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LATENT COMPONENTS IN THE ELECTROCARDIOGRAM

by

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LATENT COMPONENTS IN THE ELECTROCARDIOGRAM[†]

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A considerable amount of work has been published on the representation of a set of simultaneously recorded electrocardiogram waveforms as sums of component waveforms [1,2,3,4,5,6,7,8]. This paper describes a new technique in this class of representations, which all start with the concept that if there are N observed waveforms, $D_i(t)$, $i = 1, 2, \dots, N$, and M component waveforms, $C_j(t)$, $j = 1, 2, \dots, M$, then the $D_i(t)$ can be represented as

$$D_i(t) = \sum_{j=1}^M A_{ij} C_j(t) + e_i(t)$$

where A_{ij} are coefficients which do not vary with time, and $e_i(t)$ is the error between the representation and the observed waveform.

A Brief Classification of the Published Techniques and the Latent Component Technique

1. Specified Components Method. The most common is the Fourier series representation, where $C_j(t)$ is $\cos(j-1)\pi f_0 t$ for j odd and $\sin j\pi f_0 t$ for j even, and f_0 is the reciprocal of the time interval that is to be represented [2]. The coefficients A_{ij} can be calculated directly once the time interval and $t = 0$ point are specified. Functions other than sines and cosines have been used [3,4]. In general

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they are chosen to be an orthogonal set, which simplifies the calculations of the coefficients A_{ij} .

2. Form of Components Specified Method. An example of this representation is the use of Gaussian components, $C_j(t) = H_j e^{-[K_j(T_j-t)]^2}$ [5].

The H_j , K_j and T_j along with the A_{ij} coefficients are computed for the minimum total squared error, $E^2 = \sum_{i=1}^N \sum_{t=0}^T [e_i(t)]^2$ over the time interval analyzed, 0 to T.

3. Coefficients Specified Method. The A_{ij} coefficients are determined from a model, not directly measured for the subject, and the $C_j(t)$ computed for the minimum total squared error, E^2 [6]. The number of components M is determined by the detail of the model used. An additional constraint may be imposed on the $C_j(t)$, e.g., that they be non-negative.

4. Factor Analysis Method. The $C_j(t)$ are constrained to be orthogonal to each other. The first component, $C_1(t)$, minimizes the total squared error, E^2 , when only one component is used, i.e., when $M = 1$. Each subsequent component minimizes the total squared error with the addition of that component [7,8].

5. Latent Component Representation. This new representation places the following constraints on the components.

a. Each component $C_i(t)$ is positive in the time interval from its start to its finish; i.e. from $t = S_j$ to $t = F_j$, and is zero elsewhere.

b. The components start in sequence, and end in sequence;

$$S_1 < S_2 < S_3 < \dots < S_M, \quad F_1 < F_2 < F_3 < \dots < F_M.$$

c. No more than three components can be non-zero simultaneously.

d. The number of components is the minimum necessary to make the maximum error acceptable.

Examples of this representation are illustrated in Figure 1. Appendix A gives further information on these illustrations. Appendix B explains the decomposition algorithm.

Discussion of Uniqueness in the Absence of Noise

If the electrocardiogram is used to determine the condition of the heart, the most desirable representations would be those that have one set of features dependent on the heart and another set dependent on the other variables, such as body position, orientation of the heart, the electrical characteristics of the torso, and the electrode placement [7]. This is achieved in the Coefficients Specified Method, by including all the non-heart effects in the coefficients, and obtaining them from a model. The strength of this technique is also its weakness, i.e. the coefficients are only as accurate as the model, and it is difficult to include individual subject variations in the model, particularly for abnormal cases. The computed components are sensitive to errors in the coefficients [6].

If the components were of Gaussian shape (or any other predetermined shape) then the Form of Components Specified Method would yield a unique set of parameters associated with components, and consequently with the heart. However, the form of the basic components is not predetermined, particularly for abnormal hearts, and this representation will not separate out heart dependent parameters.

The Specified Components Method and Factor Analysis Method have the

appearance of being unique in that the mathematical algorithms are clear and well understood. However, in both cases, the resulting components and/or coefficients are dependent on the leads used and on the time interval analyzed. For example, if the P and T waves were included with the QRS wave in a factor analysis, a different set of factors would be obtained than if only the QRS wave were used. Furthermore, the shapes of these different factors would be different in the common QRS interval. This dependence of the factors on the time interval is an indication of the physiologically inconsistent mathematical constraint that the factors be orthogonal over a time interval. The component waveforms associated with sections of the heart are not orthogonal in time to one another [6].

The Latent Component Representation is unique in the sense that any set of data waveforms processed will always yield the same results, since a definite algorithm can be followed. The one used in obtaining the results shown in Figure 1 is described in Appendix B. Preliminary investigations using this algorithm indicate that the shapes of the latent components extracted are not sensitive to moderate variation in the lead positions, even when noise is present.

Noise Considerations

An observed electrocardiogram set does not reproduce identically from one beat to another, even if it is taken from the same subject with the same leads, the same body position, and the same lung volume. The differences can be called noise, and under this definition the noise would include electrode noise, amplifier noise, electrode artifacts, and true beat to beat differences in the ECG. Averaging a number of ECG's

will reduce this noise, including the observation of the true variability in the ECG. The ECG being analyzed, whether averaged or not, is indeterminate by some value. A simplistic approach is to assume that the true value of each data point of the waveform is within δ of the observed value. This ignores the frequency characteristics of the noise, and the statistical aspects of its amplitude distribution, but does express the concept that the observed values are not exactly correct. Under these assumptions we should be satisfied with a maximum error of δ for every point on every simultaneous data waveform.

The number of components used in any representation, if not predetermined (for example, from a model), should be the minimum necessary to meet the error criterion. An exception to this is when the maximum number of components obtainable still does not meet the desired error criterion, as in factor analysis when only three Frank lead data waveforms are available and the three factors derived are not adequate to obtain the desired accuracy.

Dependence of Representation on Accuracy Criteria

In each representation there is a relationship between the number of components and the error, which is data dependent. In the Specified Components Method the waveforms of the components are all known a priori. In the Form of Components Specified Method, an additional component added to the representation will alter the waveforms of the components derived for the best representation without this component. In the case of Factor Analysis, each additional component reduces the error without affecting the previously derived components. In the Latent Component

representation, an additional component will alter the waveforms of the components derived for the best representation without this component. The primary alteration observed in the cases examined was to split one component into two components which were portions of the original component, and whose coefficients straddled the coefficients of the original component. The other components were affected in a secondary way. This is similar to what happens when more detail is added to the model in the Coefficients Specified Method.

The number of components used in the Latent Component representation is, as in the other cases where the number of components is not predetermined, a function of the acceptable error. One difference in the Latent Component representation with the algorithm outlined in Appendix B is that the final error criterion used to terminate the addition of components is an acceptable maximum error at every point in time on every waveform, and not an acceptable percent total squared error. This distinction is important since the input data ECG's can include the entire P-QRS-T waveform, and the Latent Component representation for each section will be the same as when each section (P,QRS,T) is analyzed separately. This is not true for any of the other representations.

A few comments are in order concerning the baseline. When there is a common level of horizontal segment before the P wave, between the P and the QRS waves, and between the QRS and T waves, there is no problem about determining the baseline. It is in this case only that it is reasonable to analyze the P, QRS, and T waves as separate waveforms. As long as there is a horizontal segment preceding the P wave, it is reasonable to use this as the baseline. If the ECG does not return to

this baseline between the P and QRS waves and/or between the QRS and T waves, the Latent Component representation will have components that extend through these regions, but that may be associated predominantly with one of the normal segments, i.e. with the P, QRS or T segments. In fact, one way of looking at the Latent Components is that the QRS wave is now being decomposed in the same way the P and T waves have always been considered separate (non-latent) components of the complete P-QRS-T electrocardiogram, with the additional feature that the P and T waves may now also decompose. An algorithm for determining the baseline when no horizontal segment precedes the P wave remains a difficult problem for this analysis technique. It might be noted that a change in baseline will affect the results in all the representations, and has a confined effect only in the Fourier representation.

Summary

A new representation for simultaneous ECG waveforms has been described which has some similarities to existing techniques, but has the following advantages.

1. The number of components is determined by the accuracy required, and is not limited by the number of simultaneously observed waveforms.

2. The components represent sequential time events, with overlap of the events. This is similar to the descriptions of ECG phenomena used in clinical cardiology texts [9].

3. To a first order the component waveforms depend on the potentials generated by the heart, and the coefficients depend on the heart orientation, the torso electrical properties (including lung volume), and the lead placement.

4. No model is necessary to accomplish the decomposition, but the resulting representation can be related to existing models [10].

The original motivation was to develop a technique to classify the ECG waveforms in a way that might lend itself to clinical diagnosis. The usefulness of this representation for clinical purposes remains to be demonstrated.

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Appendix A

The waveforms in Figure 1 are from Ellison and Restieaux, "Vectorcardiography in Congenital Heart Disease" [11], and appear as Figure 11-9 A and B. They are Frank lead cardiograms and vectorcardiograms of a child with transposition of the great arteries. The A waveforms were taken shortly after birth, and the B waveforms at eight months of age.

The XYZ tracings were enlarged and manually digitized. Forty-five time intervals covered the QRS waveform, and the negative peak of the X waveform in A is -60 units on the vertical scale used for the digitization. The data was filtered with a Gaussian time transversal filter having half amplitude points at a separation of two time intervals, to smooth it slightly before the decomposition into Latent Components. These smoothed waveforms are considered the input data.

The computation according to the algorithm outlined in Appendix B was executed on a Control Data 6400. The smoothed data curves and the components are plotted on scales about twice that of the original data, and the coefficients are plotted in a unit circle for the Horizontal, Frontal, and Left Sagittal planes. The amplitude of the maximum error for the filtered A waveforms is 1.15 units and the maximum error for the filtered B waveforms is 1.81 units. The percent root power error is 1.4% for the filtered A and 2.8% for the filtered B. The correspondence between the coefficients displayed for the Horizontal plane and the corresponding vectorcardiogram directions is very close for both the A and B waveforms.

Figure 2 is the same Latent Component representations as in Figure 1, but with each corresponding A and B component and coefficient vector superimposed.

Appendix B

The algorithm for obtaining the Latent Components is an iterative fitting process. Figure 3 illustrates the steps involved. If we assume there is no noise, then in a notation for digital computation with sampled data

$$D_i(t) = \sum_{k=1}^M A_{ik} C_k(t) \quad .$$

The components are not derived in the sequence of their occurrence in time, but approximately in the order of their peak power. For this reason the index k is used to identify the M components. They are later relabeled to correspond to the sequence of their time occurrence, and identified by the subscript j .

If we calculate the total power, which is the squared error before we have any representation for the N observed data curves, $D_i(t)$, we obtain

$$E_o^2(t) = \sum_{i=1}^N \{D_i(t)\}^2 \quad .$$

The time of the occurrence of the maximum value of this curve, $t_{\max 1}$, is determined, and the $C_1(t_{\max 1})$ component is set to 1. The A_{i1} are determined for an exact fit at this time, i.e. $A_{i1} = D_i(t_{\max 1})$. Adjacent time values of $C_1(t)$ for these A_{i1} are determined for minimum squared error. The $C_1(t)$ curve is terminated in each time direction either

- a) when it goes negative, or

b) when the residual squared error $E^2(t)$ goes below $N\delta^2$, where δ is the maximum acceptable error, or

c) when the proportional residual error

$$\frac{\sum_{i=1}^N [D_i(t) - A_{i1} C_1(t)]^2}{\sum_{i=1}^M [D_i(t)]^2}$$

exceeds .5, since this would be the situation when two adjacent spacially orthogonal components would have equal amplitudes, i.e., would be a cross-over point of the components. By spacially orthogonal components is meant $\sum_{i=1}^N A_{ik} A_{i\ell} = 0$. This leaves a residual error $E_1^2(t)$, which is the original $E_0^2(t)$ in the time region not occupied by $C_1(t)$. The process is repeated for the remaining maximum squared error, with an additional terminating condition on extending the components in time, which is

d) that when a component intrudes into an existing adjacent component, it is extended only while at the current point in time the residual squared error using this new component is less than the error using the existing component. That is, the boundary between components is chosen to minimize the maximum squared error.

This process is iterated until the entire time interval where $E_0^2(t)$ exceeds $N\delta^2$ is covered by non-overlapping components.

The coefficients are then recalculated to reduce the total squared error. The component curves are then modified and extended in both directions in time to reduce the total squared error. The limit of this extension in time is determined by either

a) the component going negative (in which case the value is set to 0), or

b) until the time at which the power in the adjacent component (C_{j+1} or C_{j-1}) becomes less than the power in the next farther component (C_{j+2} or C_{j-2}).

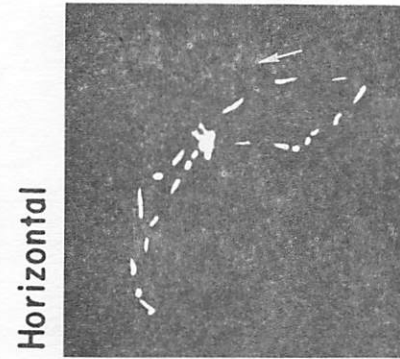
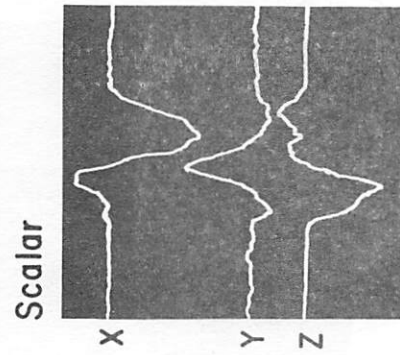
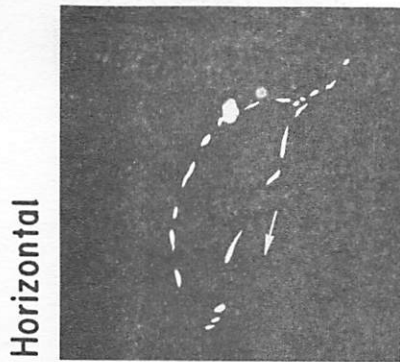
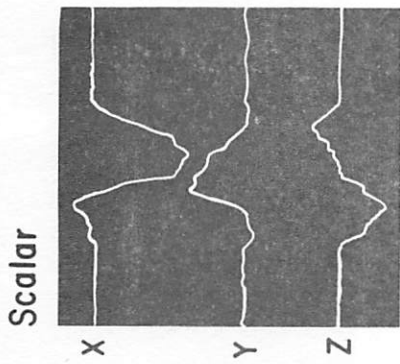
The sequence of recalculating the coefficients with the components fixed, and then the components with the coefficients fixed, is iterated, and the residual maximum error is then observed. If the residual maximum error is too large, i.e., it exceeds the acceptable error, the computation returns to the original non-overlapping components, and inserts a new component at the time where the residual error was a maximum.* The extension of this new non-overlapping component proceeds as before, and is followed by the iterations of coefficient and component calculations. The process is repeated until the maximum residual error meets the desired error criterion.

The curves are now relabeled to correspond to their occurrence in time, as identified by the subscript j . The curves are normalized by setting $\sum_{i=1}^N (A_{ij})^2 = 1$, and adjusting the amplitude of $C_j(t)$ to compensate for this normalization of the components.** The normalization allows the coefficients of each component to be considered a vector of unit length in N dimensional space. In the particular case where the data waveforms represent three spacially orthogonal ECG components, the

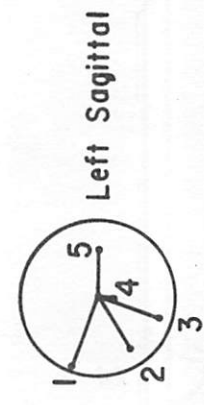
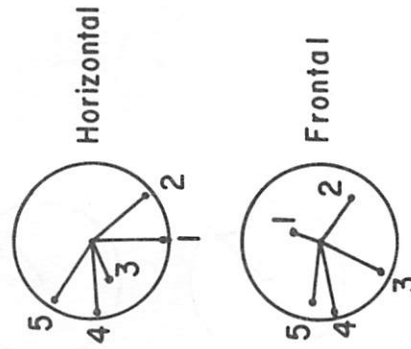
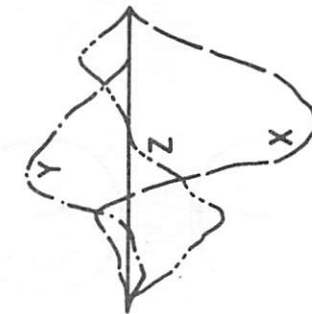
* The shift from minimizing the total squared error to using a different error measure, maximum error, as a criterion causes no difficulty, as the two are maximum at almost the same point. The square error criterion is simpler computationally, and the absolute error is a better bound on the fidelity of the representation, since the noise in the N data waveforms is uncorrelated.

** This normalization is done at every step in Fig. 3 so that all the components would appear on the same scale.

coefficients can be considered the components of a unit vector corresponding to the spacial orientation of that component [7].



A



B

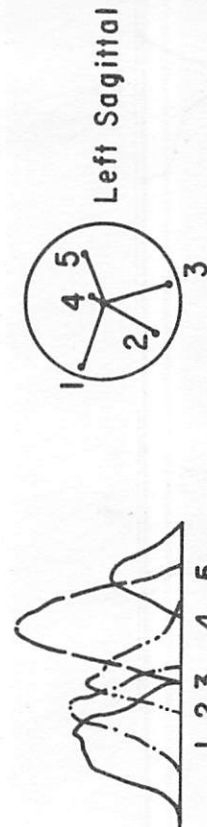
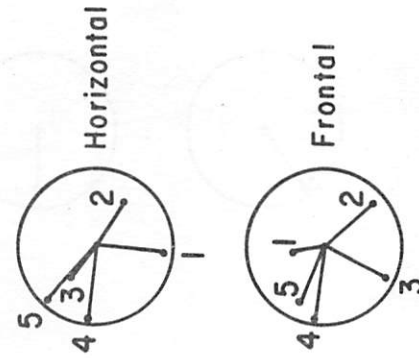
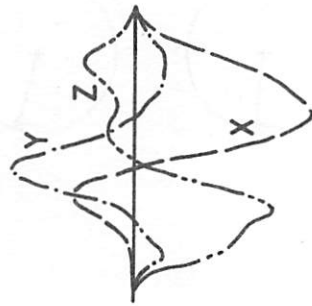


Figure 1.

A ———
 B ······

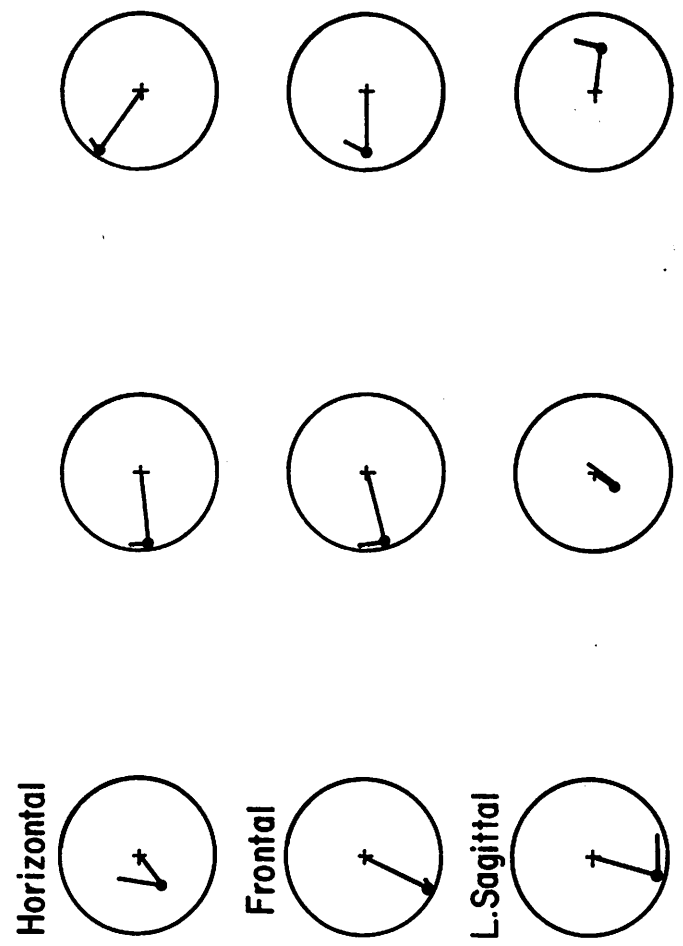
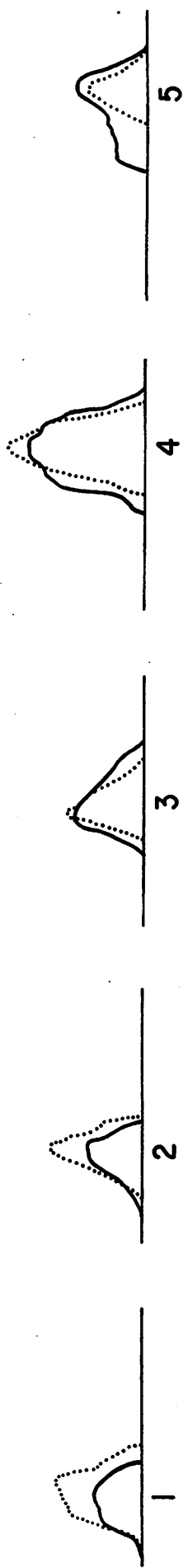



Figure 2.

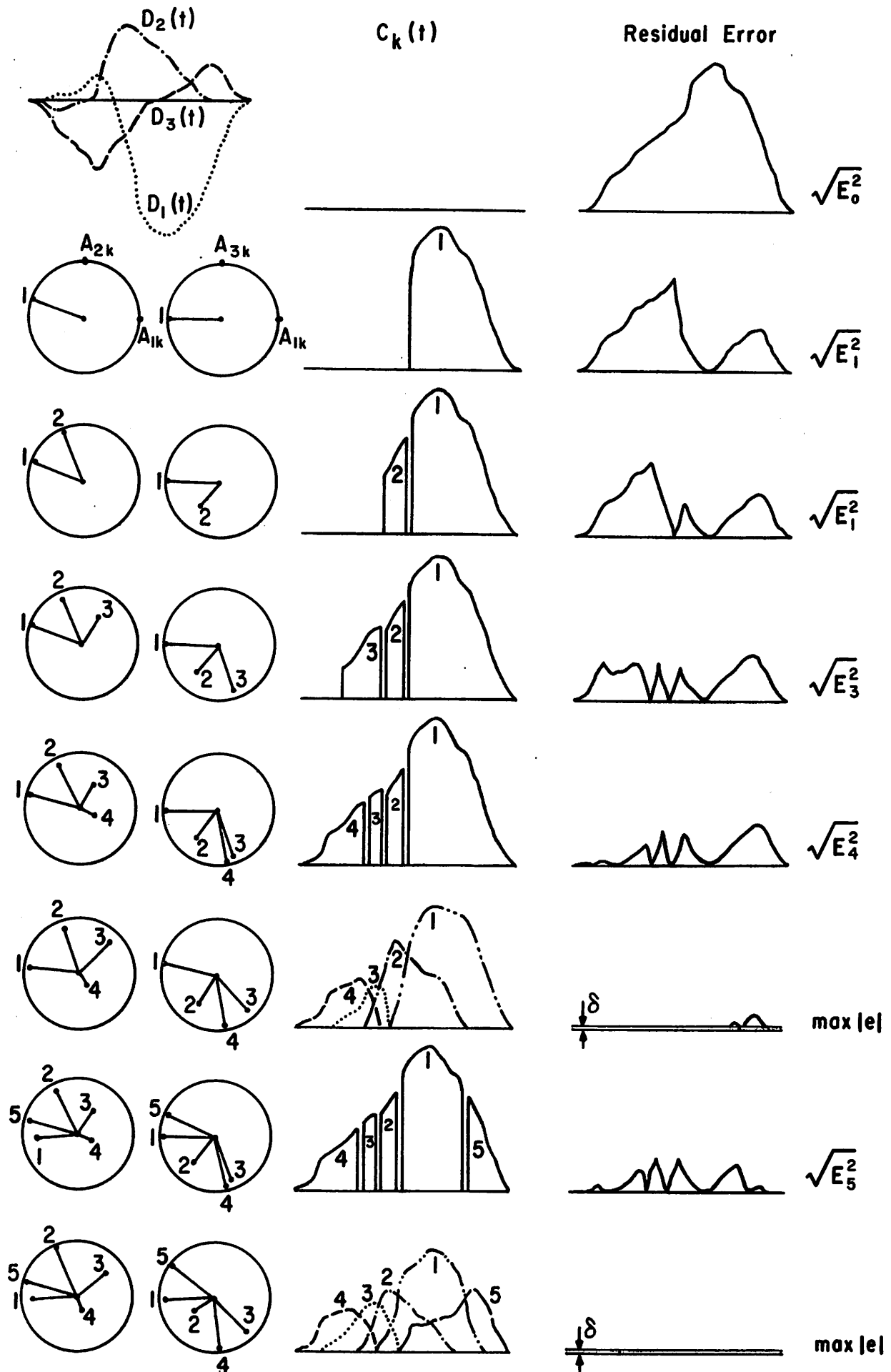


Figure 3.