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INTRODUCTION AND SUMMARY

Consider a city where production occurs at a single point - the center of a homogeneous plane - and where land is used for housing and transportation for a homogeneous labor force. Since the average transportation cost and the average cost for housing both increase with city size, the aggregate labor supply curve is increasing[1]. It follows then that whether producers hire labor in a monopsonistic or competitive manner will make a difference in terms of the size of the labor force as well as the capital/labor and output/labor ratios. Secondly, a part of the increase in housing and transportation costs are due to increases in land rents. Therefore, the shape of the labor supply curve will be dependent upon whether land is owned by a monopsonistic producer (as in a 'factory town') or whether there is an independent land market.

This brief paper examines, in a very simple framework, the different allocations arising out of the different monopsonistic/competitive production and joint/separate land ownership arrangements.

It is found that if there are non-decreasing returns to scale to the industry but constant returns to an individual firm then the polar cases of competitive production, separate land ownership arrangement gives the <u>same</u> allocation as the monopsonistic production, joint land ownership arrangement. The 'intermediate' arrangement of monopsonistic production, separate land ownership yields a <u>lower</u> city size and a <u>lower</u> capital/output ratio. Of course, the distribution of output between labor, capital and land is different in the different arrangements.

TRANSPORTATION AND HOUSING

Production of transportation and housing requires land and capital inputs and the production function is assumed to exhibit constant returns to scale. Land has no alternative use while the rental rate of capital is r. Each unit of labor has a constant demand of one unit of housing and one daily commuting trip. Now suppose that H units of capital are devoted to housing and transportation and that there are L units of labor. Then, assuming no congestion in transportation, a competitive allocation of H is efficient i.e., results in a minimum of transportation cost for the labor force. Call this minimum T(L, H). By the duality theorem of nonlinear programming it follows that T is convex, $T_L = \frac{\partial T}{\partial L} > 0$ and $T_H = \frac{\partial T}{\partial H} < 0$. Furthermore, $T(\lambda L, \lambda H) > \lambda T(L, H)$ when $\lambda>0$, $L>0$, $H>0$. Now the assumption of constant rental rate for capital implies that the housing and transportation industries will employ that amount of capital H which, for a given demand L, minimizes T(L, H) + rH. Hence H is given by $T_H(L, H) + r = 0$. The properties of the function T described above serve as a rationalization for adopting the following specific functional form for T:

$$T(L, H) = L^{\beta+\gamma+1} H^{-\beta}$$

where $\beta>0$, $\gamma>0$ are constants[2]. Then for a given L the efficient amount of H is given by

$$H(L) = \left(\frac{\beta}{r}\right)^{\frac{1}{\beta+1}} L^{\frac{\beta+\gamma+1}{\beta+1}},$$

and the resulting transportation cost is

$$T(L) = T(L, H(L)) = \frac{r}{\beta}\left(\frac{\beta}{r}\right)^{\frac{1}{\beta+1}} L^{\frac{\beta+\gamma+1}{\beta+1}},$$

so that adding the capital cost of housing and transportation we get

$$C(L) = T(L) + rH(L) = \frac{\beta+1}{\beta}\, r \left(\frac{\beta}{r}\right)^{\frac{1}{\beta+1}} L^{\frac{\beta+\gamma+1}{\beta+1}},$$

or

$$C(L) = \frac{\beta+1}{\beta}\, \delta\, L^{\frac{\beta+\gamma+1}{\beta+1}} , \qquad (1)$$

where $\delta = r\left(\frac{\beta}{L}\right)^{\frac{1}{\beta+1}}$ is a constant.

Thus whether there is a competitive market for land and a competitive industry for housing and transportation or whether land is owned by producers and housing and transportation provided by them, in either case <u>the resource cost C(L) is the same</u>. But under the former arrangment land will fetch a rent which will be included in the supply price of housing and transportation. What will this rent be? From our assumptions of constant returns in housing and transportation production it follows quite readily that for a population of size L the total rent accruing to land will be

$$R(L) = LT_L(L, H) + HT_H(L, H) - T(L, H)$$

which upon substitution of the earlier relations for H(L) etc. gives

$$R(L) = \frac{\gamma}{\beta}\, \delta\, L^{\frac{\beta+\gamma+1}{\beta+1}} . \qquad (2)$$

Therefore, if there is an independent land market, the aggregate supply curve for housing and transportation for a labor force of size L is given by the relationship

$$\overline{C}(L) = C(L) + R(L) = \frac{\beta+\gamma+1}{\beta+1}\, \delta\, L^{\frac{\beta+\gamma+1}{\beta+1}} \qquad (3)^3$$

THE CITY'S PRODUCTION FUNCTION

A single homogeneous output is produced at the city center. Only capital and labor inputs are used. The aggregate production function is assumed to be

$$Q = AL^{1+\alpha} f(k) \tag{4}$$

where Q = output, A>0 is a constant, L = labor input, k = capital/labor ratio, f(k) is an increasing concave function i.e., f(0) = 0, f(k)>0 for k>0, f'(k)>0, f"(k)≤0, and α≥0 is a constant. If α=0 we have constant returns and if α > 0 we have increasing returns.

Two different interpretations of the production function will be employed depending upon whether production occurs in a single monopsonistic firm or in a large number of competitive firms. In the first case we take (4) to be the production firm of the single firm. In the second case it will be assumed that each individual firm has a constant-returns-to-scale production function given by

$$g = A(L)\ \ell\ f(k) \tag{5}$$

where ℓ = labor and k = capital/labor ratio employed by the firm. The coefficient $A(L) = AL^{\alpha}$, where L is the total labor force employed in the city, is to be interpreted as representing positive "agglomeration" economies. Each individual firm takes A(L) as an 'environmental' parameter[4].

COST OF LABOR

It is assumed that arbitrary amounts of labor are available at a fixed <u>real</u> wage of w units of output per unit of labor. Real wage is defined to be monetary wages (measured in output units) <u>less</u> housing and transportation cost.

We now turn to the different production and ownership arrangements mentioned in the introduction.

Case 1: Monopsonistic Producer, Separate Land Market

The cost of L units of labor is now given by $\overline{C}(L) + wL$ where $\overline{C}(L)$ is given in (3). The rental rate on capital is r. The firm's production function is given by (4), its decision variables are k, L and it maximizes[5]

$$\pi = AL^{1+\alpha} f(k) - rkL - wL - \frac{\beta+\gamma+1}{\beta} \delta L^{\frac{\beta+\gamma+1}{\beta+1}} . \tag{6}$$

The first order conditions $\pi_k = 0$, $\pi_L = 0$ respectively yield

$$AL^{\alpha} f'(k) = r , \tag{7}$$

and

$$(1+\alpha) AL^{\alpha} f(k) - rk - w \frac{(\beta+\gamma+1)^2}{\beta(\beta+1)} \delta L^{\frac{\gamma}{\beta+1}} = 0 \tag{8}$$

It is clear from (6) that to guarantee a finite maximum π we must assume that $\alpha < \frac{\gamma}{\beta+1}$. Unfortunately, this is still not sufficient and some conditions on f are also needed. Instead of elaborating on these aspects we simply assume that (6) has a unique local and global maximum. Now (7) implies that k is a non-decreasing function of L (recall $f'' \leq 0$). The left-hand side of (8) is a concave function in L^{α} for fixed k and hence there are at most two values of L for each value of k which satisfy (8), and furthermore, the second order conditions for a maximum rule out the smaller value of L. The typical behavior of (7), (8) is illustrated in Fig. 1.

Let k_1, L_1 denote the optimum values, let $q_1 = \frac{Q}{L_1}$ denote the output/labor ratio, and let $s_1 = (1+\alpha) A L_1^{\alpha} - w - rk_1$. Then the maximum profit π_1 is found by substitution in (6) to be

$$\pi_1 = \left[\frac{\gamma}{\beta+\gamma+1} s_1 - \alpha q_1\right] L_1 \tag{9}$$

whereas the rent $R_1 = R(L_1)$ accruing to land is obtained from (2),

$$R_1 = \left[\frac{\gamma(\beta+1)}{(\beta+\gamma+1)^2} s_1\right] L_1 \tag{10}$$

Therefore

$$\frac{\pi_1}{R_1} \leq \frac{\beta+\gamma+1}{\beta+1} \tag{11}$$

with equality holding for the case $\alpha=0$. For the values $\gamma=1/2$, $\beta=1$ (see footnote 2) we get the estimate

$$\frac{\pi_1}{R_1} \leq 1.25$$

We can define the "surplus" per unit of labor as $\sigma_1 = \dfrac{\pi_1 + R_1}{L_1}$,

$$\sigma_1 = \frac{\gamma}{\beta+\gamma+1}\left[1 + \frac{\beta+1}{\beta+\gamma+1}\right] s_1 - \alpha q_1 \tag{12}$$

Case 2: Monopsonist Producer Owning Land

The cost of labor to the firm is now $C(L) + wL$ so that the expression for profit is

$$\pi = A\, L^{1+\alpha} f(k) - rkL - wL - \frac{\beta+1}{\beta} \delta\, L^{\frac{\beta+\gamma+1}{\beta+1}} \tag{13}$$

Maximization of π leads to the relations

$$A\, L^{\alpha} f'(k) = r \tag{14}$$

which is the same as (7), and

$$(1+\alpha)\, A\, L^{\alpha} f(k) - rk - w - \frac{\beta+\gamma+1}{\beta} \delta\, L^{\frac{\gamma}{\beta+1}} = 0. \tag{15}$$

After defining the quantities π_2, q_2, etc. analogous to the previous case we obtain the relation

$$\sigma_2 = \frac{\pi_2}{L_2} = \frac{\gamma}{\beta+\gamma+1} s_2 - \alpha q_2 \ . \tag{16}$$

The typical behavior of (14), (15) is sketched in Fig. 1.

Comparison of Cases 1 and 2

It is clear from the figure and can be shown quite readily that

$$L_2 > L_1 \, , \quad k_2 \geq k_1 \, . \tag{17}$$

If $k_1 = k_2$ then it is easy to see that $L_2 > L_1$ since in the second case the marginal cost of labor is smaller. It is interesting to know that the inequalities still persist when we allow variable capital/labor ratio. The following inequality follows from the definition,

$$\pi_2 > \pi_1 + R_1 \, .$$

However, in the case that $k_1 = k_2$[6] it can be shown that

$$\sigma_2 < \sigma_1 \tag{18}$$

For the case $\alpha=0$ we have in fact $s_1 = s_2$ so that from (12), (16) we get

$$\frac{\sigma_1}{\sigma_2} = 1 + \frac{\beta+1}{\beta+\gamma+1}$$

which is equal to 1.8 for $\beta=1$, $\gamma=1/2$. Next from (8) and (15)

$$\frac{(1+\alpha)\ A\ L_1^{\alpha}\ f(k_1) - rk_1 - w}{(1+\alpha)\ A\ L_2^{\alpha}\ f(k_2) - rk_2 - w} = \left(\frac{\beta+\gamma+1}{\beta+1}\right)\left(\frac{L_1}{L_2}\right)^{\frac{\gamma}{\beta+1}} \tag{19}$$

From (7) and (14) the left-hand side is equal to

$$\frac{(1+\alpha)r\ (f'(k_1))^{-1}\ f(k_1) - rk_1 - w}{(1+\alpha)r\ (f'(k_2))^{-1}\ f(k_2) - rk_2 - w}$$

which is bounded from above by 1 since $(1+\alpha)\ r(f'(k))^{-1}\ f(k) - rk$ is an increasing function of k and since $k_1 \leq k_2$. Therefore, from (19) we get the estimate

$$\left(\frac{L_1}{L_2}\right)^{\frac{\gamma}{\beta+1}} \leq \frac{\beta+1}{\beta+\gamma+1} \tag{20}$$

with equality if $k_1 = k_2$. For our choice of parameters $\gamma=1/2$, $\beta=1$ this gives

$$\frac{L_1}{L_2} = (0.8)^4 \approx 0.41$$

As a final point, assuming $\alpha=0$ so that $k_1 = k_2$ and $s_1 = s_2$, we can compare the total surplus under the two cases using (9), (10), (16) and (20),

$$\frac{\pi_1 + R_1}{\pi_2} = \left(1 + \frac{\beta+1}{\beta+\gamma+1}\right) \frac{L_1}{L_2} = \left(1 + \frac{\beta+1}{\beta+\gamma+1}\right)\left(\frac{\beta+1}{\beta+\gamma+1}\right)^{\frac{\beta+1}{\gamma}}$$

$$\approx 0.79 \quad \text{for} \quad \gamma=1/2 \;, \quad \beta=1.$$

Case 3: Competitive Production, Separate Land Market

As discussed earlier this arrangement is interpreted to mean that production is organized among many firms each of which has the production function (5) and such that each firm take the price of labor as given. If the city's labor force is L then the unit price of labor is given by p_L,

$$p_L = \frac{1}{L}C(L) + w = \frac{\beta+\gamma+1}{\beta+1}\,\delta\, L^{\frac{\gamma}{\beta+1}} + w \,. \tag{21}$$

Therefore, each firm will adopt a capital/labor ratio k, and will hire amount of labor ℓ so as to maximize

$$A(L)\;\ell\; f(k) - P_L^{\ell} - rk\ell.$$

The first order conditions are

$$A(L)\; f'(k) = r \;, \tag{22}$$

$$A(L)\ f(k) = P_L + rk \quad . \tag{23}$$

Hence the profit of each firm is zero. To obtain the equilibrium size of the labor force we must equate the supply and demand equations (21) and (23) which leads to

$$A(L)\ f(k) - rk = \frac{\beta+\gamma+1}{\beta+1}\ \delta\ L^{\frac{\gamma}{\beta+1}} + w \tag{24}$$

We note that (22) and (24) are identical to (14) and (15) respectively. Hence, Case 2 and Case 3 yield the <u>same</u> allocation of labor and capital. The only difference is that in Case 3 profits are zero and all 'surplus' accrues to land.

CONCLUSIONS

This paper has analyzed within a static framework some of the implications of differences in the allocation of labor and capital in a city due to differences in monopsony power and land ownership. Somewhat surprisingly the two polar cases, 2 and 3, yield the same allocation although the distribution of the output is different in the two cases. Some reasonable <u>a priori</u> estimates of the labor supply curve indicate that the difference in the allocations between case 1 and case 2 or 3 is substantial. For the special case where $\alpha=0$, so that $k_1=k_2=k$ say, and $s_1=s_2=Af(k) - rk - w = s$ say, the results can be summarized as in Table I.

REFERENCE

Hoch, I. (1972). Income and City Size. Urban Studies, Vol. 5, No. 3: 299 - 328.

FOOTNOTES

[1]Assuming that the real wage for labor is constant. The monetary wage will increase also because of an increase in the cost-of-living which in turn is due to increases in housing and transportation costs. See for example (Hoch, 1972).

[2]If average population density does not increase very rapidly with L, then the average commuting distance will grow at a slightly slower rate than $L^{1/2}$ so that a reasonable a priori estimate for $\gamma=1/2$. Similar argument will lead to an estimate of β somewhat larger than 1.

[3](Irving Hoch, 1972, p.310) obtains the following estimate for cost-of-living C as a function of the population size P, $\log(C-80) = 1.0538 + 0.0938 \log P + 0.0208\ (NE) - 0.0254\ (NC) - 0.2390\ (S)$ where NE, NC, S are dummy variable representing North-East, North-Central and South. This regression would lead us to an estimate of $\frac{\gamma}{\beta+1} = 0.0938$, considerably smaller than our a priori estimate of $\gamma = 0.25$. However, the money wage increases roughly in proportion to $P^{0.20}$ (Hoch, 1972, Table 9), which is closer to our estimate.

[4]This seems an attractive way of combining competitive assumptions with increasing returns to scale.

[5]Thus we are assuming that the output is sold at a fixed price of unity.

[6]This is the case if the production function allows only fixed capital/labor ratio or if $\alpha=0$ and $f''<0$ because then (7), (14) give a unique k.

Per unit of labor / Case	Output	Real Wages	Housing and Transport Capital Cost	Production Capital Costs	Transport Costs	Land Rent	Profits to Producers
1	Af(k)	w	$\frac{\beta(\beta+1)}{(\beta+\gamma+1)^2}\,s$	rk	$\frac{\beta+1}{(\beta+\gamma+1)^2}\,s$	$\frac{\gamma(\beta+1)}{(\beta+\gamma+1)^2}\,s$	$\frac{\gamma}{\beta+\gamma+1}\,s$
2	Af(k)	w	$\frac{\beta}{\beta+\gamma+1}\,s$	rk	$\frac{1}{\beta+\gamma+1}\,s$	0	$\frac{\gamma}{\beta+\gamma+1}\,s$
3	Af(k)	w	$\frac{\beta}{\beta+\gamma+1}\,s$	rk	$\frac{1}{\beta+\gamma+1}\,s$	$\frac{\gamma}{\beta+\gamma+1}\,s$	0

Table I

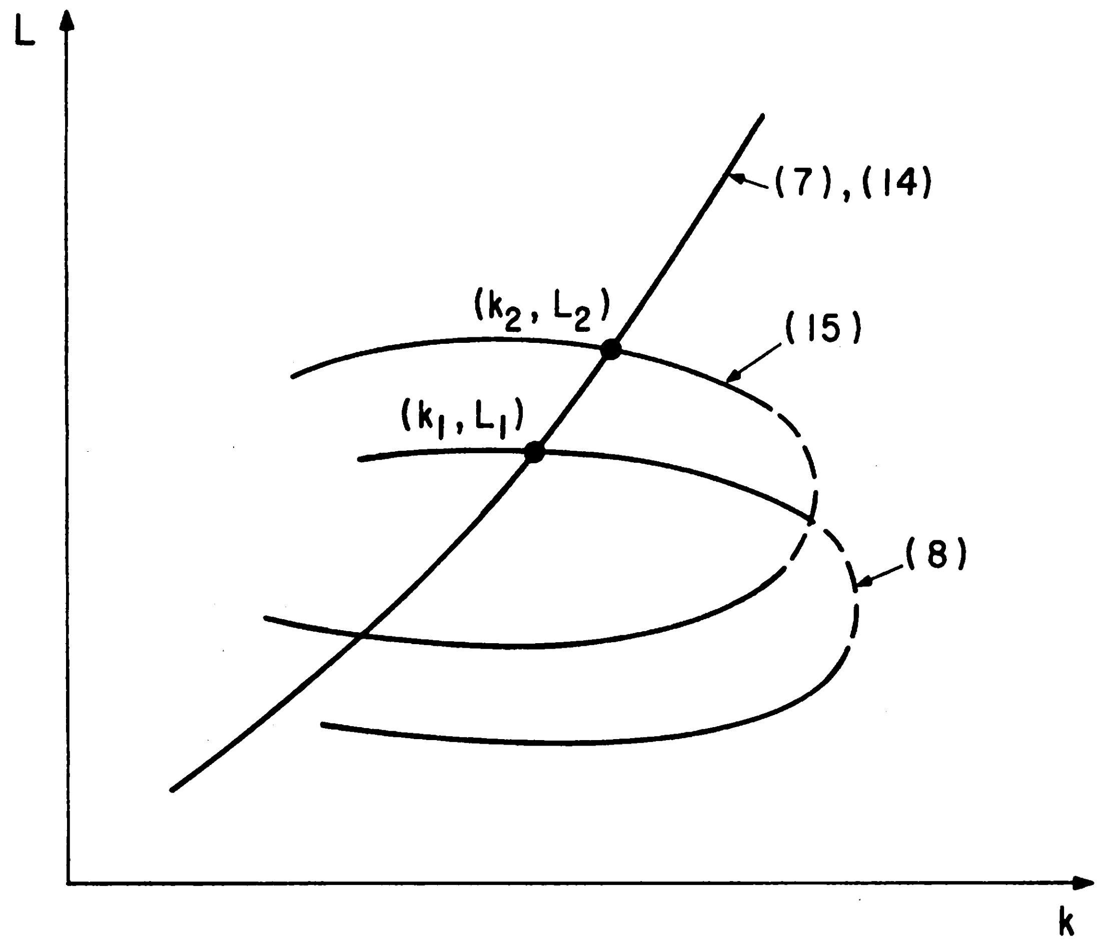

Fig. 1. First order conditions for Cases 1 and 2.