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# EXCITATION CONTROL 

 OF SYNCHRONOUS GENERATORSby

Dale E. Mesple, Thomas K. K. Tam and Otto J. M. Smith

Memorandum No. UCB/ERL M337

9 June 1972

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## ELECTRONICS RESEARCH LABORATORY <br> College of Engineering University of California, Berkeley 94720

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## ACKNOWLEDGMENT

The authors wish to thank the Pacific Gas and Electric Company for the grant which supported this research project.

## ABSTRACT

The growing need for electricity in the U.S. has brought forth the question of reliability of our power systems. With the development of such power sources that would require the need for longer transmission lines the stability problem is even greater. System disturbances can and will cause widespread power outages if these systems are not controlled.

This paper considers the development of an optimal excitation control scheduler. The controller built here was designed for a 15 KVA synchronous generator. The control variables are acceleration, velocity and angle error. These are operated on by the optimal weighting vector found by computer optimization to give the optimal excitation schedule. The controller delivers a deadbeat response for the design case and removes all oscillations after the first cycle for other operating conditions. During this deadbeat control the controller reduces the initial velocity peak by $43 \%$ and the maximum increase in field voltage was $35 \%$.

A target angle predictor and a torque angle sensor were developed for this control and for state variable recording.

The controller was designed by a sequence of two optimizations. The first determined the open-loop excitation as a function of time after a transient to produce a prescribed "best" or "optimal" recovery from the transient. The second optimization determined the closed-loop device to generate the excitation from the set of measurable state variables. The first optimization was experimented
and the second optimization was a digital computer program, with experimental modification to adapt it to a variety of transients.

## CHAPTER I

EXPERIMENTAL RESULTS

This chapter considers the results of the optimal excitation control algorithm. The control variables used are the acceleration of the rotor, $\ddot{\delta}$, velocity of the rotor, $\dot{\delta}$, and the error in the angle of the rotor, $-\delta$ taken with respect to the target state of the angle §. It has been shown by Jones(2) and Mottershead(3) that these state variables are highly desirable for transient suppression in synchronous machines.

An optimal excitation function was found by an experimental point by point optimization technique that was equivalent to the Pontryagin method(11). This function was then realized by an optimal weighting vector operating on the state variables $\pm \delta, \pm \dot{\delta}$ and $\pm(-\Delta \delta)$ The optimal weighting vector was found by linear programming.

This optimal exitation control algorithm was implemented on a 15 KVA synchronous machine connected to an infinite bus through two parallel transmission lines. (See Appendix $C$ for complete system).

The first control considered was that using the optimal computer results for the case of increasing line reactance (loss of line). The weighting vector found by the optimization routine was:

$$
\underset{\sim}{X}=\left[\begin{array}{l}
x_{1} \\
x_{2} \\
x_{3} \\
x_{4} \\
x_{5} \\
x_{6}
\end{array}\right]=\left[\begin{array}{l}
0.0 \\
0.442 \\
0.0 \\
0.56 \\
2.6 \\
0.0
\end{array}\right]
$$

where
$x_{1}$ is the negative weighting of negative angle error
$x_{2}$ is the negative weighting of positive angle error
$x_{3}$ is the positive weighting of positive velocity
$x_{4}$ is the positive weighting of negative velocity
$x_{5}$ is the positive weighting of positive acceleration
$x_{6}$ is the positive weighting of negative acceleration
Note that these weightings correspond to the fraction of each signal required by the optimal schedule. For example: $x_{2}=0.442$ implies that the correction is a negative signal derived from a positive angle error of $0.442(-\delta-|\delta|) / 2$, which is $0.442(-\delta)$ passed through a negative diode.

The controlled and uncontrolled transients for loss of line and regaining line are shown in Figure 3.4. One can see that the uncontrolled case is lightly damped and very oscillatory and that the controlled case has only one cycle of oscillation. In attempt to further improve the weighting vector the next case was considered.

By trial error the weighting were adjusted to the following values:

$$
\underset{\sim}{\mathrm{X}}=\left[\begin{array}{l}
0.04 \\
0.47 \\
0.04 \\
0.59 \\
2.80 \\
0.3
\end{array}\right]
$$

The controlled and uncontrolled transients for loss of line and regaining line are shown in Figure 1.1. The controlled response to the loss of line was deadbeat ( $1 / 2$ cycle oscillation). Note that the final optimal weighting vector compares quite well with the computer weighting vector except for the negative acceleration component. This is due to the initial negative pulse needed to under excite the generator in the case of regained line. Using this optimal weighting vector, other operating conditions were also considered.

Figures 1.1 to 1.6 show the controlled and uncontrolled transient responses for the following operating conditions.

CASE 1 (Design case)
Power $=3.0 \mathrm{KW}$
Capacitive load, underexcited generator
$I_{f}=9.4 \mathrm{Amp}$.
$\mathrm{V}_{\mathrm{f}}=35.5 \mathrm{~V}$.
Prime mover current $I_{d c}=44.4$ Amp.
Maximum increase in $\mathrm{V}_{\mathrm{f}}=42.5 \%$
See Figure 1.1

CASE 2 (Figure 2.2)
Power $=5.1 \mathrm{KW}$

Inductive load, overexcited generator
$I_{f}=11.05 \mathrm{Amp}$.
$\mathrm{V}_{\mathrm{f}}=42.8 \mathrm{~V}$.
Prime mover current $I_{d c}=61.0$ Amp.
Maximum increase in $V_{f}=35.0 \%$

CASE 3 (Figure 1.3)
Power $=2.4 \mathrm{KW}$
Inductive load, overexcited generator
$I_{f}=10.5 \mathrm{Amp}$.
$\mathrm{V}_{\mathrm{f}}=40.5 \mathrm{~V}$
Prime mover current $I_{d c}=35.0$ Amp.
Maximum increase in $\mathrm{V}_{\mathrm{f}}=37.0 \%$

CASE 4 (Figure 1.4)
Power $=7.74 \mathrm{KW}$
Capacitive load, underexcited generator
$I_{f}=8.0 \mathrm{Amp}$.
$\mathrm{V}_{\mathrm{f}}=32.0 \mathrm{~V}$.
Prime mover current $I_{d c}-$ 86.5 Amp.
Maximum increase in $\mathrm{v}_{\mathrm{f}}=202.0 \%$

CASE 5 (Figure 1.5)
Power $=3.2 \mathrm{KW}$
Capacitor load, underexcited generator
$I_{f}=7.0 \mathrm{Amp}$.
$V_{f}=26.2 \mathrm{~V}$.
Maximum increase in $\mathrm{V}_{\mathrm{f}}=96.0 \%$

This optimal control was tested in the case of momentary
outage of the transmission line. Figure 1.6 shows the result of this experiment. The outage was approximately $1 / 2$ second.

A second stage amplifier (Fig. 3.3) was used to amplify the


#### Abstract

desirable control signal from the summing amplifier. Identical transients but with different amplification on the control signal were performed. One can see there exists an optimal amplification such that the rotor was brought to target in $1 / 2$ cycle without any further oscillation (no over-shoot or under-shoot). Figure 1.7. However, this optimal amplification is subjected to the constraint of the capability of the exciter unit. The stability limit for this experiment was a maximum gain of 80 in the second stage amplifier and excellent performances were observed at the gain of 25 .


SUMMARY
The control function developed in this report has been shown very effective in quenching electromechanical transients for the case of a single 15 KVA machine connected to an infinite bus. This control algorithm can be developed for used in a general machine system (Chapter 7).

For the design case in this report, the transient was quenched in 0.5 second while the machine resonant frequency was approximately 1 Hz . (i.e. $1 / 2$ cycle of oscillation only). Also, this control algorithm has shown great effectiveness in controlling transients at various operating conditions, even at a power level of more than twice the design level.

The effectiveness of this control algorithm is the result of the highly reliable and noise-free control variables developed in this report. To improve this control algorithm further a voltage regulator is used in addition to the control variables discussed earlier. These additional results are presented in Chapter 9.

CHAPTER II

OPTIMAL EXCITATION

Almost every method used in controlling power system transient has to deal with redistribution of stored energy throughout the system. Optimal excitation control is such a method which redistributes the stored energy in a most efficient manner such that the transient is damped in some "best" manner. For example, minimum transient energy in the first quarter period followed by minimum velocity of the opposite sign and no further reversals of velocity and maximum transient time after the first velocity zero is one "best" control. Starting with the most basic equation for the steady state power flow between a generator and infinite bus one has

$$
P_{m}-P_{1}=V_{1} V_{2} Y \sin \delta
$$

where

$$
\begin{aligned}
\mathrm{P}_{\mathrm{m}}= & \text { mechanical power input } \\
\mathrm{P}_{1}= & \text { power loss in generation } \\
\mathrm{V}_{1}= & \text { generated voltage } \\
\mathrm{V}_{2}= & \text { voltage of infinite bus } \\
\delta= & \text { phase angle between } \mathrm{V}_{1} \text { and } \mathrm{V}_{2} \\
\frac{1}{\mathrm{Y}}=\mathrm{X}= & \text { reactance of line between the machine and } \\
& \text { the infinite bus. }
\end{aligned}
$$

Under normal operating conditions, the steady state power torqueangle curve is shown by curve (A) in figure 2.1.

Suppose the disturbance in the system is caused by an increase of line reactance (for example: losing one of the two parallel transmission lines), the transmitted power will be reduced due to this
increase of line reactance. This is shown by curve ( $B$ ) in figure 2.1. The new operating condition is now at point $A^{\prime}$, where the torque-angle $\delta$ and the mechanical power input to the rotary system remain the same as before the disturbance. However, because of the sudden gain in acceleration, the rotor speeds up and the kinetic energy increases linearly with speed. Without any means of damping, the rotor would continue to speed up until the rotor reaches point B (figure 2.1). At this instant the acceleration of the rotary system is zero. But the rotor has gained so much momentum during its advance from point $A$ to point $B$ that the rotor would continue to advance but at a negative decrement (acceleration is negative) until it reaches point $C$. At this point the velocity will start to advance in the negative direction because the kinetic energy deviation has gone to zero but the acceleration is still negative because the transmitted power is higher than the input power. With some type of damping such as provided by the damper windings, the rotor would oscillate about point $B$ and finally settle down at the new steady state point B. The above discussion assumes that the stability limit is not exceeded. A case where the system becomes unstable after a disturbance is shown in figure 2.2.

## FIELD EXCITATION

The power torque-angle curve of a one-machine and infinite bus system under normal field excitation is shown by curve $A$ in figure 2.3. When the field excitation is suddenly increased the power delivering capacity of the generator is increased; that is, higher power operated at the same torque angle (as the normally excited machine). This is
shown by curve $B$ in figure 2.3. A lower than normal excitation would produce an opposite effect as shown by curve $C$ in figure 2.3.

OPTIMAL FIELD EXCITATION
DEAD-BEAT CONTROL
A steady state power torque-angle curve of a one-machine system is shown in figure 2.4. Assume this system has both parallel transmission lines in service and delivers power $P_{m}$ at an angle of $\delta_{o}$ (shown by point A). When one of the transmission lines is switched out the operating point will drop to a new lower power A'. From the last section, if there is no control, the system may gajn enough momentum during its advance from $\delta_{0}$ to $\delta_{t}$ that the system would be unstable. However, if a proper permissible higher positive field voltage is applied immediately after the disturbance, the positive rotor acceleration would be reduced because the drop in transmitted power is reduced. The rotor would advance in speed as described in the last section. If the generator field remains at the over-excited level (higher positive field voltage than normal operation), the rotor would gain velocity in the negative direction immediately after the rotor reaches point C. In figure 2.4 , the excitation current is suddenly reduced to normal by a large negative voltage pulse at $\delta_{t}$, and the transient disappears. However, such an algorithm may appear to be theoretically sound, the practicality is questionable because the field mmf and exciter ceiling may not be sufficient for dead-beat half-cycle control.

ONE OVER SHOOT CONTROL
One way to prevent this oscillation is to increase the kinetic
energy of the rotor before $\delta$ gets to its maximum angle $\delta_{1}$ (Fig. 2.5) (i.e. to speed up the rotor again in the positive direction before the rotor velocity reaches zero). This can be done by changing the excitation to a lower than normal field voltage. If the switching of excitation is chosen properly and the exciter can provide the required voltages, the rotor can be brought to the new steady state operating point D without further oscillation.

One may choose to have more than one switching for the following reasons:

1. Huge voltage switching may produce an undersirable effect on the system.
2. The exciter may not deliver the required voltages.
3. Driving the exciter into saturation is highly undesirable for economical reasons.

In case of having more than one switching, the optimal control is shown in figure 2.5. The field voltage applied here is less than the case of having only one switching. Therefore, an over shoot of target is unavoidable. A less than normal excitation is applied before the rotor reaches state $C$. This switching does not require the system to return to the steady state at this instant, but rather the rotor velocity is permitted to go in a slightly negative direction. At state $C^{\prime}$, the field voltage is returned to normal and the rotor will be brought to the target without further oscil1ation.

## CHAPTER III

## REALIZATION OF OPTIMAL EXCITATION FUNCTION

Chapter 2 introduces the fundamentals of optimal excitation control of synchronous machines in terms of machine field voltages. Since the field is supplied by the exciter, the main concern would be the control of the exciter.

This chapter is concerned with laboratory implementation and realization of the excitation function described in Chapter 2.

OPTIMAL EXCITATION FUNCTION (For the case of increase of line reactance)

When a positive signal appears at the control field of the exciter it produces a positive incremental generator field voltage. A negative signal produces a decrement of field voltage. A pulse generator was built for the purpose of generating the control signals for the exciter (Fig. 3.1). This pulse generator is designed to be triggered on at the instant of the disturbance.

From Chapter 2, a positive pulse is highly desirable immediately after one of the two parallel transmission lines is switched out. (2 and 3) Therefore, a single positive pulse (of pulse duration $1 / 10$ second) of various amplitudes was tried for identical transients with the same operating condition. And the "best" amplitude (for the best transient control) was then selected using this best first pulse up to the time 0.1 seconds, a succeeding best second pulse up to the time 0.2 seconds was obtained by the same search procedure. Continuing this step by step optimization procedure for successive times
up to 1.2 seconds, the optimal excitation function was obtained and is shown in Fig. 3.2a. The machine response for a particular case \#1 controlled transient is shown in figures 1.1a and $c$. The same case \#1 disturbance controlled by the optimal excitation function (open loop) is shown in figures 3.2a and $b$. (Note that the controlled transient is an open-loop control). The controlled transient is clearly superior to the uncontrolled one.

OPTIMAL WEIGHTING FUNCTION
The optimal excitation function must be realizable and must be produced by a composite of the state variables. As described earlier the control variables used in this report are positive and negative angle, positive and negative velocity, and positive and negative acceleration. In order to utilize these state variables, a weighting vector on these variables has to be determined, which will produce the optimal excitation function.

Computing at each pulse instant, a weighting vector $\underset{\sim}{X}$ operating on a vector of state variables a should give the desired control function $v$ at that instant (Appendix A).

For example:

$$
\begin{equation*}
a_{i i} x_{i}+a_{i 2} x_{2}+a_{i 3} x_{3}+a_{i 4} x_{4}+a_{i 5} x_{5}+a_{i 6} x_{6}=v_{i} \tag{3.1}
\end{equation*}
$$

where

$$
\begin{aligned}
& x_{1}=\text { weighting on }+ \text { value of negative angle } \\
& x_{2}=\text { weighting on }- \text { value of negative angle } \\
& x_{3}=\text { weighting on }+ \text { velocity } \\
& x_{4}=\text { weighting on - velocity }
\end{aligned}
$$

```
    \(x_{5}=\) weighting on + acceleration
    \(x_{6}=\) weighting on - acceleration
\(a_{i j}=\) signal amplitude of variable \(j\) at time \(i\)
        \((i=1,2,3, \ldots, N)\) and \((j-1,2,3, \ldots, 6)\)
    \(\mathrm{N}=\) number of pulse instants
    \(v_{i}=\) the desired signal at time \(i\)
```

Therefore, a system of N linear equations (or constraints) are obtained where every weighting satsfies the desired signal at each instant. One may attempt to solve these equations at this point, but a careful inspection of these $N$ equations in only 6 unknowns shows that a solution does not exist because they are mutually inconsistant. Therefore, a linear programming technique was used to determine the best solution for vector $X$ from this system of equations.

In order to apply the linear programming technique a cost function has to be defined. That is, a cost is assigned arbitrarily to each variable such that the main objective is to minimize this cost function subject to the linear constraints.

Let $f_{0}=$ cost function
Then the problem can be reformulated as follows:

$$
\text { Minimize } f_{0}=C_{\sim}^{T} X
$$

Subject to $A X=V$
where

$\underset{\sim}{A}=\left[\begin{array}{llll}a_{11} & a_{12} & \cdots & a_{16} \\ a_{21} & & \cdots & a_{26} \\ & & & \\ a_{N I} & & \cdots & a_{N 6}\end{array}\right] \quad a_{i j}$ as defined previously


The Simplex method is usually used to the linear programming problems. For this paper a computer subroutine Alphac was used (13).

LABORATORY REALIZATION OF THE OPTIMAL WEIGHTING FUNCTION
After the weighting of the state variables were obtained, a summing circuit was built to realize this weighting in the laboratory (Fig. 3.3). From the summing circuit

$$
v_{0}=-R_{f}\left[\frac{E_{1}}{R_{1}}+\frac{E_{2}}{R_{2}}+\frac{E_{3}}{R_{3}}+\frac{E_{4}}{R_{4}}+\frac{E_{5}}{R_{5}}+\frac{E_{6}}{R_{6}}\right]
$$

Then

$$
\begin{aligned}
& \mathrm{x}_{1}=\mathrm{R}_{\mathrm{f}} / \mathrm{R}_{1} \\
& \mathrm{x}_{2}=\mathrm{R}_{\mathrm{f}} / \mathrm{R}_{2} \\
& \mathrm{x}_{3}=\mathrm{R}_{\mathrm{f}} / \mathrm{R}_{3} \\
& \mathrm{x}_{4}=\mathrm{R}_{\mathrm{f}} / \mathrm{R}_{4} \\
& \mathrm{x}_{5}=\mathrm{R}_{\mathrm{f}} / \mathrm{R}_{5} \\
& \mathrm{x}_{6}=\mathrm{R}_{\mathrm{f}} / \mathrm{R}_{6}
\end{aligned}
$$

The machine transient response (with this optimal control function) is presented in Fig. 3.4. This transient response shows an improvement over the uncontrolled transients in Figure 1.1a and $c$, and also shows an improvement over the open-loop optimal control in Fig. 3.2. However, a trial and error method was used in the laboratory to improve the tail end of the transient, and the final "optimal" result was presented in Chapter I, case \#1, Figure 1.1e.

## TORQUE ANGLE PREDICTOR

The prediction of the future target state of the machine rotor makes it possible to use the error in the angle as a feedback element in the optimal excitation control algorithm. Previous work has shown that the angle error is a definite aid in the control algorithm. This work was based on knowing the future state of the machine rotor. By having a future angle predictor the knowledge of the future state is not needed before hand. This makes it possible to control any transient. The torque angle predictor designed here is designed for the worst case transient, that of full power loss of line. Note that full power in this case is one half the machine rating. It is also based on the velocity and acceleration signals. Chapter 6 gives a description of how these signals are obtained.

Based on the assumption that the machine being controlled is connected by a reactance to a zero-impedance bus of infinite inertia, a circle or an ellipse can be drawn in the $\delta$ vs $\delta$ phase plane centered on the average angle $\delta_{a}$, such that any operating state within the ellipse is considered normal, and any operating state outside of the ellipse is considered a transient. The trajectories resemble logarithmic spirals or circles when the $\dot{\delta}$ coordinate has units of

$$
\begin{equation*}
w \sqrt{J \Omega / P_{\max } \cos \delta_{a}} \tag{4.1}
\end{equation*}
$$

and the $\delta$ coordinate has units of radians.

$$
\begin{equation*}
P_{\max }=V_{1} V_{2} / X \tag{4.2}
\end{equation*}
$$

$$
\begin{aligned}
& \mathrm{J}=\text { moment of inertia } \\
& \delta_{a}=\text { average operating phase of the rotor } \\
& \Omega=2 \pi f
\end{aligned}
$$

The shaft speed in electrical radians/second is

$$
\begin{equation*}
(\Omega+w)=\Omega+\dot{\delta} \tag{4.3}
\end{equation*}
$$

Let

$$
\begin{equation*}
K=\sqrt{J \Omega / P_{\max } \cos \delta_{a}} \tag{4.4}
\end{equation*}
$$

A circle of radius $r$ in the $K w$ versus $\delta$ phase plane is given by

$$
\begin{equation*}
r^{2}=(K w)^{2}+\delta^{2} \tag{4.5}
\end{equation*}
$$

The condition that a transient exist is when $r>r_{\min }$ and

$$
\begin{equation*}
(K w)^{2}+\delta^{2}-r_{m}^{2}>0 \tag{4.6}
\end{equation*}
$$

where $r_{m}$ is obtained from operating experience.
When a transient occurs the target state will deviate from the initial state. If there is no damping and if the angular changes are small, the trajectory is approximately a circle as shown in figure 4.1. The center is at $\delta_{t}$ and the radius is $R$. The slope is given by:

$$
\begin{equation*}
(\mathrm{d}(\mathrm{Kw}) / \mathrm{dt}) /(\mathrm{d} \delta / \mathrm{dt})=\mathrm{K}(\mathrm{dw}) / \mathrm{wdt}=\mathrm{K}\left(\mathrm{~d} \log _{\mathrm{e}} \mathrm{w} / \mathrm{dt}\right) \tag{4.7}
\end{equation*}
$$

From the geometry this slope is also equal to:

$$
\begin{equation*}
\left(\delta_{t}-\delta\right) / K w=K d w / w d t \tag{4.8}
\end{equation*}
$$

or

$$
\begin{equation*}
\delta_{t}=\delta+k^{2} \dot{\delta} \tag{4.9}
\end{equation*}
$$

When the assumptions are relaxed, there is some error in the above calculations. To correct the signal $\delta_{t}$ for the transient decrement, the velocity signal can be added into equation (4.9) as
shown below:

$$
\begin{equation*}
\delta_{t}=\delta+k^{2} \ddot{\delta}+k_{2} \dot{\delta} . \tag{4.10}
\end{equation*}
$$

Let

$$
\begin{equation*}
K_{3}=k^{2} \tag{4.11}
\end{equation*}
$$

then

$$
\begin{equation*}
\delta_{t}=\delta+K_{2} \dot{\delta}+K_{3} \ddot{\delta} \tag{4.12}
\end{equation*}
$$

Five transients were run for data. They are shown in figures 4.2-4.6.

A computer program was written to solve equation (4.12). The data was taken each 0.05 seconds for 100 points. The program is based on the instrumental variable method. Appendix B gives a description of the program and a listing of the program used. The results are also shown in appendix $B$.

The results are given below
case 1. (worst case)
The transients are shown in figure 4.2

$$
\begin{aligned}
& K_{2}=1.1878 \\
& K_{3}=31.096
\end{aligned}
$$

case 2.
The transients are shown in figure 4.3
$K_{2}=-0.76$
$K_{3}=34.7987$
case 3.
The transients are shown in figure 4.4
$K_{2}=0.0326$
$K_{3}=28.8843$
case 4.
The transients are shown in figure 4.5

$$
\begin{aligned}
& \mathrm{K}_{2}=2.81 \\
& \mathrm{~K}_{3}=35.3342
\end{aligned}
$$

case 5.
The transients are shown in figure 4.6
$\mathrm{K}_{2}=0.6752$

$$
K_{3}=74.1615
$$

The error in the angle is defined as:

$$
\begin{equation*}
\delta-\delta_{t}=\Delta \delta \tag{4.13}
\end{equation*}
$$

where $\delta$ is the actual angle, $\delta_{t}$ is the target angle, and $\Delta \delta$ is the error. From equation (4.12) it can be seen that

$$
\begin{equation*}
\Delta \delta=-\left(K_{2} \dot{\delta}+K_{3} \ddot{\delta}\right) \tag{4.14}
\end{equation*}
$$

The circuit used to implement $\Delta \delta$ is shown in figure 4.7. For plotting purposed $-\delta$ and $-\delta_{t}^{\prime}$ are also used as outputs.
Note that $\delta_{t}^{\prime}=\delta_{t} / 2$.
The predicted angle for the first three cases described above are shown in figures 4.8-4.10. The coefficients used were those found by the computer.

By trial and error the above predictions were improved by adjusting the coefficients. Designing for best results at the worst case the final coefficients are:

$$
\begin{aligned}
& K_{2}=4.0 \\
& K_{3}=29.0 .
\end{aligned}
$$

The predicted angle and the error $\Delta \delta$ is shown for the first three cases described above in figures 4.11-4.13. Note that the
coefficients are the same for all three cases. These results are quite good for all three cases.

## Summary :

The torque angle predictor developed here provides the angle error signal needed for the optimal excitation control algorithm. The predictor works very well for the three over-excited cases when the coefficients are held constant.

## CHAPTER V

TORQUE ANGLE SENSOR

The purpose of building an angle sensor is to be able to have a continuous signal that is proportional to the shaft angle of the machine. This signal gives a direct measure of the rotor angle with respect to an infinite bus. Under transient conditions this signal can be combined with the target or post fault angle of the rotor and used in feedback to aid in the optimal excitation control signal.

The torque angle sensor developed here has a sensitivity of 17.8 volts/radian. The peak output voltage is 19.0 volts and the signal to noise ratio is 64 db .

The design of the angle sensor is based on the idea of a phase discriminator and a square law rectifier. Figure 5.1 shows that generator voltage $V_{I}$, the reference voltage $V_{2}$, and the resultant voltage $\underline{V}_{\mathrm{r} 1}$ and $\underline{\mathrm{V}}_{\mathrm{r} 2}$. The reference voltages are obtained from the vector addition of $\underline{V}_{1}$ and $\underline{V}_{2}$. The angle $\delta$ is the phase difference between $\underline{V}_{1}$, the generator voltage and $\underline{V}_{2}$, the reference voltage. The difference $\underline{V}_{r 1}-\underline{V}_{r 2}$ varies as $\cos \delta$ which gives a signal dependent only on the angle $\delta$. This can be seen from the following:

$$
\begin{align*}
& \underline{V}_{\mathrm{r} 1}=\mathrm{V}_{1}+\mathrm{V}_{2}(\cos \delta+j \sin \delta)  \tag{5.1}\\
& \mathrm{V}_{\mathrm{r} 2}=\underline{V}_{1}+\underline{V}_{2}(\cos \delta-j \sin \delta)  \tag{5.2}\\
& \underline{\mathrm{V}}_{\mathrm{Oa}}^{\prime}=\left|\mathrm{V}_{\mathrm{r} 1}\right|-\left|\underline{\mathrm{V}}_{\mathrm{r} 2}\right|=\sqrt{\left(\mathrm{V}_{1}-\mathrm{V}_{2} \cos \delta\right)^{2}+\left(\mathrm{V}_{2} \sin \delta\right)} \\
&=\sqrt{\left(\mathrm{V}_{1}+\mathrm{V}_{2} \cos \delta\right)^{2}+\left(\mathrm{V}_{2} \sin \delta\right)^{2}} \tag{5.3}
\end{align*}
$$

Because $\mathrm{V}_{1}>\mathrm{V}_{2}$, this is approximately equation (5.4)

$$
\begin{equation*}
v_{o a}^{\prime}=\left|v_{r 1}\right|-\underline{v}_{r 2}|\tilde{m}-2| \underline{v}_{2} \mid \cos \cos \delta \tag{5.4}
\end{equation*}
$$

In the old design $V_{o a}^{\prime}$ was a constant with a 60 Hz ripple due to rectification of $\left|\mathrm{v}_{\mathrm{r} 1}\right|$. Adding the outputs of the three phases together.

$$
\begin{equation*}
v_{\mathrm{J} .}^{\prime}=v_{o a}+v_{o b}^{\prime}+v_{o c}^{\prime}=-6 v_{2} \cos \delta . \tag{5.5}
\end{equation*}
$$

$V_{o}^{\prime}$ contains a 180 Hz ripple.
The new form of phase discriminator uses the difference of the squares. To get a D.C. voltage without ripple that varies as cos $\delta$ the signals $\underline{V}_{r 1}$ and $V_{r 2}$ are square law rectified and added to corresponding signals from the other two phases also square law rectified. In one phase only

$$
\begin{align*}
& \mathrm{v}_{\mathrm{oa}}=\left|\underline{\mathrm{v}}_{\mathrm{r} 1}\right|^{2}-\left|\underline{v}_{\mathrm{r} 2}\right|^{2}=\left(\mathrm{V}_{1}-\mathrm{v}_{2} \cos \delta\right)^{2}+\left(\mathrm{V}_{2} \sin \delta\right)^{2} \\
& =\left(\mathrm{V}_{1}+\mathrm{V}_{2} \cos \delta\right)^{2}-\left(\mathrm{V}_{2} \sin \delta\right)^{2} \\
& =\mathrm{v}_{1}^{2}-2 \mathrm{v}_{1} \mathrm{~V}_{2} \cos \delta+\mathrm{v}_{2}^{2} \cos ^{2} \delta-\mathrm{v}_{1}^{2}-2 \mathrm{v}_{1} \mathrm{~V}_{2} \cos \delta-\mathrm{v}_{2}^{2} \cos ^{2} \delta \\
& \mathrm{~V}_{\mathrm{oa}}=-4 \mathrm{~V}_{1} \mathrm{~V}_{2} \cos \delta \text {. } \\
& \mathrm{V}_{\mathrm{oa}} \text { is again a constant, but when } \mathrm{V}_{\mathrm{r} 1}^{2} \text { and } \mathrm{V}_{\mathrm{r} 2}^{2} \text { are each determined } \\
& \text { by 3-phase square law rectification, there is no ripple. Then } \\
& \mathrm{v}_{\mathrm{o}}=(3 / 2)\left|\mathrm{v}_{\mathrm{oa}}\right|=-6\left|\mathrm{v}_{1} \mathrm{v}_{2} \cos \delta\right|  \tag{5.8}\\
& \text { This follows from: }
\end{align*}
$$

$$
\begin{align*}
& v_{r 1} \text { in phase } A=\left|v_{r 1}\right| \cos (\omega t)  \tag{5.9}\\
& v_{r 1} \text { in phase } B=\left|v_{r 1}\right| \cos (\omega t+2 \pi / 3) \tag{5.10}
\end{align*}
$$

$$
\begin{equation*}
v_{r 1} \text { in phase } c=\left|v_{r 1}\right| \cos (\omega t-2 \pi / 3) \tag{5.11}
\end{equation*}
$$

If the voltages in equations (5.(9) through (5.11) are each squared the results are:

$$
\begin{align*}
& \left(v_{r 1} \text { in phase } A\right)^{2}=\left|v_{r l}\right|^{2}\left(\frac{1}{2}+\frac{1}{2} \cos (2 \omega t)\right)  \tag{5.12}\\
& \left(v_{r 1} \text { in phase } B\right)^{2}=\left|v_{r 1}\right|^{2}\left(\frac{1}{2}+\frac{1}{2} \cos (2 \omega t+2 \pi / 3)\right)  \tag{5.13}\\
& \left(v_{r 1} \text { in phase } C\right)^{2}=\left|v_{r 1}\right|^{2}\left(\frac{1}{2}+\frac{1}{2} \cos (2 \omega t-2 \pi / 3)\right) \tag{5.14}
\end{align*}
$$

When equation (5.12) through (5.14) are added the result is the ripplefree voltage $V_{1}$.

$$
\begin{equation*}
\mathrm{v}_{1}=\sum_{3 \text { phase }}\left(\mathrm{v}_{\mathrm{r} 1}\right)^{2}=\left|\mathrm{v}_{\mathrm{r} 1}\right|^{2}\left(\frac{3}{2}+0\right)=\left(\frac{3}{2}\right)\left|\mathrm{v}_{\mathrm{r} 1}\right|^{2} \tag{5.15}
\end{equation*}
$$

In a similar manner, the sum of the three square law rectified signals $\mathrm{V}_{\mathrm{r} 2}$ from the three phases is ripple-free.

$$
\begin{equation*}
\mathrm{v}_{2}=\frac{3}{2}\left|\mathrm{v}_{\mathrm{r} 2}\right|^{2} \tag{5.16}
\end{equation*}
$$

The output voltage is

$$
\begin{equation*}
\mathrm{v}_{\mathrm{o}}=\mathrm{v}_{1}-\mathrm{v}_{2}=\frac{3}{2}\left(\left.\mathrm{v}_{\mathrm{r} 1}\right|^{2}-\left|\mathrm{v}_{\mathrm{r} 2}\right|^{2}\right)=-6\left|\mathrm{v}_{1} \mathrm{v}_{2} \cos \delta\right| \tag{5.17}
\end{equation*}
$$

Thus the output of the rectifier is a constant which depends upon the rotor angle $\delta$, but not the frequency $\omega$. From this type of design one can theoretically get a ripple-free signal.

Note that the square-law phase discriminator has an output linearly proportional to each input voltage, i.e., it is sensitive to the generator voltage. The old rectifier phase discriminator had an output proportional only to the smaller voltage $\mathrm{V}_{2}$ when $\mathrm{V}_{1} \gg \mathrm{~V}_{2}$, and
therefore when $V_{1}$ was obtained from the generator terminal voltage, the phase calibration was not a function of terminal voltage. The new square-law phase discriminator does not have this desirable feature, but the ripple-free output is a significant improvement.

The design of the rectifier is based on the following assumptions:
i) As $\underline{V}_{r 1}$ increases the admittance should act as a square law as shown in Figure 5.8.
ii) $V_{0}=40$ volts D.C.
iii) $R_{\text {load }}=5 \mathrm{k}$ ohms.
iv) $\frac{V_{1}}{1}=110$ volts, $\underline{v}_{2}=26$ volts. $V_{1}$ is obtained from $\mathrm{a}^{1}$ second generator on the main rotor. It has an output voltage of 240 volts three phase. $\underline{V}_{2}$ is the infinite bus voltage. The line voltage is ${ }^{-2} 240$ volts. Both voltages are stepped down accordingly.

Based on the above assumptions the following designs are considered. Design 1:

Shown in figure 5.2 is the circuit for the first design considered.

$$
\begin{equation*}
I_{\text {averl }}=40 \mathrm{v} / 5 \mathrm{k}=8 \mathrm{ma} . \tag{5.18}
\end{equation*}
$$

or on a per phase basis

$$
\begin{equation*}
I_{\text {averl }}=8 \mathrm{ma} . / 3=2.7 \mathrm{ma} \tag{5.19}
\end{equation*}
$$

From Figure 5.9 the average voltage across $r_{L}$ is:

$$
\begin{equation*}
\mathrm{V}_{\mathrm{ave}}=0.636 \times \text { peak } \times 6 / 7=0.545 \times \text { peak } \tag{5.20}
\end{equation*}
$$

This implies from figure 5.2 that:

$$
\begin{equation*}
((114 \mathrm{v} \times 1.4 \times 0.545)-40 \mathrm{~V}) / \mathrm{R}_{1}=2.7 \mathrm{ma} \tag{5.21}
\end{equation*}
$$

or

$$
\begin{equation*}
\mathrm{R}_{1}=45 \mathrm{~V} / 2.7 \mathrm{ma} .=20 \mathrm{k} \text { ohms } \tag{5.22}
\end{equation*}
$$

Figure 5.10 shows $R_{1}$ plotted on the $I$ vs $V$ curve. Note that $R_{1}$ is not a very good match to the desired square law curve. This is
wxpected since $R_{1}$ is linear. Because of this poor match too much ripple was produced so design 2 was considered.

Design 2:
Figure 5.3 shows the circuit for design 2. If a tangent point is picked at 140 volts on figure 5.11 this makes slope 2 equal to $2 \times$ slope 1 or

$$
\begin{equation*}
G_{1}=G_{2} \tag{5.23}
\end{equation*}
$$

This implies $R_{1}=20 k$ and $R_{2}=20 k$, which inturn implies that

$$
\begin{equation*}
V_{\text {zener }}=2 \times 40 \mathrm{v}=80 \mathrm{v} \tag{5.24}
\end{equation*}
$$

This design also does not match the square law curve as closely as desired. Therefore, design 3 is considered.

Design 3:
On figure 5.12 pick a tangent at 140 volts. The previous slope $S_{1}$ was equal to $G_{1}+G_{2}=I / V=4$; i.e. $G_{1}+G_{2}=2 G_{1}$. For design 3 make slope 2 equal to

$$
\begin{equation*}
S_{2}=\left[(15 / 16) \times S_{1}\right] /[(30 / 80) /(40 / 80)]=(7 / 4) \times S_{1} \tag{5.25}
\end{equation*}
$$

This implies

$$
\begin{equation*}
G_{1}+G_{2}=1.75 \tag{5.26}
\end{equation*}
$$

or

$$
\begin{aligned}
& \mathrm{R}_{1}=20 \mathrm{~K} \\
& \mathrm{R}_{2}=30 \mathrm{k}
\end{aligned}
$$

and

$$
\begin{equation*}
v_{\text {zener }}=105-40=65 \mathrm{v} \tag{5.27}
\end{equation*}
$$

Design 4:
A fourth design was considered to reduce the ripple to a smaller amount. This is shown in figure 5.4. Changing $V_{\text {test }}$ from $0-30$ volts
made no significant change in the ripple content of the output voltage.
The final design is shown in figure 5.3. $\mathrm{R}_{\text {load }}$ was optimized to give $\mathrm{V}_{\mathrm{o}}=40$ volts as assumed. The optimal value is 8.06 k ohms.

When the entire circuit was built (shown in figure 5.5) the resultant ripple content was 125 mv at 60 hertz. In reducing the ripple a limit was reached on the size of $C_{1}$. When $C_{1}$ became greater than $4 \mu \mathrm{~F}$ the output phase started to shift. The value of $2 \mu \mathrm{~F}$ was chosen for $\mathrm{C}_{1}$ as a trade off between minimum noise and zero phase shift. With this limitation the final results are:

$$
\begin{aligned}
& \mathrm{V}_{\text {omax }}=19 \text { volts } \mathrm{D} . \mathrm{C} \\
& \text { sensitivity }=17.8 \text { volts/radian } \\
& \text { signal to noise ratio }=64 \mathrm{db}
\end{aligned}
$$

The angle sensor is also sensitive to changes in the generator voltage $\underline{V}_{1}$. This sensitivity is $=3.75 \times 10^{-2}$ degrees per volt.

Figure 5.13 shows the output of the angle sensor as the phase of $\mathrm{V}_{2}$ is changed via the potential phase shifter and as the phase of $\underline{V}_{1}$ is changed via changes in the rotor angle.

As can be seen from figure 5.13 the output voltage $\mathrm{V}_{\mathrm{o}}$ is not linear near the peak of the curve. Since a linear relationship is needed between $V_{0}$ and the torque angle $\delta$ of the rotor some trade offs are necessary. The most important case is when the maximum angle is reached. At this point it imperative that the correct angle be known. Therefore it is necessary to back up the zero power point on the angle sensor. This is done by shifting the position of the armature of the second generator. The generator setting was put at an angle of -35 degrees. This gives a larger linear swing for angle changes. Figure 5.7 shows the torque angle
sensor output under transient conditions. 7a. is for loss of line and 7 b . is for gain of the same line.

## Summary:

The torque angle sensor developed here provides the required signal needed for an accurate reading of the rotor angle. This signal has a sensitivity of 17.8 volts/radian and a signal to noise ratio of 64 db .

The signal delivered is quite adequate for recording purposes. Note that the actual angle signal is not needed for the angle feedback signal. This angle signal was invaluable in obtaining the linear transformation of the acceleration and velocity signals needed to obtain the angle error signal.

The final circuit is shown in figure 5.5 along with a parts list.

## CHAPTER VI

## ACCELERATION AND VELOCITY SIGNALS

The network shown in figure 6.1 computes the net torque power from the generator voltage and current.

The potential transformers $\mathrm{P}_{\mathrm{TA}}, \mathrm{P}_{\mathrm{TB}}, \mathrm{P}_{\mathrm{TC}}$ induce voltage proportional to the output voltage of the three phases $A, B, C$ of the generator. $C_{T A}$ and $C_{T B}$ current transformers measure the line currents $A$ and $B$ of the generator. The different impedances $Z, X_{f}$ and admittance $g$ are representing the evaluation of the internal losses, scaled by the turns ratio of the transformers.

The final currents and voltages are fed to a two-element three phase Noller watt transducer.
$Z$ corresponds to the armature winding resistance and transient (1 to 2 Hz ) quadrature axis reactance $Z_{c}=R+j X$
$Z=\left(Z_{c}\right)(n)(1.5) / m$
$X_{f}$ corresponds to quadrature axis reactance less leakage;
$X_{f}=\left(X_{q}-X_{c}\right) n / m=0$
$g$ corresponds to the equivalent eddy-current conductance G of the core alternator;
$g=(G)(m)(h) / 2 n$

The compensations for copper losses, core losses, and prime mover losses are adjusted experimentally to obtain an accurate representation for the range of power in which the system is operated. 1. Copper Loss

The copper loss compensation is adjusted to yield the same

Noller output at both high generator current and at low generator current for zero power factor current. During the adjustmnet the prime mover input is kept constant.

## 2. Core Loss

With a high reactance power line, the core loss compensation is adjusted to yield the same Noller output for minimum and maximum field current and terminal voltage when prime mover input is held constant.
3. Prime Mover

The prime mover compensation is adjusted to yield the same (Noller + Compensation) signal at high power as well as low power prime mover inputs.

The above transducer delivers a signal proportional to the torque power. Since one is only interested in the slip acceleration the d-c component of the signal must be suppressed. This is done by inserting $5 \mu \mathrm{~F}$ capacitor in series with the output.

The noise level at the output is 70 mV with the main frequency components at 120 Hz and 60 Hz . A twin-T filter is used to remove this noise. This filtered signal is then amplified with a Burr Brown operational amplifier, whose d-c gain is 1.5.

The final signal has been calibrated at 0.0137 V -sec $2 / \mathrm{rad}$. The noise level is approximately 8 mV peak to peak at 60 Hz .

The basic 2 element Noller Watt-transducer delivers 2 volt per kilwatt 3 phase with the current coils in series (5-ampere connection). With 20 to 1 current transformers and 2 to 1 potential transformer, the accelerometer delivered 50 microvolts per watt high side, or 50 millivolts per kilowatt high side. At a nominal rating of 6 kw for the
generators, the sensitivity was 0.3 volts per per-unit power change.

$$
\begin{equation*}
K_{a}=0.30 \quad \text { v/per-unit power } \tag{6.1}
\end{equation*}
$$

From Figure 4-2, $a$ and $b$

$$
\begin{align*}
\mathrm{K}_{\mathrm{ac}} & =\frac{\mathrm{Kv}}{2 \pi 1.07} \text { (ratio of envelopes) } \\
& =\frac{(\mathrm{Kv})}{6.72}(0.412)  \tag{6.2}\\
\mathrm{K}_{\mathrm{ac}} & =0.0614 \mathrm{Kv} \text { volts } /\left(\text { radian } / \mathrm{sec}^{2}\right) \tag{6.3}
\end{align*}
$$

The velocity signal is derived from a six phase, no brush, permanent magnet generator on the main rotor of the system. This signal is then square law rectified giving a signal proportional to the rate of change of the rotor angle. Figure 6.2 shows the circuit. diagram for the square law rectifier. This rectifier theoretically will give a ripple free signal if the resistors are matched. In this design there is a large IR drop causing the resistors to drift. To compensate for the resultant ripple a twin-T 12 Hz rejection filter was implemented. The sensitivity of the velocity signal is $0.224 \mathrm{~V}-\mathrm{sec} / \mathrm{rad}$.

The angle sensitivity $=17.8$ volts/radian. From figure 4.2, $b$ and $c$,
$\mathrm{Kv}=\left(\frac{17.8}{2} 1.07\right)\left(\frac{\mid \text { velocity } \mid}{\mid \text { Angle } \mid}\right)=2.644(0.0847)=0.224 \mathrm{v}-\mathrm{sec} / \mathrm{rad}(6.4)$
The velocity transducer rectifier delivers 75 v at $377 \mathrm{rad} / \mathrm{sec}$ of the rectifiers were perfect square law, then the differential sensitivity was designed to be approximately

$$
\begin{equation*}
\mathrm{Kv}=\frac{150}{377}=0.398 \mathrm{v}-\mathrm{sec} / \mathrm{rad} . \tag{6.5}
\end{equation*}
$$

The sensitivity before the isolating amplifier was measured to be

$$
\begin{equation*}
\mathrm{Ku}=0.224 \tag{6.6}
\end{equation*}
$$

Using this value in (6.3), the acceleration constant for the output of the Noller is

$$
\begin{equation*}
\mathrm{K}_{\mathrm{ac}}=0.01375 \mathrm{~V}-\mathrm{sec}^{2} / \mathrm{rad} \tag{6.7}
\end{equation*}
$$

## CHAPTER VII

GENERAL DESIGN CONSIDERATIONS

DESIGN FOR THIS REPORT

In this chapter the optimal design is completely characterized. From this characterization the desing considerations for any machine are given such that all one needs to know is the inertia of the machine, the power level and the resonant frequency for the worst case condition.

From figure 7.1 one can see that the following is true

$$
\begin{equation*}
V_{0}=R_{f}\left[\frac{E_{1}}{R_{1}}+\frac{E_{2}}{R_{2}}+\frac{E_{3}}{R_{3}}+\frac{E_{4}}{R_{4}}+\frac{E_{5}}{R_{5}}+\frac{E_{6}}{R_{6}}\right] \tag{7.1}
\end{equation*}
$$

Where the minimum gain resistor $R_{x}$ is included in each resistance value (i.e. $R_{1}=R_{x}+R_{1}^{\prime}$ ). Also $V_{0}$ can be related to the field voltage of the generator by the following equation

$$
\begin{equation*}
\Delta V_{f}=\left(k_{1}\right) \Delta V_{o} \tag{7.2}
\end{equation*}
$$

Where $K_{1}$ is the gain of the system from $V_{o}$ to the field voltage $V_{f}$. By laboratory experiment it was found that

$$
K_{1}=R_{y} K_{a m p} 100,000=\left\{\begin{array}{l}
65 \text { optimum gain } \\
200 \text { limit of stability }
\end{array}\right. \text { (7.3) }
$$

When $K_{a m p}$ is the gain of the amplidyne which is equal to 2.5 . Therefore for the optimal case

$$
\begin{equation*}
\Delta V_{f}=R_{f} K_{1}\left[\frac{E_{1}}{R_{1}}+\frac{E_{2}}{2}+\frac{E_{3}}{R_{3}}+\frac{E_{4}}{R_{4}}+\frac{E_{5}}{R_{5}}+\frac{E_{6}}{R_{6}}\right] \tag{7.4}
\end{equation*}
$$

By separating equation (7.4) into the sum of six equations each part can be analyzed separately i.e.

$$
\begin{equation*}
\Delta V_{f}=\sum_{n=1}^{6} V_{f n}=R_{f} K_{1} \sum_{n=1}^{6} E_{n} / R_{n} \tag{7.5}
\end{equation*}
$$

If equation (4) is divided by the nominal field voltage or base field voltage $\mathrm{V}_{\text {fo }}$ the result is percent change in field voltage i.e.

$$
\begin{equation*}
\frac{\Delta V_{f}}{V_{f o}}=\frac{R_{f} K_{1}}{V_{f o}} \sum_{n=1}^{6} E_{n} / R_{n} . \tag{7.6}
\end{equation*}
$$

Starting with the positive acceleration signal equation (5) gives

$$
\begin{equation*}
\frac{\Delta V_{f 5}}{V_{f o}}=\frac{R_{f} K_{1} E 5}{V_{f o} R_{5}} \tag{7.7}
\end{equation*}
$$

But

$$
\begin{equation*}
\mathrm{E}_{5}=+\mathrm{V}_{\ddot{\delta}}=\mathrm{K}_{\mathrm{ac}}(+\ddot{\delta})=\frac{\Omega^{K_{a}}}{\mathrm{~K}_{3}}(+\ddot{\delta}) \tag{7.8}
\end{equation*}
$$

For our circuit, $\mathrm{P}_{\mathrm{b}}=6000$ watts

$$
K_{a}=0.30 \text { on a } 6 \mathrm{KW} \text { base constant }
$$

$$
K_{3}=\frac{P_{b}}{J}=\text { reciprocal perunit inertia for }
$$ our machine

$$
=\frac{\mathrm{K}_{\mathrm{a} \Omega}}{\mathrm{~K}_{\mathrm{ac}}}=\frac{(.30)(377)}{(0.01375)}=8210.0
$$

$$
K_{n}=\text { volts } / K W
$$

$$
=K_{a} \frac{1000}{P_{b}}=0.05 \text { volts } / \mathrm{kw} \text { high side }
$$

Also

$$
\begin{equation*}
\ddot{\delta}=\Delta \mathrm{P} / \mathrm{J} \Omega \tag{7.9}
\end{equation*}
$$

where
$\Delta \mathrm{P}$ is the change in power
$J$ is the inertia of the system

$$
\Omega \text { is } 2 \pi 60=377
$$

Therefore equation (7.7) becomes

$$
\begin{equation*}
\left(\Delta V_{f 5} / V_{f 0}\right)=\left(R_{f} k_{1} k_{a} / V_{f o} K_{3} R_{5}\right)\left(\frac{\Delta P}{J}\right) \tag{7.10}
\end{equation*}
$$

If equation (7.10) is multiplied and divided by $P_{b} / P_{b}$ the result is per unit change in field voltage in terms of perunit change in power i.e.

$$
\begin{equation*}
\left(\Delta V_{f 5} / V_{f o}\right)=\left(R_{f} K_{1} K_{a} / V_{f o} K_{3} R_{5}\right)(P / J)\left(\Delta P / P_{b}\right) \tag{7.11}
\end{equation*}
$$

or

$$
\begin{equation*}
\left(\Delta \mathrm{V}_{\mathrm{f} 5} / \mathrm{V}_{\mathrm{fo}}\right)=\left(\mathrm{K}_{1} \mathrm{~K}_{2} \mathrm{~K}_{3}^{1}\right)(\Delta \mathrm{P} / \mathrm{P}) \tag{7.12}
\end{equation*}
$$

Where

$$
\begin{align*}
& K_{2}=\left(R_{f} K_{a} / V_{f o} R_{5} K_{3}\right)=\text { constant }  \tag{7.13}\\
& K_{3}^{1}=P / J . \tag{7.14}
\end{align*}
$$

It can be seen that for the negative acceleration the only change
is in $K_{2}$ i.e.

$$
\begin{equation*}
\left(\Delta V_{f 6} / V_{f o}\right)=K_{1} K_{4} K_{3}^{1}\left(\Delta P / P_{b}\right) \tag{7.15}
\end{equation*}
$$

Where

$$
\begin{equation*}
K_{4}=\left(R_{f} K_{a} / V_{f o} K_{3} R_{6}\right)=\text { constant. } \tag{7.16}
\end{equation*}
$$

By using the relation that

$$
\dot{\delta}=\ddot{\delta} / \omega
$$

one can obtain the relationships needed for the positive and negative velocity signals. Starting with the positive velocity equations gives

$$
\left(\Delta V_{f 3} / V_{f o}\right)=\left(K_{1} R_{f} E_{3} / V_{f o} R_{3}\right)
$$

But

$$
\begin{equation*}
E_{3}=K_{v}(+\dot{\delta})=K_{v}(\ddot{\delta} / \omega)=K_{v}(1 / \omega)(P / J \Omega) . \tag{7.19}
\end{equation*}
$$

Then

$$
\begin{equation*}
\left(\Delta v_{f 3} / v_{f o}\right)=\left(K_{1} R_{f} K_{v} / v_{f o} R_{3}\right)(1 / \omega)(\Delta P / J \Omega) . \tag{7.20}
\end{equation*}
$$

If (13) is then put in terms of percent change in power the result is

$$
\begin{equation*}
\left(\Delta V_{f 3} / V_{f o}\right)=\left(R_{f} K_{v} K_{1} / \Omega V_{f o} R_{3}\right)(1 / \omega)(P / J)(\Delta P / P) \tag{7.21}
\end{equation*}
$$

or

$$
\left(\Delta \mathrm{v}_{\mathrm{f} 3} / \mathrm{v}_{\mathrm{fo}}\right)=\left(\mathrm{K}_{1} \mathrm{~K}_{5} \mathrm{~K}_{6} \mathrm{~K}_{3}^{1}\right)(\Delta \mathrm{P} / \mathrm{P})
$$

Where

$$
\begin{align*}
& K_{5}=\left(R_{f} K_{v} / \Omega V_{f o} R_{3}\right)=\text { constant }  \tag{7.23}\\
& K_{6}=1 / \omega \tag{7.24}
\end{align*}
$$

For the negative part of the velocity the result is

$$
\begin{equation*}
\left(\Delta \mathrm{V}_{\mathrm{f} 4} / \mathrm{V}_{\mathrm{fo}}\right)=\mathrm{K}_{1} \mathrm{~K}_{7} \mathrm{~K}_{6} \mathrm{~K}_{3}^{1}(\Delta \mathrm{P} / \mathrm{P}) \tag{7.25}
\end{equation*}
$$

Where

$$
\begin{equation*}
K_{7}=R_{f} K_{V} / S_{f o} R_{4}=\text { constant } \tag{7.26}
\end{equation*}
$$

If one now integrates the velocity the angle relationship can
be found. the angle error is given by

$$
\begin{equation*}
\delta=\dot{\delta} / w=\ddot{\delta} / \omega^{2} \tag{7.27}
\end{equation*}
$$

For the positive angle error one has the following:

$$
\begin{equation*}
\left(\Delta V_{f 1} / V_{f o}\right)=K_{1} R_{f} E_{1} / V_{f o} R_{1} \tag{7.28}
\end{equation*}
$$

But

$$
\begin{equation*}
E_{1}=K_{e} \delta=K_{e} \ddot{\delta} / \omega^{2}=K_{e}\left(1 / \omega^{2}\right)(\Delta P / J \Omega) \tag{7.29}
\end{equation*}
$$

or

$$
\begin{equation*}
\left(\Delta V_{f 1} / V_{f o}\right)=\left(k_{1} R_{f} K_{e} / \Omega V_{f o} R_{1}\right)(\Delta P / J)\left(1 / \omega^{2}\right) \tag{7.30}
\end{equation*}
$$

If (20) is put in terms of percent power change the result is

$$
\begin{equation*}
\left(\Delta V_{f 1} / V_{f o}\right)=\left(K_{1} R_{f} K_{e} / \Omega V_{f o} R_{1}\right)(P / J)\left(1 / w^{2}\right)(\Delta P / P) \tag{7.31}
\end{equation*}
$$

or

$$
\begin{equation*}
\left(\Delta \mathrm{V}_{\mathrm{f} 1} / \mathrm{V}_{\mathrm{fo}}\right)=\mathrm{K}_{1} \mathrm{~K}_{8} \mathrm{~K}_{9} \mathrm{~K}_{3}^{1}(\Delta \mathrm{P} / \mathrm{P}) \tag{7.32}
\end{equation*}
$$

Where

$$
\begin{equation*}
R_{8}=R_{f} K_{e} / \Omega V_{f o} R_{1} \tag{7.33}
\end{equation*}
$$

Note that the $K_{e}$ used here is one tenth the actual calibration of the angle errior signal. See appendix $E$.

$$
\begin{equation*}
K_{9}=1 / \omega^{2} \tag{7.34}
\end{equation*}
$$

For the negative angle error signal the only change is in $\mathrm{K}_{8}$.
Therefore

$$
\begin{equation*}
\left(\Delta V_{f 2} / V_{f o}\right)=K_{1} K_{10} K_{9} K_{3}^{1}(\Delta \mathrm{P} / \mathrm{P}) \tag{7.35}
\end{equation*}
$$

Where

$$
\begin{equation*}
\left(K_{10}=R_{f} K_{e} / \Omega V_{f o} R_{2}\right. \tag{7.36}
\end{equation*}
$$

Therefore the total percent field voltage change is given by

$$
\begin{align*}
\left(\Delta V_{f} / V_{f o}\right)= & K_{1} K_{3}^{1}\left(K_{2}+K_{4}+K_{5} K_{6}+K_{7} K_{6}+\right. \\
& \left.K_{8} K_{9}+K_{10} K_{9}\right)(\Delta P / P) \tag{7.37}
\end{align*}
$$

Where the $K_{i}$ 's are defined above.
For the optimal design considered here the following parameters were used:

$$
\begin{aligned}
& \mathrm{R}_{\mathrm{f}}=43 \times 10^{3} \text { ohms } \\
& \mathrm{K}_{1}=2.5\left(\mathrm{R}_{\mathrm{y}} / 10^{5}\right)=255 \text { Maximum for stability } \\
& \mathrm{V}_{\mathrm{fo}}=35 \text { Optimum value } \\
& \Omega=2 \pi \times 60 \text { Volts } \\
& \mathrm{w}=2 \pi \times 1 \text { (resonant frequency) }
\end{aligned}
$$

There are two ways to define the power base and inertia of the system. One can either use the values on the high side of the potential and current transformers or the low side. When the latter
is used the values need to be modified by the turns ratios of the transformers. For this design the high side values of the power and inertia are used. The power base used is 6 KW . The reason for this value is that although the generator is a 15 KVA machine for laboratory safety and to perm.it severe transient experiments, it was rerated at $7.5 \mathrm{KVA}, 6 \mathrm{KW}$, and the transmission line impedances correspondingly increased. The system was capable of a wide variety of good transient swings.

The inertia is given by:

$$
\begin{equation*}
J=K_{a c} K_{p} K_{c} / K_{n} \tag{7.38}
\end{equation*}
$$

Where

$$
\begin{aligned}
& \mathrm{K}_{\mathrm{ac}}=\text { Noller output calibration }=0.0137 \mathrm{~V}-\mathrm{SEC}^{2} / \mathrm{RAD} \\
& \mathrm{~K}_{\mathrm{p}}=\text { Potential transformer turns ratio }=2 \\
& \mathrm{~K}_{\mathrm{c}}=\text { Current transformer turns ratio }=20 \\
& \mathrm{~K}_{\mathrm{n}}=\text { Noller sensitivity }=0.050 \text { Volts } / 1 \mathrm{KW} .
\end{aligned}
$$

Therefore the inertia $J$ is

$$
\begin{aligned}
& \mathrm{J}=\left(.0137 \mathrm{~V}-\mathrm{SEC}^{2} / \mathrm{RAD}\right)(40) /(0.05 \mathrm{~V} / \mathrm{KW}) \\
& \mathrm{J}=1096 \text { watt-sec}{ }^{2} / \mathrm{radian}
\end{aligned}
$$

Although the control design case power level was 3 KW , the nominal base was 6 KW . The signal calibration constants are given in appendix $E$. From these numbers the constant $K_{i}$ 's can be calculated. They are given below:

$$
\begin{aligned}
\mathrm{K}_{2} & =2.88 \times 10^{-6} \mathrm{sec}^{2} / \mathrm{rad}^{2} \\
\mathrm{~K}_{4} & =3.08 \times 10^{-7} \mathrm{sec}^{2} / \mathrm{rad}^{2} \\
\mathrm{~K}_{5} & =9.85 \times 10^{-6} \mathrm{sec}^{2} / \mathrm{rad}^{2} \\
\mathrm{~K}_{7} & =9.85 \times 10^{-6} \mathrm{sec}^{2} / \mathrm{rad}^{2}
\end{aligned}
$$

$$
\begin{aligned}
& \mathrm{K}_{8}=5.7 \times 10^{-6} \mathrm{sec}^{2} / \mathrm{rad}^{2} \\
& \mathrm{~K}_{10}=6.85 \times 10^{-5} \mathrm{sec}^{2} / \mathrm{rad}^{2} \\
& \mathrm{~K}_{3}^{1}=2.74 \mathrm{sec}^{2} / \mathrm{rad}^{2} \\
& \mathrm{~K}_{3}=0.3 \mathrm{sec}^{2} / \mathrm{rad}^{2} \\
& \mathrm{~K}_{6}=0.159 \mathrm{sec}^{2} / \mathrm{rad}^{2} \\
& \mathrm{~K}_{9}=0.0253 \mathrm{sec}^{2} / \mathrm{rad}^{2}
\end{aligned}
$$

This gives the entire characterization of the control system. DESIGN FOR OTHER SYSTEMS

In designing for other systems one must pick $R_{1}-R_{6}$ with respect to a chosen $R_{f}$ such that the $K_{2}, K_{4}, K_{5}, K_{7}, K_{8}$, and $K_{10}$ remain constant. To do this the base field voltage must be specified and the signal calibration constants must be known then the resistor values can be calculated.

The $K_{3}$ variable will vary with each particular system and power level. $K_{6}$ and $K_{9}$ also will vary with machine and system conditions. For the design that one would use, the $\omega$ and $P$ should be the worst case that could occur.

OPTIMAL CONTROL DESIGN SYMMARY
Parameters of the specific machine are:

$$
\begin{align*}
& \mathrm{K}_{3}=\mathrm{Pb} / \mathrm{J}=\text { reciprocal perunit inertia } \\
& \mathrm{K}_{6}=1 / \omega  \tag{7.24}\\
& \mathrm{K}_{9}=1 / \omega^{2}=\mathrm{K}_{6}^{2} . \tag{7.34}
\end{align*}
$$

Fixed Design constants are:

$$
\begin{aligned}
K_{1} & =65 \\
K_{2} & =2.88 \times 10^{-6} \\
K_{4} & =3.08 \times 10^{-7}
\end{aligned}
$$

$$
\begin{aligned}
& \mathrm{K}_{5}=6.7 \times 10^{-7} \\
& \mathrm{~K}_{7}=9.85 \times 10^{-6} \\
& \mathrm{~K}_{3}=5.7 \times 10^{-6} \\
& \mathrm{~K}_{10}=6.85 \times 10^{-5} \\
& \mathrm{R}_{\mathrm{f}}=3.43 \mathrm{~K} \\
& \mathrm{v}_{\mathrm{fo}}=35.5 \\
& \mathrm{~K}_{11}=\frac{\mathrm{R}_{\mathrm{f}} \mathrm{~K}_{1}}{\mathrm{~V}_{\mathrm{fo}}}=76 \times 10^{3}
\end{aligned}
$$

The optimal controller is given by:

$$
\begin{equation*}
\frac{\mathrm{AV}_{f}}{\mathrm{~V}_{\mathrm{fo}}}=\frac{\mathrm{R}_{\mathrm{f}} \mathrm{~K}_{1}}{\mathrm{~V}_{\mathrm{fo}}}\left[\sum_{\Lambda=1}^{\mathrm{m}} \frac{\mathrm{E}^{\mathrm{n}}}{\mathrm{R}_{\mathrm{n}}}\right]=76 \times 10^{3}\left[\sum_{\mathrm{n}=1}^{\mathrm{m}} \frac{\mathrm{E}_{\mathrm{n}}}{\mathrm{R}_{\mathrm{n}}}\right] \tag{7.6}
\end{equation*}
$$

Where

$$
\begin{align*}
& R_{6}=\frac{R_{f} K_{a}}{K_{4} V_{f o} K_{3}}=\left(1.395 \times 10^{11}\right)\left(\frac{K_{a}}{V_{f o}}\right)\left(\frac{J}{P_{b}}\right)  \tag{7.16}\\
& R_{5}=\frac{K_{a} R_{f}}{K_{2} V_{f o} K_{3}}=\left(1.495 \times 10^{10}\right)\left(\frac{K_{a}}{V_{f o}}\right)\left(\frac{J}{P_{b}}\right)  \tag{7.13}\\
& R_{4}=\frac{R_{f} K_{v}}{\Omega V_{f o} K_{7}}=\left(4.37 \times 10^{9}\right) \frac{K_{v}}{V_{f o}}  \tag{7.26}\\
& R_{3}=\frac{R_{f} K_{v}}{\Omega V_{f o} K_{5}}=\left(6.42 \times 10^{10}\right) \frac{K_{v}}{V_{f o}}  \tag{7.23}\\
& R_{2}=\frac{R_{f} K_{e}}{\Omega V_{f o} K_{10}}=\left(6.28 \times 10^{8}\right) \frac{K_{e}}{V_{f o}}  \tag{7.36}\\
& R_{1}=\frac{R_{f} K_{e}}{\Omega V_{f o} K_{8}}=\left(7.55 \times 10^{9}\right) \frac{K_{e}}{V_{f o}} \tag{7.33}
\end{align*}
$$

ANGLE ERROR PREDICTOR CHARACTERIZATION
From figure 7.2 one can see that the output voltage of the angle error predictor is given by:

$$
\begin{equation*}
v_{3}=\left(-R_{3} / R_{1}\right) v_{1}+\left(-R_{3} / R_{2}\right) v_{2} \tag{7.39}
\end{equation*}
$$

Given the three calibration constants of the respective transducers, one has the following:

$$
\begin{align*}
v_{1} & =K_{a c} \ddot{\delta}  \tag{7.40}\\
v_{2} & =K_{v} \dot{\delta}  \tag{7.41}\\
v_{3} & =K_{e}(+\Delta \delta) . \tag{7.42}
\end{align*}
$$

Where

$$
\begin{aligned}
& \ddot{\delta}=\text { acceleration in radian/second } \\
& \\
& \dot{\delta}=\text { velocity in radians/second } \\
& \Delta \delta=\text { angle error in radians } \\
& \mathrm{K}_{\mathrm{ac}}=\text { acceleration calibration constant } \\
& \mathrm{K}_{\mathrm{v}}=\text { velocity calibration constant } \\
& \mathrm{K}_{\mathrm{e}}=\text { angle error calbiration constant }
\end{aligned}
$$

$K_{a c}, K_{v}$, and $K_{e}^{\prime}$ are described in appendix $E$.
Noller accelerometer:

$$
\begin{equation*}
V_{1}=K_{n} K_{p} K_{c} P=\text { output of noller } \tag{7.43}
\end{equation*}
$$

Where $K_{p}, K_{c}, K_{n}$, and $P$ are as defined above. Therefore

$$
\begin{equation*}
K_{a c}=K_{n} K_{p} K_{c} J \tag{7.44}
\end{equation*}
$$

where $J$ is the system inertia as defined above.
Now assume the following:

$$
\begin{equation*}
\delta=\left(-K_{s} / \omega\right) \dot{\delta}+\left(-\mathrm{K}_{\mathrm{ss}} / \omega^{2}\right) \ddot{\delta} \tag{7.45}
\end{equation*}
$$

Where $K_{s}$ and $K_{s s}$ are symmetry constants needed to make the trajectories in the phase plane circles. Then

$$
\begin{equation*}
V_{3}=-K_{e}^{\prime}\left(K_{s} / \omega\right) \dot{\delta}-K_{e}^{\dagger}\left(K_{s s} / \omega^{2}\right) \ddot{\delta} \tag{7.46}
\end{equation*}
$$

or

$$
\begin{equation*}
v_{3}=-K_{e}^{\prime}\left(\left(V_{2} K_{s} / \omega K_{v}\right)+\left(V_{1} K_{s s} / \omega^{2} K_{a c}\right)\right) \tag{7.47}
\end{equation*}
$$

or

$$
\begin{equation*}
V_{3}=-K_{e}^{\prime} K_{s} V_{2} / \omega K_{v}-K_{e}^{\prime} K_{s s} V_{1} / \omega^{2} K_{a c} . \tag{7.48}
\end{equation*}
$$

From equations (26) and (35) one has the following:

$$
\begin{align*}
& R_{3} / R_{1}=K_{e}^{\prime} K_{s} / \omega K_{v}  \tag{7.49}\\
& R_{3} / R_{2}=K_{e}^{\prime} K_{s s} / \omega K_{a c} . \tag{7.50}
\end{align*}
$$

From equations (36) and (37) one can find the symmetry constants
$K_{s}$ and $K_{s s}$. They are

$$
\begin{aligned}
& \mathrm{K}_{\mathrm{s}}=27 \text { radians } \\
& \mathrm{K}_{\mathrm{ss}}=0.109 \text { radians }
\end{aligned}
$$

DESIGN FOR OTHER SYSTEMS
With the above constants one can design for the general case.
The ratio $R_{2} / R_{1}$ is given by:

$$
\begin{equation*}
R_{2} / R_{1}=\left(K_{s} / K_{s s}\right)\left(1 / K_{v}\right)\left(K_{n} K_{p} K_{c} J\right) \tag{7.51}
\end{equation*}
$$

or

$$
\begin{equation*}
\left.R_{2} / R_{1}=K_{s} / K_{s s}\right)\left(K_{a c} / K_{v}\right) \tag{7.52}
\end{equation*}
$$

The ratio $R_{3} / R_{1}$ is given by equation (7.49). From these equations (7.49) and (7.52) one has the following:

$$
\begin{align*}
& R_{3}=\left(K_{s} K_{e} / K_{v}\right)(1 / \omega) R_{1}  \tag{7.53}\\
& R_{2}=\left(K_{s} K_{a c} / K_{s s} K_{v}\right) R_{1} \tag{7.54}
\end{align*}
$$

With equations (7.53) and (7.54) one can design and angle error predictor by specifying $R_{1}$ and inserting the values of the calbration constants. These along with the constants $K_{s}$ and $K_{s s}$ as defined above can then be used to solve for $R_{3}$ and $R_{2}$. Putting these resistor values into the circuit shown in figure 7.2 one has the required angle error predictor.

We have selected a base of $6 \mathrm{KW}, 7.5 \mathrm{KVA}$, and 20 amperes.

However, we used a current transformer rated 100 amperes to 5 ampere. If this $C T$ were reconnected for 50 amperes to 5 amperes, then the Noller output and $K_{a}$ would each be increased by two to $K_{a}=0.60$ volts per perunit power change. It is desirable that the CT be conservatively rated so that it does not saturate during transients.

CHAPTER 8

PRIME MOVER TORQUE CONTROL

The present D.C. motor being used as a prime mover is a constant speed motor. To simulate a steam turbine the prime mover should have a constant torque. The constant torque control is developed here.

The torque of the D.C. motor is given by:

$$
\begin{equation*}
I_{\text {armature }} \times I_{\text {field }}=\text { Torque } \tag{8.1}
\end{equation*}
$$

or

$$
\begin{equation*}
\mathrm{T} \propto \mathrm{~V}_{\mathrm{RA}} \times \mathrm{V}_{\mathrm{F}} \tag{8.2}
\end{equation*}
$$

The torque $T$ should be constant for correct operation. If the armature current changes then the field current should make the appropriate change to keep the torque constant. The circuit used to obtain this result is shown in figure 8.1. The control operates on the field of the prime mover when a change in armature current is sensed. When the torque control $R_{t}$ is set to the desired value, $I_{a}$ increases until $V_{R A}$ equals the bias set by $R_{t}$. If $I_{a}$ changes then $\mathrm{V}_{\mathrm{RA}}$ changes giving a difference at the input of the operational amplifier and thus changing the field voltage by operating on the amplidyne. This keeps the produce of $I_{a} \times I_{f}$ constant (i.e. the torque is kept constant). Figure 8.3 shows a block diagram of the control in terms of field voltage, the voltage across $R_{a}$, and the gain constants.

The spark gap across the field circuit is for protection of the field. The gap is set at 1 mil and started to spark across at 800 V and stopped at 600 V .

There was a 0.5 VAC noise across $R_{a}$ with a frequency of 60 hertz.

Since the amplidyne's frequency limit is approximately 20 hertz there was no need to filter the signal $V_{R A}$.

Since the prime mover is now a constant torque device it will run away when the generator goes out of step (i.e. when the generator goes out of synchronism). To guard against this problem an over speed trip-out is necessary. Figure 8.2 shows the circuit used for this purpose. The trip-out speed can be set by adjusting $R_{j}$. It is now set at 1390 RPM. This is approximatel $15 \%$ over speed trip-out. This high setting is due to the $133 A-B$ lab having trouble synchronizing the generator and continually tripping the relays. A normal setting would be $2-5 \%$ over speed.

Conclusion:

The constant torque controller works quite well. It solves the problem of getting constant torque out of the prime mover. It also solves the problem of drift that the prime mover had previously. T This had been a major problem. The proof that the prime mover is now constant torque device is that it does definitely run away when the generator goes out of synchronism.

CHAPTER 9

VOLTAGE REGULATOR

Since all generator systems have voltage regulators one was implemented in this system. The 3-phase terminal voltage was transformed to 6 phase and square law rectified giving only the changes in terminal voltage $\Delta V$.

Without a phase lead network in the voltage loop the system went unstable at a gain of $K_{a m p}=2.2$. When the phase lead network shown in figure 9.1a was implemented the maximum gain was $K_{a m p}=25$. This was using all the voltage signal. Shown in figure 9.2 is the response for case 1 using the previously defined optimal weighting on the acceleration, velocity, and angle error and all of the voltage signal. The gain $K_{a m p}=7.5$.

Shown in figure 9.1b is another type of phase lead network. The feedback is taken from the 820 ohm resistor giving current feedback or a $90^{\circ}$ phase lead. With this feedback added in, case 1 was run again with a gain of $K_{a m p}=15$. Figure 9.3 shows the results. One can see from these results that the controller is no longer optimal. This is expected since 2 new state variables were added to the weighting vector Using these new control variables a new nearly optimal weighting vector was found by trial and error.

The results are given below: "
Panel dial readings

$$
\begin{aligned}
\mathrm{R}_{1} & =10 \\
\mathrm{R}_{2} & =2.10 \\
\mathrm{R}_{3} & =4.0
\end{aligned}
$$

$$
\begin{aligned}
& R_{4}=1.8 \\
& R_{5}=1.0 \\
& R_{6}=7.0 \\
& R_{7}=1.0 \\
& R_{8}=5.0
\end{aligned}
$$

where $R_{1}$ through $R_{6}$ are defined previously and
$R_{7}$ weights the positive swing of negative voltage.
$R_{8}$ weights the negative swing of positive voltage. Figure 9.4 a and b shows the results of the new controller.

A different phase-lead network (Fig. 9.5) was tried and the results are presented in Fig. 9.6(a and b). Without the voltage $V_{t}$ feedback, $\dot{\delta}$ is quenched in $\frac{1}{2}$ cycle of oscillation (lose line) as discussed in Chapter I, case \#1 (Fig. 9.6a). However, the voltage drifts about its normal level and the voltage deviation is quite large during transient disturbances. The control including $V_{t}$ feedback seems to regulate the voltage fairly well for the price of a more oscillatory $\delta$ (Fig. 9.6b). For this particular control, the following weighting vector was used

$$
\underset{\sim}{X}=\left[\begin{array}{l}
0.04 \\
0.47 \\
0.04 \\
0.59 \\
2.80 \\
0.50 \\
0.15 \\
0.08
\end{array}\right]
$$

There seems to be an optimal trade off between the voltage regulation and the magnitude of the mechanical oscillation control.

This is a suitable topic for additonal study, research, and optimization. (chapter 10)

Summary:
Even when the voltage regulator was added to the system excellent results were still obtained. With further research one should be able to adjust the system to a more typical loop for the voltage loop. Having done this, the optimization procedure could be rerun and the optimal weighting vector redefined.

SUGGESTIONS FOR FUTURE RESEARCH

1) Obtain a mathematical model for the system and run computer simulations to obtain the optimal weighting vector. Once the computer simulations are giving correct results design a control system for a larger machine ( 300 MW ).
2) Design a phase lead network to compe nsate for the field and amplidyne phase lags. After this phase lead network is implemented obtain the new weighting vector.
3) Do a complete design of the phase lead network used in the voltage regulator loop to obtain a gain that is more typical of voltage regulators. With the regulator working use the plus $V$ and minus $V$ as control signals and obtain the new optimal weighting vector.
4) Design the circuitry for the statistically optimum transducer signals.
5) Make the phase meter output independent of the magnitudes of the input voltages $V_{1}$ and $V_{2}$ by some type of voltage regulator or clipping the voltages with appropriate compensation in the square-law device, or compensating by dividing by the product of the voltages.

## APPENDIX A

## CALCULATION OF THE OPTIMAL WEIGHTING VECTOR

This appendix presents the procedure of setting up the linear constraints for calculating the optimal weighting vector.

From past experience, a cost function $f_{0}$ is choosen arbitraily. Take

$$
\begin{equation*}
f_{0}=0.3 x_{1}+0.3 x_{2}+x_{3}+0.3 x_{4}+0.1 x_{5}+0.5 x_{6} \tag{A-1}
\end{equation*}
$$

And calculate at each pulse instant, for example; at the first pulse instant (Fig. A-1)

$$
\begin{equation*}
-0.25 x_{2}+0.12 x_{5}=1.16 \tag{A-2}
\end{equation*}
$$

That is, only $\Delta(-\delta)$ and $+\ddot{\delta}$ can contribute to the control signal at the first instant. For the second pulse instant,

$$
\begin{equation*}
-0.125 x_{2}+0.075 x_{3}+0.1 x_{5}=1.16 \tag{A-3}
\end{equation*}
$$

In this case $\Delta(-\delta),+\dot{\delta},+\ddot{\delta}$ can contribute to the desirable signal. Continue to calculate at each instant, twelve linear constraints are formed and summarized as follows;

| Minimize $\mathrm{f}_{0}$ |  |  |  |
| :---: | :---: | :---: | :---: |
| Subject to |  |  |  |
|  | $-2.5 x_{2}$ | $+0.12 x_{5}$ | $=1.16$ |
|  | $-1.25 \mathrm{x}_{2}+.075 \mathrm{x}_{3}$ | $+0.1 \mathrm{x}_{5}$ | $=1.16$ |
| $.75 \mathrm{x}_{1}$ | $+0.2 \mathrm{x}_{3}$ | $+0.05 x_{5}$ | $=0.13$ |
| $2.0 \mathrm{x}_{1}$ | $+.25 x_{3}$ | $-.02 x_{6}$ | $=-.39$ |
| $2.0 \mathrm{x}_{1}$ | $+.2 \mathrm{x}_{3}$ | $-.07 \mathrm{x}_{6}$ | $=-.28$ |
| $.75 \mathrm{x}_{1}$ | $+.08 \mathrm{x}_{3}$ | -. $1 \mathrm{x}_{6}$ | $=.52$ |

$$
\begin{array}{rlrl}
-0.1 x_{4} & -.02 x_{6} & =-.065 \\
-1.25 x_{2} & -0.22 x_{4}+0.06 x_{5} & -.07 x_{6} & =-.22 \\
-2.5 x_{2} \div .03 x_{3}-0.23 x_{4}+0.1 x_{5} & -.1 x_{6} & =-.0 .0 \\
-2.3 x_{2}--.125 x_{4}+0.1 x_{5} & -.05 x_{6} & =0.25 \\
-1.25 x_{2}-.02 x_{4}+0.08 x_{5} & & =0.195 \\
0.125 x_{3} & -.02 x_{6} & =0.0
\end{array}
$$

Using the computer subroutine Alphac (13) to solve this system of equations, the problem is first formulated into the prescribed computer input format as shown in Listing One. This computer subroutine uses the Simplex method in solving the optimization problem.

The optimal weighting calculated by the computer is

$$
\underset{\sim}{X}=\left[\begin{array}{l}
0.0 \\
.442 \\
0.0 \\
.56 \\
2.6 \\
0.0
\end{array}\right]
$$

Then this weighting can be realized by the input and feedback resistors of the summing amplifier.

$$
\begin{aligned}
& x_{1}=R_{f} / R_{1} \\
& x_{2}=R_{f} / R_{2} \\
& x_{3}=R_{f} / R_{3} \\
& x_{4}=R_{f} / R_{4} \\
& x_{5}=R_{f} / R_{5} \\
& x_{6}=R_{f} / R_{6}
\end{aligned}
$$

For this report, $R_{f}=43 \mathrm{~K} \Omega$, and from the resistive graph (Fig. A.2):

$$
\begin{aligned}
& R_{1}=\infty \\
& R_{2}=97 \mathrm{~K} \\
& \mathrm{R}_{3}=\infty \\
& \mathrm{R}_{4}=77 \mathrm{~K} \\
& \mathrm{R}_{5}=16.5 \mathrm{~K} \\
& \mathrm{R}_{6}=\infty
\end{aligned}
$$

Panel resistor dial readings:

$$
\begin{aligned}
& \mathrm{R}_{1}=\infty \\
& \mathrm{R}_{2}=2 \\
& \mathrm{R}_{3}=\infty \\
& \mathrm{R}_{4}=1 \\
& \mathrm{R}_{5}=0 \\
& \mathrm{R}_{6}=\infty
\end{aligned}
$$

This is the first vector used, and was excellent for lost line, but since it was not designed for a gained line, it was not optimal for that application. After laboratory experimentation with both types of transients, the vector was modified by experience to provide the optimal weighting given on page 2 and in the circuit in Figure C-3.

## APPENDIX B

ANGLE PREDICTOR PROGRAM
The prediction of the future state of the rotor angle is based upon the three available signals (actual angle, velocity, and acceleration). To get the predicted angle from these signals they must be weighted such that they give the desired constant voltage output or the error between $\delta$ and $\delta_{t}$. This is given below:

$$
\begin{equation*}
\delta_{t}=k_{1} \delta+k_{2} \dot{\delta}+k_{3} \ddot{\delta} \tag{B.1}
\end{equation*}
$$

In the steady state the velocity and the acceleration are equal to zero. Therefore the future angle is the actual angle. Thus the coefficient of the angle $k_{1}$ is set equal to 1.0 .

The instrumental variable method is used to solve equation (B.1) for the coefficients $k_{2}$ and $k_{3}$. 100 data points were used covering the first five seconds of the transients. Only loss of line was considered.

Equation (B.1) can be represented by

$$
\begin{equation*}
S K=W \tag{B.2}
\end{equation*}
$$

where
$\underline{K}$ is the coefficient vector
$\underline{S}$ is the signal matrix
$s_{1 j}=\delta_{j}$
$s_{2 j}=\dot{\delta}_{j}$
$s_{3 j}=\ddot{\delta}_{j}$
W is the target vector.

To get a solution to equation (B.2) one must average the equations. This is done by multiplying by $\underline{S}^{T}$, thus giving:

$$
\begin{equation*}
\underline{S}^{\mathrm{T}} \underline{\mathrm{SK}}=\underline{\mathrm{S}}^{\mathrm{T}} \underline{W} . \tag{B.3}
\end{equation*}
$$

This gives an averaged system of three equations and three unknowns. With $k_{1}$ being known equation (B.3) is modified to give:

$$
\begin{equation*}
\underline{S}^{\prime} \underline{S}^{\prime} \underline{K}^{\prime}=\underline{S}^{\prime T} \underline{W}^{\prime} \tag{B.4}
\end{equation*}
$$

where

$$
\begin{aligned}
& s_{2 j}=\underline{S}^{\prime} 1_{j} \\
& s_{3 j}=\underline{s}^{\prime} 2_{j} \\
& \underline{K}^{\prime}=\left(K_{2}, K_{3}\right)^{T} \\
& \underline{W}^{\prime \prime}=\underline{W}-s_{1 j} .
\end{aligned}
$$

To emphasize the more critical points of the transients a weighting vector was used. The most important points being given a weighting of 1.0 and the least important 0.3 .

The program used to implement this is given in listing two.
The results of the computer runs are given in output one.
The signal $\Delta \delta$ or the feedback signal is labeled "error signal" in the plotted outputs.

APPENDIX C

SYSTEM SIMULATION

The system used to simulate an actual power plant is described here. Table C. 1 gives the name plate values and figures C.1-C.4 show the system block diagram and excitation circuits.

TABLE C. 1

| Main generator Auxiliary generator | 3 phase <br> 15 KVA | $\begin{aligned} & \mathrm{f}=60 \mathrm{~Hz} . \\ & 220 \mathrm{~V} \end{aligned}$ | $\begin{aligned} & 1200 \mathrm{RPM} \\ & 39.4 \mathrm{~A} / \text { Terminal } \end{aligned}$ |
| :---: | :---: | :---: | :---: |
| Main generator |  |  |  |
| exciter amplidyne | 220V | 1725 RPM | 60 Hz . |
|  | 1.5 KVA | 115V | 12 AMP. |
|  | $\mathrm{F}_{1} \mathrm{~F}_{2}$ | 980 OHMS | 0.120 AMP. |
|  | $\mathrm{F}_{3} \mathrm{~F}_{4}$ | 980 OHMS | 0.120 AMP. |
|  | $\mathrm{F}_{5} \mathrm{~F}_{6}$ | 43 OHMS | 0.598 AMP. |
|  | $\mathrm{F}_{7} \mathrm{~F}_{8}$ | 43 OHMS | 0.598 AMP. |

Prime mover

| DC motor -shunt- | 20 HP | 115 V |
| :--- | :--- | :--- |
|  | 1150 RPM | 148 AMP. |


|  |  |  |  |
| :--- | :--- | :--- | :--- |
| Torque contro1 <br> amplidyne | 115 V | 13 AMP | 60 Hz. |
|  | 250 V | 3 AMP. | 0.75 KW |
|  | $\mathrm{F}_{1} \mathrm{~F}_{2}$ | 436 OHMS | 0.301 AMP. |
|  |  |  |  |
| Amplifiers | Burr Brown | $1542 / 25$ | $3009 / 15 \mathrm{c}$ |

## APPENDIX D

LINE SIMULATION

A long line is simulated by the use of a variable auto-transformer and inductive coils on the secondary of the transformer. The figure below show the circuit for the line simulation on a one phase basis. With breakers

$$
\begin{aligned}
& \mathrm{L}_{1}=1 \mathrm{mh}, 0.024 \text { ohms } D C \\
& \mathrm{~L}_{2}=2.8 \mathrm{mh}, 0.08 \text { ohms } D C \\
& \mathrm{~L}_{3}=\text { variable auto-transformer }
\end{aligned}
$$

$B_{1}$ and $B_{2}$ closed and breaker $B_{3}$ open the equivalent impedance is;

$$
\mathrm{z}_{\mathrm{eq}}=1.28 \text { ohms. }
$$

This is the normal operating condition. With breakers $B_{1}$ and $B_{3}$ open and breaker $B_{2}$ closed the condition is that of lost line and the equivalent impedance is;

$$
z_{\mathrm{eq}}=4.06 \text { ohms. }
$$



APPENDIX E

## CALIBRATION OF SIGNALS

From figure 2 chapter 4 the following relationships hold.
Change in $\delta=20.1$ degrees electrical
Change in $V_{0}$ of the angle sensor $=6.21 \mathrm{~V}$
This implies that the calibration constant of the angle is

$$
\begin{equation*}
K_{\delta}=6.21 \mathrm{~V} / 20.1^{\circ}=0.304 \mathrm{~V} / 1^{\circ} \tag{E.1}
\end{equation*}
$$

or

$$
\mathrm{K}_{\delta}=17.45 \mathrm{~V} / \mathrm{radian}
$$

also

$$
\begin{equation*}
K_{\delta} / K_{v}=v_{\delta} \omega / v_{v} \tag{E.2}
\end{equation*}
$$

where

$$
\begin{aligned}
& \mathrm{K}_{\delta}=17.45 \mathrm{~V} / \text { radian } \\
& \mathrm{V}_{\delta}=3.8 \mathrm{~V} \quad \text { from figure } 2 \text { chapter } 4 \\
& \mathrm{~V}_{\mathrm{v}}=0.300 \quad \text { from figure } 2 \text { chapter } 4
\end{aligned}
$$

Therefore

$$
K_{v}=\left(\frac{17.8}{(2 \pi 1.07)}\right)\left(\frac{v_{v}}{v_{\delta}}\right)=2.644(0.0847)=0.224
$$

Also

$$
\begin{equation*}
K_{v} / K_{a c}=V_{v} \omega / v_{a c} \tag{E.3}
\end{equation*}
$$

where

$$
\begin{aligned}
& \mathrm{K}_{\mathrm{v}}=0.224 \mathrm{~V} \text {-second/radian } \\
& \mathrm{V}_{\mathrm{v}}=0.300 \mathrm{~V} \quad \text { from figure } 2 \text { chapter } 4 \\
& \mathrm{v}_{\mathrm{ac}}=0.125 \mathrm{~V} \quad \text { from figure } 2 \text { chapter } 4
\end{aligned}
$$

Therefore

$$
\mathrm{K}_{\mathrm{ac}}=0.01375 \mathrm{~V}-\text { seconds }^{2} / \mathrm{radian}
$$

From figure 4.11a one can see that the change in target
angle is

$$
\Delta \delta_{t}=(-0.4)-(-5.86)=5.46 \text { volts. }
$$

The change in angle for the same transient is

$$
\Delta \delta=16.61^{\circ}=0.288 \text { radians } .
$$

Therefore the angle error calibration constant is

$$
K_{e}^{\prime}=5.46 \mathrm{~V} / 0.288 \text { radians }=19 \text { volts } / \text { radian } .
$$

In the nonlinear adder only one tenth of this value was used.
Therefore the new calibration is

$$
\mathrm{K}_{\mathrm{e}}=0.1 \times \mathrm{K}_{\mathrm{e}}^{\prime}=1.9 \mathrm{volts} / \mathrm{radian}
$$

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(a)

(b)


FIGURE 1.1 (one of three)
CASE \# 1 - DESIGN CASE
$\delta=1$ volt/in.
$\delta=5$ mvolts/in.

(d)


## (e)



FIGURE 1.1 (three of three)
(a)


FIGURE 1.2 (one of four)


FIGURE 1.2 (two of four)

(three of feur)
(xnof Jo Jnof) $Z^{\bullet} \tau$ gynols



§
ลิ
©


## FIGURE 1.4 (one of three)





§


LOSE LINE UNCONTROLLED


$$
\begin{aligned}
& \dot{\delta}=2 \mathrm{mv} / \mathrm{in} \\
& \delta=1 \mathrm{v} / \mathrm{in}
\end{aligned}
$$



MOMENTARY OPEN THEN RECLOSE
OPEN TIME $: 1 / 2$ second

FIGURE 1.6


MIN $=7$
GAIN $=2.2$

$$
\dot{\delta}=20^{\mathrm{mV}} / \mathrm{in}
$$

$$
\delta=1 \mathrm{~V} / \mathrm{in}
$$





Fl6. 2.2

FIG 2.3


Fig 2.4


FIg 2.5



Fig. 3.2


Fig 3.3

(b)

Fig. 3.4



2b

a. Acceleration
b. Velocity
c. Angle

Power $=7.53 \mathrm{KW}$
Power factor $=0.81$
Lose Line
Resonant frequency $=1.07 \mathrm{~Hz}$

8.vs. $\delta$

2d


$$
\begin{gathered}
\circ^{\circ} \cdot v s . \delta \\
2 \mathrm{e}
\end{gathered}
$$



FIGURE 2 (three of three)


> 3 c
> FIGURE 4.3 (one of three)

## a. Acceleration

b. Velocity
c. Angle

Power $=5.25 \mathrm{KW}$
Power factor $=0.95$
Lose Line
Resonant frequency $=1.00 \mathrm{~Hz}$

© .vs. $\delta$
3d

$$
\dot{0}^{\circ}=0.2 \mathrm{v} / \mathrm{in} .
$$

$$
8^{\circ} . \text { vs } . \delta^{\circ}
$$

$3 e$

FIGURE 463 (two of three)


FIGURE 4.3 (three of three)
2H TS6 ${ }^{\circ} 0=$ Kouenbexj дuruosey a. Acceleration
b. Velocity
c. Angle
Power : 2.4 KW
Power factor $=0.8$
Lose Line-


$\delta=2.0 \mathrm{~V} / \mathrm{in}$.

© . vs. $\delta$
4d

$$
\delta^{\circ}=0.2 \mathrm{~V} / \mathrm{in} .
$$

$$
\ddot{\delta}=0.2 \mathrm{~V}
$$ /in.

" 8 . Vs. $\delta^{\circ}$
$4 e$

$$
\delta=2.0 \mathrm{v} / \mathrm{in} .
$$



$$
\begin{gathered}
-\delta^{\circ} \text {.vs. } \delta \\
4 \mathrm{f}
\end{gathered}
$$

FIGURE 4.4 (two of two)



5b


5
FIGURE 4.5
a. Acceleration
b. Velocity
c. Angle

Power $=2.4 \mathrm{KW}$
Power factor $=0.8 \mathrm{lag}$
Lose Line
Resonant frequency $=0.941 \mathrm{~Hz}$
Resonant frequency $=0.73 \mathrm{~Hz}$

## Lose Line



$\stackrel{0}{0}$



FIGURE 4.7






FIGURE 4all



FIGURE 4,13


FIGURE 5.1


FIGURE 5.2


NOTE THIS IS ONLY ONE HALF OF THE CIRCUIT PER PHASE.

FIGURE 5.3


FIGURE 5.4


## PARTS LIST

```
R1}=22.6k 2
R2 = 43.2k 1/2W
R3}=8.06k 2
NOTE ALL RESISTORS 1%
Cl = 2uF 100WVDC----WMF 02W2
D1 = 1N2071
D2 = 1N3041B
T1 = STANCOR TP-2
T2 = TRIAD N-68X
Vm}=\pm20VD
```


$7 a$.


## 7b.

## FIGURE 5.7

## 7a. Lose line.

7b. Gain line.
Power $=7.35 \mathrm{~kW}$
Power factor $=0.82$
$\mathbf{V}_{\mathbf{a}}=-6.7 \mathbf{v}$
$V_{b}=-0.5 v$







(

CONTROL
FIGURE 8.2

OVER SPEED TRIP-OUT

$K_{1}$ is gain of DC motor
$K_{2}$ is gain of op amp
$K_{3}$ is gain of Amplidyne
FIGURE 8.3

(a)

(b)


## Fig. 9.2



Fig. 9.3


(a)

Figure 9.4

(b)

Figure 9.4


FIG. 9.5


| WT |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  |
|  |  | - |  |  |  |  |
|  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |
|  | +3, |  |  |  |  |  |
|  |  |  |  |  |  |  |
|  | $\bigcirc$ | - |  |  |  |  |
|  | $\bigcirc$ |  |  |  |  |  |
|  | 1-1. | - |  | $\cdots$ |  |  |
|  |  | , |  |  |  |  |
|  |  |  |  |  |  |  |
|  |  | + |  | - | ) | ) |
|  |  | , |  |  |  |  |
|  |  |  |  |  |  |  |
|  | T. | , |  |  |  |  |
|  | 1. |  |  |  |  |  |
|  | - |  |  |  |  |  |
|  | $\cdots$ |  |  |  |  |  |
|  | -1... |  |  | - |  |  |
|  | ( ${ }^{\text {a }}$ |  |  | $\cdots$ |  | Ti |
|  | 1- |  |  |  |  |  |



Figure A-1





Figure A-2

FIGURE C. 1

NONLINEAR ADDER CIRCUIT WITH OPTIMAL RESISTOR VALUES
values given are for the optimal vector x on page 2

VOLTS DC
MAIN GENERATOR EXCITATION SYSTEM
Figure c. 2

FIGURE $\subset .4$

INITIALZ
CONVERT
SETUP
PRIMAL
END
THIS program calculates the optimal weighting vecctor 1361 NAME ROWS

| N | ORJ |
| :---: | :---: |
| F | ROW 1 |
| F | ROW2 |
| F | ROW3 |
| F | ROW4 |
| F | ROW5 |
| E | ROW6 |
| F | ROW7 |
| E | ROW8 |
| F | ROW9 |
| F | ROW10 |
| E | ROW 11 |
| F | ROW 12 |

## COLUMNS

$x 1$
$\times 1$
$\times 1$
$\times 2$
$\times 2$
$\times 2$
$\times 2$
$\times 3$
$\times 3$
$\times 3$
$\times 3$
$\times 4$
$\times 4$
$\times 4$
$\times 5$
$\times 5$
$\times 5$
$\times 5$
$\times 6$
$\times 6$
$\times 6$
RHS
$R$
$R$
$R$
$R$
$R$
$R$

ENDATA

| ORJ | .3 |
| :--- | :--- |
| ROW4 | $\bullet$. |
| ROW6 | $\bullet 075$ |
| ORJ | .3 |
| ROW2 | -.125 |
| ROW9 | -.25 |
| ROW11 | -.125 |
| ORJ | 1. |
| ROW3 | .2 |
| ROW5 | .2 |
| ROW9 | .03 |
| ORJ | .3 |
| ROW8 | -.22 |
| ROW10 | -.125 |
| ORJ | .1 |
| ROW2 | .1 |
| ROW8 | .06 |
| ROW10 | .1 |
| ORJ | .5 |
| ROW5 | -.07 |
| ROW 7 | -.05 |
| ROW1 | 1.16 |
| ROW3 | .13 |
| ROW5 | -.28 |
| ROW7 | -.065 |
| ROW9 | 0. |
| ROW11 | .195 |

0 OBJ

ROW 1

| ROW 3 | 0.075 |
| :---: | :---: |
| ROW 5 | $\bigcirc$ ? |
| ROW 1 | -. 25 |
| ROW8 | -. 125 |
| ROW 10 | -. 23 |
| ROW 7 | . 075 |
| ROTV 4 | . 25 |
| ROWG | $\bigcirc 08$ |
| ROW 12 | 012 |
| ROW7 | -0.n |
| ROW9 | -. $? 3$ |
| ROW 11 | -.02 |
| ROW 1 | -12 |
| ROW2 | 0.5 |
| ROW9 | 01 |
| ROW 11 | 007 |
| ROW4 | -.02 |
| ROWn | -. 1 |
| ROW 12 | -.0n |


| ROW | 1.16 |
| :--- | :--- |
| ROW4 | -.39 |
| ROW6 | .52 |
| ROW8 | -.72 |
| ROW10 | 0.25 |
| ROW 12 | 0. |

ROW12 X6
LISTIN $B$ TWO
PRUGRA.i AEWANI (INPUT,OUTPUT)

```COMNON/CONST/iIN, ANGCONS, FRROR1(1J0)
```



```COMVUN/MAT2/ANGTARG(1UO), FRROR (1U0), S(1U0, 3), ST(3, 100), W(100), WAYV1ECT(100)
            UIMENSION ANG(100),VEL(100),ACC(100)
    C MIO IS THE NUMOER OF COLUMNS IN S. AND N IS THE NUMBR OF DOWS IN S
        A:vUCONS IS THL LOEFFICIENT OF the ANGLE
            READ IO,M,N,ANGCONS
            PRINT 9,it,N,ANGCONS
            DO 20 I=1,N
            !(I)=n.0
            DO 20 J=1,M
            S(I,J)=0.0
            ST(J,I)=0.0
            20 conttnuja
            ERBANX=0.0
            ERMiN N=0.0
            IT=0
C LHU OF INITIALIZATION
            n) }50\textrm{J}=1,\textrm{A
            KEAD GO,(S(i,J),J=1,:`),N(I)
            5n CONTINIIF
            nu 52 J=1,N
            <MAD 6?,WAYVFCT(J)
                    -2 cuintinue
            FRINT 63
            Du 51 I=1,i
            P!रINT GI,(S(I,J),J=1,M),W(I),:#AYVECT(I),I
=1 Continue
            no 70 I=1,\because
            00 70) J=1,0
            ST(I,J)=S(J,I)
            70 covtinulif
            DO 26 J=1,N
            ANG(J)=S(J,l)
            ACC(J)=S(J,?)
            26 CONTINIF
            CALL Mult
            CALL SOLVF
            CALL CHECK
            CALL GRAF(ANGTARG,ERPOR,ANG,ERROR1,ACC,N)
            O3 CONTINJE
            IT=IT + I
            IF(IT.GT.)IGO TO 75
            PRINT 29,IT
            ERNAAX=R:IAX(FRROR,N)
            ERMTN=RMIN(FRROR,N)
            IF(ERMAX.GT•O.?)GO TO 27
            IF(ERMIN.LT•-O. .) FOO TO 27
            GO TO 75
            27 CONTIMUE.
            CALL MF:IVFCT
            C.ALL M!JLT
            CALL SOLVE
            CALL EHFCK
```

(ALL GRAF (ANGTARG,ERROR, ANG,ERROR1, ACC, N)
GO TO 28
75 CONTINUE
10 FORMAT(2I5,F10.4)
○ FORMAT $(q x, *$ THE NUMBER $\cap F$ VARIABLES IS $=*, q X, I 5,3 X, *$ THE NUMBER OF T
 3/1)

60 FORMAT (4F10.4)
51 FORMAT(3F10.4, 1UX,F1U.4, ?UX,F10.4,15X,I3)
G2 FORMAT(F10.4)
 2,12X,*TIAF POINT*)
STOP
END
SIFEROIIT INF CHECK



1F:CT(100)
10 $21 \mathrm{~J}=1$, N !
ANGTARG(J) $=0.0$
FRROR (J)=0.0
rRRORI (J) = = . 0
$\rightarrow 1$ CONTINIIF
$X Y Z(1)=$ ASGrONS
$X Y Z(ح)=R(T)$
$X Y Z(2)=B(2)$
ก.) $430 \mathrm{~T}=\mathrm{j}$, N
DO $430 \mathrm{~J}=1,4$
AMUTARG(I)=ANUTARG(I) + S(I,J)\#XYZ(J)
430 CONTINGF
C ERLOR IS THF ERROR IN PREDICTION.

- Lraod is the brrop betwetn the actual anule and the target angle. DO $435 \mathrm{~J}=5, \mathrm{~N}$ LRROR (J)=AMGTARG(J) - V(J)
435 comtinue
DO $436 \quad \mathrm{~J}=1, \mathrm{~N}$
ERPROR1 $(J)=S(J, 1)-\operatorname{ANGTARC:~}(J)$
136 CONTINIJF
RFT! TRM!
END
GUUROUTINE GRAF (AV,GV,CV,DV,EV,MAXT)
UIIE!
©IAENSION AA(3UU), $B B(300), C(1300), D D(30).), E F(300)$
INTEGER PLOT(130)
$T_{1}: \therefore=130$
$1 \mathrm{l}=1$.
SCALE $=24.0$
$N=: A \Delta X T$
DO $101 \mathrm{~J}=1, \operatorname{MAXT}$
$A \cap(J)=A V(J)$
$B P(J)=B V(J)$
$C C(J)=C V(J)$
$2(J)=n v(J)$
EE(J) = FV:J)
101 CONTINiJe

C FIND THF MIN OF ALL VECTORS
AMIN $=\operatorname{RiAIN}(A A, N)$
BMIN $=\operatorname{RMIN}(B R, N)$
CMIN $=\operatorname{RMIN}(C C, N)$
DMIN=RMTN(OD,N)
EMIN $=$ R:MIN(FE,N)
DO $120 \mathrm{~J}=1, \mathrm{~N}$
$A A(J)=A A(J)-A M \cdot I N$
$B R(J)=B B(J)-B M I N$
$C C(J)=C C(J)-C M I N$
DD (J) =DD(J)-DMIN
EE(J)=EE(J)-EMIN
120 CONTINUE
FIND MAX OF ALL THE MODIFIED VECTORS NOW
$A M A X=\operatorname{RMAX}(A A, N)$
BMAX $=\operatorname{RMAX}(R B, N)$
$C M A X=R M A X(C C, N)$
DMAX=RMAX(nD,N)
EMAX $=\operatorname{RMAX}(E E, N)$
C INITIALIZF TO ZFRO PLOT VECTOR
DO $150 \mathrm{~J}=\mathrm{IZ}, \mathrm{IM}$
$150 \operatorname{PLOT}(J)=1 \mathrm{H}$
PRINT 380
PRINT 390
PRINT 400
P $\because A X A=A, \operatorname{MAX}+$ AMI $N$
$P: A A X B=B M \wedge X+B M I N$
PMAXC $=$ CMAX + CMIN
PMAXD= חivA $X+$ OMIN
PHAXF =FMAX FF.MIN
PRINT 4GU,AMIN,PMAXA,BMIN,PMAXB,CMIN,PMAXC,DMIN,PMAXD,EMIN,PMAXE
PRINT $45 \cup, A M I N$, PMAXA,BMIN, PMAXB, CMIN, PMAXC, DMIN, PMAXD, EMIN, PMAXE
$\operatorname{IF}(A M A X \cdot E Q \cdot 0.0)$ AMAX $=1.0$
$\operatorname{IF}(B M A X \cdot E Q .0 .0)$ BMAX $=1.0$
$\operatorname{IF}(C M A X \cdot E \cap .0 .0)$ CMAX $=1.0$
$\operatorname{IF}($ DMAX.EO.0.0) DMAX $=1.0$
$\operatorname{IF}(E M A X \cdot E Q .0 .0)$ EMAX $=1.0$
DO $160 \mathrm{~J}=1, \mathrm{~N}$
$A=A A(J) * S C A L E / A M A X+1.0$
$B=B B(J) * S C A L E / B M A X+1.0$
$C=C C(J) * S C A L E / C M A X+1.0$
$D=D D(J) * S C A L E / D M A X+1.0$
$E=E E(J) * S C A L E / E M A X+1.0$
C PLOTS LIE IN 2-26,28-52,54-78,90-104,106-130
$I A=I N T(A)$
IF(IA.EQ•O) IA=1
$I R=I N T(R)+76$
$I C=I N T(C)+52$
$I D=I N T(D)+78$
$I E=I N T(E)+104$
$\operatorname{PLOT}(I A)=1 H^{*}$
$\operatorname{PLOT}(I R)=1 \mathrm{H}$.
PLOT (IC) $=1 \mathrm{H}^{\circ}$
PLOT (IN) $=1$ H* $^{*}$
PLOT (TE) $=1 \mathrm{H}$.
PRINT 20U,(PLOT(J), J=1, 120 )
C ERASF

```
            PLOT(IA)=1H
            PLOT(IB)=1H
            PLOT(IC)=3H
            PLOT(TD) = IH
            PLOT(IF) = IH
IGO CONTINUE
            DO 205 J=1,N
            PRINT ?1O,AV(J),BV(J),CV(J),DV(J),EV(J)
    205 CONTINUF.
200 FORMAT(1HZ,1X,130A1)
    210 FORMAT(OX,5(F1O.4,14X))
2aO FORMAT(fHJ,//g&X,*PLOT OF TRANSIENT WAVEFORMS AND PREDICTED ANGLE*
        1)
    ?OU FORMAT(gHus5X,*PREDICTEU ANGLE*,OX,*ERROR OF PREDICTION*,OX,*ACTUA
```



```
400 FORMAT(っX,弓(*MINIMUM*, 
450 FORMAT(つX,5(F7.4,1)X,F7.4, >X))
4ヶO FORMAT(2X, 5(EO.?,&X,EO.2,2X))
            RETURN
                    FNn
            SUBROUTINF MULT
            COMMON/CONST/M,N,ANGCONS,ERRORq(\etaOU)
```




```
            1ECT(10O)
            ACCCON5=20.0
            wi-1(1) = 0.0
            S%(1,1)=0.0
            00 80 J=1,N
            SM(1,1)=SM(l,1) + ST(7,J)*S(J,つ)
            wM(1)=WM(1) + ST(っ,J)*(w(J) - S(J,]) - S(J,2)*ACCCONS)*WAYVFCT(J)
        80 CONT INUE
C RESULT OF ST*S AND ST*W
            PRINT OQ
            PRINT 100, Si4(1,1), 利(1)
                    RFTURN
```



```
    100 FORMAT(F10.4,35X,F10.4)
            FND
            S!JRROUTINF NFWVFCT
            COMMON/ CONST/M,N, ANGCONS,FRROR1(100)
            COMMON/|AT / /ANGTARG(1U0),ERROR(100),S(100,2),ST(2,100),W(100), !AYV
            1ECT(100)
                X=0.0
C THIS sUOROUTINL EVALUATËS ERROR AND READJUSTS THE wEIGHTING TO ELIMINA
C THL FIRST TWO TIME POINTS ARE NOT READJUSTED.
            PRINT 61
            NO 40 J=5,N
            X=FRROR(J)
            IF(X.GT.1.O.OR.X.LT.-1.0)GO TO 20
            IF(X\bulletGT•O.,.OR•X\bulletLT•-0.5)GO TO 20
            IF(X.GT•O.1.OR•X•LT.-U.1)GO TO pO
            GO TO 40
            20 WAYVECT(J)=WAYVFCTT(J) + O.1
            GO TO 40
    20 NAYVECT(J)='NAYVECTTJ) + 0.3
            GO TO 40
```

```
    20 जAYVECT(J)=NAYVECT(J) + U.5
    ^O PRINT &O,WAYVECT(J),J
    &O FORMAT(F10.4,I5)
```



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        1CTOR IS mUJUSTEU W•R\bulletT. THE ERROR FUNCTION*,///,1X,*THE NEW WEIGHT
        ?INO VECTOR IS*,//, 
            RFTIJRN
        ENn
        FUNCTION RIAAX(X,N)
        REAL X(N)
        R`IAX=X(1)
        DO 10 I= .,N
        IF(RMAX.LT.X(I)) RMAX=X(I)
        10 CONTINUE
        RFT!JRN
        「\ר
            F\NCTION R`IN(X,N)
            RrAL X(N)
            R.`!N=x(1)
            C10 I= 1,N
            IF(R"IH.GT.X(I)) RMIN=X(I)
        10 co:!TIN:!F
            OFTIRO
            =?n
            OL3BROUT IME SNLVF
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```
            PRT:T 110
            A(1, ])= S`(1, 1)
            S(1)={!:O(1)/A(1,1)
            B(2)=2n.0
            PRINT 10%
            PRINT ?OO,A\GしO\S,(B(J),J=1,つ)
            RET!R^!
!l rorual(1x,///g5X,*THE FOLLOWING IS THE WEIGHTEU AVERAGED SOLTION!
            2OF THE FJUATIOA (SMI)(K1)=(WM) - ANGIFF - ACCEI_FRATIONX!O2*)
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    IGRA:A.*,///////lx,** K? K? K? *)
~ON FORMAT( 3F?O.4)
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