ON THE ANALYSIS OF LARGE-SCALE SYSTEMS

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Abstract

In recent years, systems analysis has become a widely used technique for dealing with problems relating to the design, management and operation of large-scale systems.

A thesis advanced in this paper is that the conventional techniques of systems analysis are of limited applicability to societal systems inasmuch as such systems are, in general, much too complex and much too ill-defined to be amenable to quantitative analyses. It is suggested that the applicability of systems analysis may be enhanced through the use of the so-called linguistic approach, in which words rather than numbers serve as values of variables. The basic elements of this approach are outlined and illustrated by examples.

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1. Introduction

One of the most significant concomitants of technological progress has been--and continues to be--the accelerating growth in the degree of interdependence both within and across all strata of modern society.

This phenomenon--which is becoming increasingly pervasive in its manifestations--affects individuals, groups, organizations, industrial enterprises, cities, countries and continents. Thus, the events in Kuwait may have a profound impact on the jobs of workers in Detroit and Dodge City; the strike of copper miners in Chile may curtail the production of electric motors in Schenectady; and the latest developments in Washington may be the subject of informed discussions in the cafes of Teheran, Vienna and Rio de Janeiro.

Reduced to its basic factors, the growth in the degree of interdependence in modern society may be viewed as a confluence of several distinct and yet interrelated developments: (a) The major advances in our ability to communicate information at high speed, low cost and over long distances; (b) the decline in cost combined with the increase in the speed of transportation of raw materials and manufactured products; (c) the growing mobility of people brought about by the enhancement in speed, comfort and economy of traveling; (d) the sharing of energy and information resources through power distribution networks, oil pipelines, radio and television networks, data banks, computer networks, etc.; (e) the drive toward greater efficiency which impels a centralization of decision-making and exerts a pervasive pressure to merge small units into bigger ones; (f) and last, but not least important, the growth in population density, especially in the vicinity of urban centers.

In the case of physical systems, it is a well-known empirical fact that unrestrained increase in the degree of feedback between the components of a system leads to instability, oscillations and, eventually, catastrophic failures. There is a warning in this that cannot be ignored: our societal systems, too, are likely to become unstable if the growth in the degree of interdependence within them is not accompanied by better planning, coordination, and--what might be much less palatable--restraints on our freedoms.[1] Indeed, in some of the advanced societies with libertarian traditions--in which there is an understandable aversion to planning and control--we are already witnessing the manifestations of what might be diagnosed as the _crisis of undercoordination_: vehicular and air traffic congestion, deterioration in the quality of municipal services, decay of urban centers, power blackouts, air and water pollution, shortages of energy, unemployment, etc. And these may well be just the precursors of far more serious stresses and strains which lie ahead--stresses which may test to the limit the endurance of our democratic institutions.

Does it follow that the price of survival is the loss of the large measure of freedom which we enjoy today in deciding on where to live, for whom to work, where to travel, etc? Hopefully, the answer is in the negative. But it is clear that in order to be able to survive without a significant erosion of our freedoms, it will be necessary to develop a much better understanding of the forces which shape--and the dynamics which

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1 This is implicit in Skinner's controversial book "Beyond Freedom and Dignity" [1]. See also H. Wheeler's "Beyond the Punitive Society" [2].

govern--the evolution of the highly interdependent large-scale system which constitutes our modern society. For, it is only through such understanding that we may be able to achieve that degree of coordination which might be needed to preserve the equilibrium of our societal structures without creating an oppressive environment in which an individual has few of the freedoms which we take for granted at present.

Although we have made a great deal of progress during the past two decades in our ability to analyze the behavior of small-scale systems whose behavior is governed by differential, difference or integral equations, there are still many difficult problems which stand in the way of our ability to comprehend, much less predict, the behavior of large-scale societal systems. In part, the difficulties in question relate to the fact that societal systems are orders of magnitude more complex than the well-structured mechanistic systems which constitute the major part of the domain of applicability of classical system theory. More importantly, we still have a far from adequate understanding of the basic issues relating to conflict resolution, aggregation of preferences, choice of time-horizons, decision-making under uncertainty, and many other problems which arise when we deal not with machines--as in classical system theory--but with human judgements, perceptions and emotions.

To make system theory more relevant to societal problems, we may have to make radical changes in our basic approaches to systems analysis. In particular, we may have to accept what to some may be a rather unpalatable conclusion, namely, that in the case of societal systems relevance is incompatible with precision. In more specific terms, this means that we may have to accept much lower standards of rigor and precision in our analyses of societal systems than those which prevail in system theory--

if we wish such analyses to have substantial relevance to real-life problems.

In retreating from precision in the face of the overpowering complexity of societal systems, it is natural to explore the use of what might be called linguistic variables, that is, variables whose values are not numbers, but words or sentences in a natural or artificial language. The motivation for the use of such variables is that, in general, verbal characterizations are less precise than numerical ones, and thus serve the function of providing a means of approximate description of phenomena which are too complex or too ill-defined to admit of analysis in conventional quantitative terms. Actually, we use such variables in ordinary discourse, e.g., when we assert that "Paris is very beautiful," "David is a highly intelligent man," "Harry loves Ann," "The economy is in a state of recession," "Reduce speed if the road is slippery," etc. In these assertions, the italicized words may be viewed as the values of linguistic variables, with each such value representing a label of a fuzzy subset of a universe of discourse. In this sense, a statement such as "Stella is young" may be interpreted as the assignment of a linguistic value young to a linguistic variable named Age, and a collection of such statements constitutes a system of assignment equations from which other assignment equations may be deduced by a process of logical inference.

What we suggest is that in the case of societal systems the use of linguistic variables may offer a more realistic--if less precise--means of systems analysis than the conventional approaches based on the use of quantified variables. In effect, our contention is that the classical techniques of systems analysis--which were conceived and developed for dealing with systems which have well-structured mathematical models--are unsuitable for the analysis of the behavior of societal systems. To cope with such systems, we must forsake our veneration of precision and be content with answers which are linguistic rather than numerical in nature. This, in essence, is the spirit of what might be called the linguistic approach--an approach in which the concept of a linguistic variable is employed both as a means of approximate characterization of system behavior and as a basis for approximate reasoning involving the use of a fuzzy logic with linguistic truth-values.

In what follows, we shall present in a summarized form some of the basic concepts which play a central role in the linguistic approach. More detailed expositions of these concepts may be found in [3] and [4].

2. Elements of the Linguistic Approach

As was stated in the Introduction, linguistic variables serve as a means of approximate characterization of phenomena which are too complex or too ill-defined to be amenable to description in numerical terms. More concretely, a linguistic variable is characterized by a quintuple $(\mathcal{X}, T(\mathcal{X}), U, G, M)$ in which $\mathcal{X}$ is the name of the variable; $T(\mathcal{X})$ is the term-set of $\mathcal{X}$, that is, the set of names, X, of linguistic values of $\mathcal{X}$; and M is a semantic rule for associating a meaning, M(X), with each name (linguistic value) in $T(\mathcal{X})$. Generally, M(X) will be assumed to be a fuzzy subset of U.

As an illustration, consider a linguistic variable named Age. The term-set, T(Age), or simply T, of Age may be represented as follows

T(*Age*) = *young* + *very young* + *not young* + *very very young* + *not very young* + ... + *old* + *very old* + *not old* + ... + *not very young and not very old* + ... *extremely young* + ... + *more or less young* + ... (1)

in which + denotes the union rather than the arithmecic sum.

The universe of discourse for *Age* may be taken to be the interval [0,100], with the numerical variable u which ranges over U = [0,100] constituting the *base variable* for *Age*. Then, a value of *Age*, e.g., *young* may be viewed as a name of a fuzzy subset of U which is characterized by its *compatibility function*, $c : U + [0,1]$, with c(u) representing the *compatibility* of a numerical age u with the label *young*.[2] For example, the compatibilities of the numerical ages 22, 28 and 35 with *young* might be 1, 0.7 and 0.2, respectively. The meaning of *young*, then, would be represented by a graph of the form shown in Fig. 1, which is a plot of the compatibility function of *young* with respect to the base variable u.

A typical linguistic value in (1) contains one or more *primary terms*, e.g., *young*, *old*, whose meaning is both subjective and context-dependent and hence must be defined a priori; connectives such as *and* and *or*; the negation *not*; and linguistic hedges such as *very*, *more or less*, *extremely*, *quite*, etc. The syntactic rule may be represented as a context-free grammar which generates the terms (linguistic values) in T(*Age*) (see [3]), while the semantic rule is a procedure for computing the meaning of a

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[2]The compatibility of u with *young* is identical with the grade of membership of u in the fuzzy set labeled *young*.

linguistic value from the knowledge of the meanings of its components. Thus, if very is defined to be an operator which squares the compatibility function of its operand, then the compatibility function of very young may be expressed as (see Fig. 2)

$$c_{\underline{\text{very young}}}(u) = (c_{\underline{\text{young}}}(u))^2 \tag{2}$$

Similarly, if more or less X, where X is a primary term, is defined by

$$c_{\underline{\text{more or less}}\ X}(u) = (c_{\underline{\text{young}}}(u))^{\frac{1}{2}} \tag{3}$$

then

$$c_{\underline{\text{more or less young}}}(u) = (c_{\underline{\text{young}}}(u))^{\frac{1}{2}} \tag{4}$$

The negation and the connectives are defined by the relations

$$c_{\underline{\text{not}}\ X}(u) = 1-c_X(u) \tag{5}$$

$$c_{\underline{X \text{ and } Y}}(u) = c_X(u) \wedge c_Y(u) \tag{6}$$

$$c_{\underline{X \text{ or } Y}}(u) = c_X(u) \vee c_Y(u) \tag{7}$$

where X and Y are labels of fuzzy subsets of U, and $\vee$ and $\wedge$ stand for Max and Min, respectively. As an illustration, if

$$c_{\underline{\text{young}}}(35) = 0.2 \tag{8}$$

$$c_{\underline{\text{old}}}(35) = 0.1 \tag{9}$$

then

$$c_{\underline{\text{very young}}}(35) = 0.04 \tag{10}$$

$$c_{\underline{very}\ \underline{old}}(35) = 0.01 \tag{11}$$

$$c_{\underline{not}\ \underline{very}\ \underline{young}}(35) = 0.96 \tag{12}$$

$$c_{\underline{not}\ \underline{very}\ \underline{old}}(35) = 0.99 \tag{13}$$

and

$$c_{\underline{not}\ \underline{very}\ \underline{young}\ \underline{and}\ \underline{not}\ \underline{very}\ \underline{old}}(35) = 0.96 \wedge 0.99 = 0.96 \tag{14}$$

What this example shows is that once we have defined the meaning of and, or, not and the linguistic hedges, the meaning of any value of the linguistic variable which can be generated by the syntactic rule can be computed from the knowledge of the meaning of the primary terms. In effect, it is this structured nature of linguistic variables that makes them so convenient to use in the approximate characterization of complex phenomena.

Another point that helps to clarify the significance of a linguistic variable is that if the values of a conventional variable are represented as points in a plane, then the values of a linguistic variable may be likened to ball-parks with fuzzy boundaries. Furthermore, the compatibility function which defines the meaning of a linguistic value may be regarded as the membership function of a fuzzy restriction on the values of the base variable. This implies that a linguistic variable has a hierarchical structure which is illustrated in Fig. 3.

If $\mathcal{X}$ and $\mathcal{Y}$ are linguistic variables with respective universes of discourse U and V, then a linguistic function from U to V is defined to be a function from $T(\mathcal{X})$ to $T(\mathcal{Y})$. To illustrate, suppose that $U = V = (-\infty,\infty)$ and the function from $T(\mathcal{X})$ to $T(\mathcal{Y})$ is defined by the table

X	Y
very small	very very small
small and not very small	very small
not small and not large	small
large	not very small

Then the fuzzy graph, f, of the function in question is the union of the cartesian products[3]

$$f = \text{very small} \times \text{very very small} + \text{small and not very small} \times \text{very small} + \text{not small and not large} \times \text{small} + \text{large} \times \text{not very small} \tag{15}$$

As an additional illustration, suppose that $\mathcal{X}$ and $\mathcal{Y}$ represent, respectively, the input and output of a memoryless system which is characterized by a linguistic input-output function, f, whose table has the form shown below

X	Y
"u_1"	"v_1"
"u_2"	"v_2"
---	---
"u_n"	"v_n"

[3] The cartesian product of a fuzzy subset A of U and a fuzzy subset B of V is a fuzzy subset AxB of UxV whose compatibility function is related to those of A and B by $c_{A \times B}(u,v) = c_A(u) \wedge c_B(v)$. The union of A and B is denoted by A+B (or, more conventionally, by $A \cup B$) and is defined by $c_{A \cup B}(u) = c_A(u) \vee c_B(u)$.

where "u_i", $i = 1, \ldots, n$, denotes a fuzzy subset of $(-\infty,\infty)$ labeled <u>approximately</u> u_i, and likewise for "v_i". In more concrete terms, "u_i" may be defined by an expression such as

$$"u_i" = \int_{-\infty}^{\infty} \left(1 + \left(\frac{u - u_i}{a} \right)^2 \right)^{-1} / u \tag{16}$$

in which $\left(1 + \left(\frac{u - u_i}{a} \right)^2 \right)^{-1}$ is the compatibility function of "u_i", with a being a parameter which may depend on u_i. (The notation

$$A = \int_U \mu(u)/u \tag{17}$$

means that A is a fuzzy subset of U which may be represented as the union of fuzzy singletons $\mu(u)/u$. Thus, the integral sign in (17) denotes the union; u is the base variable; and $\mu(u)$ is the grade of membership of u in A or, equivalently, the compatibility of u with A.)

In the case under consideration, the graph of f is given by (Fig. 4)

$$f = "u_1" \times "v_1" + \ldots + "u_n" \times "v_n" \tag{18}$$

where "u_i" $\times$ "v_i", $i = 1, \ldots, n$, denotes the cartesian product of the fuzzy sets "u_i" and "v_i". More generally, in the case of a system with memory, the dependence of s^{t+1}, the state at time t+1, on s^t, the state at time t, and u^t, the input at time t, may be expressed approximately as a fuzzy graph g

$$g = \sum_i "s_i^{t+1}" \times "s_i^{t}" \times "u_i^{t}" \tag{19}$$

where s_i^t is a point in the state space of the system; "s_i^t" is a fuzzy subset of the state space labeled <u>approximately</u> s_i^t; u_i^t is a point in the input space; "u_i^t" is a fuzzy subset of the input space labeled <u>approximately</u> u_i^t; and $\sum$ denotes the union rather than the arithmetic sum.

Among the various concepts that can be dealt with linguistically, there are two that are of particular relevance to the analysis of societal systems, namely, <u>Truth</u> and <u>Probability</u>. In the case of <u>Truth</u>, its term-set may be assumed to be (compare with (1))

T(<u>Truth</u>) = <u>true</u> + <u>very</u> <u>true</u> + <u>not</u> <u>true</u> + <u>very</u> <u>very</u> <u>true</u> + <u>not</u> <u>very</u> <u>true</u> + ... + <u>false</u> + <u>very</u> <u>false</u> + ... + <u>not</u> <u>very</u> <u>false</u> <u>and</u> <u>not</u> <u>very</u> <u>true</u> + .. <u>more</u> <u>or</u> <u>less</u> <u>true</u> + ... (20)

while that of <u>Probability</u> might be

T(Probability) = <u>likely</u> + <u>very</u> <u>likely</u> + <u>not</u> <u>likely</u> + <u>very</u> <u>very</u> <u>likely</u> + <u>unlikely</u> + <u>very</u> <u>unlikely</u> + ... + <u>more</u> <u>or</u> <u>less</u> <u>likely</u> + "0" + "0.1" + ... + "1" + ... (21)

The primary term in <u>Truth</u> is <u>true</u>, which is defined to be a fuzzy subset of the unit interval [0,1]. A convenient approximation to the compatibility function of <u>true</u> is provided by the expression

$$
\begin{aligned}
c_{\underline{true}}(v) &= 0 \quad \text{for} \quad 0 \leq v \leq a \\
&= 2\left(\frac{v-a}{1-a}\right)^2 \quad \text{for} \quad a \leq v \leq \frac{a+1}{2} \\
&= 1 - \left(\frac{v-1}{1-a}\right)^2 \quad \text{for} \quad \frac{a+1}{2} \leq v \leq 1
\end{aligned}
\tag{22}
$$

which has $v = \frac{1+a}{2}$ as its <u>crossover</u> <u>point</u> (i.e., the point at which $c(v) = 0.5$). Correspondingly, the compatibility function of <u>false</u> is given by (see Fig. 5)

$$c_{\underline{false}}(v) = c_{\underline{true}}(1-v) , \qquad 0 \le v \le 1 \tag{23}$$

The same applies to Probability, with likely and unlikely being analogous to true and false, respectively.

The linguistic truth-values in (21) serve the function of providing an approximate assessment of the truth of a possibly fuzzy assertion. As an illustration, consider the proposition "George is very intelligent," which, as pointed out earlier, may be interpreted as an assignment of the linguistic value very intelligent to the intelligence of George. Now, the assignment of a truth-value, say very true, to the proposition in question results in the composite proposition

" "George is very intelligent" is very true "

As shown in [4], such a proposition may be approximated by a simple proposition of the form

" George is X "

where X is a linguistic value of Intelligence which depends on the compatibility functions of the primary terms intelligent and true.

In the case of a societal system, S, we would usually be concerned with a collection of propositions such as:

A_1 : " S is quite stable "

A_2 : " S is highly mobile "

A_3 : " S is experiencing rapid growth "

A_n : " S is in a state of depression "

Suppose that the propositions $A_1, \ldots, A_n$ are assigned the linguistic truth-values $T_1, \ldots, T_n$, respectively. The collection of possible truth-values that can be assigned to $A_1, \ldots, A_n$ is called a *truth-value distribution*. For example, for n = 3, we may have the truth-value distribution

$$T = (\underline{true}, \underline{quite}\ \underline{true}, \underline{very}\ \underline{true}) + (\underline{false}, \underline{false}, \underline{true}) + (\underline{very}\ \underline{true}, \underline{more}\ \underline{or}\ \underline{less}\ \underline{true}, \underline{not}\ \underline{true}) + (\underline{true}, \underline{true}, \underline{false}) \tag{24}$$

which means that there are four possible assignments of truth-values to A_1, A_2, A_3 expressed by the terms of (24). These assignments reflect the relationships between the truth-values of A_1, A_2, A_3, which are induced by the underlying dependencies between the propositions in question.

The use of linguistic truth-values leads to a *fuzzy logic* in which not only the truth-values but also the rules of inference are fuzzy in nature. A simple example of such inference is the following: (x and y are real numbers)

premiss: x is small

premiss: x and y are approximately equal

---

approximate conclusion: y is more or less small

The difference between fuzzy logic and the classical two-valued logic is illustrated by the familiar syllogism:

premiss: All men are mortal

premiss: Socrates is a man

---

conclusion: Socrates is mortal

In fuzzy logic, an analogous syllogism would be premiss: Most men shave

premiss: Lakoff is a man

approximate probabilistic conclusion: It is very likely that Lakoff shaves

The idea here is that most and likely may be defined as fuzzy subsets of the unit interval (Fig. 6). Thus, if the compatibility function of very likely is an acceptable approximation to that of most, we can assume that the conclusion "It is very likely that Lakoff shaves," follows approximately from the premisses "Most men shave" and "Lakoff is a man."

A basic rule of inference in fuzzy logic is the compositional rule [3]. In a simplified form, this rule may be stated as follows. Let A : "u is P" be a proposition in which P is a value of a linguistic variable, e.g., "u is small." Let B : "u and v are Q" be another proposition in which Q is a fuzzy relation expressed in linguistic terms, e.g., "u and v are approximately equal." Then, from A and B we can infer C:

A : u is P

B : u and v are Q

C : v is $P \circ Q$

where $P \circ Q$ is the composition[4] of the unary relation P with the binary relation Q.

[4] The compatibility function of the composition of P and Q is given by $c_{P \circ Q}(u,v) = \vee_u (c_P(u) \wedge c_Q(u,v))$, where $\vee_u$ denotes "maximum over u."

As a simple illustration of this rule assume that we have

A : u is small

B : u and v are approximately equal

where small is defined as a fuzzy subset of the universe of discourse (+ means union)

$$U = 1 + 2 + 3 + 4 \tag{25}$$

which is expressed as

$$\underline{small} = 1/1 + 0.6/2 + 0.2/3 \tag{26}$$

where a term of the form μ/u signifies that the grade of membership of u in small (or, equivalently, the compatibility of u with small) is μ. Similarly, approximately equal is defined as a fuzzy relation by

$$\begin{aligned}\underline{approximately}\ \underline{equal} = {} & 1/(1,1) + 1/(2,2) + 1/(3,3) + 1/(4,4) \\ & + 0.5/(1,2) + 0.5/(2,1) + 0.5/(2,3) \\ & + 0.5/(3,2) + 0.5/(3,4) + 0.5/(4,3)\end{aligned} \tag{27}$$

To compute the composition small $\circ$ approximately equal it is sufficient to form the max-min matrix product of small and approximately equal as shown below

$$[1 \quad 0.6 \quad 0.2 \quad 1] \begin{bmatrix} 1 & 0.5 & 0 & 0 \\ 0.5 & 1 & 0.5 & 0 \\ 0 & 0.5 & 1 & 0.5 \\ 0 & 0 & 0.5 & 1 \end{bmatrix} = [1 \quad 0.6 \quad 0.5 \quad 0.2]$$

Thus,

$$\underline{small} \circ \underline{approximately}\ \underline{equal} = 1/1 + 0.6/2 + 0.5/3 + 0.2/4 \tag{29}$$

which may be approximated linguistically as more or less small. It is this result that provides the basis for the statement made earlier to the effect that from A and B we can infer (approximately) that v is more or less small.

A special and yet important case of the compositional rule of inference is one where the proposition B has the form

B : If u is R then v is S

where R and S are values of linguistic variables, e.g., "If u is small then v is very large." It can be shown [4], that in this case B may be replaced by the equivalent proposition

B* : u and v are Q

where (+ denotes the union and V is the universe of discourse associated with S)

$$Q = R \times S + \underline{\text{not}}\ R \times V \tag{30}$$

Then, the application of the compositional rule of inference to A and B* yields what might be called the generalized modus ponens:

A : u is P

B : If u is R then v is S

C : v is $P \circ (R \times S + \underline{\text{not}}\ R \times V)$

Examples of the application of this rule may be found in [3] and [4].

Just as linguistic truth-values can be used to provide a basis for the assignment of approximate truths to assertions about a complex system, so can linguistic probabilities be employed to assert approximate likelihoods of various events. For example, in response to the question "What is the probability that there will be a recession next year," we may respond with a fuzzy ball-park estimate <u>very likely</u>, with the understanding that <u>very likely</u> is a linguistic probability-value whose meaning can be computed once the compatibility function of <u>likely</u> has been defined. The way in which this can be done as well as the techniques for computing with linguistic probabilities are discussed in greated detail in [4].

3. Concluding Remarks

Our main objective in the foregoing discussion was to outline some of the basic aspects of the concept of a linguistic variable and suggest that the "ball-park" nature of the values of linguistic variables may make them an appropriate tool for the analysis of societal systems. Clearly, the task of applying the linguistic approach to the solution of specific problems relating to the behavior of societal systems is certain to be nontrivial. Nevertheless, the ideas immanent in the linguistic approach appear to be of sufficient relevance to the analysis of societal systems to justify further exploration of their utility in the characterization of the behavior of large-scale systems.

References

1. B. Skinner, Beyond Freedom and Dignity, A. Knopf, New York, 1971.

2. H. Wheeler, ed., Beyond the Punitive Society, W. H. Freeman Co., San Francisco, 1973.

3. L. A. Zadeh, "Outline of a New Approach to the Analysis of Complex Systems and Decision Processes," IEEE Trans. on Systems, Man and Cybernetics, vol. SMC-3, pp. 28-44, January 1973.

4. L. A. Zadeh, "The Concept of a Linguistic Variable and Its Application to Approximate Reasoning," Memorandum M 411, Electronics Research Laboratory, University of California, Berkeley, October 1973.

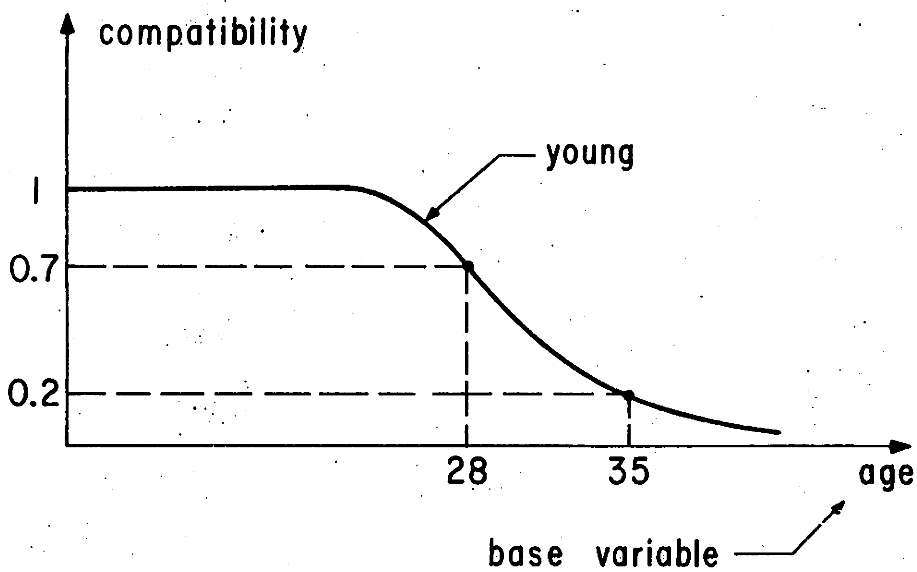

Fig. 1 Compatibility function for young

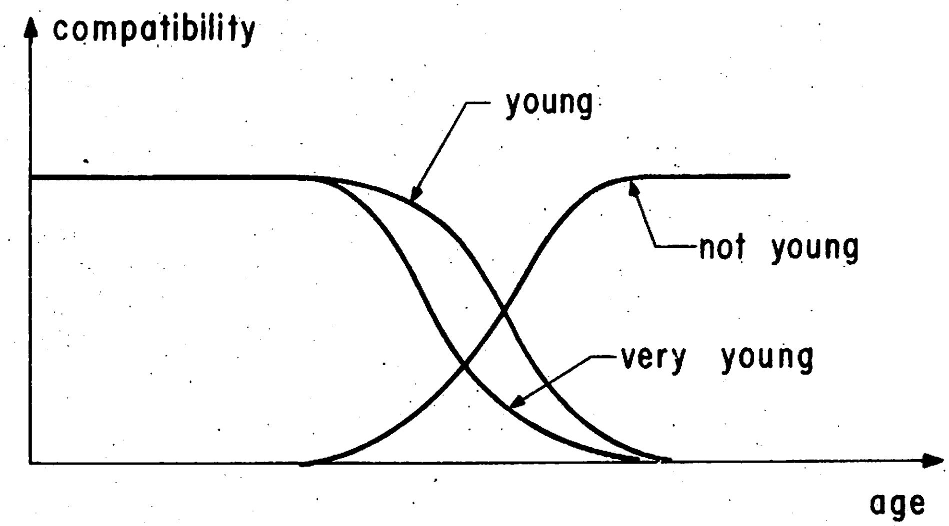

Fig. 2 Compatibilities of <u>young</u>, <u>not young</u>, and <u>very young</u>

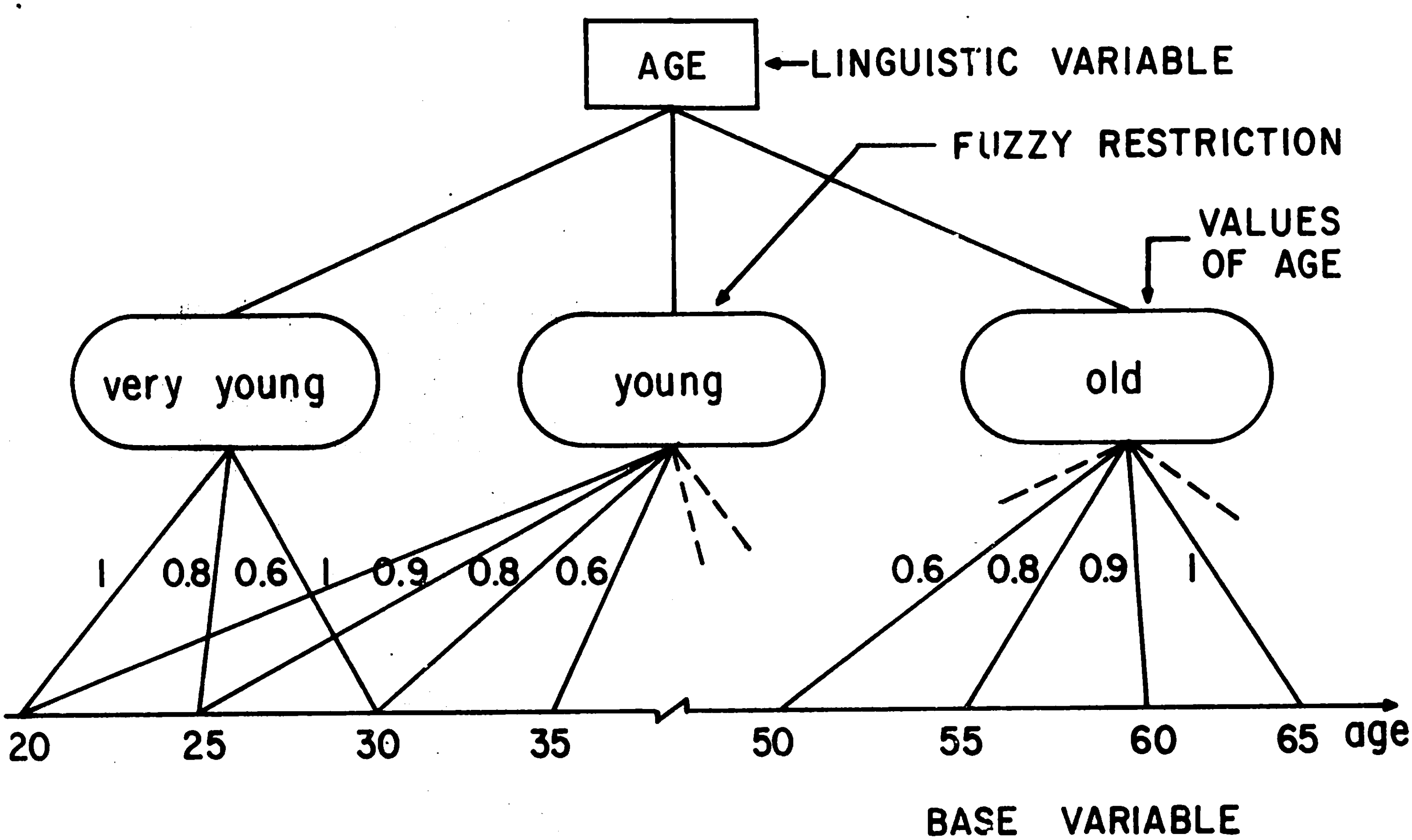

Fig. 3 Hierarchical structure of a linguistic variable

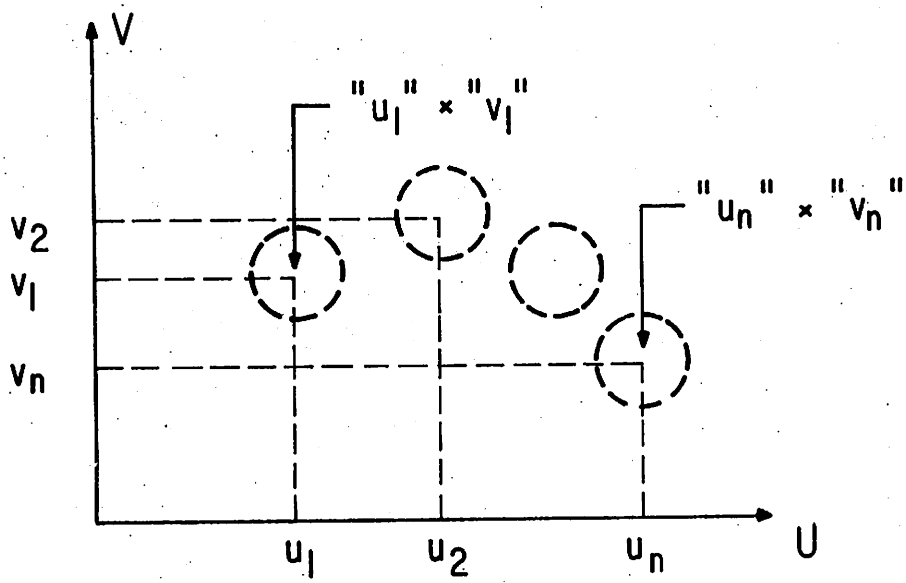

Fig. 4 Representation of a fuzzy graph as a union of fuzzy points

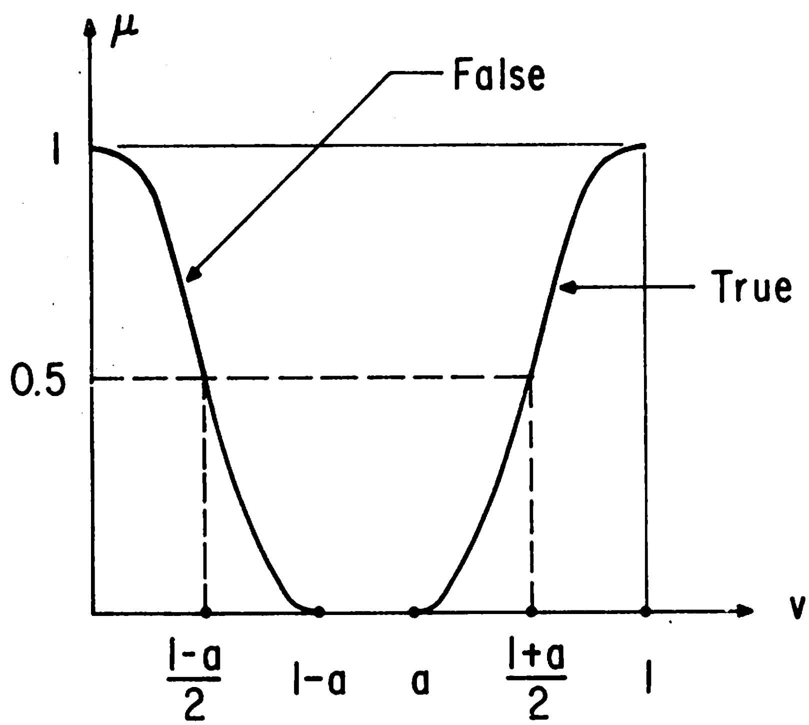

Fig. 5 Compatibility functions of linguistic truth-values true and false

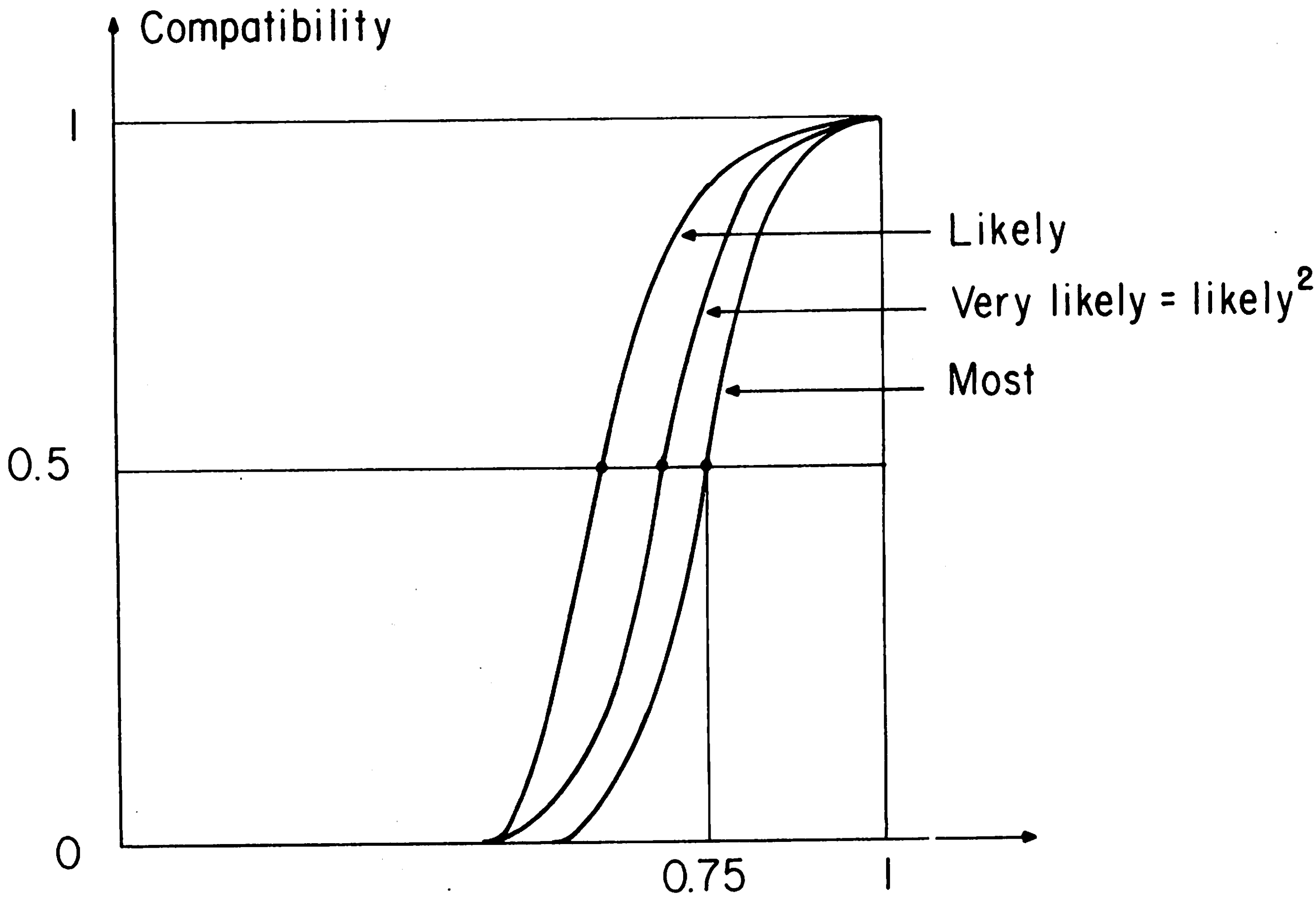

Fig. 6 Compatibility function of most as an approximation of that of very likely