LOCAL PUBLIC GOODS: A REEXAMINATION OF THE TIEBOUT MODEL

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ABSTRACT

The paper examines Tiebout's theory of local expenditures in the context of a simple model in which the allocation mechanism and the residence choice of voter-consumers is specified mathematically. The case of a public good and a publicly provided private good are distinguished. It is shown that an efficient state is unlikely to be sustained in general.

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1. Introduction

Samuelson (1954) has argued that in an economy with public goods an efficient allocation may not be sustainable by a competitive system since individuals have an incentive to not reveal their preferences. Starting from the observation that many public goods are locally produced and consumed, Tiebout (1956) has argued that consumers do reveal their preferences by relocation ("voting by foot"), and that if sufficiently many bundles of public goods were offered by local communities, then an efficient allocation will result.

In this brief note we reexamine Tiebout's proposition in the framework of a very simple mathematical model keeping all the assumptions made by him, and we compare the efficient allocations with those obtained when individuals locate so as to maximize their own preferences. Two different cases are considered: the case where the good produced is a pure public good in the Samuelsonian sense, and the case where it is a publicly provided private good which, for brevity, we call public service. Both cases are considered since it is clear from Tiebout's paper, and the succeeding literature, that both kinds of goods are being referred to.

Our main finding is that in both cases in this simple model, individual location decisions do not generally lead to an efficient allocation, although a simple revenue sharing scheme can be designed which sustains efficiency. Finally, in Sections 4, 5, we also compare allocation achieved by a decision rule which reflects majority voting.

2. Case of Pure Public Goods

To take the simplest possible case compatible with Tiebout's assumptions consider a 2 x 2 x 2 economy: there are two goods, money and the public good, two cities denoted by subscript $j = 1, 2$, and a population of fixed size partitioned into two homogeneous groups denoted by superscript $i = 1, 2$. The cardinal utility function of an individual of group i (hereafter an i-individual) is denoted U^i, his income is w^i, and there are N^i i-individuals N^i_j of whom live in j. Suppose j produces the public good at a quality (or quantity) q_j at a money cost $F(q_j)$ financed through a (proportionate) income-tax of rate t_j. Then an i-individual living in j enjoys utility $u^i_j = U^i(c^i_j, q_j)$ where $c^i_j = (1 - t_j)w^i$ is his disposable income.[1]

Further simplification is achieved by the following specification:

$$U^i(m,q) = m + b^i q \quad i = 1, 2 \tag{1}$$

$$w^1 < w^2, \quad b^1 > b^2, \tag{2}$$

$$F(q) = q^\alpha, \tag{3}$$

Where the b^i, w^i and α are positive constants and $\alpha > 1$. Thus 2-individuals are richer and have a lower preference for the public good than 1-individuals. To complete the model it is necessary to describe j's manager's decision role and the movements of the consumers. J's manager selects t_j and q_j so as to maximize a simple welfare function subject to a balanced budget constraint:

$$\begin{aligned} &\underset{t_j,\, q_j}{\text{Maximize}} \quad \sum_i N^i_j U^i(c^i_j, q_j) \\ &\text{Subject to} \quad c^i_j = (1-t_j)w^i, \; i = 1, 2 \\ &\qquad\qquad\quad q^\alpha_j = t_j \sum_i N^i_j w^i \end{aligned} \tag{4}$$

[1] With Tiebout it is assumed that income and preferences do not depend on community of residence.

For fixed N_j^i, (4) triviality leads to a unique (local and global) optimum denoted $q_j^*(N_j^1, N_j^2)$, $t_j^*(N_j^1, N_j^2)$ which upon substitution into the U_j^i leads to the derived utility of an i-individual living in j,

$$u_j^{i*} = w^i - \left[\frac{1}{\alpha}\left(N_j^1 b^1 + N_j^2 b^2\right)\right]^{\frac{1}{\alpha-1}} \left[\frac{w^i}{\alpha}\,\frac{N_j^1\, b^1 + N_j^2\, b^2}{N_j^1\, w^1 + N_j^2\, w^2} - b^i\right] \quad (5)$$

The aggregate net movement of i-individual is assumed described by the differential equation

$$\dot{N}_1^i = \delta^i\,[\,u_1^{i*} - u_2^{i*}\,], \quad (6)$$

$$\dot{N}_2^i = \delta^i\,[\,u_2^{i*} - u_1^{i*}\,], \quad (7)$$

where $\delta^i > 0$ is constant. The equations imply that in the aggregate individuals move to that city where they receive higher utility. Since (5) and (6) preserve $N_1^i + N_2^i = N^i$, it follows that (5), (6), (7) describe a dynamical system in two variables only, namely $x^i = N_1^i$, $i = 1, 2$. In this notation $N_2^i \equiv N^i - N_1^i \equiv N^i - x^i$. While detailed study of the dynamical system is possible it is not so illuminating as the analysis of its behavior in the neighborhood of two distinguished states: the integrated state $(x^1, x^2) = (N_1^1, N_1^2) = (0,0)$ where the entire population lives in one city and the segregated state $(x^1, x^2) = (N^1, 0)$ where the two groups live in different cities. Evidently the states $(x^1, x^2) = (N^1, N^2)$ and $(x^1, x^2) = (0, N^2)$ are symmetric cases.

The local behavior near these two states is best seen in the phase portraits of Figure 1. The behavior depends mainly upon α with α near 1 and α much larger than 1 giving qualitatively different behavior and $\alpha = 2$ as a "transition" region. Before commenting on the Figure we first note that the only efficient state is the integrated state, since we are concerned with a (pure) public good.

Now for $\alpha \sim 1$, the integrated state is unstable whereas the segregated state is stable. The reason for this is that the richer, 2-individuals have a lower preference for the public good, and for $\alpha \sim 1$ the benefits from cost-sharing are insufficient to outweigh the increased taxes due to increased quantity of the public good which would be produced because of the poor, 1-individuals in the integrated city.

On the other hand, for α sufficiently large the savings from cost-sharing induce the rich to join with the poor so that the efficient state i.e., the integrated city, is stable.

Finally, for $\alpha = 2$, the integrated city is stable only if a) $N^1 \geq N^2$ i.e., the poor dominate the rich numerically and b) $\frac{b^1}{b^2} < 2 \frac{w^1}{w^2}$ i.e., the poor's preference for the public good is not much larger than that of the rich.

In summary the efficient state is sustained in this model only if the savings (for the rich) from cost-sharing are substantial i.e., if the production of the public good exhibits a large degree of decreasing returns. Consider now a "revenue sharing" design in which a planner determines an "optimal" tax rate unifrom over i and j and redistributes the revenue on a per capita basis. The tax rate is chosen to

$$\underset{t}{\text{Maximize}} \quad \sum_i N_j^i \, U^i \, ((1-t)\, w^i, q_j)$$

$$\text{Subject to} \quad q_j^\alpha = t(N_j^1 + N_j^2) \quad j = 1, 2$$

If we carry out this maximization to obtain the derived utilities u_j^{i*} and if we retain the population movement equations (6) and (7) then the resulting phase portrait is as shown in Figure 2. The efficient, integrated state is now the only stable state, independent of α and the relative sizes of the two groups. There is also a family of unstable equilibrium forming the locus, $x^2 = \frac{1}{2}(N^1 + N^2) - x^1$.

3. Case of Public Services

In this section the good being publicly produced is a purely private good which we call a public service. q_j now represents the amount of the service being produced (and consumed) <u>per capita</u> in j.[2] We retain the specification (1), (2), (3). In addition it is being assumed that for fixed q production costs are linearly homogeneous with respect to population size.

With these assumptions the budget constraint in (4) is replaced by $q_j^\alpha \sum_i N_j^i = t_j \sum_i N_j^i w^i$. In place of (5) the derived utilities are now given by

$$u_j^{i*} = w^i - \left[\frac{1}{\alpha}\ \frac{N_j^1 b^1 + N_j^2 b^2}{N_j^1 + N_j^2}\right]^{\frac{1}{\alpha-1}} \left[\frac{w^i}{\alpha}\ \frac{N_j^1 b^1 + N_j^2 b^2}{N_j^1 w^1 + N_j^2 w^2} - b^i\right] \quad (8)$$

The population dynamics (6), (7) are assumed unchanged so that together with (8) they form a new dynamical system.

Once again we only examine the system behavior in a neighborhood of the integrated and segregated states. But this time since we are dealing with a private good and since the two population are assumed to

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2 For some purposes such services as public education and police protection can be considered as public services in the sense used here. Indeed the common practice of using per capita expenditures for measuring these services rests on the assumption that they are private goods.

have different preferences, the only <u>efficient state is the segregated state</u>. Again the qualitative properties of the behavior depend on α as shown in Figure 3. The behavior does not depend upon the relative sizes of the population groups. For α small relative preference differentiation outweighs any redistributive advantage that the poor group would receive by joining the rich and the efficient, segregated state is stable. However for large α the redistribution effects become significant and the poor try to join the rich. The same forces propel the latter to escape from the poor. Neither state is then a possible equilibrium.

Finally suppose we have a revenue-sharing scheme by which a common tax was imposed and redistributed on a per capita basis. Evidently then both cities would produce the same amount of service per capita, so that there would be no incentive for anyone to move and <u>every</u> state would be stable. Furthermore, the efficient allocation would <u>never</u> be reached since it requires the different communities to produce different amounts of service per capita.

4. Public Good with Majority Voting

In this section and the next one we briefly reexamine the case of public good and public service respectively, with a crucial modification in the allocation rule (4). It is assumed now that in each city the allocation chosen is the one which is most beneficial to the majority. Thus rule (4) is replaced by (9).

$$\begin{aligned} &\underset{t_j,\, q_j}{\text{Maximize}} \quad N_j^i\, U^i(c_j^i,\, q_j) \\ &\text{subject to} \quad c_j^i = (1-t_j)w^i \end{aligned} \tag{9}$$

$$q_j^{\alpha} = t_j \sum_i N_j^i w^i$$

where $i = 1$ or 2 according as $N_j^1 >$ or $< N_j^2$. The desired utilities change accordingly. If we retain the movement dynamics (6), (7) then the qualitative behavior of the system is as shown in Figure 4.[3] α is again a crucial parameter but this time the behavior depends also upon whether $N_j^1 > N_j^2$ or $N_j^1 < N_j^2$. The following comments explicate Figure 4 in some more detail:

(i) The majority voting rule partitions the phase space into areas determined by which group is in the majority. In the Figure area I(II) corresponds to group 1 (2) forming a majority in city 1(2) and group 2(1) forming a majority in city 2(1), whereas III(IV) corresponds to group 2(1) forming a majority in both cities.

(ii) The line segment $I_1 = I_2$ characterizes city compositions at which the city incomes are identical ($I_j = \sum_i N_j^i w^i$). It follows that in areas III and IV this line segment is a set of unstable equilibria. In areas I and II (6) and (7) vanish along the segments labelled $\dot{x}^1 = 0$ and $\dot{x}^2 = 0$ respectively, and since these segments do not intersect we can have no equilibrium in the interior of regions I or II. It follows then that a stable equilibrium can occur only in the segregated or integrated state. We discuss these next.

(iii) First for $\alpha \sim 1$ the efficient, integrated state is unstable whereas the segregated state which is inefficient is always unstable. Thus the situation here is the same as in Figure 1 and for the same rea-

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[3] The results of this and the next section hold if (2) is replaced by the weaker condition $\frac{w^1}{w^2} < \frac{b^1}{b^2}$.

sons namely, cost-sharing advantages are insuffficent to overcome tendencies to segregate because of differences in preferences and incomes. For the case $\alpha >> 1$ the efficient, integrated state is always stable, again as in Figure 1. However, unlike Figure 1, the segregated, inefficient state may also be stable. This occurs if and only if $(N^1,0)$ is in area I.2 and $(0,N^2)$ is in area II.2 i.e., if the segregated state lies between the lines $\dot{x}^1 = 0$ and $\dot{x}^2 = 0$, and a tedius computation reveals that in turn this is possible exactly when $1 < \frac{N^1b^1}{N^2b^2} < 2$. If we interpret N^ib^i as in the aggregate preference of i-individuals for the public good, then we have the seemingly paradoxical result that the segregated state is stable if the aggregate preferences of the two groups are not too dissimilar.

5. Public Service with Majority Voting

The tax rate t_j and amount of public service per capita q_j in city j is now given by the optimal solution of the following problem,

$$\begin{aligned} \underset{t_j,\, q_j}{\text{Maximize}} \quad & N_j^i\, U^i(c_j^i,\, q_j) \\ \text{subject to} \quad & c_j^i = (1-t_j)w^i. \\ & q_j \sum_k N_j^k = t_j \sum_k N_j^k\, w^k, \end{aligned}$$

where i is 1 or 2 according as $N_j^1 >$ or $< N_j^2$. The resulting derived utilities and dynamics (6), (7) define a dynamical system. The system behavior is portrayed in Figure 5 for the case $\alpha \sim 1$ only. The behavior again depends upon which group forms a majority and it depends also upon which city has a larger income per capita (In the Figure H_j = per capita income in city j). Upon comparing Figure 5 with Figure 3 some dramatic changes are evident. First of all, when $N^1 \geq N^2$ the majority voting rule eliminates all stable equilibria. Secondly, when $N^1 \leq N^2$ the inefficient, integrated state becomes stable whereas under the welfare rule of Figure 3 it has the efficient, segregated rule which was stable. The reason behind the instability in the case $N^1 \geq N^2$ is straightforward: the 1-individuals have a larger demand for the public service relative to their income (recall that $\frac{b^1}{w^1} > \frac{b^2}{w^2}$), and they can always migrate to a city to form a majority and impose a higher tax rate; the only alternative available to the 2-individuals is to try to avoid the tax by migrating.

6. Conclusion

The argument suggesting that localized production and consumption of public goods combined with consumer-taxpayer mobility leads to an efficient allocation has been examined in the context of a very simple formal model. The case of a pure public good and a publicly provided private good have been considered separately. Two allocating rules, maximization of a utilitarian welfare criterion and a (selfish) majority voting mechanism, were investigated.

The main conclusion is a negative one namely that the efficient allocation is stable only under exceptional circumstances which depend primarily upon the relative magnitudes of the benefits of cost-sharing and the differences in preferences and incomes. It was also shown that in the public good case a revenue-sharing scheme leads to an efficient stable allocation. This suggests that in some instances inter-city benefit spillovers may lead to a stable efficient state and the absence of spillovers destroys stability.

The analysis depends crucially upon the assumption (also implicit in Tiebout's argument) that an individual receives no direct (psychic) benefits from an increase in the utility of others, in particular, no "merit" goods are considered. Empirical evidence suggests that this is not so.[4] It is evident that such inter-utility dependencies would often act as stabilizing influences.

The assumption of linearity in the utility functions was used merely for analytical convenience. However, if utility functions are nonlinear but differentiable, then all of the preceding is still valid as "local" analysis.

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[4] See e.g. Greene, _et. al._, (1974).

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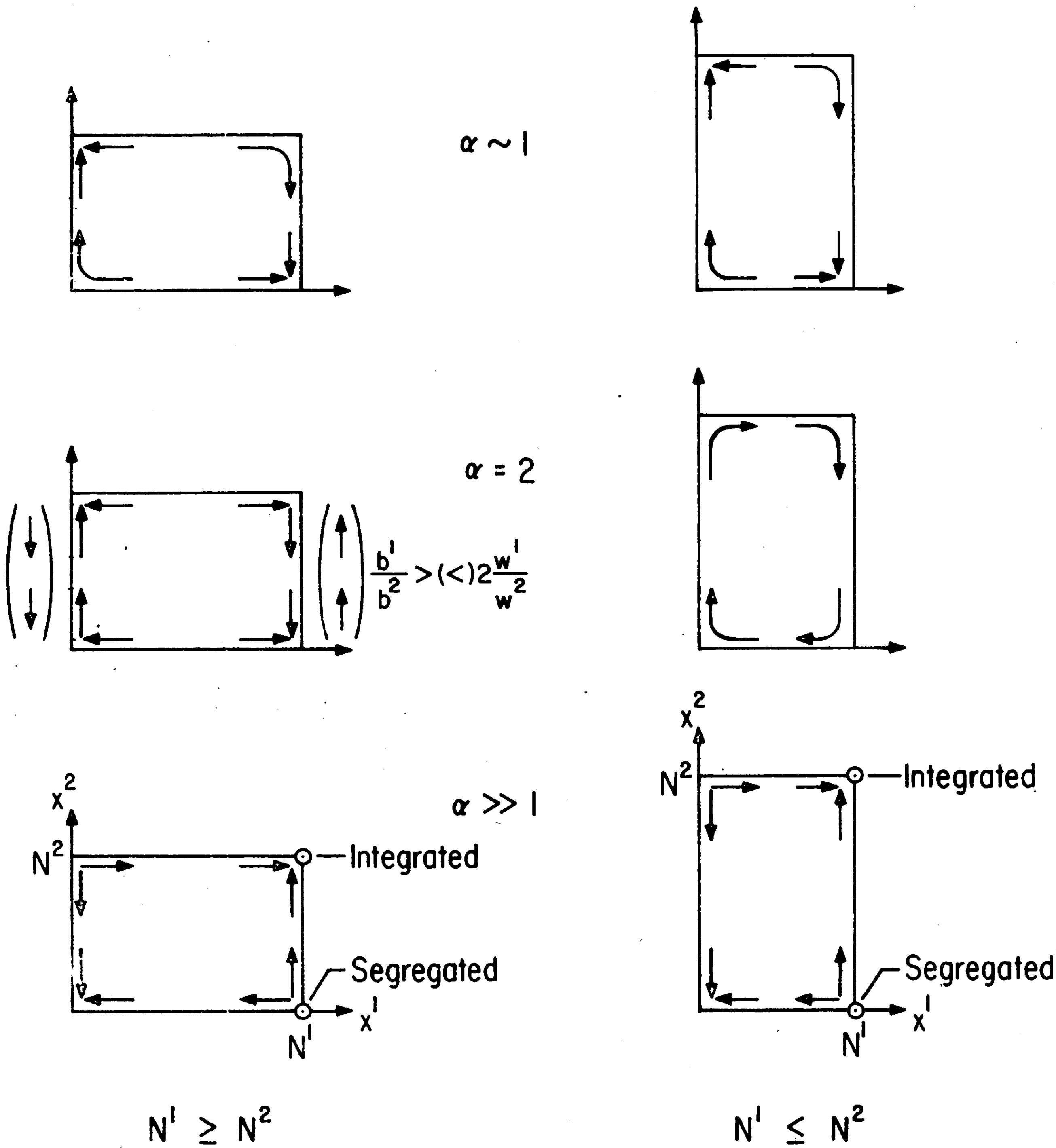

Fig. 1. Behavior in the public good case.

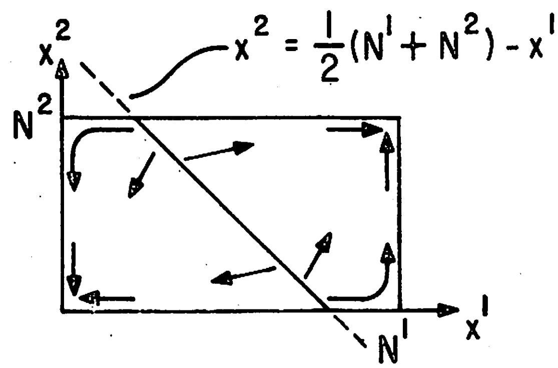

Fig. 2. Public good with revenue sharing.

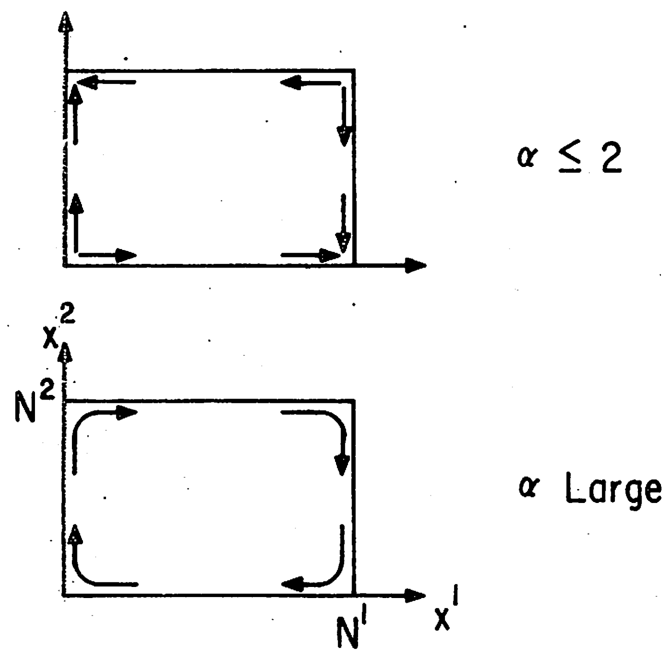

Fig. 3. Behavior in the public service case.

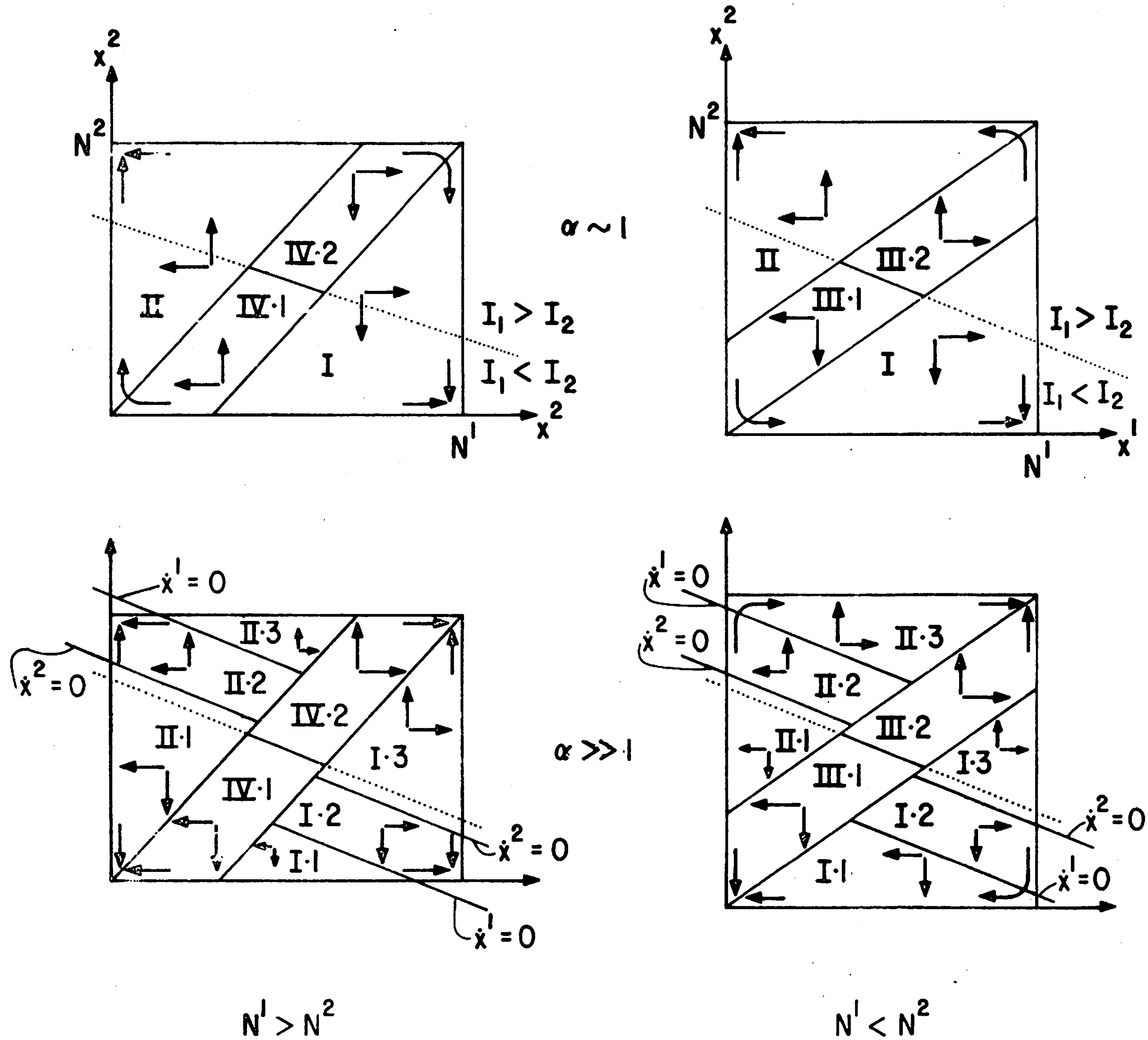

Fig. 4. Behavior in public good case under majority voting.

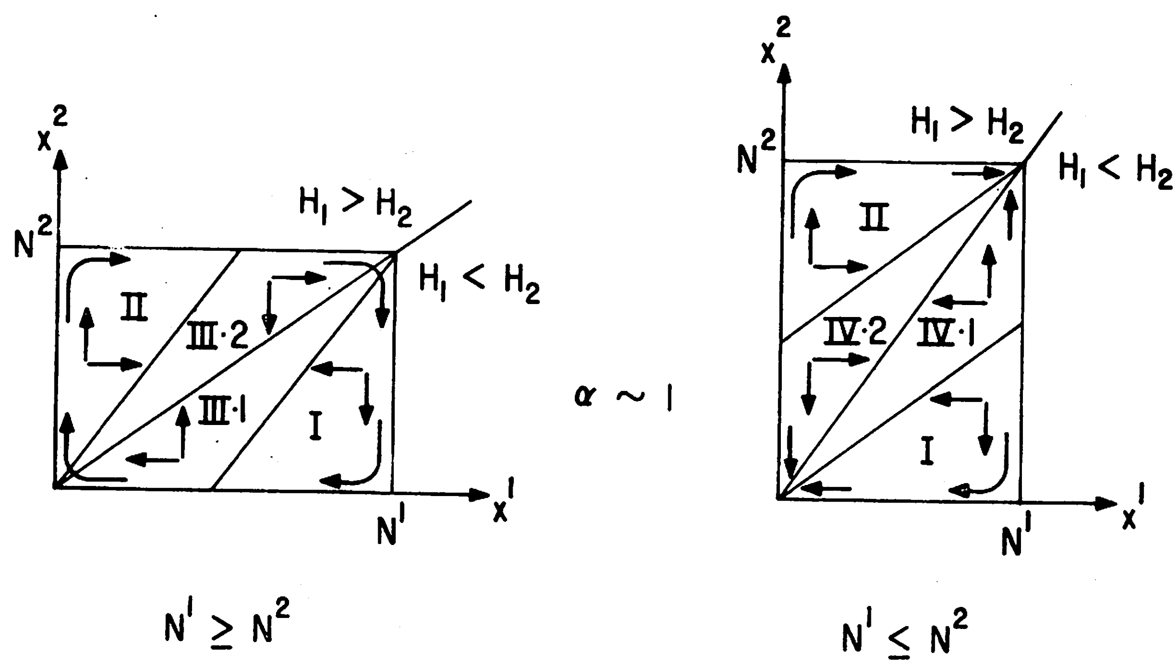

Fig. 5. Behavior in public service case with majority voting.