NON-ADIABATIC AND STOCHASTIC MECHANISMS FOR CYCLOTRON RESONANCE TRAPPING AND HEATING IN MIRROR GEOMETRIES

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Non-adiabatic and Stochastic Mechanisms for Cyclotron Resonance Trapping and Heating in Mirror Geometries

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ABSTRACT

Numerical orbit calculations indicate very efficient trapping of injected particles by an r.f. field with a frequency corresponding to the cyclotron frequency within a magnetic mirror. For an initially isotropic velocity distribution the orbit theory indicates that the trapping process is non-adiabatic and consequently that the particles will remain trapped for very long periods of time and be stochastically heated to high energies. The orbit theory also predicts a transition to adiabatic behavior for large initial velocity anisotropies, in agreement with results from Hamiltonian perturbation theory, explaining the inability to heat highly anisotropic plasmas in this manner. A statistical theory has been developed to calculate both the degree of heating and the trapping times. The predicted

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distribution function is Gaussian in energy rather than Maxwellian. Experiments on r.f. cyclotron resonance capture and heating have been performed. Previously reported work has been extended and considerably higher temperatures (~ 100 keV at peak compression of 45 kG) have been found. A clear transition between the Gaussian and Maxwellian energy distribution is found, as predicted by the theory. The absolute magnitude of the stochastic heating (~ 8 keV at the end of the heating pulse at cyclotron resonance of 3.7 keV) is in reasonable agreement with the theoretical prediction. The number of trapped particles is found to be greater than 10^{12}.

I. INTRODUCTION

For static magnetic-mirror confining fields, for which the magnetic moment $\mu = \frac{1}{2} m v_{\perp}^{2}/B$ is a constant of the motion, a particle entering a mirror field (within the loss cone) will leave it after a single transit. This short containment time is not useful, and consequently various schemes have been proposed for introducing a perturbation in the magnetic field which results in variations of the magnetic moment. For example, a small oscillation of the magnetic field of the form $B = B_0 (1 + h \sin kz)$ can introduce large perturbations in the magnetic moment for particles with a resonant parallel velocity $v_{\parallel} = \omega_c/k$, where $\omega_c = \frac{e B_0}{m}$ is the cyclotron frequency in the absence of the perturbation. This system has been investigated by Sinelnikov et al (1960), and by Laing and Robson (1961), by Dreicer et al (1962), and by

Dunnet et al (1965). Such a perturbation will lead to resonant loss of contained particles, but not severly on the first reflection because the interchange of energy between $v_{\parallel}^2$ and $v_{\perp}^2$ initially shifts the particle from resonance. If the perturbation is helical, rather than azimuthally symmetric, the initial rate of trapping versus loss is more favorable, since particles traveling in the reverse direction are not in resonance. The helical perturbation was proposed by Wingerson (1961), analyzed theoretically by Wingerson, Dupree and Rose (1964), and experimentally by Demirkhanov et al (1964). The experimental results agree qualitively with the theoretical predictions, but are generally less optimistic. Demerkhanov et al (1964) found that a helical perturbing field $B \sim 0.25$ Gauss in an approximately 100 Gauss guide field could, on the average, convert 40 percent of the longitudinal energy to transverse energy. Measurements of the trapping time showed that approximately 45 percent of the particles were trapped in a long-lived component of about $N = 50$ longitudinal transits, which is in reasonable agreement with $N \cong 100$ calculated from the theory. Liouville's theorem predicts that for a mirror magnetic field the maximum ratio of trapping time to the transit time is equal to the ratio of the total momentum space available to the particles, to the momentum space of the loss cone. A more efficient trapping mechanism leads to more rapid charge buildup, but eventually to a more rapid loss. This situation has been treated statistically by Robson and Taylor (1965), and found to agree with the results of the numerical calculations and experiments.

It is also possible to trap particles in a time varying magnetic field. If the magnetic field increases with time, $v_\perp$ increases faster than $v_\parallel$. Thus some particles will be stably trapped in a rising field if they initially enter the field at the edge of the loss cone. However, if the adiabatic condition is well satisfied, the change in $v_\perp$ during one longitudinal transit is small, and consequently the percentage of trapped particles is also small. Since the process of injection into an increasing magnetic field is necessarily a transient one it is difficult to trap large particle densities. If the initial trapping field is small enough, or the rate of increase of the field large enough, there is additional trapping arising from the non-conservation of magnetic moment. Trapping in this transition region is the mainstay of injection into mirror compression experiments. If, for example, we take $(1/\omega_c B)(dB/dt) = 1/10$ to be the criterion for the onset of non-adiabatic effects, we can calculate the value of dB/dt at a given B, or alternatively the value of B at a given dB/dt, which satisfies the criterion, and compare with the results of trapping in various experiments. If we take for hot electron experiments (Post, 1958), a characteristic $dB/dt = 200$ Gauss/μsec, then we find the adiabatic condition is not satisfied at $B = 10$ Gauss. At significantly higher fields (say a factor of 10) one finds that trapping efficiency does, indeed, fall off. For the low magnetic field, required for efficient trapping, energetic ions are not magnetically confined, which limits the final values of ion energies obtainable by magnetic compression.

From the discussion above, one arrives at the conclusion that efficient trapping and long containment time are mutually exclusive with a time independent static field perturbation. A time varying field, on the other hand, is not sufficiently flexible to obtain all of the desired injection characteristics. The results suggest, however, that a perturbation that can be turned on until a steady state density is reached and then turned off, can achieve simultaneously efficient trapping and long containment. A perturbation which changes the particle energy, and thus enlarges the available phase space, may also achieve both efficient trapping and long containment. In this latter case the containment is not necessarily independent of the trapping mechanism.

One perturbation with both of these trapping features is a pulse of r.f. energy at the cyclotron frequency. For injection at low energy with high r.f. power almost all phases of the r.f. field with respect to the particle lead to trapping. The r.f. can then be turned off after the injection process is complete. An additional advantage in using r.f. energy is the resultant particle heating. Besides the obviously beneficial effect of producing a hotter plasma, the heating also increases the containment time by enlarging the phase space available to the particles. In order to effectively trap and heat particles the r.f. pulse must produce either continuous or random acceleration. A time oscillating acceleration, characteristic of an r.f. interaction off resonance, will rapidly return particles to the loss cone. For magnetic mirror confined particles with small longitudinal transits Seidl (1965) has shown from perturbation theory, that a new adiabatic invariant exists which defines a unique relation between energy and the average phase of the particle with respect to the r.f. wave. The existence of the invariant leads to energy oscillations about an average value; in this case there is no continuous heating of the particles, and they also return to the loss cone within one energy oscillation. Numerical solution of the exact equations of motion by Tuma and Lichtenberg (1967) confirmed the existence of this invariant, but also showed that for large longitudinal excursions the energy changes at resonance appeared random. Their interpretation was that either the energy no longer oscillated or that the oscillation period was sufficiently long that it was not observable. In either case there would be effective trapping and heating of the plasma. In the next section we demonstrate numerically a transition from an ordered phase energy relation to a random one, allowing for both trapping and heating.

If the phase between the particle and the wave is random at each resonance crossing then the particles will be stochastically heated. The theory of stochastic heating has been developed, to treat the acceleration of particles by a turbulent wave spectrum. Stix (1964) considered the fields to be coherent for a given length of time and then to experience a random phase change. This approach led naturally to the development in terms of a random walk, in which the step length was determined directly from the coherence time. A different approach was used by Sturrock (1965), who assumed that the distribution function was described by the Fokker-Planck equation (Wang and Uhlenbeck, 1945), in which the coefficients are determined in terms of the second-order correlation functions of the spectrum. In either approach a major difficulty is in determining the appropriate spectrum of the turbulent fields. Using an approach similar to that of Sturrock, Puri (1966) considered the simpler problem of determining the acceleration of particles in an applied stochastic field with a known spectrum. In the situation treated here, the random accerating step is reasonably well known, and thus the direct method, similar to that used by Stix, is applicable. In Section II B we develop the statistical theory which is compared with the experiemntal results in Section III. In addition to predicting efficient heating, the statistical theory also indicates that particles will be contained for very long periods of time, as would be expected from the previous qualitative discussion.

II. THEORY

A. Single Particle Calculation

The simplest case of synchronous acceleration is the resonance between a circularly polarized electromagnetic wave with angular frequency ω and a particle orbiting in a constant magnetic field with the cyclotron frequency $\omega_c = \omega$. With the initial conditions $v_x = 0$ and $v_y = v_{\perp i}$ at $t = 0$, the secular solution for non-relativistic motion is

$$v_x = \frac{eE_0}{m} t \sin(\omega t + \phi) + v_{\perp i} \sin \omega t$$

$$v_y = \frac{eE_0}{m} t \cos(\omega t + \phi) + v_{\perp i} \cos \omega t \tag{1}$$

$$v_\perp^2 = v_x^2 + v_y^2$$

where the electric field is given by

$$\underline{E} = \hat{\underline{u}}_x E_0 \sin(\omega t + \phi) + \hat{\underline{u}}_y E_0 \cos(\omega t + \phi)$$

The transverse kinetic energy of the particle is, after squaring and adding velocity components,

$$W_\perp = \frac{1}{2} m v_\perp^2 = \frac{1}{2} m v_{\perp i}^2 + \frac{1}{2} m \left(\frac{E_0}{B}\right)^2 (\omega_c t)^2 + m \frac{E_0}{B} v_{\perp i} \omega_c t \cos \phi \tag{2}$$

The second and third terms give the secular energy change. The third term dominates initially leading to either an increase or a decrease in energy, depending on the phase of the particle with respect to the wave. At later times the second term dominates leading to an ultimate increase in energy. For a mirror confined particle t is replaced by a time T such that $\omega_c T/2\pi$ is the effective number of cyclotron orbits spent in the resonance region. For our experiment $(kT_e)_{\text{injected}} \approx 30$ eV, $E_0 \approx 10^5$ V/m, and from the numerical calculation (see Fig. 2) $\omega_c T \approx 100$, giving for the 2nd and 3rd terms in Eq. (2), after the initial pass through resonance, $W_\perp = 2200$ eV and $W = 230 \cos \phi$ eV respectively. We see that for the first resonance the 2nd tern dominates and the energy always increases. For subsequent resonant passes the initial transverse velocity is much larger and the third term will usually dominate, the two terms being equal (order of magnitude) when $v_{\perp i} = E_0 \omega_0 T/2B$, or, numerically $W_\perp \cong 500$ eV. We shall compare these values with numerical calculations and experiments in the subsequent sections.

In a previous paper (Tuma and Lichtenberg, 1967) we numerically solved the exact relativistic equations of motion for an electron in a magnetic mirror with the field on the axis given by

$$B_z = B_0 \left[1 - \left(\frac{R_m - 1}{R_m + 1} \right) \cos \frac{2\pi z}{L} \right]$$

where R_m is the mirror ratio. We found that for particles which do not penetrate deeply towards the magnetic mirrors, the energy oscillated in time, as predicted by the Hamiltonian perturbation theory of Seidl (1965). This implies the existence of an invariant of motion averaged over many longitudinal periods. The existence of this invariant will lead to an ordered relationship between the particle energy and its phase with respect to the electric field, at a fixed longitudinal position. We therefore have numerically calculated, $v_{\perp}/c$, the normalized velocity perpendicular to B, versus the phase difference between the particle and the r.f. electric field, at successive resonance crossings. Results for a case of little penetration and for a case of deep penetration into the mirror field, in Fig. 1a and 1b, respectively, show the difference between an ordered and a random energy-phase relation. We conclude that, provided the longitudinal particle motion penetrates deeply enough into the mirror field, (a situation which exists in the experiments described in Section III) a random phase assumption can be used to statistically calculate the trapping and energy change of particles. It should be pointed out, however, that if heating is attempted late in an adiabatic compression cycle, or on particles that have been injected at the mirror midplane with primarily transverse energy, (i.e., cases of small excursions from the midplane), the invariant exists, and only those few particles with almost <u>all</u> transverse energy can continually gain energy (Seidl, 1965).

In order to compare theory and experiment the orbits of a number of particles at various initial phases were calculated for the mirror field of Eq. 3, with the parameters corresponding to the experiment. The results are shown in Fig. 2. We see as predicted from Eq. (2), after an initial number of cyclotron periods in which the particle either gains or loses energy (corresponding to the third term in Eq. (2)) the energy always increases to a value of the order of $W_\perp = 2000\,\mathrm{eV}$. For succeeding resonance crossings there are large changes in energy with the amount and sign of the change depending on the effective phase. Off resonance the energy oscillates with an angular frequency given approximately by $[\omega_c(z) - \omega]$, as expected from theory. Because $\Delta W_\perp \gg kT_{e\parallel}$ after the initial resonance, particles are very effectively trapped. In fact, trapping is effected until $W_{\perp\,\mathrm{Resonance}}\left(\dfrac{B_{max}}{B_{resonance}} - 1\right) \lesssim (kT_{e\parallel})_{resonance}$. We demonstrate this situation for a single injection phase in Fig. 3. Here $v_\perp/c$ and the normalized cyclotron frequency are also shown. The electron just escapes from the mirror but is captured by a second mirror, which is provided for in the computer program.

B. Stochastic Treatment

We consider that the following heirarchy of energies applies

$$W_{\perp LC} \ll W_0 \ll (W_0 < W_\perp >)^{1/2} \ll < W_\perp > \tag{3}$$

where $W_{\perp LC}$ is the maximum perpendicular energy to a particle that will lie in the loss cone, $W_0 = \frac{1}{2}\, m\,(E_0\,\omega_c\,T/B)^2$ is the energy gained in the first transit after injection (2nd term in Eq. 2), $(W_0 < W_\perp >)^{1/2}$ is the average energy gained on succeeding transits (average of 3rd term in Eq. 2), and $\langle W_\perp \rangle$ is the average particle energy. We shall define these terms more exactly, below. The first and second inequalities follow from the numerical studies of the previous section. The third inequality embodies the usual statistical assumption that the evolution of the distribution function has taken place over a large number of random energy changes.

We assume that the electrons return to the resonance region with a random phase with respect to the electric field and, using the second inequality with a perpendicular velocity large enough such that the resonant change in energy depends only on the phase dependent term in Eq. (2). For this situation we have

$$\Delta W_\perp = W_{\perp n+1} - W_{\perp n} = m \frac{E_0}{B} \left(\frac{2 W_{\perp n}}{m} \right)^{1/2} \omega_c T \cos \phi \qquad (4)$$

where both $W_{\perp n}$ the perpendicular energy upon entering the resonance region, and ϕ are independent random variables. The number of effective orbits within the resonance region, $\omega_c T$, is considered constant, as suggested by the numerical calculations. Equation (4) represents a Markovian process, as the successive values of $W_\perp$ depend only on the previous values. In this situation it can be shown (Wang and Uhlenbeck, 1945) that, in the limit of small changes in energy for each transit ($\Delta W_\perp \ll W_\perp$), the probability distribution $f(W_\perp)$ is governed by the

Fokker-Planck equation

$$\frac{\partial}{\partial n} f(W_\perp) = \frac{\partial}{\partial W_\perp}[A(W_\perp) f(W_\perp)] + \frac{1}{2}\frac{\partial^2}{\partial W_\perp^2}[B(W_\perp) f(W_\perp)] \tag{5}$$

where $A(W_\perp)$ and $B(W_\perp)$ are the first and second moments of the change in energy

$$A(W_\perp) = \int \langle \Delta W_\perp \rangle_\phi f(W_\perp)\, dW_\perp, \qquad B(W_\perp) = \int \langle (\Delta W_\perp)^2 \rangle_\phi f(W_\perp)\, dW_\perp \tag{6}$$

and $\langle\ \rangle_\phi$ is an average over phase ϕ. We assume that $f(W_\perp)$ is a Gaussian

$$f(W_\perp) = \left(\frac{2}{\pi}\right)^{1/2} \frac{1}{\langle W_\perp^2 \rangle_{W_\perp}^{1/2}} \exp\left[\frac{-W_\perp^2}{2\langle W_\perp^2\rangle_{W_\perp}}\right] \tag{7}$$

where

$$\langle W_\perp^2 \rangle_{W_\perp} = \int W_\perp^2 f(W_\perp)\, dW_\perp$$

With this assumption the first and second moments of the change in energy are given, for $\Delta W_\perp \ll W_\perp$ by,

$$\langle W_{\perp\, n+1} \rangle_{\phi, W_\perp} - \langle W_{\perp n} \rangle_{\phi, W_\perp} = \langle \Delta W_\perp \rangle_{\phi, W_\perp} \tag{8}$$

and

and

$$\langle (W_{\perp n+1})^2 \rangle_{\phi, W_\perp} - \langle W_{\perp n}^2 \rangle_{\phi, W_\perp} = \langle (\Delta W_\perp)^2 \rangle_{\phi, W_\perp}$$

Since $W_\perp$ and ϕ are independent random variables $\langle W_\perp \rangle_{\phi, W_\perp} = \langle W_\perp \rangle_{W_\perp}$ and thus

$$\langle W_\perp \cos \phi \rangle_{\phi, W_\perp} = \langle W_\perp \rangle_{W_\perp} \langle \cos \phi \rangle_\phi$$

Averaging over all values of ϕ we find that the change in the average value is zero, while the change in the second moment is given approximately by

$$\langle (W_{\perp n+1})^2 \rangle_{\phi, W_\perp} - \langle W_{\perp n}^2 \rangle_{\phi, W_\perp} = m \left(\frac{E_0}{B} \omega_c T \right)^2 \langle W_{\perp n} \rangle_{W_\perp} \tag{9}$$

where we substituted for $\Delta W_\perp$ from (4) and used $\langle (\cos \phi)^2 \rangle_\phi = \frac{1}{2}$.
Replacing $\langle W_\perp^2 \rangle_{W_\perp}$ by U and determining from the definitions that

$$\langle W_\perp \rangle_{W_\perp} = (\frac{2}{\pi})^{1/2} \langle W_\perp^2 \rangle_{W_\perp}^{1/2} \tag{10}$$

then for large n Eq. (9) can be written, approximately, in differential form

$$\frac{dU}{dn} = (\frac{2}{\pi})^{1/2} m \left(\frac{E_0}{B} \omega_c T \right)^2 U^{1/2}$$

Solving for U we have

$$U^{1/2} = \langle W_\perp^2 \rangle_{W_\perp}^{1/2} = \frac{m}{(2\pi)^{1/2}} \left(\frac{E_0}{B} \omega_c T \right)^2 n \tag{11}$$

Equation (11) gives the stochastic heating which we shall compare with experiments in Section III. Since the change in the second moment in Eq. (9) is itself proportional to $(n)^{1/2}$, the evolution of $\langle W_\perp^2 \rangle^{1/2}$ is seen to be proportional to n, rather than $n^{1/2}$.

We now show that with $\langle W_\perp^2 \rangle^{1/2}$ given by (11) and $f(W_\perp)$ by (7) that (5) is satisfied. From (6), (8) and (9) we determine that,

$$A(W_\perp) = 0 \tag{12}$$

and

$$B(W_\perp) = m \left(\frac{E_0}{B} \omega_c T \right)^2 \langle W_{\perp_n} \rangle \tag{13}$$

where we have dropped the $W_\perp$ subscript on $\langle \; \rangle_{W_\perp}$. Substituting these results together with (10) into the Fokker-Planck equation we have

$$\frac{\partial}{\partial n} \left[\frac{2^{1/2}}{\pi^{1/2} \langle W_\perp^2 \rangle^{1/2}} \exp\left(- \frac{W_\perp^2}{2 \langle W_\perp^2 \rangle^{1/2}} \right) \right] = \frac{1}{2} \frac{\partial^2}{\partial W_\perp^2} \left[m \left(\frac{E_0}{B} \omega_c T \right)^2 \right.$$

$$\left. \cdot \left(\frac{2}{\pi} \right)^{1/2} \langle W_\perp^2 \rangle^{1/2} \left(\frac{2}{\pi} \right)^{1/2} \frac{1}{\langle W_\perp^2 \rangle^{1/2}} \exp\left(- \frac{W_\perp^2}{2 \langle W_\perp^2 \rangle} \right) \right]$$

which is satisfied for $<W_{\perp}^2>^{1/2}$ as given by (11). Thus we have shown that the assumption of a Gaussian distribution is consistent with the particular Markovian process we are considering, i.e., if the distribution function is initially Gaussian it will remain so. For arbitrary initial conditions the Fokker-Planck equation must be used, an approach taken by Sturrock (1966). However, the many approximations necessary to reduce the equations to tractable form leave doubt as to the increased accuracy of the method over the assumption of an initial Gaussian energy distribution. In Part III the experimental distribution of energies is found to be approximated by a Gaussian, thus indicating that the experimental time is sufficient for the distribution function to relax to the form we are considering.

Since $W_{\perp}$ takes on only positive values we must also assume a perfectly reflecting barrier at $W_{\perp} = 0$ in order to maintain the form of the energy distribution (Chandrasekhar, 1943). The actual barrier is somewhat fuzzy and includes absorption, as we have shown in II A. This mainly affects the absolute density without appreciably affecting the form of $f(W_{\perp})$ as a consequence of the first inequality in (3). The equation that describes the decay of the particles trapped during a single transit is

$$\frac{d\eta_j}{dn} = -\eta_{\ell j} \tag{14}$$

where η_j is the number of contained particles remaining from the j th injection and $\eta_{\ell j}$ is the number of particles lost from that class per transit. The total number of trapped particles after n transits with continuous injection is then

$$N = \sum_{j=1}^{n} \eta_j = \int_1^n \eta(j)\, dj \tag{15}$$

where we have replaced the sum with an integral for large n. In order to solve (14) we must obtain a relation for η_ℓ, which we can write approximately in the form

$$\eta_\ell = \int_{W_{\perp LC}}^{W_0} \eta f(W_\perp) \frac{\Delta\phi}{2\pi}\, dW_\perp \tag{16}$$

where $\Delta\phi/2\pi$ is the fraction of the r.f. phases that take a particle from $W_\perp$ into the loss cone, and the limits of integration are those values of $W_\perp$ for which $\Delta\phi/2\pi$ exists. Ignoring the second term in (2) we find $\Delta\phi$ from the relations

$$v_{\perp f}^2 = v_\perp^2 + \frac{E_0}{B} v_\perp \omega_c T \cos\phi \tag{17}$$

and

$$0 < v_{\perp f} < v_{\perp LC} \tag{18}$$

where $v_{\perp LC}$ is the maximum $v_\perp$ in the loss cone. Finding $\cos\phi$ from (17) at the limits of $v_{\perp f}$ in (18), and expanding for small $\Delta\phi$ we obtain

$$\Delta\phi = \frac{v^2_{\perp LC}}{v_\perp (v_0^2 - v_\perp^2)^{1/2}} = \frac{W_{\perp LC}}{W_\perp^{1/2}(W_0 - W_\perp)^{1/2}} \tag{19}$$

Substituting Eqs. (19) and (7) in (16) we obtain

$$\eta_\ell = \int_{W_{\perp LC}}^{W_0} \frac{\eta}{2^{1/2}\pi^{3/2}\langle W_\perp^2\rangle^{1/2}} \, e^{-\frac{W_\perp^2}{2\langle W_\perp^2\rangle}} \frac{W_{\perp LC} \, dW_\perp}{W_\perp^{1/2}(W_0 - W_\perp)^{1/2}} \tag{20}$$

Using the inequalities of (3), i.e., by neglecting terms of the order $\frac{W_0}{\langle W_\perp^2\rangle^{1/2}}$ and $\frac{W_{\perp LC}}{W_0}$ compared to unity the integral can be evaluated. The final result is,

$$\eta_\ell = \frac{\eta}{2^{1/2}\pi^{1/2}} \frac{W_{\perp LC}}{\langle W_\perp^2\rangle^{1/2}} \tag{21}$$

and substituing (16) and (6) into (9) we have

$$\frac{d\eta}{dn} = -\frac{1}{2} \frac{W_{\perp LC}}{W_0} \frac{\eta}{n} \tag{22}$$

Integrating Eq. (22) we obtain the number of particles remaining, from those injected during a single transit, after n additional transits of the resonance region

$$\eta = \eta_0 n^{-C} \tag{23}$$

where $C = \frac{W_{\perp LC}}{2 W_0}$ and η_0 is the number of particles trapped in a single transit. For continuous injection the total number of trapped particles is from (15)

$$N = \frac{\eta_0}{1 - C} [n^{1-C} - 1] \tag{24}$$

For $C \ll 1$, which is generally the case for low energy injection and high r.f. fields, the decay of a single transit is slow and the buildup for continuous injection rapid. The containment time of a particle can also be calculated. If we measure the containment by the half life of particles injected during a single transit then from (23) with $\eta/\eta_0 = \frac{1}{2}$ we obtain

$$n = 2^{1/C}$$

If the time for a single transit $(n=1)$ is τ_ℓ, and substituting for C, the half life of containment is

$$\tau = \tau_\ell \, 2^{2W_0/W_{\perp LC}} \tag{25}$$

For our example $W_0/W_{\perp LC} > 10$ giving $\tau > 10^6 \tau_\ell$. Thus a particle is contained, on the average, on the order of 10^6 transits. From Fig. 3 we find that $\omega_c \tau_\ell \cong 500$ and thus for 3 cm r.f. waves which we are considering, $\tau_\ell \cong 10^{-8}$ and $\tau \cong 10^{-2}$. This is longer than the decay time of the experiment in the absence of heating and thus, if the probabilistic solutions were valid over this long time period, we might expect little change in lifetime due to application of continuous r.f. heating. Actually, the energy calculated from (11) would show that the particle would become highly relativistic, invalidating the theory. In practice, as we shall see in Section III, only a short r.f. pulse of the order of a microsecond duration is used, during which the decrease in trapped particles is negligible. For subsequent times normal collisional decay prevails. In the various approximations we have made, particularly in the simplified form of (19), we have overestimated the particle loss, and thus (23) and (24) are lower bounds on the confined density.

III. EXPERIMENT

The experimental configuration shown schematically in Fig. 4 is similar to the Table Top device (Post, 1958). The plasma is generated by a deuterated-titanium washer-stack source, which injects plasma into a rising field having a mirror ratio of 1.5:1. The field rises to its maximum midplane value (up to 100 kG) in 500 μsec and decays with a time constant of 20 msec. The high field region of the vacuum chamber (base pressure 2×10^{-7} Torr) is 10 cm diameter and 30 cm long. There is an initial, uniform dc bias magnetic field of 20 G with the same field direction as the compression field.

The experiment is usually operated with compression from an initial field somewhat in excess of 20 G to fields between 30 and 60 kG. At 50 kG the plasma electron temperature is 75 keV; the density greater than $10^{12}/cm^3$, the diameter about 5 mm, and the plasma decays slowly with a time constant of about 5 msec. For other values of magnetic field, the temperature and density of the electrons are as expected from the adiabatic compression law. The plasma electron temperature is measured from the energy spectrum of the x-ray Bremsstrahlung emitted from the plasma. The size and location of the hot electron component are determined from the photographs of a phosphor located normal to the magnetic axis. The total number of radiating particles is obtained either from the absolute calibration of the intensity of the synchrotron radiation spectrum (Sesnic, 1968), or from a cavity perturbation technique. The source of the r.f. power is a magnetron, nominally producing a 250 kW, 10 Gc (3 cm), 0.5 μsec pulse, which is transmitted into the midplane of the plasma chamber through a side port. The magnetron pulse is generally initiated slightly after source injection.

From the statistical analysis we determined that, for those particles whose excursions are large enough to be stochasticly heated, the energy distribution after r.f. heating pulse would be Gaussian rather than Maxwellian, as given by Eq. (7). In Fig. 5 we plot, on semi-log paper, the relative intensity of Bremsstrahlung x-rays versus the square of the photon energy. The radiating electrons are injected with a delay of 35 μsec after initiating the magnetic field, corresponding to cyclotron resonance within the mirror. For energies above $<W_{\perp}>$ the exponential behavior dominates, and the straight line dependence gives the predicted behavior with $2<W_{\perp}^2> = 19500\,(\text{kev})^2$ or $<W_{\perp}^2>^{1/2}\Big|_{B\simeq 45\text{kG}} \simeq 100\,\text{keV}$. In Fig. 6 the time dependence of the magnetic field is plotted at the midplane and at the magnetic mirror, indicating the range of times within which cyclotron resonance occurs somewhere within the mirror. In Fig. 7 the equivalent plasma perature $(kT_e = <W^2>^{1/2})$ is plotted vs. delay time between the r.f. pulse and initiation of the magnetic field. The temperature and density fall rapidly above 38 μsec delay. The increase in temperature at short delay times is due to adiabatic compression coupled with some non-resonant r.f. heating. We see that there is a close correspondence between resonance and resultant trapping and heating.

Coincident with the magnetron pulse the light eminating from the plasma increases. We interpret this phenomena as an increase in background ionization. The newly ionized particles can also be trapped and heated, thus increasing the total density of contained hot plasma. This result can be compared with earlier experimental results, shown in Fig. 8, in which the source is fired in the normal trapping mode, early in time (Sesnic et al, 1968). The increase in the intensity of synchrotron radiation is due to an increase in radiating electrons; x-ray pulse height analysis indicates there is no significant heating, consistent with the prediction that an anisotropic velocity distribution remains adiabatic. Most electron cyclotron heating experiments use the r.f. power both to create the plasma by ionization and then heat it, e.g. Becker et al (1962), Ferrari and Kuckes (1965) and Fessenden (1966).

Using a nominal value of $\langle W_{\perp}^{2}\rangle^{1/2}\Big|_{B=45\,kG} = 100\,keV$, and assuming adiabatic compression, the mean energy at cyclotron resonance of 3.7kG is $\langle W_{\perp}^{2}\rangle^{1/2}\Big|_{B=3.7\,kG} \cong 8\,keV$. We can compare this value of electron energy with that expected from the calculations. A typical average time between resonance, as found from Fig. 3, is $\tau_{\ell} = 500/\omega_c$ and for 3.7kG magnetic field $\tau_{\ell} \cong 10^{-8}$ sec. The effective pulse length is measured to be $\tau_p \cong 2 \times 10^{-7}$ sec. resulting in an approximate number of resonance crossings of

$$n = \tau_p/\tau_{\ell} \cong 20.$$

Substituting into Eq. (11), this value of n, an electric field $E = 10^{5}$ V/m (derived, approximately from the peak power) and $\omega_c T \approx 100$ (with $B = 3.7\,kG$) the statistical theory predicts that $\langle W_{\perp}\rangle\Big|_{B=3.7\,kG} \cong 30\,keV$.

The discrepancy of a factor of four between the measured and calculated values may be due to a number of causes. (1) The electric field strength within the plasma may be overestimated due to shielding. A calculation of the plasma frequency of the hot component at 3.7kG obtained from the measured density at peak compression gives $f_p = 4GH_z$. Additional cold plasma production can also add to the shielding, such that ω_p may be greater than ω during part of the heating pulse. Such factors as impedance mismatch, wall losses, and mode effects have also not been considered. (2) There may be higher order correlations, not taken into account by the Markovian assumption. Although no energy-phase plots gave smooth curves, for the injection parameters of the experiment, in a few calculations there did appear to be some correlation between neighboring resonance crossings. (3) The equivalent time in the resonance region may decrease as the perpendicular energy increases. This phenomenon appears physically plausible and there is some evidence for it from the computations. However, the numerical computations have not been performed over a sufficient number of resonances to unambiguously confirm the effect. It is interesting to note that the existence of this phenomenon, while limiting the heating, has the beneficial effect of counteracting diffusion into the mirror loss cone.

Despite the degradation in mean energy from what might be taken as an optimum energy transfer, the results are still very optimistic as to the energies and densities obtainable by non-adiabatic r.f. trapping and heating. In addition, ionization and concommitant trapping and heating serve to increase the final density. In Table 1 we present a comparison of the plasma characteristics achieved with the usual injection at low magnetic fields and the injection with the assistance of the r.f. pulse. Both energy and density are comparable in the two situations. However, with the trapping at the resonance field of B = 3.7 kG, it is also possible to contain hot ions within the vacuum chamber.

REFERENCES

Becker, M., Dandl, R., Eason, H., England, A. and Ard., W. (1962) Nuclear Fusion Suppl., Part I, p. 345.

Chandrasekhar, S. (1943) Rev. Mod. Phys. 15, p. 1.

Demerkhanov, R. A., Kursanov, Yu. V., Baratov, D. C. and Kharin, G. V. (1965) Soviet Phys. Tech. Phys. 9 p. 45.

Dreicer, H., Karr, H. J., Knapp, E. A., Phillips, J. A., Stoval, E. J. and Tuck, J. L. (1962) Nuc. Fus. Supplement Part I, p. 299.

Dunnett, D. A., Laing, E. W. Roberts, S. J. and Robson, A. E. (1965) Plasma Phys. (J. N. E. Pt. C) 7 p. 359.

Ferrari, L. and Kuckes, A., (1965), Phys. Fluids, 8, p. 2295.

Fessenden, T. J., (1966) M. I. T. Research Laboratory of Electronics, Tech. Rpt. 442.

Laing, E. W. and Robson, A. E. (1961) Plasma Phys. (J. N. E. C. Pt. C) 3 p. 146.

Post, R. F. (1958) Proc. 2nd Int. Conf. on the Peaceful Uses of Atomic Energy 32, p. 245.

Puri, S. (1966) Phys. Fl. 9 p. 1043.

Roberts, C. and Buchsbaum, S. (1964) Phys. Rev. 135 A 381.

Robson, A. E., and Taylor, J. B. (1965) Phys. Fl. 8 p. 2026.

Seidl, M. (1965) Plasma Physics (J. N. E. Pt. C.) 6 p. 597.

Sesnic, S., Tuma, D. T., Lichtenberg, A. J., Trivelpiece, A. W., (1968) "Emission and Absorption of Cyclotron Radiation by a Hot Electron Plasma," Phys. of Fl. (to be published).

Sinelnikov, K. D., Fedorchenko, V. D., Rutkevich, B. N., Cherni, B. M. and Safronov, B. G. (1960) Soviet Phys. Tech. Phys. 5 p. 236.

Stix, T. (1964) Phys. Fl. 7 p. 1960.

Sturrock, P. A. (1966) Phys. Rev. 1 p. 186.

Tuma, D. T., and Lichtenberg, A. J. (1967) Plasma Physics 9 p. 87.

Wang, M. C. and Uhlenbeck, G. E. (1945) Rev. Mod. Phys. 17 p. 323.

Wingerson, R. C. (1961) Phys. Rev. Letters 6 p. 446.

Wingerson, R. C., Dupree, T. H. and Rose, D. J. (1964) Phys. Fl. 7, p. 1475.

TABLE 1

	Normal Trapping	R. F. Trapping
B_f	45 kG	45 kG
B_i	30 G	3.7 kG
T_{ef}	60 keV	100 keV
T_{ei}	40 eV	8 keV
	Assuming adiabatic compression	
N_f	10^{12}	2.10^{12}
D_f	0.5 cm	0.5 cm (assumed)
n_f	$2.10^{12}/cm^3$	$4.10^{12}/cm^3$

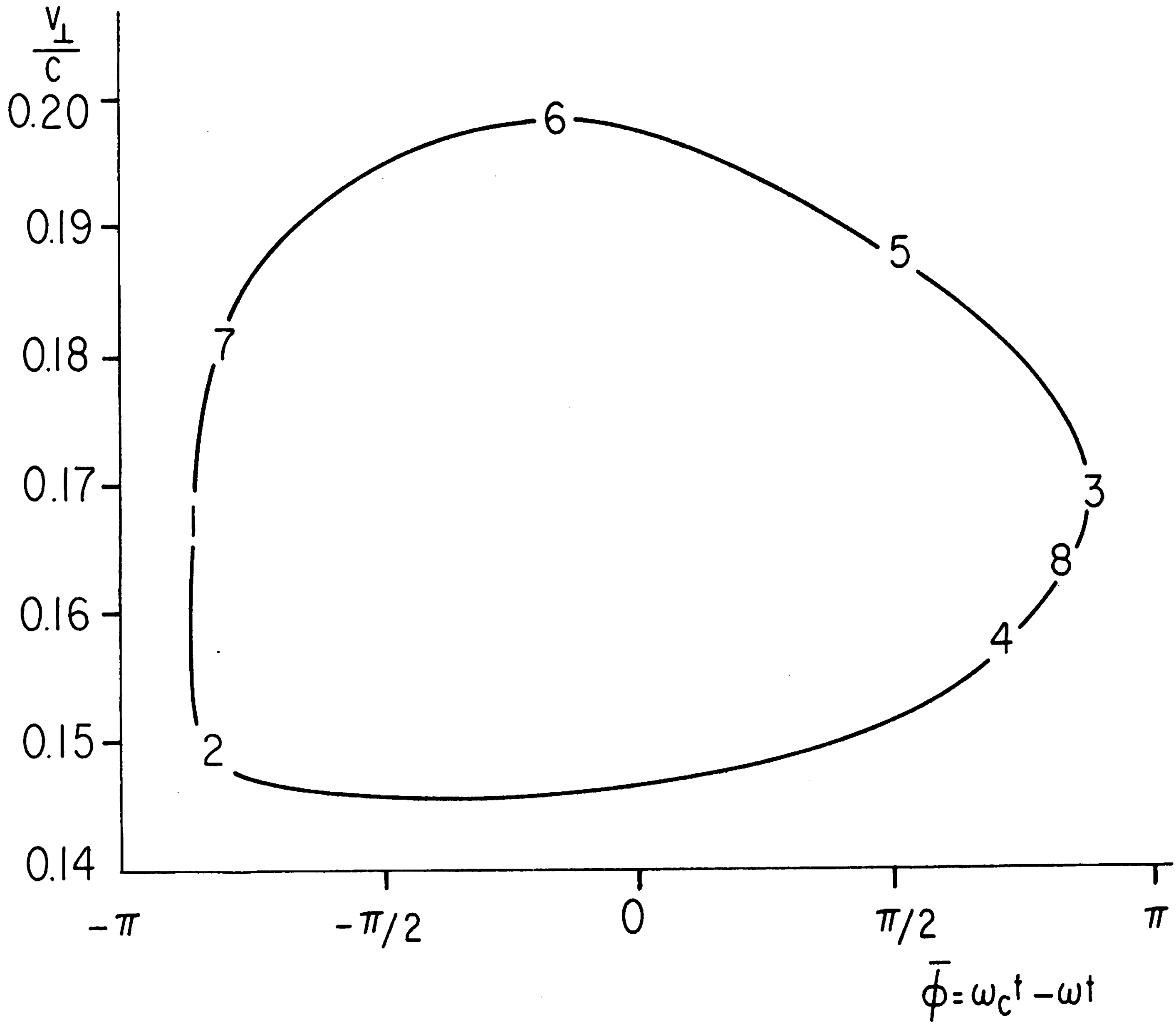

Fig. 1 (a). Energy-phase diagram for small mirror penetration in which an adiabatic invariant exists.

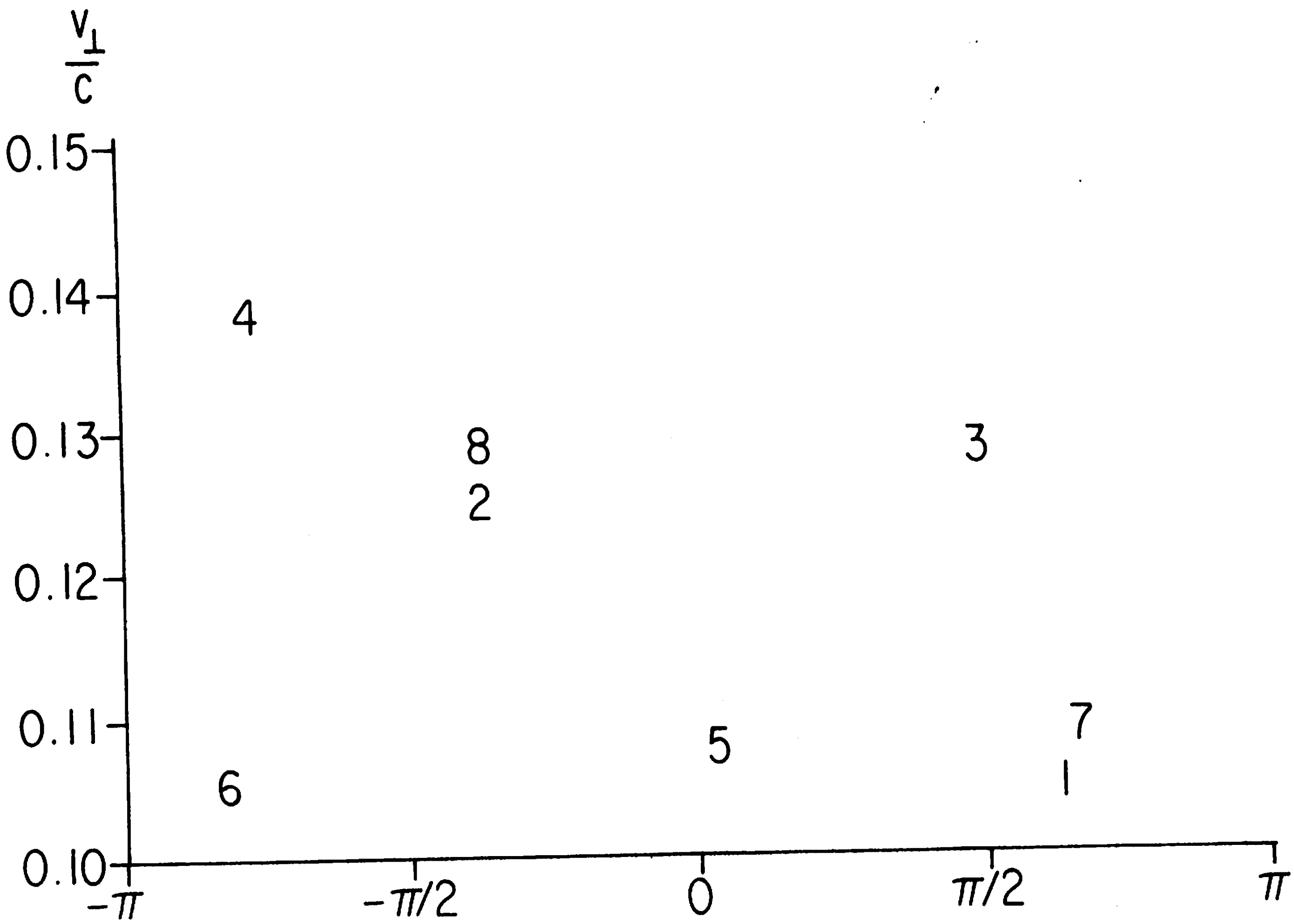

Fig. 1 (b). Energy-phase diagram for large mirror penetration in which an invariant does not exist. Numbers denote successive resonance crossings.

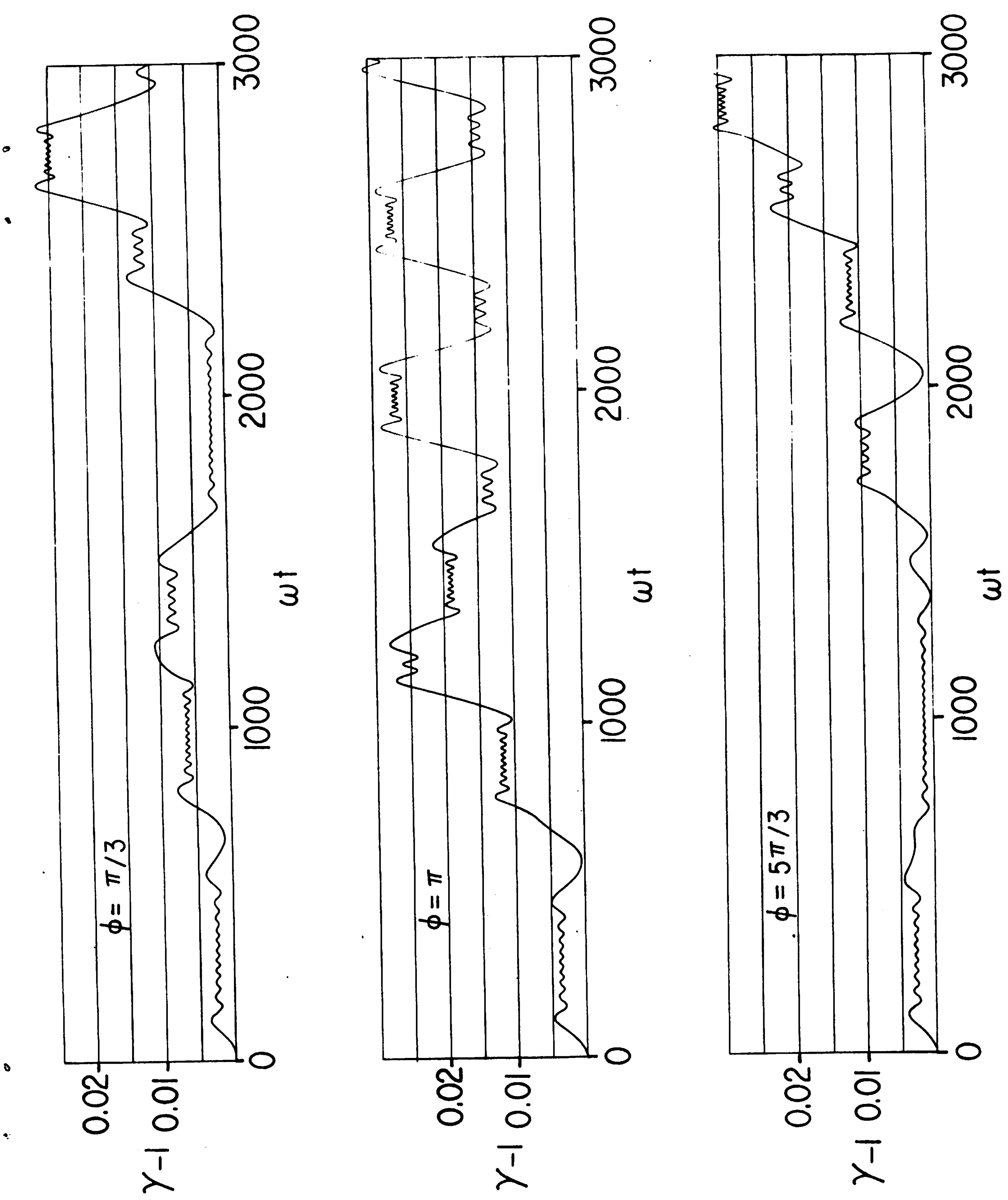

Fig. 2. Variation of energy with normalized time for injection with parameters used in the experiment. Three injection phases are shown.

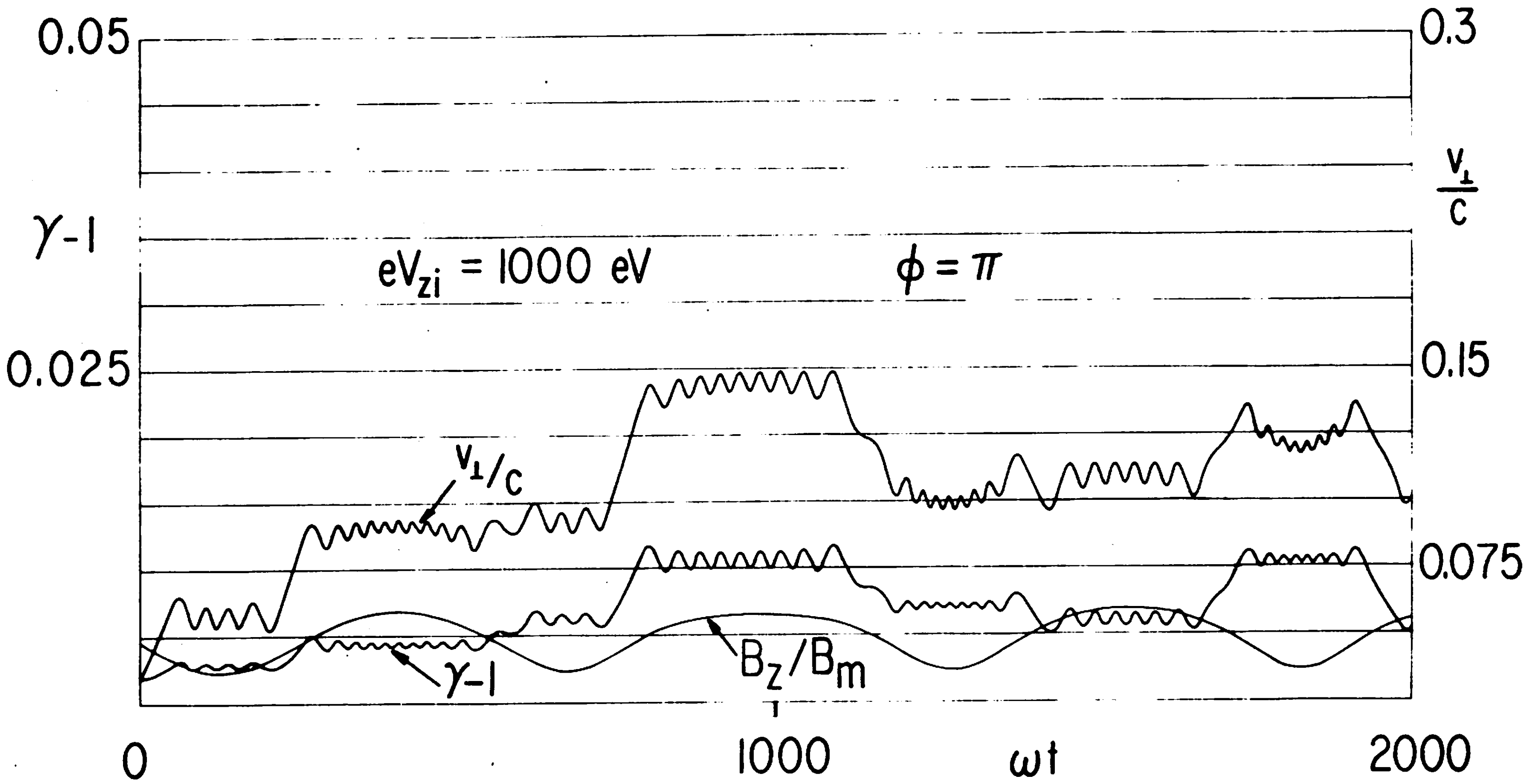

Fig. 3. Investigation of particle trapping with initial longitudinal energy of the same order as the gain in perpendicular energy during the first traversal through resonance.

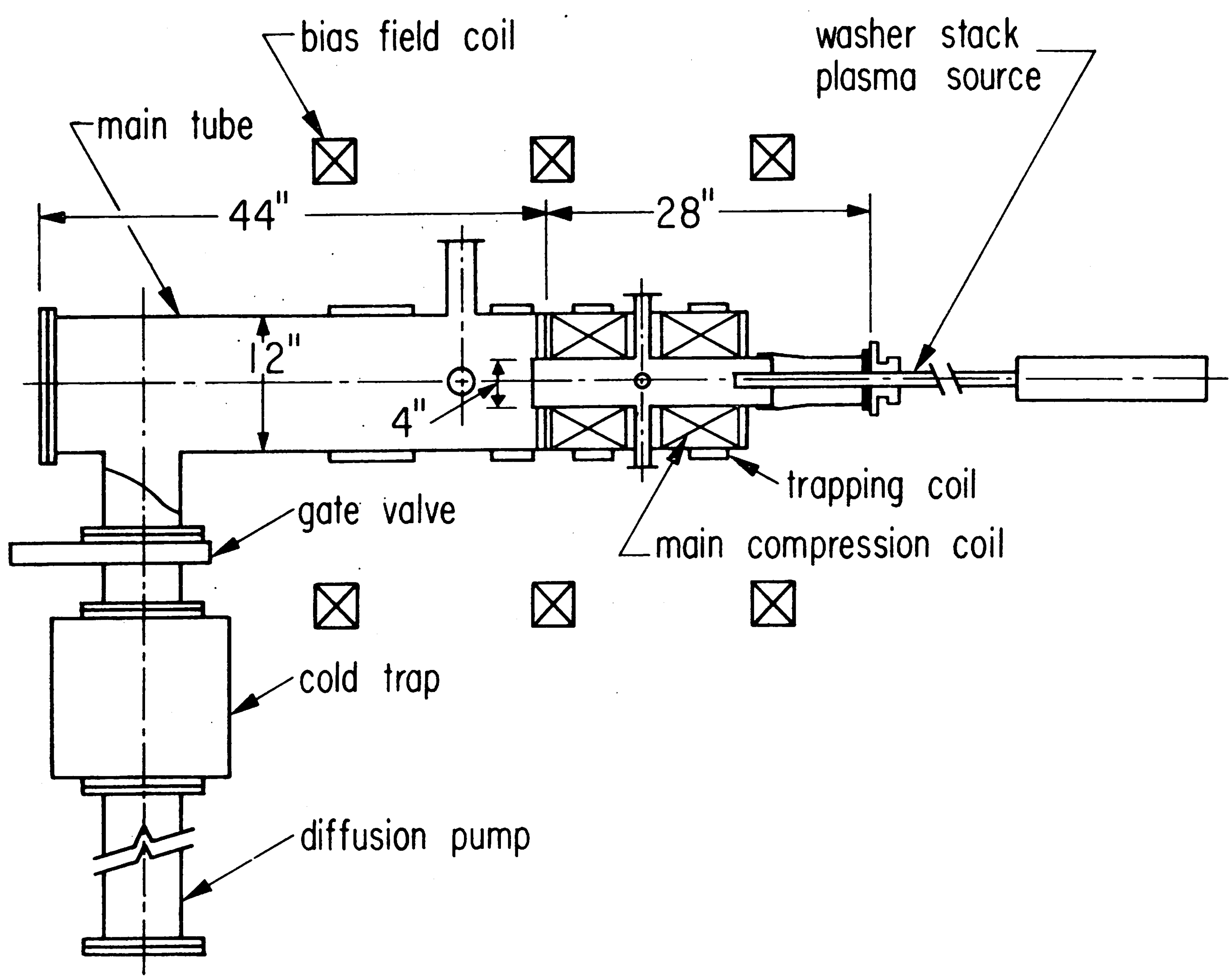

Fig. 4. Schematic diagram of mirror machine.

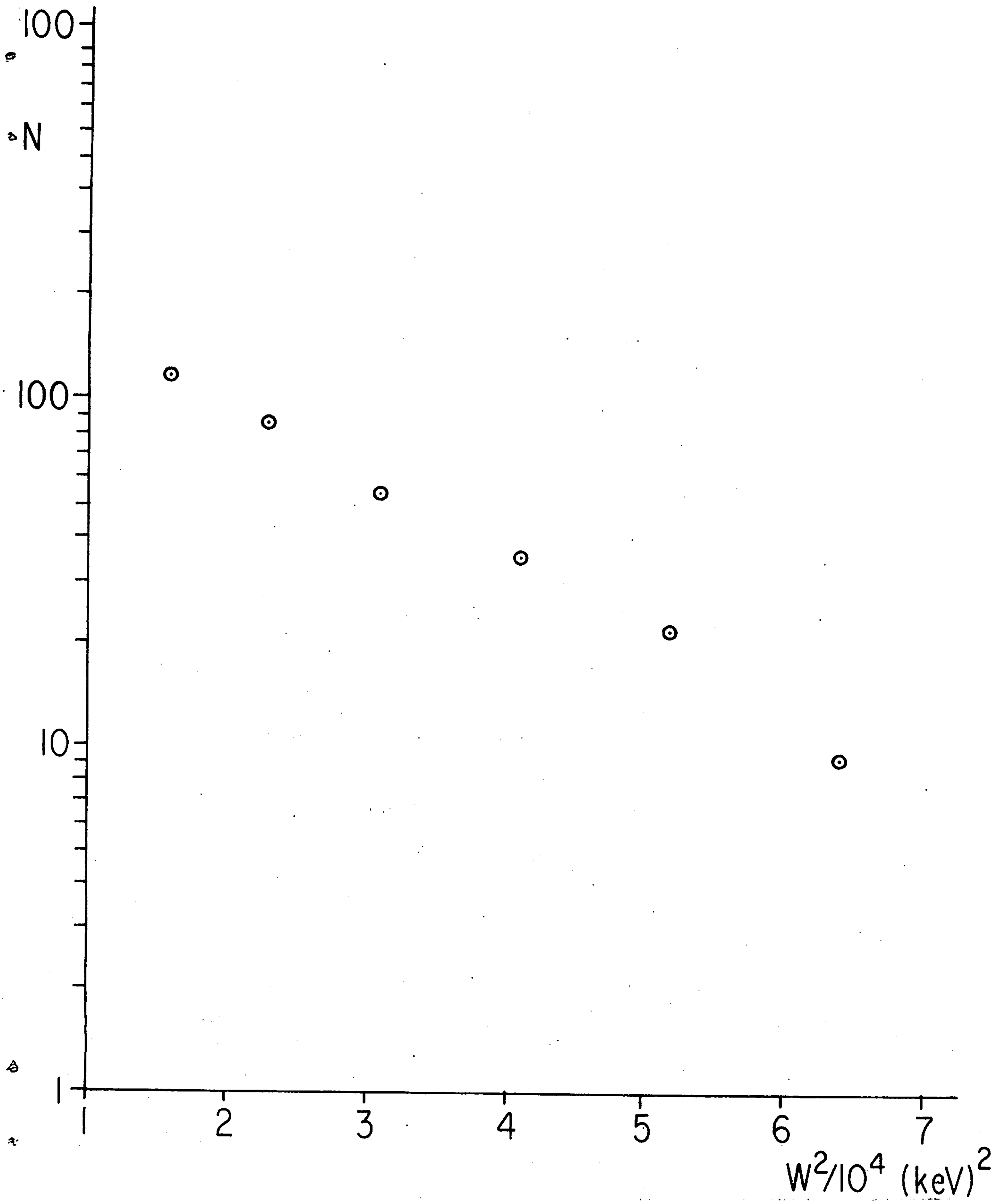

Fig. 5. X-ray pulse height distribution giving $\langle W_\perp^2 \rangle^{1/2} \cong 100\,\mathrm{keV}$.

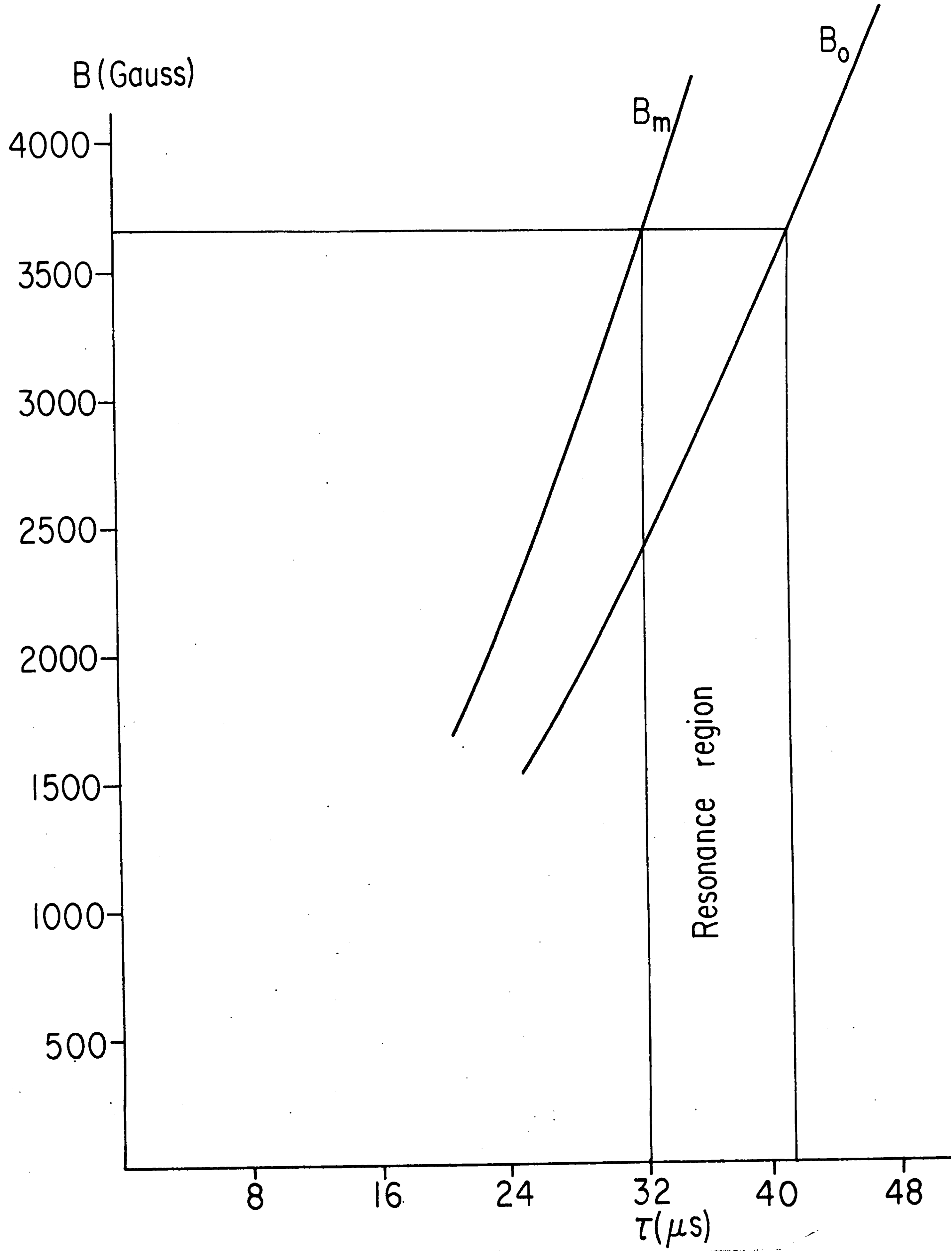

Fig. 6. Variation of magnetic field with time at midplane and at mirror, for B = 47 kG peak field, illustrating time during which there is resonance between microwave frequency and cyclotron frequency.

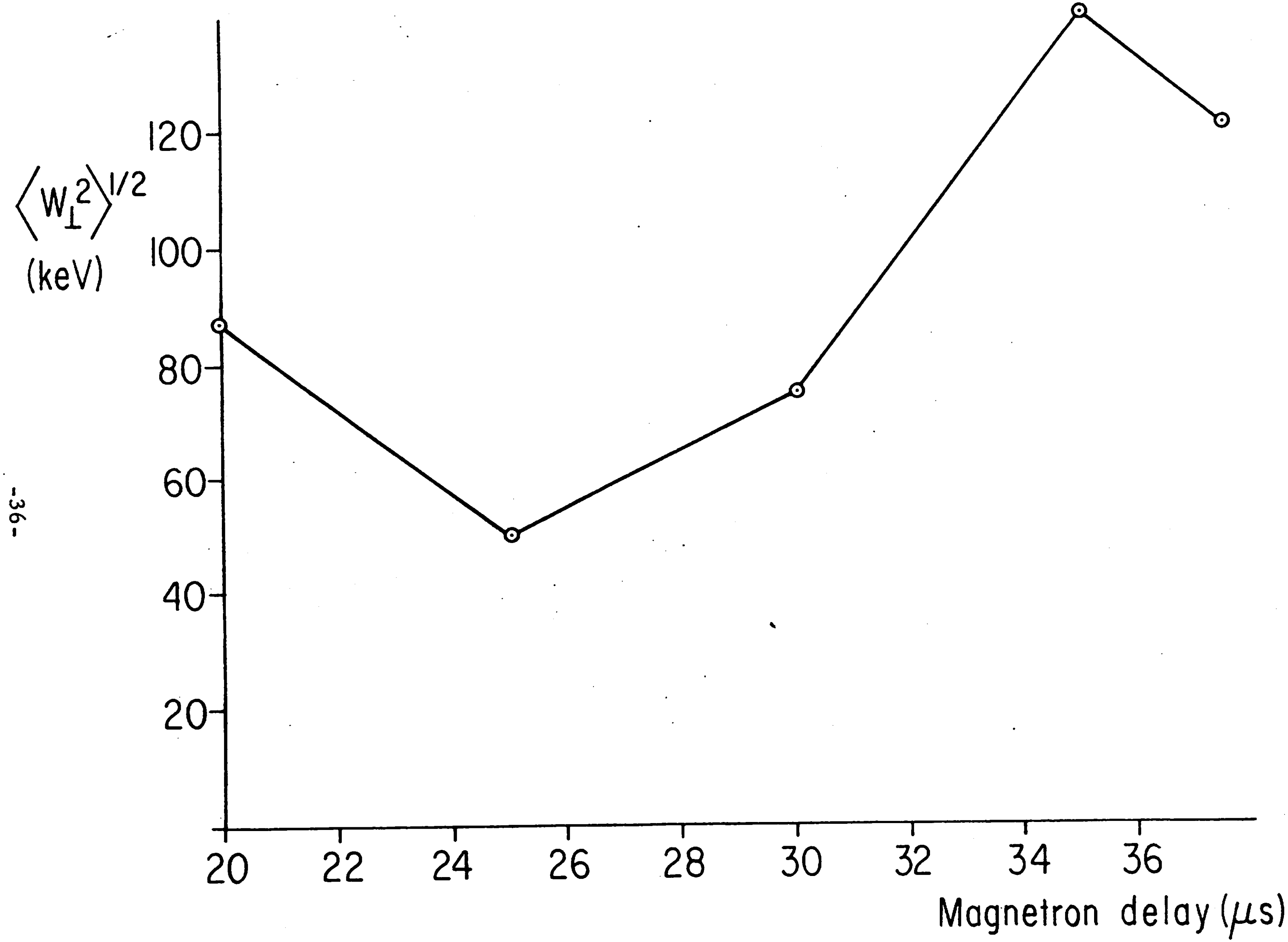

Fig. 7. Average particle energy versus delay time between initiating the magnetic field and the microwavc pulse.

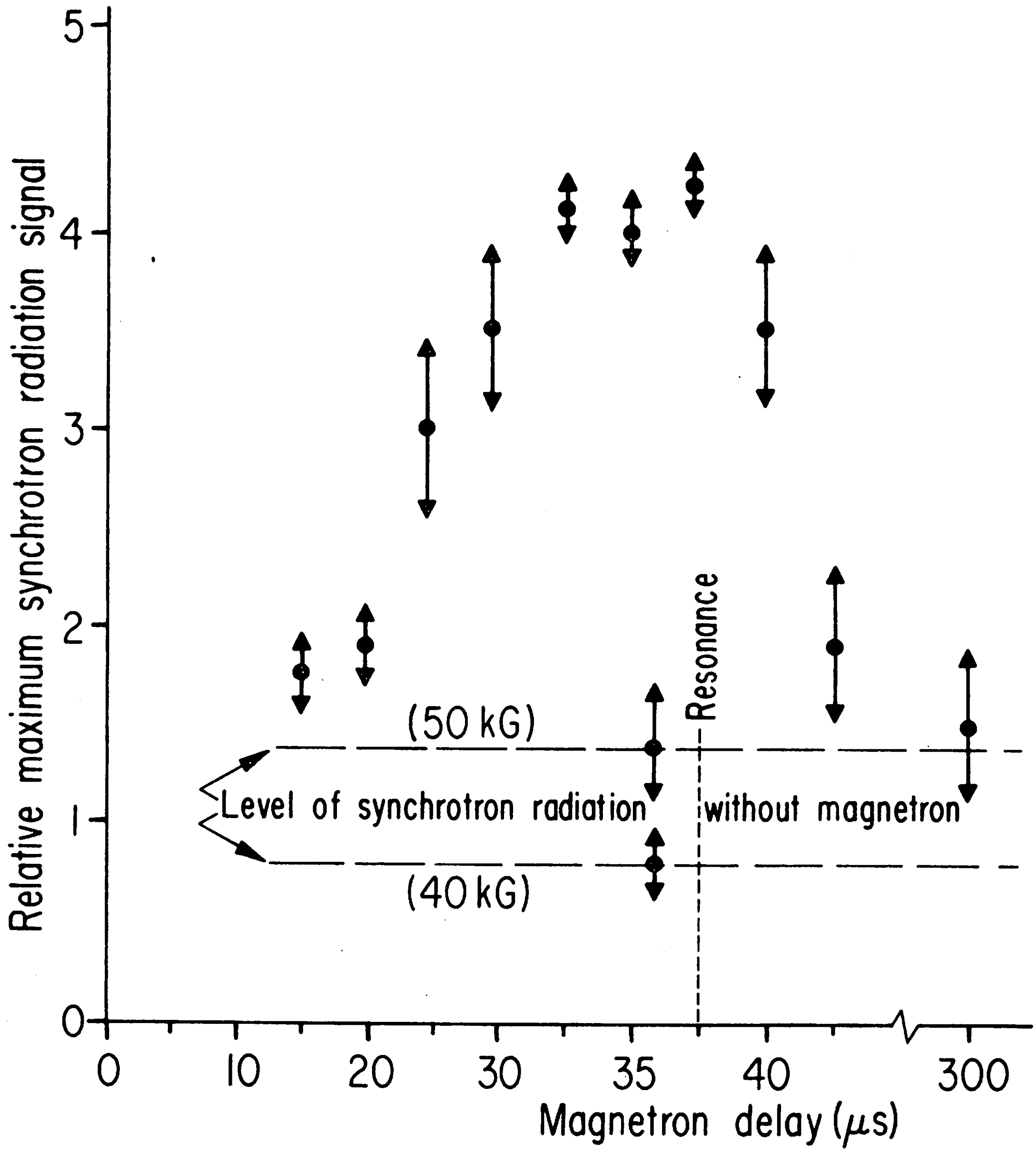

Fig. 8. Comparison of synchrotron radiation with and without application of microwave energy. The normal injection mode is used with the source fired essentially simultaneously with the magnetic field.