ECONOMIC THEORIES AND EMPIRICAL MODELS ON LOCATION CHOICE AND LAND USE: A SURVEY

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Roland Artle and Pravin Varaiya

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ELECTRONICS RESEARCH LABORATORY

College of Engineering
University of California, Berkeley
94720

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ABSTRACT

This is a survey of the theoretical and empirical models of location choice and land use in cities. The survey is conducted from a historical perspective and the models are critically evaluated in terms of their logical structure. Special attention is given to the technical difficulties created when the assumption of the convexity of the production technology is dropped. It is indicated why further progress in this area requires a fundamental rethinking of the way in which we conceptualize the behavior of individual location choice.

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1. PREFACE

Our objective is to present a unified and reasonably self-contained survey of the economics of locational choice and land use in urban areas.

The existing knowledge on location and land use is not satisfactorily integrated into central economic theory. In order to identify as precisely as possible the nature and causes of this lack of integration we propose to evaluate location and land use theory from the perspective provided by the Arrow-Debreu framework of general equilibrium, and our rationale for choosing the Arrow-Debreu framework is as follows. If, on the one hand, such an evaluation places location theory within the main body of general equilibrium theory, then the researcher would be able to exploit the mathematical structure of the Arrow-Debreu framework. If, on the other hand, the evaluation reveals such intrinsic properties of the location and land use problems as to vitiate fundamental assumptions of the Arrow-Debreu theory, it could still serve a constructive purpose by its very identification of such properties.

This frame of reference imposes some severe limitations on the scope of the present study. First, we assume that locational choices are made in the context of a capitalist economy, where all resources, including land, are privately owned, and where all transactions occur in competitive markets. Secondly, governmental or, more generally, collective action is excluded. Hence, we do not consider the literature covering the location of such public facilities as hospitals and fire stations nor the literature which is motivated by the efficiency and equity issues intimately connected with such programs as urban renewal and public housing. We also ignore the literature concerned with planned communities, whether privately or publicly owned, as well as that which examines the impact on land use of various institutional arrangements such as political jurisdictions. Since our concern is with the location and land use problems within a single urban area, we ignore the work on systems of urban areas, including central place and urban hierarchy models.

The lack of integration of locational analysis into the main body of economic theory can be traced in part to the way in which location theory has evolved historically. To shed some light on this evolution, we devote the next section to a survey of the historical development of the significant literature. Major stress will be put on the contributions by Ricardo, Thünen, and Weber, since they have profoundly influenced nearly all the subsequent efforts to explain or describe the patterns of urban land use and rent. At the end of that section it will be shown that from the point of view of the Arrow-Debreu framework, this literature can be characterized as a series of explorations of special cases, and as partial analysis. A few studies have explored the possibility of relaxing the assumption of convexity in the Arrow-Debreu model, specifically by way of introducing indivisibilities into the locational analysis.[1] These studies will be reviewed in Section 3. In Section 4, we survey a number of noteworthy and significant empirical studies which have been motivated by the analytical work reviewed in earlier sections. Finally, in Section 5 we discuss the challenging problems which arise when one of the most basic assumptions of the Arrow-Debreu framework is violated, namely the assumption of the existence of all markets.

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[1] Compare T. Koopmans, (1957), p. 154: "...without recognizing indivisibilities -- in the human person, in residences, plants, equipment, and in transportation -- urban location problems, down to those of the smallest village, cannot be understood."

2. NOTES ON THE HISTORICAL DEVELOPMENT OF LOCATION THEORY

Location theory presupposes the individuality of each decision-making unit, or "agent", in the economy. Like other branches of micro-economics, it uses simplifying assumptions about the motivation and behavior of these agents. For example, the business firm is assumed to select a site for its activity, and to delimit supply and market areas, in such a way as to maximize its profit. Similarly, households are considered as preference, or utility, maximizers in their locational decisions. All such agents are assumed to make locational choices, as well as other decisions with spatial implications, on the basis that all prices in the economy are given and beyond their individual control. Land rents and transportation prices ("costs") take on special significance in the locational context, and they as well as all other prices are assumed to be determined in markets, through the balancing, or equilibrating, of the demand for and the supply of each resource or good. Hence, the markets serve as a means of interaction between all the decision-making units, or agents. A principal role of location theory is then to analyze and describe systematic distributions in space - "locational patterns" - which arise out of the interactions among business firms, households, and other agents.

From a different standpoint, location theory attempts to describe how land is used and to explain why a particular area is utilized in a particular manner. This was in fact the way in which a location theory was first formulated. It arose from a need to explain agricultural land use, and landlords were the principal agents of this theory. Two different strands of ideas were developed, one by D. Ricardo, in his "Principles of Political Economy and Taxation" (1817), and the other by J. V. Thünen, in his

"Isolated State" (1826).[2] Ricardo's preoccupation was not with a location and land use theory as such, but rather with a much more far-reaching theory of value and distribution, that is to say, with a theory of how the values of all commodities are determined, and of how wages, profits, and rents accrue to the owners of resources. There are, generally speaking, intricate relations between location and land use, on the one hand, and the division of labor in a society, on the other. In a predominantly agrarian society, those relationships are relatively much simpler. With the immobility of the soil, the relations are reduced to problems regarding which, if any, products to cultivate or extract, and regarding which other resources, such as labor and capital, and what quantities of these, to combine with the land or natural resources. Ricardo simplified this agricultural setting even further by assuming that only one product, "corn", was to be produced. Land, in his treatment, was not only limited in quantity (in relation to the demand), but varied in quality. The quality variations arose because of differences in the fertility in the soil, <u>or</u> because of differences in location, in relation to the market. The quality differences played a key role in Ricardo's theory of value, which was a <u>labor</u> theory of value. The poorest quality of soil in use required the most labor for the production (and distribution to market) of a given quantity of corn. In Ricardo's theory, it was precisely the value of this labor input on marginal land which determined the price of corn on the market. Hence, the richer or better located land yielded a surplus value and this formed the basis for the determination of land rent. In this

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[2] Both authors drew inspiration from A. Smith's "Wealth of Nations", whose concept of land rent, however, was criticized by both of them for being improperly (impurely) defined.

sense, then, Ricardo's theory of rent gave an explanation for differential rents, and it denied the existence of an absolute rent.

As Palander[3] stressed, it was unfortunate from the point of view of a meaningful development of location and land use theory among English-speaking economists, that Ricardo treated variations in the intrinsic quality of the soil and variations in the distance from the market as equivalent factors. Except under the most simplistic and uninteresting assumptions, it turns out that different parcels of land cannot be consistently order _a priori_ according to "quality" in the Ricardian sense. More importantly, his treatment obscured and blurred, for a long time to come, the significance of location and distance in economic analysis.

Some of the deficiencies in Ricardo's work, from the standpoint of a theory of location and land use, were remedied by his contemporary, J. v. Thünen. But Thünen was an agriculturist, and not considered to be an economist, and he wrote in German, and not in English or French -- and both circumstances acted as communication barriers.

Just as Ricardo, Thünen was concerned with the development of a theory of rent. But whereas for Ricardo rent and its determination was merely a building block in a larger theoretical structure, Thünen's aim was much closer to land rent itself, namely with the optimal use of agricultural land. For the purpose of his analysis, he developed a remarkable conceptual framework. He summarized its main features and purposes as follows: "Imagine a very large town, at the centre of a fertile plain which is crossed by no navigable river or canal. Throughout the plain the soil is capable

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[3] T. Palander, (1935), pp. 63-70.

of cultivation and of the same fertility. Far from the town, the plain turns into an uncultivated wilderness which cuts off all communication between this State and the outside world." - "There are no other towns on the plain. The central town must therefore supply the rural areas with all manufactured products, and in return, it will obtain all its provisions from the surrounding countryside." - "The problem we want to solve is this: What pattern of cultivation will take shape in these conditions?; and how will the farming system of the different districts be affected by their distance from the Town?"[4] In order to answer these questions, Thünen found that he had to pose and solve the additional problem of how and why land rent arises and what determines its level.

Thünen's method combined both deductive and inductive elements. He used the accounting data assembled on his own estate (Tellow) as the empirical basis for his work, but since the assumptions underlying his analysis were quite abstract, the numerical results obtained could best be described as numerical examples. In addition to the assumption of a completely homogeneous plain surrounding the single central town, Thünen built his analysis on a number of premises. These are scattered throughout his book, but could perhaps be summarized as follows:

1) The spatial extent of, and the locational arrangement within, the central Town was disregarded by Thünen, and he concentrated all his attention on the surrounding land; hence, the town itself was considered as a point.

2) All agricultural land was utilized in a "rational" manner, implying

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[4] J. H. v. Thünen, (1826), pp. 11-12.

by this that each landlord maximized his profits. All other land uses are neglected.

3) All landlords had complete information about the available methods of cultivation of all the different possible crops.

4) The central Town functioned as a market-place for all products, and all prices were fixed and known by all agents (landlords and farm laborers in particular);[5] landlord and labor markets exist at every point in space. With one exception, Thünen used the price of rye as the unit (numeraire) in terms of which all other prices were expressed.

5) The per-mile price, or cost, of transporting one bushel of rye, and the equivalent quantity of all other products, was also fixed and known to all agents.

6) There were no intermediate goods of production, hence no linkages between different production units.

7) All labor was assumed to be perfectly mobile, such that the "real"

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[5] In the generally very interesting and well-written Introduction to the recent translation of Thünen's book, the reader is given an impression that product prices in the Thünen framework are not constants but variables. On p. xxiv, it is said that the market price of grain "is determined conjointly by the size of the town demand and by the costs of the marginal producer (the farmer who just finds it profitable to get his grain on the market)." - "Now Thünen has the farm price (and) he can study the output of the farm and the costs which must be balanced against price." See P. Hall (1966). However, this masks the essential simultaneity of the problem: Where the marginal production of a particular crop will be located, can only be determined when all crops are optimally located. That is to say, only when the answer to the basic problem that Thünen set out to solve is known, can prices be calculated. But, in Thünen's formulation, the problem cannot be solved unless these prices are known. It is true that Thünen's procedure for calculating transportation costs - the feed of the horses and the food of the drivers had to be carried along - meant that there existed an upper-bound on the distance for each product. But that distance, beyond which the entire load would be consumed by horses and drivers, was for most crops too large to provide a unique determination of the marginal locations.

wage (the purchasing power of the wage) was everywhere the same.

The last-mentioned assumption about a uniform real wage raised some difficulties in Thünen's multi-product framework, especially since he excluded from the calculations the transportation costs involved in shipping city-produced goods to the rural areas. Thünen resolved this difficulty by an approximation; he estimated that on the average three fourths of the farm worker's wage was paid and consumed in natura (grain) and one fourth was paid in money and spent on other products. This meant that production costs, for any given crop, decreased with distance from the town since farm products fetched a higher price in the town. To sum up, for each agricultural product, the revenue obtainable per acre of land decreased with distance from the central town, but so did the total production cost. Thünen defined land rent as the difference between these revenues and costs. Although for most of the products he investigated he found that the corresponding land rents decreased monotonically with distance, it should be clear from the above that there is no intrinsic reason for this to occur.

The land rent thus calculated became the criterion by which Thünen <u>ordered</u> all the possible crops. A landlord located at a specific distance would thus choose the crop, or product, which yielded the highest rent per acre. Furthermore, since Thünen linked his theory of rent with a marginal-productivity theory of remuneration of labor, the <u>intensity</u> of cultivation of any crop at any distance was also determined.[6] This was

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[6] Such determinations of intensity levels were not formalized by Thünen, who instead used numerical examples. For an increasing formalization, see M. Beckmann, (1972). In one numerical example, Thünen expresses the intensity variation in terms of a density measure. He finds that in the agricultural zone closest to the market place, 69 persons per sq. km. obtained their income from agriculture, whereas the corresponding density was only one tenth thereof in the zone farthest from the town. J. v. Thünen, (1826).

the way in which Thünen derived his famous "rings", or concentric zones, surrounding the central Town. Each ring had its own particular crop, cultivated at the optimal intensity level.

W. Isard (1956) appears to have been the first to clearly recognize the applicability of Thünen's theory of agricultural land use to urban location and land use. In such an application the Central Business District (CBD) plays the role of a single, fixed market place thus serving as a focus analogous to Thünen's central Town. The surrounding circular space would be devoted to production of housing and other goods with land rent and land use intensity decreasing with distance from the CBD. These ideas were systematically explored by R. Muth (1961), L. Wingo (1961), W. Alonso (1964), and E. Mills (1967).

A simple model will serve to illustrate the nature of the resulting theory. In this model the CBD is a point and only one good is produced in the city. The following assumptions are made:

(i) Production of the good requires land and capital as inputs. There are J different (linear) production techniques and if x units of the good are produced using the jth technique then $a_j^L x$ and $a_j^K x$ units of land and capital inputs respectively are necessary, where a_j^L, a_j^K are non-negative constants.

(ii) Transportation of one unit of the good over a unit distance costs \$c, while capital can be transported without cost. Of course land is immobile.

(iii) The good can be sold at the CBD only at an a priori fixed per unit price of \$p, whereas capital services cost \$R per unit. A unit area of land at distance u from the CBD rents for \$r(u). This rent function r(·) is to be determined within the model.

(iv) New firms enter into production as soon as there is an opportunity for positive profits so that in equilibrium there will be only zero profits.

From assumptions (i), (ii) and (iii) the profit per unit of output resulting from adopting technique j at distance u from the CBD is

$$\pi_j(u) = (p-cu) - (a_j^L r(u) + a_j^K R)$$

so that using assumption (iv) we can conclude that, in equilibrium, r(u) will be the rent function and technique j will be adopted at u, if and only if

$$\pi_j(u) = 0 , \qquad \pi_i(u) \leq 0 \qquad i \neq j$$

Solving for r(u) from these relations leads us to the rent function and land use pattern shown in Figure 2.1., where the techniques

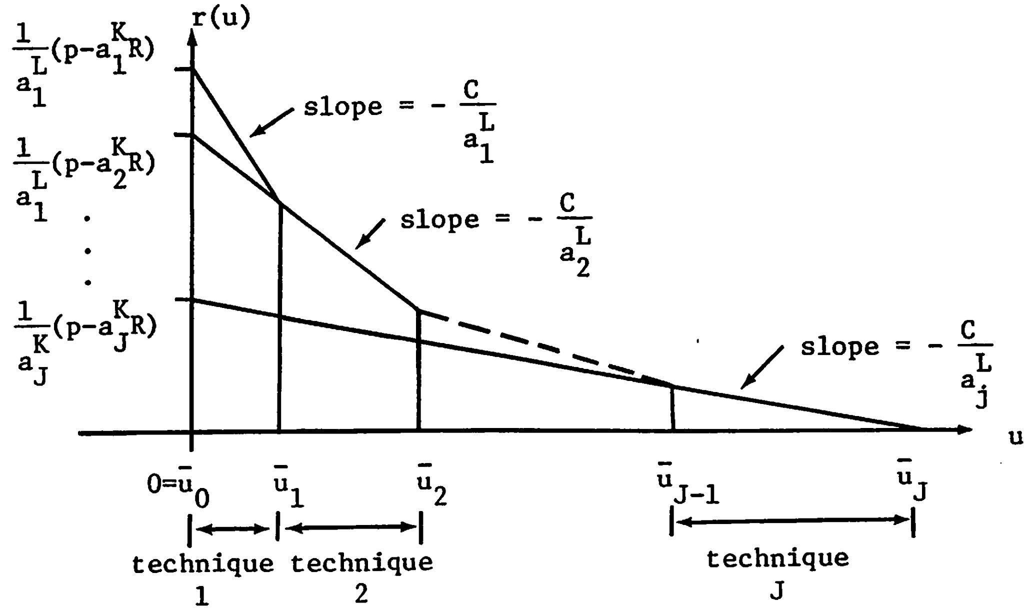

Figure 2.1: Rent and land use

have been relabelled so that

$$\frac{1}{a_1^L}(p-a_1^K R) \geq \dots\dots \geq \frac{1}{a_J^L}(p-a_J^K R) \ ,$$

$$\frac{C}{a_1^L} \geq \dots\dots \geq \frac{C}{a_J^L}$$

Even this simple model yields interesting conclusions. The first thing to observe that rent decreases with distance $\frac{dr}{du} < 0$, and $\frac{dr}{du}$ is nondecreasing with u. Next, since in equilibrium, the techniques must be so ordered that $\frac{1}{a_1^L} \geq \dots \geq \frac{1}{a_J^L}$, we can conclude that the output/land ratio must decline with distance. Thirdly, since $\frac{1}{a_j^L}(p-a_j^K R) - \frac{C}{a_j^L}\bar{u}_j = r(\bar{u}_j) = \frac{1}{a_{j+1}^L}(p-a_{j+1}^K R) - \frac{C}{a_{j+1}^L}\bar{u}_j$ and $\frac{1}{a_j^L} \geq \frac{1}{a_{j+1}^L}$, therefore, $a_j^K \leq a_{j+1}^K$, hence the capital/output ratio declines with distance. The last two conclusions together imply that the capital/land ratio declines with distance also. A slightly more careful analysis will reveal that an "isoquant" drawn through the techniques must yield a convex curve as shown in Figure 2. It is evident from this

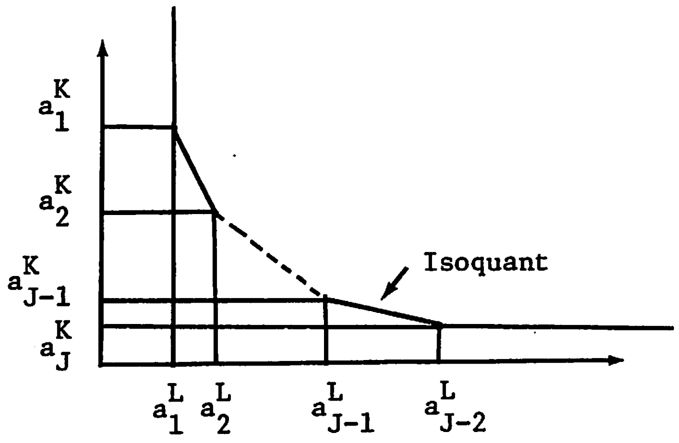

Figure 2.2: Isoquant generated by production techniques.

Figure that if the finite number of production techniques were replaced by a smooth "neo-classical" production function we would have obtained smooth curves in Figures 2.1, 2.2.

As indicated, it took a very long time for economists to absorb, and to further explore, Thünen's ideas. Much better known was A. Weber's analysis of the location of industries, first published in 1909.[7] The type of industry studied by Weber was "urban" industry (manufacturing), and one of the important locational determinants in his framework was a so-called agglomeration factor. It may appear surprising, therefore, that students interested in the internal spatial structure of cities did not turn to Weber's framework. An inspection of the weberian analysis and its underlying assumptions will reveal the reasons. Just like Thünen, Weber assumed that the location of the markets and the prices of all goods were fixed and known to all agents. However, Thünen's single market (Town) and homogeneous land Weber replaced by several fixed markets and land which was inhomogeneous with respect to the distribution of various factors of production. Weber's interest centered on the locational factors, or forces, which could help to explain a producer's choice of location. He stressed in particular so-called regional factors and agglomerative (deglomerative) factors. All these factors were thought of as cost advantages (or disadvantages). The agglomerative (deglomerative) factors were used to explain concentration (dispersion) of plants and industries. While Weber stressed their importance, his formal analysis was primarily limited to the effect of regional factors on the location of a single plant. Weber singled out

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[7] English translation in C. Friedrich (1929).

two regional factors as the important ones, namely regional variations in labor costs; and transportation costs.[8] He assumed that labor was immobile -- the opposite, incidentally, to the corresponding assumption made by Ricardo and Thünen. Therefore, (real) wages could differ between different places or localities, but at any such place the supply of labor was assumed to be infinitely elastic ("unlimited at constant cost," see Friedrich (1929), p. 211). No explanation was given why such labor supplies existed in the first place. Weber's assumption that the prices of finished products, the prices of raw materials, etc. were also fixed and independent of the production level is perfectly compatible with the economic theory of general equilibrium as long as he was dealing with the location of a single agent. But Weber ultimately aimed his whole analysis at understanding the location of entire industries. At the level of the industry his assumptions are not compatible with any economic theory. By contrast, Thünen solved this problem for his most important variable, namely the land rent, and in a very ingenious manner, indeed.

In his theoretical analysis, Weber used geometrical constructions and analogies with mechanical forces in states of equilibrium.[9] The simple analytical tools available to him forced him to introduce some rather far-reaching assumptions, among them an assumption that all relevant locations were points rather than areas. This enabled Weber to construct so-called locational triangles for the simple case of a firm which used two trans-

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[8] While Weber also considered prices of capital, raw material, fuel and intermediate goods as a third factor which varied with location he assumed that this could be included as variations in transportation costs.

[9] The approach was first used by W. Launhardt, (1882), who derived points of minimum transportation costs by means of locational triangles.

portable inputs, located at two points, to produce one output to be sold in one market located at a third point. The distances between these three points were assumed to be known, and Weber further assumed a fixed-coefficient input-output technology.[10] Assuming that transportation costs were proportional to weight and distance, Weber derived, geometrically, the plant's optimal location, defined as the point which minimized transportation costs.

So far, in his analysis Weber had disregarded the influence of variations in labor costs at different possible locations, as well as the differences in agglomeration advantages. To introduce these factors, Weber devised a set of so-called "isodapanes" each of which defined as the locus of points with equal transportation costs and was assumed to form a closed curve around the point of minimum transportation costs. Possible points on these level curves which offered labor or agglomeration cost advantages in comparison with the point of minimum transportation costs, were then analyzed to see whether a better location point could be found.

One of the difficulties with this kind of geometrical approach is seen when it is recognized that raw material sources and final markets are often better represented as areas rather than points. The simplicity of the isodapanes is then lost. Another problem with the weberian analysis is that rents are conspicuously absent, except as a deglomerative force, hence in this analysis land-market clearing conditions cannot even be formulated.[11]

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[10] L. Moses (1958) relaxed the assumption of fixed coefficients and showed that the location of the plant then depended on the scale of production.

[11] R. Fales and L. Moses (1972) have recently attempted to introduce site rents into a weberian model, but the linkage between their model and empirical study is rather obscure. Furthermore, and except under the most simplified assumptions, it is not clear that the isodapanes and "isotims" suggested have the required monotonicity property.

We are now in a position to evaluate the theories surveyed above within the Arrow-Debreu general equilibrium model.[12] Such a model consists of the following elements:

(i) a finite set of commodities all of which are mobile except one, land, which is immobile; transportation of commodities consumes commodities;[13]

(ii) a finite set of firms which take some commodities, including land, as inputs and produce others as outputs with the possible input-output combinations or techniques available to a firm being described by a fixed production function;

(iii) a finite set of households or consumers who exchange some commodities (e.g. labor) for others which they consume with the preferences of each household being described by a fixed utility function.

These elements interact with each other in competitive markets as described next:

(iv) there are a finite set of (locationally) fixed market places at which all the mobile commodities are exchanged i.e., bought and sold at prevailing prices;

(v) all firms and all land is owned by the consumers.

Within this framework a competitive equilibrium is a triple consisting of (a) a production technique for each firm, (b) a consumption vector for each household and (c) a vector of prices for the mobile commodities at each market place and a rent for land at each location, such that (d) each firm taking prices and rents as given maximizes profits among all techniques

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[12] The general theory of the Arrow-Debreu model first appeared in K. Arrow and G. Debreu (1954), more details are available in G. Debreu (1959) and K. Arrow and F. Hahn (1971).

[13] Note that we cannot talk about transportation costs since prices are to be determined within the model.

available to it at the assigned technique, (e) each household maximizes its utility function at the given consumption vector among all consumption vectors which satisfy its budget constraint[14] and (f) total supply of each commodity (including land) equals the total demand for it.

It can be proved that if the production function of each firm and the utility function of each consumer are concave then a competitive equilibrium exists.[15]

We recognize here some assumptions from the locational models discussed earlier, namely that the market places are fixed and that the convexity condition on production and consumption holds. However, all of the earlier models are _partial_ in that they assume that prices of some of the commodities are fixed _a priori_ and all rents and payments for transportation flow out of the city.[16] It is indeed surprising that urban economists have for so long been content to work in such a partial framework, and our only explanation is that they have followed the historical development of the field instead of rigorously analyzing the logical structure of their theories.

In the next section we retain the partial context but examine the implications of relaxing the assumption of concave production functions. Section 5 briefly discusses what happen when market places are not fixed _a priori_.

3. NON-CONVEX PRODUCTION TECHNOLOGY

The difficulties arising in the analysis above when these convexity

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[14] The funds(budget) available to each household equals the value of any commodities (such as labor) which it sells plus its share from its ownership in firms or lands.

[15] For a proof see M. Ripper and P. Varaiya (1972).

[16] Rent flows are significant in that they may account for up to 25% of household budgets.

conditions are not satisfied can be discussed using a simple, yet conceptually powerful model due to Koopmans and Beckman (1957). As shown in the next section this model is also a useful device for evaluating empirical studies.

The study region is partitioned into n *tracts* or *lots* of equal area indexed $i=1,\ldots,n$. n (physical) plants dedicated to various activities (e.g. housing, commercial, industrial), and indexed $k=1,\ldots,n$, are to be located on these lots.[17] The plants are *indivisible* since their size cannot be varied and exactly one plant can occupy one lot. Thus the convexity assumption of the previous section is violated. Let x_{ik} be the variable which takes on the value 1 or 0 according as plant k is or is not assigned to lot i, so that an assignment $\{x_{ik}\}$ is *feasible* if and only if

$$\sum_{k} x_{ik} = 1 , \qquad i = 1,\ldots,n, \tag{3.1}$$

$$\sum_{k} x_{ik} = 1 , \qquad k = 1,\ldots,n, \tag{3.2}$$

$$x_{ik} = 0 \text{ or } 1 , \quad i,k = 1,\ldots,n \tag{3.3}$$

We wish to investigate two questions. First, in a planning or normative approach, what is the optimal assignment? Secondly, what kind of assignment are likely to be observed empirically? The answer to the first question depends upon the criterion of optimality. To begin with, suppose that the contribution to social welfare obtained by assigning plant k to lot i i.e., choosing $x_{ik} = 1$, is *independent* of the assignment of the

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[17] The argument below trivially extends to the situation where the number of lots is different from the number of plants by adding "dummy" plants or lots.

other plants.[18] Let the monetary value of this contribution be a_{ik} and call it profit. Then the optimal assignment is a solution of the problem

$$\text{maximize} \qquad \sum_i \sum_k a_{ik}\, x_{ik}\ , \tag{3.4}$$

$$\text{subject to} \qquad (3.1),\ (3.2),\ (3.3)$$

Now it is a celebrated result (See Koopmans, Beckmann (1957) or the assignment problem in Dantzig (1963)) that the optimal solution to the integer programming problem (3.4) is also the optimal solution to the linear programming problem obtained by replacing (3.3) by the linear constraint,

$$0 \le x_{ij} \le 1 \qquad\qquad i,j = 1,\ldots,n \tag{3.5}$$

Consider any assignment. After relabeling, it can be expressed as $x^*_{ik} = 1$ if $i=k$, $x^*_{ik} = 0$ if $i \neq k$. By the Duality Theorem of linear programming this assignment is optimal if and only if there exist sets of numbers $\{r^*_i\}$, $\{p^*_k\}$, such that

$$r^*_i + p^*_i = a_{ii}\ , \qquad i = 1,\ldots,n \tag{3.6}$$

$$r^*_i + p^*_k \ge a_{ik}\ , \qquad i,k = 1,\ldots,n \tag{3.7}$$

These optimality conditions[19] can be given two positive interpretations in a competitive market economy. Firstly, suppose that the plants consist

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[18] This assumption of lack of locational interdependence among the activities is satisfied in the following partial equilibrium context similar to the one adopted by Thünen. Suppose that each plant i produces per unit time one unit of commodity i which is to be sold at a fixed marketplace at a fixed price p_i. Suppose that total production costs c_{ik}, including transportation to the marketplace, depend only upon the activity k and its location i. Finally, suppose that the profit $a_{ik} = p_i - c_{ik}$ is a measure of contribution to welfare.

[19] Note that the numbers $\{r^*_i\}$, $\{p^*_k\}$ are not unique. In particular, for any number π, $\{r^*_i - \pi\}$, $\{p^*_k + \pi\}$ also satisfy (3.6), (3.7).

of capital equipment whose rental value is determined outside the study region (say in the national market), and suppose that the kth plant's rental value is p_k^*. Similarly, suppose that for all i the rent of the ith lot is determined exogenously and equals r_i^*. Then the profit-maximizing manager of the kth plant will locate on lot i so as to maximize $a_{ik} - p_k^* - r_i^*$, and by (3.6) and (3.7) this occurs for i = k i.e., for the optimal assignment. Thus, if $\{p_k^*\}$ and $\{r_i^*\}$ are indeed the rents prevailing in the economy, then decentralized profit-maximizing decisions on the part of managers lead to a socially optimum assignment. We now come to the second interpretation. Suppose as before that the kth plant has an exogeneously derived rental value p_k^*. Suppose that the lots are owned by landlords who will rent them out to the highest bidder. Finally, suppose that each plant k is a member of a perfectly competitive industry so that zero-profit conditions prevail throughout. Then the kth plant's manager will offer an amount $(a_{ik} - p_k^*)$ as rent for the ith lot. Since the owner of the ith lot will accept the maximum offer, the final rent will be $r_i = \max\{a_{ik} - p_k^* \mid k = 1,\ldots,n\} = r_i^*$ from (3.6) and (3.7), and once again the equilibrium assignment will be socially optimum. This interpretation forms the basis of the currently orthodox theory of the urban land market.[20] A classical interpretation is also suggested. Since r_i^* is the shadow price of the land constraint, $\sum_k x_{ik} = 1$, it can be interpreted as the "marginal product" of land.

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[20] The standard reference is Alonso(1964), also see Stevens (1968). Instead of the zero-profit condition, Alonso permits a more general constant-profit condition. From footnote 19 above we observe that this makes no difference as far as rent differentials $r_i^* - r_j^*$ are concerned. However, there appears to be a deficiency in Alonso's argument since he seems to make the assumption that the constant (non-zero) profit level is <u>independent</u> of the scale of the plant (recall that in our treatment the plant scale is fixed).

Summarizing the discussion so far we can assert that even in the presence of indivisibilities, so long as the scale of activities is fixed, and there are no locational interdependencies among activities the equilibrium land use in a perfectly competitive market will be socially optimum. We now permit locational interdependence.[21] Retaining the same notation, suppose instead that production in the kth plant requires as inputs an amount $b_{k\ell}$ of the output of the ℓth plant, so that this amount must be transported between plants ℓ and k. Suppose that c_{ij}^{ℓ} is the cost of shipping one unit of the ℓth plant's output from lot i to lot j. Then the assignment $\{x_{ik}\}$ incurs the transportation cost $\sum_{k,\ell} \sum_{i,j} b_{k\ell} x_{ik} c_{ji}^{\ell} x_{j\ell}$. Hence an optimal assignment is a solution of the quadratic integer programming problem,

$$\text{maximize} \quad \sum_{i} \sum_{k} a_{ik} x_{ik} - \sum_{k,\ell} \sum_{i,j} b_{k\ell} x_{j\ell} c_{ji}^{\ell} x_{ik} \tag{3.8}$$

subject to (3.1), (3.2), (3.3)

We will show now that the optimal assignment cannot in general be sustained as an equilibrium in a competitive land market. To see that this is due solely to the indivisibility assumption (3.3) let us first permit the plants to be divisible i.e. replace (3.3) by (3.5). Defining T_{ij}^{k} to be the amount of output of plants of type k which is shipped from i to j allows us to easily reformulate the optimal (fractional) assignment problem as

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[21] Space limitations prevent us from considering the effects of a variable scale of activity in the presence of non-convexities. Very briefly, one is forced to replace the assumption of competitive behavior with some mixture of monopoly and competition or a planned economy. The literature (see Mills and Lav (1964), Stern (1972)) dealing with land use under these conditions derives from Lösch (1954) and is mainly concerned with the number and geographical configuration of plants in a given industry which would be sustained in a given spatial area.

$$\left.\begin{array}{l} \text{maximize } \sum_i \sum_k a_{ik} x_{ik} - \sum_k \sum_{i,j} c_{ij}^k T_{ij}^k \\ \text{subject to } x_{ik} + \sum_{j \neq i} T_{ji}^k = \sum_\ell b_{\ell k} x_{i\ell} + \sum_{j \neq i} T_{ij}^k, \ i,k=1,\ldots,n \\ \qquad T_{ij}^k \geq 0, \ i,j,k = 1,\ldots,n \text{ and } (3.1),\ (3.2),\ (3.5) \end{array}\right\} \quad (3.9)$$

By the Duality Theorem again, $\{x_{ik}^*\}$, $\{T_{ij}^{k*}\}$ is optimal if and only if there exist sets of numbers $\{r_i^*\}$, $\{p_k^*\}$, $\{u_{ik}^*\}$ such that

$$c_{ij}^k \geq u_{jk}^* - u_{ik}^* \quad \text{with equality if } T_{ij}^{k*} > 0 \tag{3.10}$$

$$a_{ik} + u_{ik}^* \leq \sum_{\ell \neq k} b_{k\ell} u_{i\ell}^* + p_k^* + r_i^* \quad \text{with equality if } x_{ik}^* > 0 \tag{3.11}$$

We interpret p_k^* and r_i^* as the rental value per unit of plant k capacity and per unit area of lot i respectively, and u_{ik}^* as the price per unit of product of plant k delivered at location i. Then (3.10) says that product k will be shipped from i to j only if the price differential $u_{jk}^* - u_{ik}^*$ can absorb the transport cost c_{ij}^k.[22] The right hand side of (3.11) is the total cost of producing one unit of the kth output at i, whereas the left hand side is the corresponding revenue. Thus (3.11) asserts that in an optimal assignment plant capacity of type k will be installed in lot i, $x_{ik}^* > 0$, only if production costs can be recovered. We can conclude that the optimal assignment can be sustained as a competitive equilibrium.

However, the preceding conclusion will not hold if the indivisibility restriction (3.3) holds. Consider the simple symmetric example where $n=3$, $a_{ik}=1$ for all i,k, $b_{k\ell}= 1/3$ for all $k \neq \ell$, and $c_{ij}^k=1$ for all k, $i \neq j$.

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[22] This condition has been derived by Samuelson (1952) in a more general context.

The optimal fractional assignment is $x_{ik} = 1/3$ for all i,k since then all interplant shipment can be restricted to the same lot so that total transportation cost is zero which is clearly optimal. On the other hand the symmetry implies that every integral assignment is optimal if we have the constraint (3.3). But it is clear that no set of rents $\{p_k\}$, $\{r_i\}$ and intermediate products prices $\{u_{ik}\}$ can sustain an integral assignment as a competitive equilibrium since there will always be a profit incentive for at least one plant manager to move to another lot.

In summary, we can conclude that if the technical conditions require that land be used for production in indivisible amounts, then a competitive land market is likely to cause inefficient land use patterns. These are evidently serious charges against the market as an institutional arrangement for allocating land among alternative uses. Unfortunately, there are few studies, either of an empirical or theoretical nature, which indicate either the degree or the pattern of inefficiency which would prevail in competitive land market.[23] Equally lacking is research suggesting allocating mechanisms which are more efficient than the market. The reader is referred to Serck-Hanssen (1970) for one noteworthy study which examines these issues in detail but in a restricted context.[24]

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[23] The degree of inefficiency can be crudely quantified as the amount of loss of social welfare i.e., as the reduction in the objective function of (3.8) from the optimum. The pattern of inefficiency can be crudely expressed by determining whether the market leads to a concentration or dispersal of productive activity relative to the optimum configuration.

[24] Roughly speaking, the author's main conclusion is that in a market economy production indivisibilities will lead to over-concentration of activity due to perceived savings in transportation cost (from the viewpoint of the individual plant manager but not from a social viewpoint). However, he ignores completely the cost of land so that the conclusion may be more valid for activities which are not land intensive. The debate over whether urban concentrations (cities) are too large or too small arouses the passions of many urban planners and economists but not much light has been shed on the controversy.

4. EMPIRICAL MODELS OF LAND USE

The empirical studies summarized here were motivated either as tests of the theoretical models presented above or for forecasting land use patterns.

The theoretical models closest to Thünen's imply that in a monocentric, radially symmetric city the land rent r(x) and the capital/land ratio k(x) must decrease with distance x from the center. Furthermore, if a single commodity is produced in the city with production techniques described by a Cobb-Douglass function,[25] and if transportation cost is proportional to distance, then it is not difficult to show using the simple model of Section 2 that r(x) and k(x) must have the form,

$$\log r(x) = a_r + b_r \log x, \quad \log k(x) = a_k + b_k \log x, \tag{4.1}$$

which is eminently suited for regressions. The statistical problems encountered are mainly concerned with finding data which are adequate proxies for x, k and r, and are reviewed in Mills (1969) where a summary of earlier empirical work along these lines is provided. The main findings of these studies are first that the form (4.1) provides a good fit to the data up to the year 1900; secondly, the coefficients b_k and b_r decrease over time[26]; and finally the form (4.1) fits the data progressively worse after 1900. This negative finding is largely understood to suggest that large modern cities are polycentric (dominant center with several satellite subcenters) and the monocentric assumption has become quite unrealistic.

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[25] This means that the quantities of capital K and land L technologically required to produce 1 unit of output satisfy the function $K^{\alpha}L^{1-\alpha} = 1$ where $0 < \alpha < 1$ is a constant.

[26] This would be predicted by the theory under the assumption that transportation cost decreases over time.

Basic patterns other than the concentric zones of Thünen have also been suggested, the chief contestant being the "sectoral" growth pattern of Hoyt (1939) who conducted extensive empirical studies although not of a statistical nature. However, the theories underlying these suggested pattern have never been made mathematical.

The form (4.1) is of course quite unsuitable as a model for forecasting land use in a city where one is interested in predicting how much land in each tract is devoted to each of several activities. Many different models have been independently proposed to this end and some of these have been tested.[27] The most immediately striking fact about these models is that they are large with respect to almost any definition of size. They have large data requirements, they require a great deal of computing resources to run and they forecast a large number of variables. Secondly, one is surprised by the lack of any explicit theoretical structure guiding much of the empirical effort. Instead one finds the model development marked by a sequence of more or less ad hoc decisions necessitated in part by lack of adequate data.

The Koopmans-Beckmann model provides a good framework for organizing our remarks.[28] A particular assignment can be visualized as a matrix as seen in Figure 1. On theoretical grounds and due to data availability

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[27] The most recent model is the NBER Model (Ingram, et al. (1971)) which is built primarily to study housing patterns. This and other models have been summarized in Kendrick (1972). Other important surveys are Lowry (1972), Batty (1972) and Goldner (1971). For a criticism of all this effort see Lee (1974) and subsequent issues of the J. Amer. Inst. Planners for the ensuing debate. Original sources are referenced in these surveys.

[28] A very similar approach is taken in Lowry (1972) from which the next paragraph is adapted. However, the connection with the Koopmans-Beckmann model is not noted there.

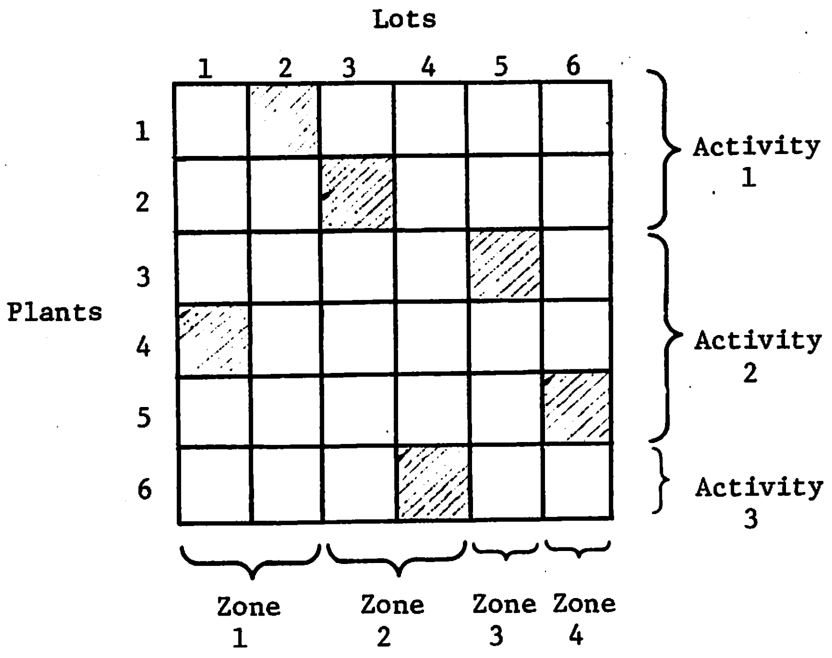

Figure 1. An assignment

individual plants are aggregated into activities i.e., by the commodity produced.[29] Data limitations and administrative or policy considerations dictate aggregations of lots into zones.[30] Column sums of the matrix then give us the land use pattern in each zone, whereas the row sum give us the locational pattern of each activity. Since we are no longer dealing with homogeneous land (as in the Thünen model), each zone is characterized by a vector different components of which corresponds to different kinds of land e.g. developed, undeveloped, unusable, as well as such legal classifications as publicly owned, commercially zoned, etc. Some models are concerned exclusively with forecasting land use patterns i.e., predicting column sums. These models will therefore preserve areal integrity so that

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[29] Thus activities may be classified according to S.I.C. codes or into such highly aggregated classes as "manufacturing", "commercial" and "residential" activities.

[30] A zone may be a planning district, a traffic zone, a school district, etc.

the amount of land allocated to different activities will add up to the total available land. If aggregate data on activity levels is available a good check on the model is to see whether they preserve row sum integrity. Other models only forecast locational patterns or row sums. A model which primarily focuses a locational patterns but does check areal integrity was first developed by Lowry (1964). Since this model has inspired a great deal of subsequent work, we will describe it in some detail. But before proceeding to this task, we return to the Koopmans-Beckmann model to appreciate the difficulties involved in constructing an empirical model.

Suppose that the actual assignment resulting in the market is optimal. Then the locations of individual activities must all be determined simultaneously and to carry out this determination the analyst must be able to quantify (in terms of costs say) the locational interdependencies among the different activities.[31] Thus for example, in order to allocate retail stores among different zones the analyst must know the transportation costs associated with different locational patterns of the supplies to the retail stores as well as those associated with the locational patterns of household who are the customers of these stores. In turn location of households depends upon the location of retail establishments as well as upon location of employment, schools, friends, etc. Evidently, such data is impossible to obtain. A drastic simplification is achieved if it is assumed that the activities are <u>hierarchically ordered</u> in the sense that the location of

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[31] In the literature, this is often called the need for access on the part of one activity to other activities. Given the location of these other activities one can construct an <u>accessibility index</u> for each zone which summarizes the access which this zone provides to these activities. Presumably then the first activity will locate in zones which have a high accessibility index. Numerous indexes have been proposed, constructed and used for locating such activities as branch banking, shopping centers, health facilities, etc.

activity 1 is exogenously specified[32] whereas that of activity k+1 depends upon the locations of the first k activities. It is still necessary to determine the costs associated with a locational pattern arising from these dependencies. An extremely crude assumption is made at this stage. Suppose the locational pattern of the first k-1 activities is determined (in terms of our earlier notation suppose $x_{i\ell}$ is known for all in and $\ell \leq k - 1$). It is then assumed that the locational pattern of the kth activity, the x_{ik}, is determined in fixed proportions by the location of the (hierarchically) preceding ones i.e., there exist constants $\gamma_{ij}^{k\ell}$ such that $x_{ik} = \sum_{j=1}^{n} \sum_{\ell=1}^{k-1} \gamma_{ij}^{k\ell} x_{j\ell}$. The empirical problem then reduces to the determination of these constants. Notice that all economic variables such as prices, rents, etc., and all economic insights such as the systematic spatial distribution of capital/land ratios disappear in this simplistic empiricism. To see what is left, we give a highly simplified example adapted from Kendrick (1972).

The city is divided into n zones. There are 3 activities, k=1 denotes the basic activity-manufacturing given exogenously, k=2 denotes labor whose (residential) location depends only upon the location of jobs, k=3 denotes retailing and its location depends only on the location of labor. Let $\ell_i^m(\ell_i^r)$ denote the number of jobs in manufacturing (retailing) provided in zone i. Let h_i = number of workers who are housed in zone i. Then, dropping subscripts to denote vectors, we have

$$h = \Gamma^h(\ell^m + \ell^r)$$
$$\ell^r = \Gamma^r h$$

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[32] The exogenously specified activities are called <u>basic</u>, the rest are <u>non-basic</u>. The reasons behind this nomenclature need not concern us here.

where Γ^h, Γ^r are constant n×n matrices whose existence was assumed above.[33] Since ℓ^1 is exogenously specified, we get

$$h = (I - \Gamma^h \Gamma^r)^{-1} \Gamma^h \ell^m$$

$$\ell^r = \Gamma^r (I - \Gamma^h \Gamma^r)^{-1} \Gamma^h \ell^m$$

If we had historical data on ℓ^m, h, ℓ^r we could estimate the coefficients of Γ^h, Γ^r and then if we had an independent projection of ℓ^m for the next period we could forecast the values of h and ℓ^r. Estimation of the matrices Γ^h and P^r is still a formidable problem. For instance, if there are 100 zones then 2×100×100 = 20,000 parameters must be estimated, a hopeless task. Another simplification is called for. It is assumed that $\Gamma^h_{ij} = f^h(\theta^h, d_{ij})$, $\Gamma^r_{ij} = f^r(\theta^r, d_{ij})$ where f^h, f^r are known functions of the unknown parameters vector θ^h, θ^r respectively and d_{ij} is the distance between zones i,j.[34] The estimation problem is then reduced to determining the considerably fewer parameters θ^h, θ^r.

The total lack of any economic indicators in the models above make them quite unsuitable for understanding the market. The NBER model is the most ambitious attempt to date to simulate the urban housing market.[35]

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[33] If we assume a linear relation between these activities and the land occupied, we obtain the location pattern. Column sum integrity may not be maintained. Basically the solution is "fudged" to achieve this.

[34] For example, a popular specification is the so-called gravity model, $f^h(\theta^h, d_{ij}) = \theta^h (d_{ij})^{-2}$. For a systematic development of these functions see Wilson (1970).

[35] A predecessor of this model is the one developed by Arthur D. Little Inc. for San Francisco the original reference for which is A. D. Little (1966), more accessible references are Robinson, et al. (1965), Wolfe and Erust (1967). The political and administrative contexts in which the San Francisco and Pittsburgh models were built and the reasons behind their "failure" are analyzed in Brewer (1973).

We give a highly abbreviated account of the model structure.[36] Activities consist of housing of different kinds (rental, owner-occupied, multiple dwelling, single family, etc.). An assignment at time t is then a $X(t) = \{x_{ik}(t)\}$ which specifies the number of units of housing of type k in zone i. There is associated with this assignment a rent matrix $R(t) = \{r_{ik}(t)\}$ with the obvious interpretation. The dwelling units are owned by profit-maximizing landlords who in year t alter the assignment from X(t) to X(t+1). The alterations are based on two sets of economic variables: an <u>estimate</u> of the rent structure $R^e(t+1)$ that will prevail in the next year and costs of making the alteration. Once these are generated, an alteration will occur if and only if it is expected to be profitable. The model then iterates forward starting now with X(t+1). The alteration costs are exogenous to the model whereas the estimates $R^e(t+1)$ are computed internally. This is perhaps the most interesting theoretical part of the model and we indicate it briefly. $R^e(t+1)$ is obtained by another assignment model as follows. The model assumes given exogenously the numbers of households z_{hj} indexed by type h (income, size, etc.) and by the zone j where the head of household is employed. These households are assigned to the existing distribution of housing units. Thus an assignment is described by numbers $z_{hj,ik}$ which give the number of households of type h employed in zone j assigned to a house of type k in zone i. Only some assignments are feasible since it is exogenously specified that only a certain fraction of households of type h will live in housing unit of type k. Within this feasible set of assignments, an optimal assignment is computed the criterion of optimality being to

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[36] A much more complete account is given in Kendrick (1973).

minimize total transportation costs of journey-to-work trips. The optimal assignment produces shadow prices $\lambda_{ik}(t)$ corresponding to the constraint

$$\sum_{h,j} z_{hj,ik} \leq x_{ik}(t)$$

and serve to indicate "demand" pressures for housing of type k in zone i. $R^e(t+1)$ is then obtained by combining R(t) with $\{\lambda_{ik}(t)\}$.[37] The actual price R(t+1) which will prevail at time (t+1) is predicted by a similar calculation.

In summary, we have shown how the Koopmans-Beckmann model can serve as a framework for formulating and analyzing empirical studies. The most difficult problems in empirical work is the determination of the criterion function and the locational interdependencies. The Lowry-type models avoid this completely and try to estimate the assignment directly. The NBER model empirically obtains the dependencies (for the housing activity) and criterion function first and then deduces an optimal assignment. Recently, it has been proposed to extend this idea to many more activities and some suggestive examples have been worked out.[38] However, no serious empirical work has been conducted along these lines.

5. NOTES FOR FUTURE RESEARCH

A general equilibrium model for land use and location which fits in the Arrow-Debreu framework requires the following two assumptions. There is a (locationally) fixed set of marketplaces where all the commodities

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[37] It should be noted that the λ_{ik} cannot be observed directly in the market. A surrogate which would be inversely correlated with λ_{ik} would be the vacancy rate. A household assignment model of this type was first proposed in Herbert and Stevens (1960).

[38] See Ripper and Varaiya (1974), and Mills (1972).

in the urban economy are traded. Secondly, markets for all forward (future) trades exist in the present. The second assumption implies for instance that a land developer who is contemplating building a housing development in a particular location two years from now would know exactly the rents that he will get when the actual construction has occurred. Clearly such forward markets do not exist, and the behavior of the land developer (more generally, the evolution of land use) cannot be adequately explained on the basis of the assumption that such markets do exist. The absence of forward markets is not peculiar to urban economies alone and creates difficulties in describing intertemporal behavior in an economy where land is not an important factor. This is an area of intense current research in mathematical economics and we do not pursue it further here. However, the first assumption raises problems which are peculiar to a spatially distributed economic system.

Suppose a retailer is planning to set up a shop in a city. By virtue of his locating at a certain point he automatically creates a new marketplace there, and excepting some trivial cases he will not know the (equilibrium) price at his location of the commodities that he plans to trade. Hence his locational choice cannot be inferred from an Arrow-Debreu framework since it cannot be a function of the equilibrium prices. Of course, after he has located the resulting prices could well be explained in an Arrow-Debreu framework, but his choice of location cannot. It seems necessary then that a theory which tries to explain our retailer's locational choice must involve a model of the way in which the retailer estimates the equilibrium prices[39] which would prevail if he locates at each one of

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[39] In other words, he must estimate the spatial distribution of the demand schedule.

several potential locations.[40] The estimates of demand at the potential locations will clearly depend upon the spatially inhomogeneous information which the retailer has. The uncertainties accompanying these estimates are quite likely to be different at different locations, so that the locational choice will depend also upon the retailer's attitude towards risk. It is easy to see that a wide variety of "rational" behavior is likely to emerge.[41] It is our conviction that systematic exploration of these behavioral patterns is one of the most important open fields of research in land use study.

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[40] The problem seems analogous to the one which arises when we try to explain behavior in an intertemporal but spaceless economy.

[41] For instance there are possibilities of monopoly power when economic activity is distributed over space, see Hotelling (1929).

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