LATENT COMPONENTS IN THE EKG

by

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Abstract

The QRS complex of a human EKG can be represented as a sum of basic components. The QRS complex of one subject observed on different leads can be represented as different linear combinations of the same basic components, which are unique to that subject. Examples of this representation given for actual EKG's in one normal and two abnormal subjects.

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Theoretical Considerations of Decomposition

An observed waveform is often represented by other than the value as a continuous function of time. There are many techniques available to do this, and the representation used may have distinct advantages over the value as a continuous function of time representation for certain specific purposes. For example, the Fourier Transform representation is useful in determining the frequency response required to amplify a waveform without distortion, and a sampled data digital representation is essential if numerical processing of the waveforms is to be used.

Three sets (U, V, and W) of six waveforms are shown in Figure 1. In each set any waveform can be represented as a linear combination of any other two waveforms in that set. For example

$$U_3(t) = 1.15\ U_2(t) + .57\ U_6(t)$$

or

$$U_3(t) = .50\ U_1(t) + .87\ U_4(t) \quad .$$

The second expression is a more compact (or economical) representation of $U_3(t)$ than the first because $U_1(t)$ and $U_4(t)$ are zero over greater portions of the interval than any of the other functions, and can be specified over their non-zero intervals, together with where their non-zero intervals are located (i.e., the starting time of the waveform).

This representation of $U_2(t)$, $U_3(t)$, $U_5(t)$, and $U_6(t)$ as linear combinations of $U_1(t)$ and $U_4(t)$ would be the most compact even if $U_1(t)$ and $U_4(t)$ were not included in the original set. If $U_2(t)$, $U_3(t)$, $U_5(t)$, and $U_6(t)$ were observed waveforms, then $U_1(t)$ and $U_4(t)$ could be derived from them and the coefficients of the linear combinations determined. For this example, if we call $U_1(t)$ and $U_4(t)$ components named C_1 and C_2, then the table of coefficients would be

	C_1	C_2
$U_2(t)$	.87	.50
$U_3(t)$	.50	.87
$U_5(t)$	-.50	.87
$U_6(t)$	-.87	.50

The decomposition of $U_2(t)$, $U_3(t)$, $U_5(t)$, and $U_6(t)$ into these components and coefficients is obvious and mathematically straightforward, since $U_1(t)$ and $U_4(t)$ each have non-zero values only when the other function is zero. Even though this is not true of the pairs $V_1(t)$; $V_4(t)$ and $W_1(t)$; $W_4(t)$, the relationship between the six $V(t)$ functions and between the six $W(t)$ functions is the same as that between the six $U(t)$ functions. In each set, the first and fourth functions are the most appropriate components for representing the other four functions. In the case of the V and W functions the components are not as obvious as in the U functions, and in such situations I call them Latent Components.

There is one parameter that is indeterminate for each component, since it could be multiplied by an arbitrary scale factor, and each coefficient associated with that component divided by that same scale factor. In the example given, if C_1 were multiplied by 10 to give C_1', and C_2 were multiplied by .1 to give C_2', then the table of coefficients would be

	C_1'	C_2'
$U_2(t)$	.087	5.0
$U_3(t)$	.050	8.7
$U_5(t)$	-.050	8.7
$U_6(t)$	-.087	5.0

This indeterminacy can be removed by the application of an arbitrary rule. Two possible arbitrary rules are:

Rule A: Scale the components such that the magnitude of the largest coefficient associated with that component is 1.00. Make the initial excursion of the component positive.

Rule B: Scale the components such that the sum of the squares of the coefficients associated with each component is 1.00. Make the initial excursion of the component positive. (In the case of three spacially orthogonal components, e.g., X, Y, and Z components of some phenomena, the coefficients then become the direction cosines of the components.)

Practical Considerations of Decomposition

When the above decomposition of observed waveforms into components and coefficients is attempted, a unique solution is obtained if the observed waveforms are really linear combinations of a set of components and there is no noise present in the observed waveforms. When noise is present, an ambiguity arises as to how many components are needed to represent the observed waveforms to within the noise indeterminacy. This point is discussed later as it applies to the EKG application.

Application of Decomposition to EKG Waveforms

This decomposition analysis has been applied to four sets of EKG waveforms.

Set 1: A twelve lead EKG of a young male. This data was taken with EKG equipment which recorded the six chest leads simultaneously on a Brush Model 260 Recorder. The recording speed was 125mm/sec and the bandwidth exceeded 100Hz. The slight 60Hz signal in the aV_F lead was removed before doing the decomposition. Since leads I, II, III, aV_F, aV_L, and aV_R are linearly related, only two of them were used in the decomposition. Leads I and aV_F were chosen because of their similarity to the X and Y Frank leads. Rule A was used for scaling the components in this case. The results are indicated in Figure 2, which shows the original EKG waveforms, the derived components and coefficients, and the synthesized waveforms which are placed as inserts near the originals.

Set 2: A Frank lead EKG of an eight year old child with a single ventricle heart. This EKG appears on page 157 of Vector-cardiography in Congenital Heart Disease by R.C. Ellison and N.J Restieaux. Figure 3 includes the illustration from the book, the derived components and coefficients, and the synthesized waveforms. Rule B was used for scaling these components.

Sets 3, 4: These are Frank lead EKGs of a child with a transposition of the great arteries. Both appear on page 142 of Vector-cardiography in Congenital Heart Disease by R.C. Ellison and N.J Restieaux. Figure 4 includes the illustration from the book for the EKG taken shortly after birth, the derived components and coefficients, and the synthesized waveforms. Rule B was used for scaling the components. Figure 5 includes the illustration from the book for the EKG taken at eight months of age, the derived components and coefficients, and the synthesized waveforms. Rule B was used for scaling the components.

Intermediate Conclusions

A comparison of the synthesized waveforms with the original waveforms in these examples shows that they are essentially the same as far as clinical interpretation is concerned, and match approximately within the noise indeterminacy. The noise includes all contributions other than the "true" EKG, e.g., amplifier noise, electrode noise, and electromyogram signals.

THIS DECOMPOSITION TECHNIQUE MAKES NO ASSUMPTIONS ABOUT ELECTRICAL MODELS OF THE HEART (e.g., about dipoles, thorax conductivity, heart orientation, etc.). IT DEALS ONLY WITH THE OBSERVED WAVEFORMS AND REPRESENTS THE DATA IN A TRANSLATED AND COMPACTED FORMAT.

Comments on Possible Uses of This Representation

An examination of the standard twelve lead case, Figure 2, seems to indicate the following:

1. The components do have the desirable characteristic of being non-zero over limited periods. They occur in an overlapped time sequence, and all have generally similar shapes, which are smooth and unipolar.

2. The coefficients of each component vary in an orderly way with the chest lead position. It should be possible to interpolate the values of the coefficients for intermediate chest lead positions. Some of the coefficients change sign as the lead placement changes from V_2 to V_3 or from V_3 to V_4. The components observed are consistent with a multiple dipole generator heart model, each dipole having its own location, orientation, and time course. The number of dipoles resolved is limited by the remoteness of the sensing electrodes from the dipoles, i.e., the electrodes are on the surface of

the body, and by the noise in the observed signals.

An examination of the single ventrical Frank lead case, Figure 3, seems to indicate the following:

1. A "simpler" heart requires fewer components in this representation than a "complex" heart.
2. The component shapes are similar to the previous case in that they occur in an overlapped time sequence, are smooth and unipolar, and are non-zero over limited periods. However, their shapes are noticeably different.

An examination of the transposed arteries Frank lead case, Figure 4 and Figure 5, seems to indicate the following:

1. The component shapes are again similar to the previous cases in that they occur in an overlapped time sequence, are smooth and unipolar, and are non-zero over limited periods. Again, the shapes are noticeably different from both previous cases.
2. The last three components changed slightly in size, shape, and direction over the eight month period. The major change is in the size and shape of the first two components.
3. There are more high frequency details in the original EKGs in this case than in the first two examples, and this information is lost in the component representation.

Future Research

1. To automate the process of obtaining this representation from observed waveforms. The components can be determined best if the observed waveforms have grossly different linear

combinations of the components. In practice this means multiple leads near the heart. A good technique would probably be to use leads near the heart to determine the components, and express the coefficients for leads remote from the heart or averaged if near the heart. Under these conditions neither the components nor the coefficients will be sensitive to lead placement.

2. To further compact the representation of the components. This might be done by having a limited dictionary of component shapes, or by extracting significant numerical values. Some possibilities are:

   a. Determine location of a component by the location of its maximum, or by its centroid.

   b. Determine width of a component by measuring its width at the half maximum points, or by taking the quotient of the area divided by the maximum height.

   c. Determine the size by the maximum height, or by the area, or by the integral of the squared values (i.e., the energy).

   It is possible to use one technique for normal EKGs, and another technique for abnormal EKGs. The high frequency information lost in the component representation might be preserved in some clinically useful way.

3. To process existing or new clinical data using the component and coefficient representation, and evaluate its clinical

usefulness. Possible applications are:

a. To use a "standard" physical heart orientation EKG for diagnosis. The observed EKG waveforms can be transformed to the "standard" by applying appropriate rotational transformation to the coefficients, and then synthesizing the set of EKG waveforms that would be observed if the heart had that transformed physical orientation.

b. To use the quantitative properties to determine normal and abnormal rates of change in normal EKGs, e.g., from year to year for use in preventive medical care.

c. To use the components and coefficients as early indicators of difficulties in cardiac care units and in surgical monitoring. The variablility from heartbeat to heartbeat may contain useful indicators.

4. The examples used were only of the QRS complex. This decomposition technique can also be applied to the P and T intervals, and should be particularly useful in quantifying phenomena in the S-T interval, such as S-T depressions.

REFERENCES

1. Lamb, Lawrence E., Electrocardiography and Vectorcardiography, Philadelphia: W.B. Saunders Company, 1965.

2. Cox, Jerome R. Jr., Nolle, Floyd M. and Arthur, R. Martin, "Digital Analysis of the Electroencephalogram, the Blood Pressure Wave, and the Electrocardiogram", Proc. IEEE, Vol. 60, No. 10, October 1972, pp. 1137-1164.

3. Ellison, R. Curtis and Restieaux, Norma J., Vectorcardiography in Congenital Heart Disease, Philadelphia: W.B. Saunders Company, 1972

4. Vectorcardiography - 1965 Symposium, I. Hoffman (ed.), Amsterdam: North-Holland Publishing Company, 1966.

5. Vectorcardiography 2 Symposium, I. Hoffman (ed.), Amsterdam: North-Holland Publishing Company, 1971.

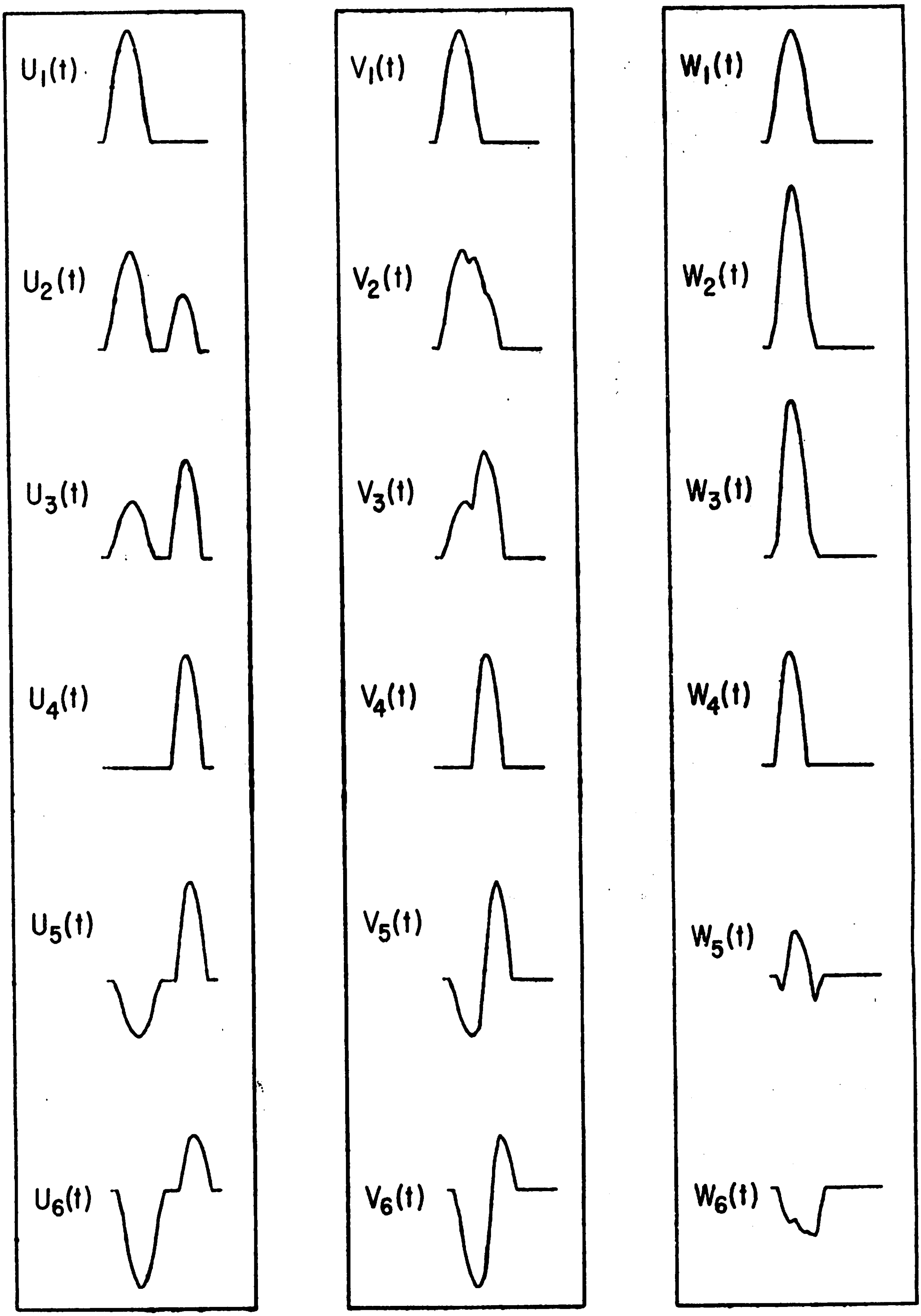
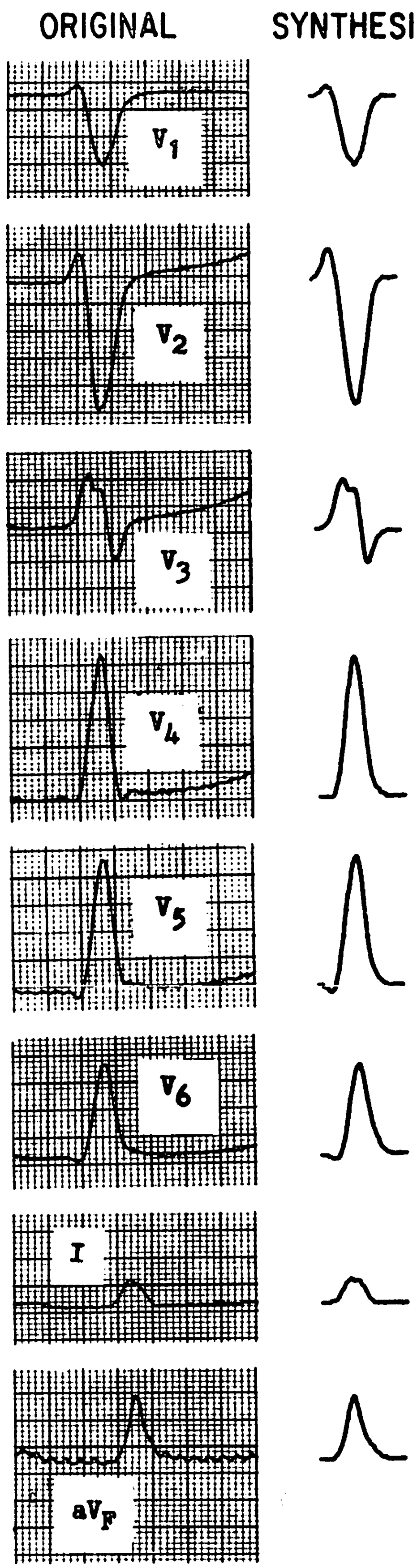
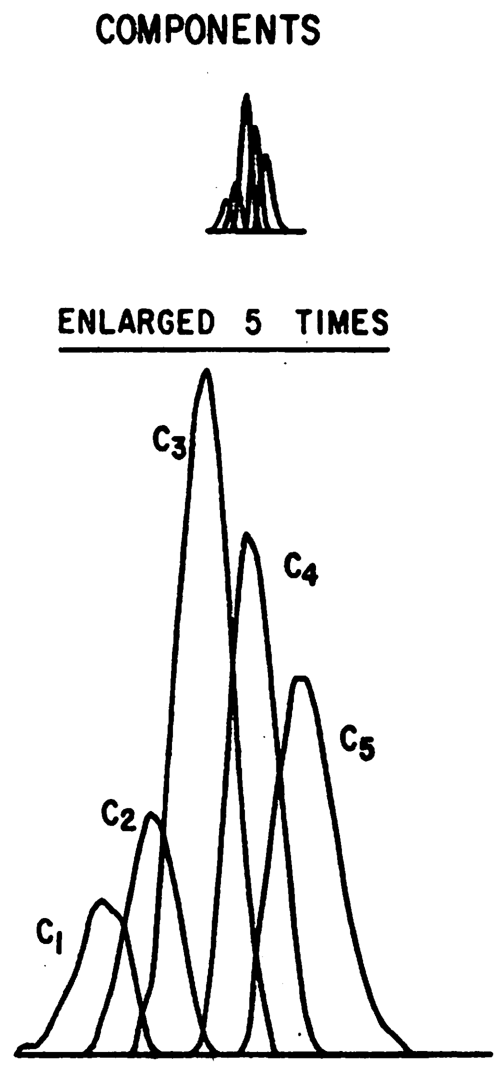

U1(t)
U2(t)
U3(t)
U4(t)
U5(t)
U6(t)
V1(t)
V2(t)
V3(t)
V4(t)
V5(t)
V6(t)
W1(t)
W2(t)
W3(t)
W4(t)
W5(t)
W6(t)

FIGURE 1



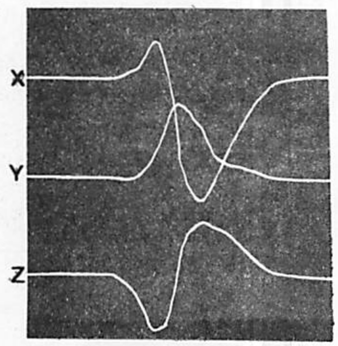


COEFFICIENTS

	C_1	C_2	C_3	C_4	C_5
V_1	.35	.03	-.38	-.53	-.73
V_2	1.00	.26	-.79	-1.00	-1.00
V_3	.31	1.00	.28	.29	-.51
V_4	.00	.97	1.00	.93	.20
V_5	-.35	.74	.85	.97	.33
V_6	-.20	.32	.55	.72	.36
I	.11	.42	.17	.10	.00
aV_F	.00	.42	.50	.23	.18

FIGURE 2

ORIGINAL



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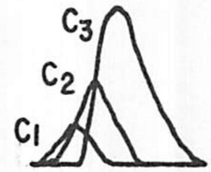

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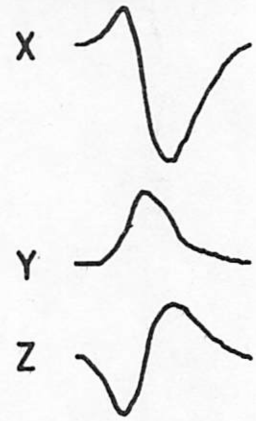
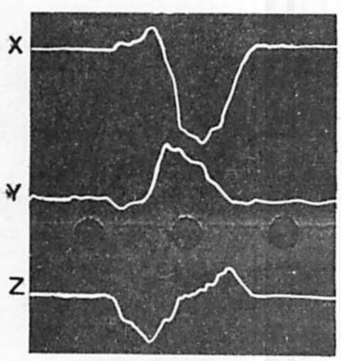


COEFFICIENTS

	C_1	C_2	C_3
X	.28	.56	-.88
Y	-.16	.68	.18
Z	-.95	-.47	.44

FIGURE 3

ORIGINAL



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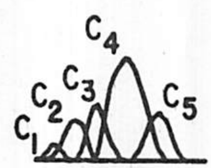

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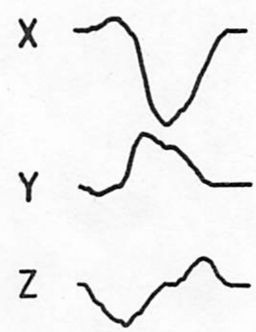
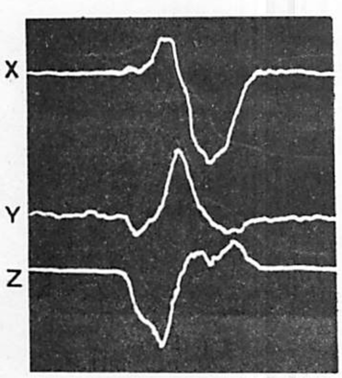


COEFFICIENTS

	C_1	C_2	C_3	C_4	C_5
X	.00	.29	-.13	-.93	-.82
Y	-.48	.00	.86	.38	.00
Z	-.88	-.96	-.49	.00	.57

FIGURE 4

ORIGINAL



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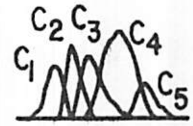

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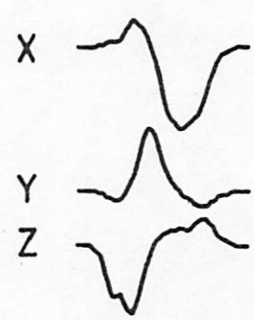

COEFFICIENTS

	C_1	C_2	C_3	C_4	C_5
X	.12	.37	.10	-.97	-.67
Y	-.19	.18	.99	-.13	-.33
Z	-.97	-.91	.00	.20	.66

FIGURE 5