ON THE DESIGN OF RENT CONTROL

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Abstract

The paper presents a procedure for designing a rent control schedule which achieves reduction in rents while it simultaneously discourages under-maintenance leading to deterioration of the housing stock. The focus of the procedure is to understand the reaction of a profit-maximizing landlord to a given rent schedule and then to design the schedule so as to induce a favorable reaction. This is formulated mathematically as an inverse problem of optimal control. Implementation requires continuous and accurate monitoring of the state of the housing stock. Some cost estimates are presented for an information system which achieves this for a city with 30,000 rental units.

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1. Introduction

Our objective is to present a method for designing a rent control scheme which appears to have a good chance of being "successful" in the following limited situation. Imagine a small city with little vacant residential land within a metropolitan area which over a short time interval becomes considerably more attractive as a place to live in comparison with neighboring cities. The increased demand drives up rents well beyond those prevailing in neighboring cities. This not only creates a redistribution of income in favor of landlords but also forces many renters to change their place of residence thereby causing a disintegration of neighborhood communities. If the inhabitants of the city find this intolerable, then one obvious countermeasure which may be available to them is the imposition of rent control, that is, the replacement of market-determined rents by an administratively determined schedule of rents. Most economists have arrived at the judgement that such an intervention in the housing market will, at least over the long run, lead to a worsening of the overall housing conditions instead of ameliorating them. Everyone familiar with the horror stories, whether of an anecdotal or statistical nature, about New York or Paris or some other city which has imposed rent control, will doubtless agree that at best rent control is a blunt instrument.

We propose here a procedure, or more accurately, a framework for carefully designing this instrument so that it will have the intended effect while simultaneously minimizing the unintended, damaging side effects for the limited situation outlined above. We view rent control as being essentially embodied in a rent schedule which determines the administered rent for each particular dwelling unit as a (mathematical) function of the bundle of housing services delivered by this unit during a fixed period of time, say one month. The problem is then transformed into that of designing this function. We assume that the owner of the unit will adjust his maintenance inputs so as to maximize his profits given the rent schedule. A particular rent schedule then elicits a determinate reaction from the landlord, so that the schedule must be so designed as to induce a reaction from the landlord which is socially desirable. Mathematically, the design procedure reduces to the so-called inverse problem of optimal control.

A crucial aspect in implementation is the cost of continuing collection and processing of information about the 'state' of each dwelling unit sufficient to determine the "bundle of services" delivered by the unit. This aspect cannot be formulated mathematically. Instead we give some cost estimates for a hypothetical city with 30,000 rental units. These estimates suggest that the scheme can be implemented at a modest cost.

The rest of the paper is organized as follows. The next section briefly summarizes the economist's main arguments against rent control. Some of these are inapplicable to the situation being considered here. Others still maintain force, whereas the remainder are effectively countered by the proposed scheme which is presented in Section 3. The implementation cost estimates are given in Section 4. Some concluding remarks of a more general nature are collected in Section 5.

2. Arguments Against Rent Control

To assess these arguments it will be useful to briefly review the theory of the rental housing market.[1] We think of each dwelling unit

[1] For further details please refer to [1-4].

in our city as being characterized by a single quality variable x, $0 \leq x < \infty$, with lower quality levels corresponding to higher values of x, so that at any time the stock of housing is described by the distribution of the number of units in each quality level.[2] Now, over a short time interval the housing stock does not change appreciably i.e., the short-run supply of housing is inelastic. Hence the rent of housing of a given quality level x, say r(x), is determined solely by the demand for housing. In particular, this market-determined rent function r(x) reflects consumer preferences so that it is an efficient rationing mechanism.

Over a larger interval of time landlords will make decisions which change the city's housing stock. These decisions are of two kinds. <u>Maintenance</u> decisions change the quality level of an individual unit: higher maintenance lowers x, poor maintenance increases x. <u>Construction</u> or demolition decisions change the total number of housing units. Since landlords seek to maximize profits their maintenance programs will depend upon maintenance costs and rent differentials among different quality levels, whereas their construction decisions depend as well upon returns to investment available outside the city's housing market.

Now suppose that rent is controlled so that the administered rent function $R(x) < r(x)$. The following damaging consequences appear evident from the theory above.

<u>C1</u> In the short run there will be inefficiencies in consumption (of housing) among renters. Renters who are willing to pay more for a

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[2] x may be regarded as the "effective" age of a unit. A well-maintained unit will have a lower effective age than an undermaintained unit constructed at the same time.

certain quality of housing than those currently occupying it will be unable to do so.

C2 Alternative rationing systems such as queues and black markets will develop as the gap $r(x) - R(x)$ increases sufficiently.

C3 There is a redistribution of income from landlords to renters.

C4 Over the long run the quality of housing stock will decline over a broad front as landlords find it unprofitable due to reduced rents to maintain housing at a high quality level. (Recall the experience of New York City.)

C5 Over the long run the supply of housing will be reduced (relative to the market-determined supply) since it is not profitable to construct new housing. This is inefficient since the community as a whole would prefer to devote more of its resources to housing as against other commodities.

C1 will remain valid if the scheme proposed below is adopted. But two points should be kept in mind before accepting C1 completely. First of all the charge of consumption inefficiency assumes implicitly that the monetary and psychic costs borne by those who have to relocate because of their inability to pay higher rents are negligible. Much of the literature on urban renewal suggests that this is not the case. Secondly, the imposition of rent control reveals that the citizenry has collectively decided to favor current residents as against potential householders who wish to locate in the city. C2 can be mitigated in part by adequate legal enforcement and in part by not permitting the gap $r(x) - R(x)$ to become very large. C3 of course remains valid and is indeed an intended effect of rent control. C4 and C5 are in some ways the most serious consequences since they allege that everyone "loses" in the long run. As will be seen C4 can be effectively eliminated by the proposed scheme. C5 carries more weight if rent control is applied throughout a nation or a large part of a metropolitan area. It is less important in the context that we are considering and, even then, the supply of new housing can be partly controlled by appropriate modification of the administered rent function R.

3. The Rent Control Scheme: Theoretical Considerations

Let $x(t)$ be the effective age (quality level) of a particular unit. In the absence of any maintenance this age increases by one year per (physical) annum. However by appropriate maintenance the aging process can be slowed down or even reversed according to the differential equation

$$\dot{x}(t) = 1 - m(x(t))\, u(t), \tag{1}$$

where, at time t, $u(t)$ is the annual rate of dollar expenditures for maintenance, and $m(x)$ is the amount of effective age reduction per dollar of expenditure when the unit is in state x. m is an exogenously specified function and has to be determined empirically [5].

Let $R(x)$ be the rent schedule. Then the profit-maximizing landlord will program his maintenance expenditures $u(t)$, $t \geq 0$, so as to

$$\underset{u(\cdot)\geq 0}{\text{Maximize}} \int_0^\infty e^{-\delta t}[R(x(t)) - u(t)]\, dt \tag{2}$$

Here δ is the rate of interest so that the integral in (2) is the present value of the program $u(\cdot)$. Let capital letters denote optimal values of the variables. By the Maximum Principle there exists a function $P(t)$, $t \geq 0$, satisfying the adjoint equation

$$\dot{P}(t) = -\frac{\partial H}{\partial x}\,(t,X(t),P(t),U(t)), \tag{3}$$

with boundary condition $\lim_{t\to\infty} P(t) = 0$, such that for all t

$$H(t,X(t),P(t),U(t)) = \underset{u\geq 0}{\text{maximum}}\; H(t,X(t),P(t),u), \tag{4}$$

where the Hamiltonian H is given by

$$H(t,x,p,u) = e^{-\delta t}[R(x)-u] + p[1-m(x)u] \tag{5}$$

If instead of the price function $P(t)$ we work with the ("current" price) function $Q(t) = e^{\delta t}P(t)$, we obtain from (3) the equation

$$\dot{Q}(t) = \delta Q(t) - R'(X(t)) + m'(X(t))\,U(t)\,Q(t), \tag{6}$$

while (4) yields the conditions

$$U(t) = \begin{cases} 0 & , \quad \text{if } [1 + m(X(t))\,Q(t)] > 0 \\ \in [0,\infty] & , \quad \text{if } [1 + m(X(t))\,Q(t)] = 0 \\ +\infty & , \quad \text{if } [1 + m(X(t))\,Q(t)] < 0 \end{cases} \tag{7}$$

We can now state our problem mathematically:

Design the rent schedule $R(\cdot)$ so that the corresponding optimal trajectories yield satisfactory values for the state of housing (low values of x) for a large number of initial values. Furthermore, the design and the optimal trajectories should not be very sensitive to errors in the specification of $m(\cdot)$ or in measurement of x.

In the (x,q) plane let S^+, S, S^- denote respectively the regions where $[1 + m(x)q] > 0, 0, <0$. We will impose the restriction that $R(x) > 0$, $R'(x) < 0$ for $x < \bar{x}$, and $R(x) = 0$ for $x \geq \bar{x}$ ($\bar{x}$ is the minimum acceptable quality level). We assume that $m \geq 0$. Finally we assume that there is a unique value of x, $x = x^*$, such that (see Figures 1,2)

$$\frac{m'(x)}{m^2(x)} + R'(x) + \frac{\delta}{m(x)} = 0 \tag{8}$$

We will first analyze the behavior of the optimal solution in the region S, then S^+, and finally S. Suppose that $(X(t),Q(t)) \in S$ for a non-vanishing time interval $I = [t_1,t_2]$. Then, for $t \in I$, $Q(t) = \frac{1}{m(X(t))}$ from (7) which upon differentiation yields

$$\begin{aligned}\dot{Q}(t) &= \frac{m'(X(t))}{m^2(X(t))} [1 - m(X(t))\, U(t)] \\ &= \delta Q(t) - R'(X(t)) + m'(X(t))\, U(t)\, Q(t) \text{ from (6)} \\ &= -\frac{\delta}{m(X(t))} - R'(X(t)) + m'(X(t))\, U(t)\, Q(t)\end{aligned}$$

Hence

$$\frac{m'(X(t))}{m^2(X(t))} + R'(X(t)) + \frac{\delta}{m(X(t))} = \frac{m'(X(t))}{m(X(t))} [1 + m(X(t))Q(t)]\, U(t) = 0,$$

so that

$$X(t) = x^* \quad , \quad t \in I,$$

and hence for $t \in I$

$$U(t) = \frac{1}{m(x^*)} = u^* \text{ say, } Q(t) = -\frac{1}{m(x^*)} = q^* \text{ say}$$

This is the unique singular solution. It is also the unique optimal steady-state. Its contribution during the interval I to the present value (2) is

$$\int_{t_1}^{t_2} e^{-\delta t}[R(x^*) - \frac{1}{m(x^*)}]\, dt = \frac{1}{\delta}(e^{-\delta t_1} - e^{-\delta t_2})\,[R(x^*) - \frac{1}{m(x^*)}] \tag{9}$$

Next, in the region S^+ $U(t) \equiv 0$ by (7) so that the optimal policy is to disinvest as rapidly as possible and to allow the unit to deteriorate. Under this policy X, Q obey the equations

$$\dot{X}(t) = 1 \quad , \ \delta\dot{Q}(t) = \ Q(t) - R'(X(t)), \tag{10}$$

so that in a neighborhood of S the trajectories of (10) satisfy

$$\left.\frac{\partial Q}{\partial X}\right|_S - \left.\frac{\partial Q}{\partial X}\right|_{S^+} = \{\frac{\partial}{\partial x}[-\frac{1}{m(x)}] + R'(x) - \delta q\}_S$$

$$= \frac{m'(x)}{m^2(x)} + R'(X) + \frac{\delta}{m(X)} \gtrless 0 \text{ as } X \lessgtr x^* \tag{11}$$

by (8), so that the vector field (10) has the form shown in Figure 3. Suppose $(X(t), Q(t)) \in S^+$ for $t \in I^+ = [t_1, t_2]$. Then integration of (10) leads to

$$e^{-\delta X(t)}Q(t) + \int_0^{X(t)} e^{-\delta x}R'(x)\,dx = e^{-\delta X(t_1)}Q(t_1) + \int_0^{X(t_1)} e^{-\delta x}R'(x)\,dx \tag{12}$$

If we define

$$\rho(x) = \int_0^{x} e^{-\delta y}R'(y)\,dy \tag{13}$$

then, by (12), the characteristic curves of (10) are given by

$$e^{-\delta X}Q(X) + \rho(X) = \text{constant} \tag{14}$$

From the terminal conditions $\lim_{t\to\infty} P(t) = \lim_{t\to\infty} e^{-\delta t}Q(t) = 0$. Since $R'(x) = 0$ for $x \geq \bar{x}$, we can see from (10) that the only trajectory in S^+ which hits the x axis and which satisfies this terminal condition is the one which passes through the point $(x,q) = (\bar{x},0)$. By (14), this trajectory lies on the curve

$$e^{-\delta X}Q(X) + \rho(X) = e^{-\delta\bar{x}}\cdot 0 + \rho(\bar{x}) = \rho(\bar{x}) \tag{15}$$

labelled T^+ in Figure 4. It is evident that if a portion of the optimal trajectory lies on T^+ for some time t_1 then it must remain on T^+ for all $t \geq t_1$. An obvious necessary condition for $R(\cdot)$ to be a good design is that if the initial state of the unit is x^* then it is more profitable for the landlord to maintain the unit in this state i.e., adopt $u(t) \equiv \frac{1}{m(x^*)}$, rather than disinvest i.e., $u(t) \equiv 0$. This gives us our first design condition,

$$\frac{1}{\delta}\left[R(x^*) - \frac{1}{m(x^*)}\right] - \int_{x^*}^{\bar{x}} e^{-\delta(y-x^*)}R(y)\,dy > 0$$

Integrating the second term by parts leads, after some simplification, to the equivalent condition

$$e^{-\delta x^*} q^* + \rho(x^*) > \rho(\bar{x}) \tag{16}$$

From (15) we see that (16) is equivalent to requiring that the curve T^+ intersects S at (x^+,q^+) to the right of (x^*,q^*) as shown in Figure 4. Another straightforward analysis reveals that there is only one other curve in S^+ along which an optimal trajectory can possibly lie. This is the curve belonging to the family (14) which passes through (x^*,q^*). It is given by

$$e^{-\delta X}Q(X) + \rho(X) = e^{-\delta x^*}q^* + \rho(x^*), \tag{17}$$

and is labelled T^{++} in Figure 4.

Finally we analyze the behavior in S^-. Suppose $(X(t),Q(t)) \in S^-$ for some t so that, by (7), at time t $U(\cdot)$ is a Dirac delta function, say

$$U(s) = u\delta(s-t) \tag{18}$$

where $u \geq 0$ is a constant. From (1) and (6) we see that both X and Q will then be discontinuous at t. To compute $X(t+) - X(t)$ rewrite (1) as

$$\int_t^{t_1} \frac{dX(s)}{m(X(s))} = \int_t^{t_1} \frac{ds}{m(X(s))} - \int_t^{t_1} U(s)\, ds \tag{19}$$

so that taking limits as $t_1 \to t+$ gives, using (18),

$$\lim_{t_1 \to t+} \int_{X(t)}^{X(t_1)} \frac{dx}{m(x)} = -u \tag{20}$$

If we define $\mu(x) = \int_0^x \frac{1}{m(y)}\, dy$, then (20) gives us

$$\mu(X(t+)) = \mu(X(t)) - u \tag{21}$$

from which we can solve for $X(t+)$ knowing $X(t)$ and u. To determine the jump in Q we proceed similarly by rewriting (6) as

$$\int_t^{t_1} \frac{dQ(s)}{Q(s)} = \int_t^{t_1} \delta\, ds - \int_t^{t_1} \frac{R'(X(s))}{Q(s)} + \int_t^{t_1} m'(X(s))\, U(s)\, ds \tag{22}$$

By (19) the last integral above is equal to

$$-\int_t^{t_1} \frac{m'(X(s))}{m(X(s))}\, dX(s) + \int_t^{t_1} \frac{m'(X(s))}{m(X(s))}\, ds$$

Substituting this in (22), integrating and taking limits as $t_1 \to t+$ yields $\log Q(t+) - \log Q(t) = \log m(X(t)) - \log m(X(t+))$ from which we obtain the desired relationship

$$Q(t+)m(X(t+)) = Q(t)m(X(t)) \tag{23}$$

In particular if $(X(t),Q(t)) \in S$ (the boundary of S^-), then $(X(t+), Q(t+)) \in S$ also.

It remains to determine the initial values of $X(0)$ for which the maintenance program (18) is in fact optimal and the corresponding values of $X(0+)$. First of all, if $X(0) \leq x^*$ then it is routine to show that it is better to disinvest i.e., the optimal trajectory lies along T^{++} in

Figure 4. On the other hand if $X(0) = x > x^*$ <u>and</u> if (18) is optimal then one can show that $X(0+) = x^*$ i.e., $(X(0),Q(0)) \in S$ and $(X(0+), Q(0+)) = (x^*,q^*)$. From our earlier analysis we know that if this is not optimal then the optimal trajectory must lie along T^+ in Figure 4. To determine which of these two extremal policies is more profitable for different initial conditions $X(0) = x > x^*$ we compare the corresponding contributions to (2). Under (19) this contribution is

$$J_1(x) = -\int_{x^*}^{x} \frac{1}{m(y)}\, dy + \frac{1}{\delta}\left[R(x^*) - \frac{1}{m(x^*)}\right],$$

whereas along T^+ it is

$$J_2(x) = \int_{x}^{\bar{x}} e^{-\delta(y-x)} \; R(y)\, dy$$

Some algebraic manipulations lead us to

$$J_1(x) - J_2(x) = -\int_{x^*}^{x} \frac{1}{m(y)}\, dy + \frac{1}{\delta}\,[R(x^*) + q^*] - \frac{1}{\delta}\,[R(x)+Q(x)] \tag{24}$$

where $Q(x)$ is given by (15), the curve describing T^+. Evidently J_1-J_2 decreases with x. Hence the investment policy (18) is optimal for $X(0) < x^-$ whereas disinvestment is optimal for $X(0) > x^-$ where x^- is defined by

$$J_1(x^-) - J_2(x^-) = 0 \tag{25}$$

The behavior of the optimal trajectories is displayed in Figure 5.

We can now deduce the ranges within which R will be a "good" design. First of all the optimal steady-state x^* can be chosen by the city subject to its defining characteristics (8). Note that it depends upon the values of R' i.e., upon rent differentials only and not upon the absolute values of R as long as the necessary condition (16) is met. Thus (8) and (16) establish the maximum range over which consequence C4 can be avoided. Of course, if R is reduced so that (16) is violated, then massive deterioration must result. To reduce x^* (i.e., to have a better steady-state) (8) suggests that R should be increased. Also, to maintain x^* in the face of exogenous changes in the interest rate δ and maintenance costs m appropriate compensatory changes in R must be made.

Secondly, a "good" design should be such that the state x^- given by (25) must be as large as possible. From (24) we see that this can be achieved by increasing $R(x^*) - R(x)$ for $x > x^*$ since that will reduce profits from disinvestment relative to maintenance. If $R(x^*)-R(x)$ is increased by raising $R(x^*)$, this may nullify the purposes of rent control to some extent; on the other hand if it is increased by reducing $R(x)$ there will be a relative increase in demand for housing of lower quality levels. In any case we can see the amount of flexibility which is possible in choosing R. Any further specification of R would require detailed consideration of intended redistribution effects among households.

Finally we suggest how to cope with C5. Since the steady-state net revenue per annum is $R(x^*) - \frac{1}{m(x^*)}$, new housing will be created only if this yields a return which is favorable in comparison with investment opportunities outside the city's housing market. One way to encourage new construction is to make R a function of x and t, $R = R(x,t)$, where t denotes the date of construction of the unit. This will permit adjustments in R according to changes in construction costs.

In summary, equations (8), (16) and (25) are the most important theoretical relations which should be considered in designing R, and these will also reveal the sensitiveness of the design to errors in the specification of m and in measurement of x.

4. Considerations in Implementation[3]

Three different sets of "inputs" enter into the production of the bundle of services that we call "housing." The first can be called the structure of the unit. Normally the only requirements imposed on this is that the structure meet the building code regulations. The second set of inputs is such capital equipment as appliances, furniture etc. whose value can be determined independently of the unit. Finally we have the set of inputs which are usually classified as "maintenance and operating" expenditures. Some of these such as janitorial services, painting, repairing, gardening, and some management services are directly related to output, and these are the empirical correlates of the function m of the previous section; other inputs in this category such as taxes and finance costs are not directly connected with output.

In the previous section we saw that a good design depends upon continuously updated information regarding maintenance costs as well

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[3] The author is indebted to D. Pessel and M. Stonebraker for the estimates presented here.

as the state of housing (so that any misspecification of m can be detected). Of course information on all other costs is also necessary to arrive at the final determination of rents. The total information can be broken down into the following categories: structural data, capital equipment -furnishings and appliances, optional services such as parking, maintenance and operating costs, financing costs such as taxes, etc. We estimate that this will account for not more than 300 bytes of data. Hence a city with 30,000 rental units requires an on-line access to a data base of 10^7 bytes of data. The rental cost of a system providing such a capacity should not exceed $12,000 per year. The other major costs are software development, data acquisition and reliability of data. Data reliability can be ensured by having the information available to both tenants and landlords with appropriate procedures to decide when conflicting claims are made. The data should be updated at least annually and if appropriate forms are designed these can be filled out by landlords at minimal costs, although the collective cost incurred by them may still be substantial (estimate: $1 per unit per year x 30,000 = $30,000 per year). Software development should not exceed six man-months, or $12,000. Thus the costs incurred in implementing an adequate information system necessary to support the proposed scheme would be $42,000 per year,[4] and a software development cost of $12,000. Since the administrative costs of rent control doubtless exceed these estimates by a huge margin, the cost of the proposed scheme appears to be quite modest.

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[4]Note that this figure includes the "invisible" cost of $30,000 borne by the landlords.

5. Concluding Remarks

Economic theory tells us that the best way to ameliorate the hardships incurred by people facing rapidly rising rents is not to interfere with the market but rather to have direct income transfers to these people. Such income transfers should be financed through taxation preferably perhaps of the landlords who are making "windfall" profits. Cities do not have such taxing authority. Hence the proposed scheme is what is called a "second best" solution.

Doubtless this scheme is extremely naive and inefficient when compared with the perfectly functioning, decentralized, housing market of some economic textbooks. But the real housing market is not as efficient as may appear, especially the cheap housing sector of the market, and it is the consumers of this housing who are least able to absorb rent increases.

Since the "first best" solutions of the economist are often not implementable it may be worthwhile investigating realistic second best solutions both regarding their effectiveness and their costs. This paper is an exercise in bringing to bear an "engineering" approach to such problems.

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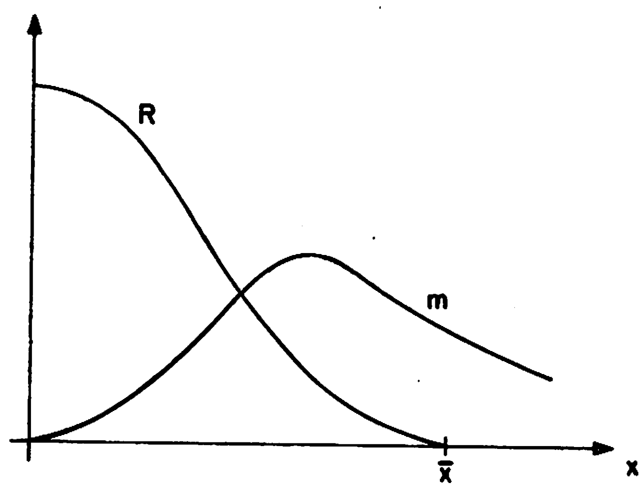

Fig. 1. Assmued behavior of R, m.

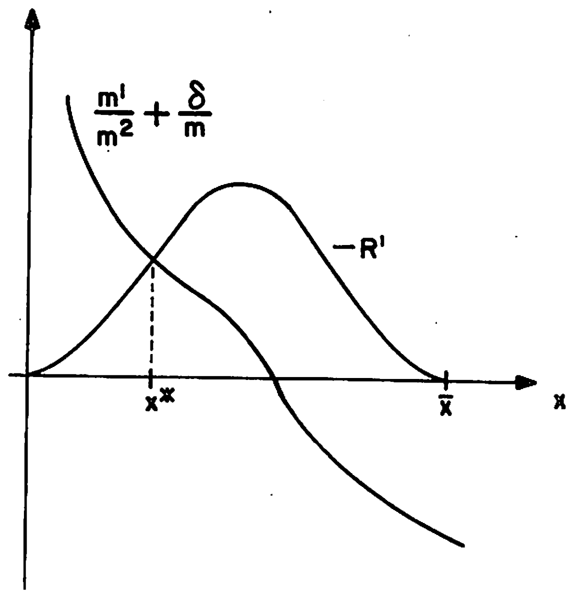

Fig. 2. The optimal steady-state x*.

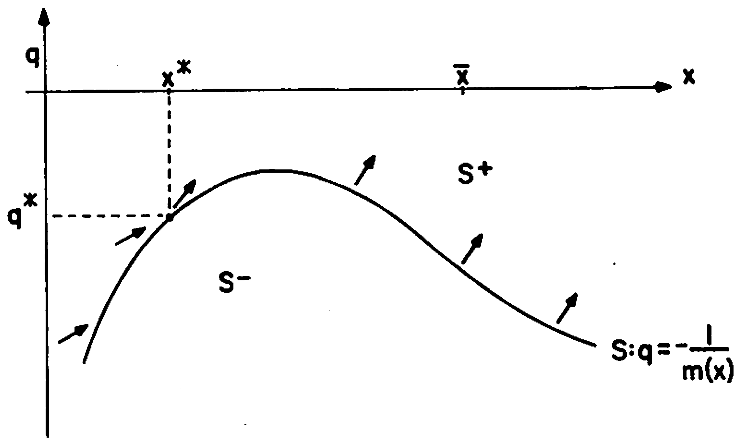

Fig. 3. Analysis in S^+.

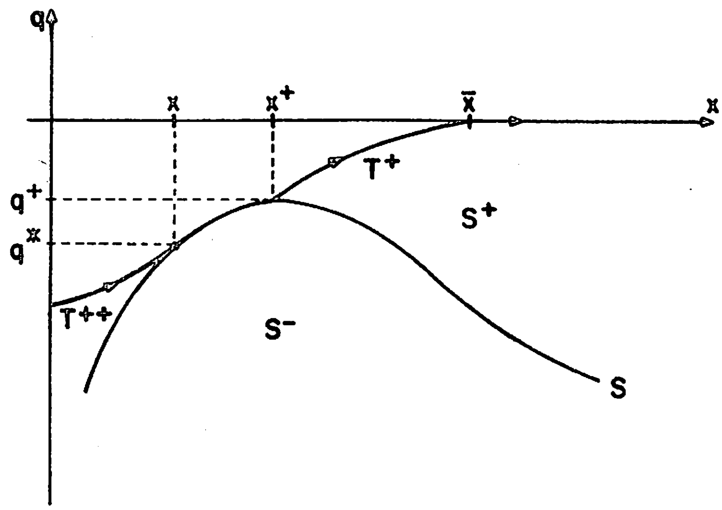

Fig. 4. Optimal trajectories in S^+.

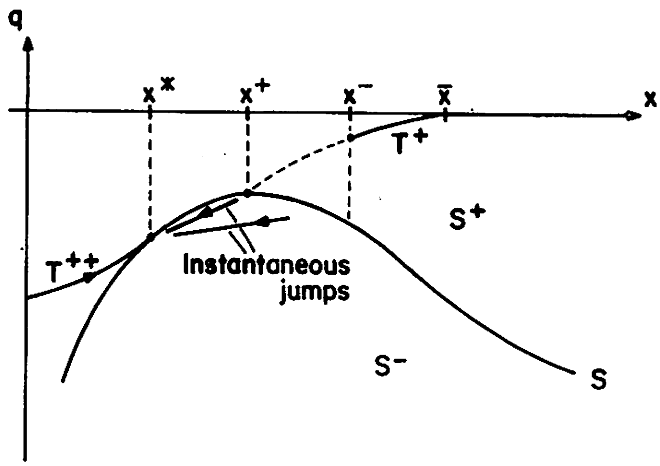

Fig. 5. The optimal trajectories.