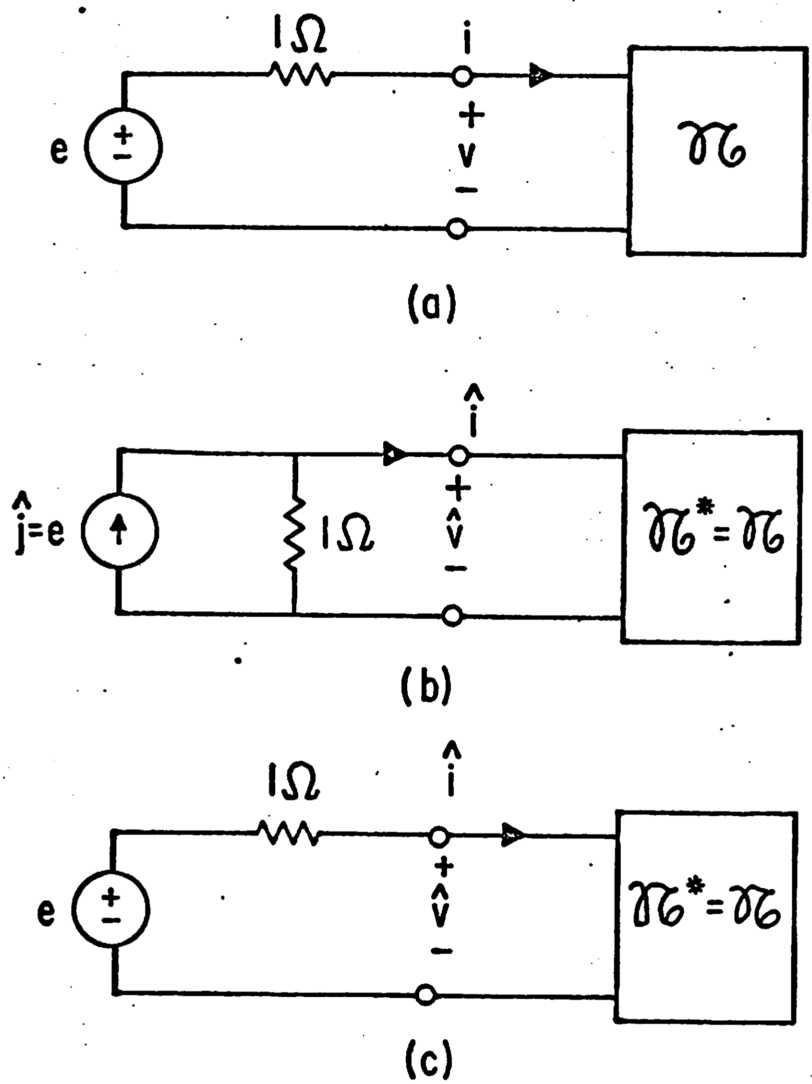

Figure 1:

(a) shows the network under consideration.

(b) is the dual of (a) and $\mathcal{N}^*$, the dual of $\mathcal{N}$ is identical to $\mathcal{N}$ by assumption.

(c) is obtained from (b) by Thévenin's theorem.