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by

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ABSTRACT

The paper builds upon earlier work by Simon upon the effects of increased productivity on the ratio of urban to rural population. The approach is based upon the analysis of excess demands rather than comparative statics. This permits a more complete portrayal of the linkages between measures of productivity change and elasticity, and shifts in the division of labor.

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1. Introduction

About 30 years ago, Herbert Simon published a paper entitled "Effects of Increased Productivity upon the Ratio of Urban to Rural Population"[6]. Its objective was to "help to reveal the anatomy of the mechanisms" behind the long-run spatial reallocation of population from rural to urban areas.

For this purpose, Simon devised an economic model consisting of two sectors, a rural sector producing "agricultural commodities", and an urban sector producing "industrial commodities". The region, or nation, considered was assumed to be a closed economy. Each sector consisted of a large number of individual producers, whose behavior was taken to be governed by profit maximization under competitive conditions. Strict concavity was assumed in each production process, and the output was produced by means of one mobile resource, labor.

The labor market was assumed to be in long-run, full employment equilibrium at the points of analysis, with the total supply of labor taken to be constant.

On the consumption side, demand functions for each commodity were derived on the basis of a "utility index", with aggregate demand being a function of the two prices and aggregate income. Simon also assumed that neither commodity was an inferior good. The aggregate consumer income, in the rural and urban areas combined, was taken to be just large enough to purchase the total output of the two commodities. (Wages were determined by marginal productivities, and as noted above the output quantities were strictly concave functions of labor inputs;

hence, there is an additional assumption implied, namely that a residual remumeration, or "surplus value", be accrued to the immobile factors, say, land and capital.)

Critical for the outcome of Simon's analysis was one further postulate, namely that the income elasticity of demand was higher for urban than for agricultural goods.

Into this framework Simon introduced technological change in production -- assumed to be multiplicative (Hicks-neutral) -- and he was able to prove the following proposition: "<u>equal percentage increases in the efficiency of production", in the two sectors, "will lead to a decrease in the quantity of labor employed in agriculture." Hence, the shift in the division of labor will lead to migration from rural to urban areas</u>.

This was a remarkable result -- and to reach it Simon used the approach of comparative statics which necessitated a long and involved series of steps. He treated the technological changes as a disturbance of an initial equilibrium. A new equilibrium would result after the effects of the increases in the efficiency of production in the two sectors had worked their way through the system. Simon's approach was to analyze the displacements of the variables implied by the new equilibrium.

Yet, Simon's paper raises an intriguing question. The conclusion about a shift in population to urban areas holds only for the case where the (percentage) rates of productivity increase in the two sectors are equal, or at least approximately equal. What happens when the rates are unequal cannot be determined, in this framework, without an additional assumption; in turn, the nature of the additional assumption is such that it cannot be supported by available empirical evidence, or "stylized facts".

There is a certain affinity between Simon's work and a more recent paper by Baumol[1], in which a distinction between a "progressive" and a "non-progressive" sector is made. In Simon's terms, this means that the technical change results in an increase in the efficiency of production in one sector and no productivity change at all in the other sector. However, as indicated above, no definite conclusion about the change in the division of labor can be made for this case, within the framework of Simon's paper, unless a different method of analysis is used. Baumol sidesteps the difficulty by assuming that aggregate demand for each good is specified exogenously and is unaffected by price or income changes.

Based on a comparison of two equilibrium positions, one before and the other after the technological change had occurred, Simon's method was to analyze the direction of the shift in the division of labor, and hence the direction of migration. In the following, we shall utilize Simon's conceptual framework, but the approach will be an excess demand analysis, rather than an equilibrium analysis. We shall set out from the initial equilibrium position described by Simon, introduce technical changes in the same way he did, and then analyze the excess demand which would arise, if the allocation of labor remained unchanged. We shall show that this disequilibrium analysis provides sufficient information to draw fairly general conclusions about the direction of the flow of labor from one sector to the other. In particular, it will be shown that Simon's proposition about the effect of equal rates of productivity increase and Baumol's case of "unbalanced growth" follow as special cases of a more general result.

One additional effect of the technological change is also analyzed. This is the change in the relative prices of the two commodities.

2. Definitions and Assumptions

Production Before technological change, the quantity supplied of the i-th consumption good is $s_i = f_i(L_i)$, $i = 1,2$, where L_i is the quantity of labor employed in sector i; it is assumed that $f_i > 0$, $f_i' > 0$, $f_i'' < 0$. After technological change, the new production functions are $s_i^N = f_i^N(L_i)$, where $f_i^N(L_i) = (1+\Delta_i)f_i(L_i)$, and where Δ_i, the percentage change in productivity in sector i, is exogenously specified; naturally $\Delta_i > -1$.

Labor The total labor force is fixed at $\bar{L}$.

Consumption All individuals have the same income. Then the aggregate demand for good i is $d_i = D_i(p_1,p_2,I)$ where p_i is the price of good i and I is aggregate income. D_i is a continuously differentiable function homogeneous of degree zero. We use the symbols e_{iI}, e_{ii}, e_{ij}, to denote, respectively, the income elasticity, the own price elasticity, and the cross-price elasticity of the demand for good i. That is

$$e_{iI} = \frac{\partial D_i}{\partial I}\,\frac{I}{d_i}\,,\quad e_{ii} = \frac{\partial D_i}{\partial p_i}\,\frac{p_i}{d_i}\,,\quad e_{ij} = \frac{\partial D_i}{\partial p_j}\,\frac{p_j}{d_i}\,.$$

From the homogeneity of the demand function it follows that

$$e_{ii} + e_{ij} + e_{iI} \equiv 0 \tag{2.1}$$

For further reference note that since there are only two sectors it follows that if $e_{ii} > -1$ and $e_{jj} < -1$, then $e_{ij} > 0$ and $e_{ji} < 0$.

Equilibrium It is assumed that before technological change the economy is in long-term, full employment equilibrium. Letting L_i^*, p_i^*, s_i^*, d_i^*, I^* denote equilibrium values, this means that

$$L_1^* + L_2^* = \bar{L}\ ,\ p_1^* f_1'(L_1^*) = p_2^* f_2'(L_2^*) \tag{2.2}$$

$$s_i^* = d_i^*\ ,\quad i = 1,2 \tag{2.3}$$

$$s_i^* = f_i(L_i^*),\quad i = 1,2 \tag{2.4}$$

$$d_i^* = D_i(p_1^*, p_2^*, I^*),\quad i = 1,2 \tag{2.5}$$

$$I^* = p_1^* s_1^* + p_2^* s_2^* \tag{2.6}$$

(2.2), (2.3) respectively represent market clearance for labor and goods. (2.6) asserts that national income equals national product. It is assumed that for each i both $p_i^* > 0$ and $L_i^* > 0$.

Disequilibrium Suppose technological change has occurred. Let L_i^{N*}, p_i^{N*} etc. be the new equilibrium values. Consider a positive (disequilibrium) price vector p_1^N, p_2^N and let L_i^N, s_i^N, d_i^N, I^N be respectively determined by the conditions (2.7), (2.8), (2.9), (2.10):

$$L_1^N + L_2^N = \bar{L}\ ,\ p_1^N f_1^{N'}(L_1^N) = p_2^N f_2^{N'}(L_2^N) \tag{2.7}$$

$$s_i^N = f_i^N(L_i^N),\quad i = 1,2 \tag{2.8}$$

$$d_i^N = D_i^N(p_i^N, p_2^N, I^N),\quad i = 1,2 \tag{2.9}$$

$$I^N = p_1^N s_1^N + p_2^N s_2^N \tag{2.10}$$

(Thus all equilibrium conditions except the one corresponding to (2.3) are satisfied).

Let

$$x_i^N = d_i^N - s_i^N, \quad i = 1,2 \tag{2.11}$$

be the excess demand for good i.

L2.1 a) $p_1^N x_1^N + p_2^N x_2^N = 0$; in particular $x_1^N \gtrless 0$ if and only if $x_2^N \lesseqgtr 0$

b) $\dfrac{p_1^{N*}}{p_2^{N*}} \gtrless \dfrac{p_1^N}{p_2^N}$ if and only if $x_1^N \gtrless 0$

Proof. $p_1^N x_1^N + p_2^N x_2^N = 0$, which is Walras' law, follows from (2.9), (2.10) and the fact that $p_1^N d_1^N + p_2^N d_2^N = I^N$; the second assertion in a) then follows since $p_i^N > 0$. b) is obvious. ◻

3. Study of Excess Demand

Consider the prices

$$p_i^N = (1+\Delta_i)^{-1} p_i^*, \quad i = 1,2 \tag{3.1}$$

L3.1 With these prices, the solutions of (2.7)-(2.10) are

$$L_i^N = L_i^*, \quad i = 1,2 \tag{3.2}$$

$$s_i^N = (1+\Delta_i) s_i^*, \quad i = 1,2 \tag{3.3}$$

$$I^N = I^* \tag{3.4}$$

$$d_i^N = D_i((1+\Delta_1)^{-1} p_1^*, (1+\Delta_2)^{-1} p_2^*, I^*), \quad i = 1,2 \tag{3.5}$$

Proof. $p_i^N f_i^{N'}(L_i^*) = (1+\Delta_i)^{-1} p_i^* (1+\Delta_i) f_i'(L_i^*) = p_i^* f_i'(L_i^*)$, so that, (3.2) follows from (2.2) and (2.7). The remaining assertions are trivial. ◻

Thus if, after the technological change, the prices (3.1) prevail, then there is no shift of labor between the two sectors. However, at these prices there is a disequilibrium in the goods markets. The excess demand for good i is

$$x_i^N = d_i^N - s_i^N = d_i^N - (1+\Delta_i)s_i^* = d_i^N - (1+\Delta_i)d_i^*,$$

or

$$x_i^N = D_i((1+\Delta_1)^{-1}p_1^*,\ (1+\Delta_2)^{-1}p_2^*, I^*) - (1+\Delta_i)D_i(p_1^*,p_2^*,I^*)$$

$$= D_i((1+\Delta_2)p_1^*,\ (1+\Delta_1)p_2^*,\ (1+\Delta_1)(1+\Delta_2)I^*) - (1+\Delta_i)D_i(p_1^*,p_2^*,I^*)$$

$$= -\Delta_i D_i(p_1^*,p_2^*,I^*) + \frac{\partial D_i}{\partial p_1}\Delta_2 p_1^* + \frac{\partial D_i}{\partial p_2}\Delta_1 p_2^* + \frac{\partial D_i}{\partial I}(\Delta_1+\Delta_2)I^* + o(\Delta_1,\Delta_2)$$

$$= \Delta_i\left[-d_i^* + \frac{\partial D_i}{\partial p_j}p_j^* + \frac{\partial D_i}{\partial I}I^*\right] + \Delta_j\left[\frac{\partial D_i}{\partial p_i}p_i^* + \frac{\partial D_i}{\partial I}I^*\right] + o(\Delta_1,\Delta_2) \tag{3.6}$$

$$= \bar{x}_i + o(\Delta_1,\Delta_2), \text{ say}$$

where $o(\Delta_1,\Delta_2)$ is a function such that $(|\Delta_1|+|\Delta_2|)^{-1}\, o(\Delta_1,\Delta_2) \to 0$ as $|\Delta_1|+|\Delta_2| \to 0$. The "first-order" term $\bar{x}_i$ in (3.7) can be expressed in terms of the various elasticities as

$$\bar{x}_i = \Delta_i d_i^*\left[-1 + \frac{\partial D_i}{\partial p_j}\frac{p_j^*}{d_i^*} + \frac{\partial D_i}{\partial I}\frac{I^*}{d_i^*}\right] + \Delta_j d_i^*\left[\frac{\partial D_i}{\partial p_i}\frac{p_i^*}{d_i^*} + \frac{\partial D_i}{\partial I}\frac{I^*}{d_i^*}\right]$$

$$= \Delta_i d_i^*[-1+e_{ij}+e_{iI}] + \Delta_j d_i^*[e_{ii}+e_{iI}]$$

$$= -d_i^*[\Delta_i(1+e_{ii}) + \Delta_j e_{ij}], \tag{3.8}$$

using the identity (2.1). From (3.8) we obtain the following result.

<u>L3.2</u> $x_i^N \gtrless 0$ if $\Delta_i(1+e_{ii}) + \Delta_j e_{ij} \lessgtr 0$

Proof. Let $x_i^N(\varepsilon)$ be the excess demand when the productivity change in the two sectors is $\varepsilon\Delta_1$, $\varepsilon\Delta_2$. Then $x_i^N = x_i^N(\varepsilon)$ evaluated at $\varepsilon = 1$. By (3.8)

$$\frac{d}{d\varepsilon}\ \frac{x_i^N(\varepsilon)}{d_i} = -\Delta_i(1+e_{ii}) - \Delta_j e_{ij}$$

and the result follows upon integration. ¤

4. Labor Migration

L4.1 After technological change labor migrates to sector i if

$$\varepsilon_i = \Delta_i(1+e_{ii}) + \Delta_j e_{ij} < 0 \tag{4.1}$$

Proof. If the condition holds then $x_i^N > 0$ by L3.2 and then $\frac{p_i^{N*}}{p_j^{N*}} > \frac{p_i^N}{p_j^N}$ by L2.1. This implies that $\frac{f_j^{N'}(L_j^{N*})}{f_i^{N'}(L_i^{N*})} > \frac{f_j^{N'}(L_j^*)}{f_i^{N'}(L_i^*)}$. Since $L_1^{N*} + L_2^{N*} = L_1^* + L_2^* = \bar{L}$, this implies $L_i^{N*} > L_i^*$. ¤

Simon's result on equal productivity changes now follows.

T4.1 (Simon) If $\Delta_1 = \Delta_2 > 0$, then labor migrates towards the more income elastic sector.

Proof. From $\Delta_1 = \Delta_2 > 0$ and L4.1, labor migrates towards sector i if $0 > 1 + e_{ii} + e_{ij} = 1 - e_{iI}$, by (2.1). But when there are only two consumption goods, $e_{iI} > 1$ if and only if $e_{iI} > e_{jI}$. ¤

In Baumol's analysis of "unbalanced growth" there is a "progressive" sector ($\Delta_i > 0$) and a "non-progressive" sector ($\Delta_j = 0$).

T4.2 In the case of unbalanced growth, labor migrates towards the progressive (non-progressive) sector if the demand for its output is elastic (inelastic) to its own price i.e. $e_{ii} < -1$ ($e_{ii} > -1$).

Proof. Immediate from L4.1 upon setting $\Delta_j = 0$, $\Delta_i > 0$. ¤

We end this section with one further application. As will be recalled, Simon in his paper used one asymmetrical assumption, namely that the income elasticity was lower for agricultural commodities than for industrial commodities. If we replace that assumption by a particular price elasticity assumption for each sector, it turns out that a much stronger conclusion about urban migration can be drawn. Letting F (for "food") and N (for "non-food") designate the two sectors, suppose we set

$$e_{FF} > -1 \quad \text{and} \quad e_{NN} < -1, \tag{4.2}$$

meaning that the demand for food is inelastic with respect to its price, and that the demand for non-food is elastic with respect to its price. Then, as noted earlier,

$$e_{NF} < 0 \quad \text{and} \quad e_{FN} > 0. \tag{4.3}$$

<u>T4.3</u> If $\Delta_N > 0$, $\Delta_F \geq 0$ and (4.2) holds then labor migrates towards the non-food sector.

Proof. From (4.1), (4.2) and (4.3) it follows that $\varepsilon_N < 0$. ◻

T4.3 can be strengthened somewhat further. Suppose there is regress ($\Delta_F < 0$) in the food producing sector. Then

$$\varepsilon_N = \Delta_N(1+e_{NN}) + \Delta_F e_{NF} < 0$$

provided that

$$\Delta_F > \frac{1+e_{NN}}{-e_{NF}} \Delta_N \tag{4.4}$$

Thus, the smaller (in magnitude) the regress is, in comparison with the technical progress in the other sector, the more likely is (4.4) to hold and the migration towards the non-food sector to continue.

5. Shift in Prices

Let $s_i = S_i(\frac{p_1}{p_2})$ be the initial supply function i.e., the solution of the relations

$$s_i = f_i(L_i), \quad p_1 f_1'(L_1) = p_2 f_2'(L_2), \quad L_1 + L_2 = \bar{L} \tag{5.1}$$

and similarly let $s_i = S_i^N(\frac{p_1}{p_2})$ be the supply function after technological change i.e. the solution of

$$s_i = f_i^N(L_i), \quad p_1 f_1^{N'}(L_1) = p_2 f_2^{N'}(L_2), \quad L_1 + L_2 = \bar{L}. \tag{5.2}$$

From (5.1), (5.2) it is straight-forward to deduce

$$S_i^N(\frac{p_1}{p_2}) = (1+\Delta_i) S_i\left(\frac{1+\Delta_1}{1+\Delta_2} \; \frac{p_1}{p_2}\right) \tag{5.3}$$

We shall calculate the excess demand after the technological change if the prices are maintained at the initial equilibrium. In sector i this excess demand is

$$x_i = D_i(\overset{*}{p}_1, \overset{*}{p}_2, I) - S_i^N(\frac{\overset{*}{p}_1}{\overset{*}{p}_2}) \tag{5.4}$$

and where

$$I = \overset{*}{p}_1 S_1^N(\frac{\overset{*}{p}_1}{\overset{*}{p}_2}) + \overset{*}{p}_2 S_2^N(\frac{\overset{*}{p}_1}{\overset{*}{p}_2}) \tag{5.5}$$

Substituting from (5.3), (5.5) into (5.4), carrying out a first-order expansion and using (2.3), gives

$$x_i = s_i^* - S_i^* + \frac{\partial D_i}{\partial I}\,[p_1^*(S_1 - s_1^*) + p_2^*(S_2 - s_2^*)] + o(\Delta_1,\Delta_2) \tag{5.6}$$

$$= \bar{\bar{x}}_i + o(\Delta_1,\Delta_2), \text{ say} \tag{5.7}$$

where $s_i^* = s_i(\frac{p_1^*}{p_2^*}) = f_i(L_i^*)$ as before and $S_i^* = S_i^N(\frac{p_1^*}{p_2^*})$

Let $\delta s_i = S_i^* - s_i^*$, $\delta I = p_1^*\delta s_1 + p_2^*\delta s_2 = I - \overset{*}{I}$; then

$$p_1^*\bar{\bar{x}}_1 + p_2^*\bar{\bar{x}}_2 = -p_1^*\delta s_2 - p_2^*\delta s_2 + (p_1^*\frac{\partial D_1}{\partial I} + p_2^*\frac{\partial D_2}{\partial I})\delta I$$
$$= -\delta I + (p_1^*\frac{\partial D_1}{\partial I} + p_2^*\frac{\partial D_2}{\partial I})\delta I$$

But since $p_1^*D_1 + p_2^*D_2 = I$, therefore $p_1^*\frac{\partial D_1}{\partial I} + p_2^*\frac{\partial D_2}{\partial I} = 1$. Hence the first order excess demands satisfy Walras' law,

$$p_1^*\bar{\bar{x}}_1 + p_2^*\bar{\bar{x}}_2 = 0 \tag{5.8}$$

and therefore we get the counterpart of L4.1:

<u>L5.1</u> After technological change, the relative price of good i increases,

$\frac{p_i^{N*}}{p_j^{N*}} > \frac{p_i^*}{p_j^*}$ if

$$\bar{\bar{x}}_i = -\delta\Delta_i + \frac{\partial D_i}{\partial I}\,\delta I > 0. \tag{5.9}$$

The counterpart of T4.1 is given next.

<u>T5.1</u> If $\Delta_1 = \Delta_2 = \Delta > 0$ then the relative price of the more income elastic good increases.

Proof. $S_i^* = (1+\Delta)s_i^*$ and so $\delta s_i = \Delta s_i^*$, $\delta I = \Delta I^*$. Also, recall that $s_i^* = d_i^* = D_i(p_1^*,p_2^*,I^*)$ and so from (5.9)

$$\frac{\bar{\bar{x}}_i}{\bar{D}_i} = \Delta[-1 + \frac{\partial D_i}{\partial I}\frac{I}{D_i}] = \Delta[-1 + e_{iI}] > 0$$

if $e_{iI} > 1$ or $e_{iI} > e_{jI}$

¤

In the remainder of this section it will be assumed that $\Delta_i \geq 0$, $i = 1,2,$ and at least one $\Delta_i > 0$. It follows that at least one $\delta s_i > 0$. It is also assumed that neither good is inferior.

<u>L5.2</u> After technological change, if the supply of good i (at earlier prices) descreases i.e., $\delta s_i \leq 0$, then its relative price increases.

Proof. Since $\Delta_1 \geq 0$, $\Delta_2 \geq 0$ and $\Delta_1 + \Delta_2 > 0$ it follows that $I > I^*$.

From (6.4)

$$x_i = D_i(p_1^*, p_2^*, I) - S_i^*$$

$$> D_i(p_1^*, p_2^*, I^*) - S_i^*, \text{ since good i is non-inferior}$$

$$= s_i^* - S_i^* \geq 0, \text{ by hypothesis,}$$

and the result follows.

¤

Suppose there is unbalanced growth with sector i non-progressive ($\Delta_i = 0$) and sector j progressive ($\Delta_j > 0$). It is then evident that $\delta s_i \leq 0$ and so the relative price of the good of the non-progressive sector must rise. From T4.2 we know that labor can migrate towards either sector depending upon demand conditions. Thus price changes and labor migration can occur in opposite directions.

<u>L5.3</u> Suppose that after technological change $\delta s_1 > 0$ and $\delta s_2 > 0$. Then the relative price of good i rises if

$$\frac{\delta s_i}{s_i}\,\frac{s_j}{\delta s_j} < \frac{e_{iI}}{e_{jI}}\,. \qquad (5.10)$$

Proof. From (5.8) $\bar{\bar{x}}_i > 0$ if $\bar{\bar{x}}_j < 0$. Hence the relative price of good i rises if

$$\frac{\bar{\bar{x}}_i}{\delta s_i} > \frac{\bar{\bar{x}}_j}{\delta s_j}$$

From (5.9) this is equivalent to

$$-1 + \frac{\partial D_i}{\partial I}\left(p_1^* \frac{\delta s_1}{\delta s_i} + p_2^* \frac{\delta s_2}{\delta s_i}\right) > -1 + \frac{\partial D_j}{\partial I}\left(p_1^* \frac{\delta s_1}{\delta s_j} + p_2^* \frac{\delta s_2}{\delta s_j}\right),$$

or

$$\frac{\partial D_i}{\partial I}\, I\left(p_i^* + p_j^* \frac{\delta s_j}{\delta s_i}\right) > \frac{\partial D_j}{\partial I}\, I\left(p_j^* + p_i^* \frac{\delta s_i}{\delta s_j}\right),$$

which is readily seen to be equivalent to (5.10).

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Denote the price elasticity of the initial supply function by

$$E_i = \frac{dS_i\left(\frac{p_1}{p_2}\right)}{d\left(\frac{p_1}{p_2}\right)}\;\frac{\left(\frac{p_1}{p_2}\right)}{S_i\left(\frac{p_1}{p_2}\right)}$$

Then it is easy to show that the first term in (5.10) is

$$\frac{\delta s_i}{s_i}\,\frac{s_j}{\delta s_j} = \frac{\Delta_i + (\Delta_1 - \Delta_2)E_i}{\Delta_j + (\Delta_1 - \Delta_2)E_j} + o(\Delta_1,\Delta_2)$$

By substituting this into (5.10) we see that if $\delta s_1 > 0$, $\delta s_2 > 0$ then the relative price of good i rises if

$$\frac{e_{iI}}{e_{jI}} > \frac{\Delta_i + (\Delta_1 - \Delta_2)E_i}{\Delta_j + (\Delta_1 - \Delta_2)E_j}\,,$$

a condition which involves both demand and supply elasticities as might be expected.

7. Concluding Remarks

In this paper, we have attempted to build upon some of Herbert Simon's early work, published in Econometrica, 1947. While remaining within his two-sector framework, we have used a different approach based on an analysis of excess demands. This has enabled us to provide a more exhaustive portrayal of the linkages between, on the one hand, measures of productivity change and elasticity and, on the other hand, shifts in the division of labor, with emphasis on rural-urban migration.

Simon's paper as well as the more recent paper by Baumol associate to an older tradition according to which all economic activity is viewed as being classifiable into "primary", "secondary", and "tertiary" sectors. Although none of the three, and especially not the tertiary sector, is homogeneously composed, certain broad generalizations are often used in describing them, [2], [4], [5]. For example, the income elasticity of demand is generally considered to be higher for the products of the tertiary sector than for the secondary sector, and higher for the products of the secondary sector than for those of the primary sector. Long-run average costs are supposed to be constant to increasing for primary industries and constant to decreasing for secondary and tertiary. Part of the tertiary sector has collective-good characteristics (exemplified by agencies and firms engaged in information processing and transmission); on the other hand, technological change is often considered to be slow or non-existent (compare Baumol) for the tertiary sector, relative to the rates of technological change in primary and secondary industries.

As a final example, the tertiary sector is regarded as labor intensive, while the primary and secondary sectors are taken to be land and/or capital intensive.

Characterizations such as these contain various suggestions for extensions of the present paper. On the input side, considerations of capital mobility [3] and of land use suggest themselves. Issues of income distribution also arise. In the economic development literature, a bimodal distributions is commonly used -- agricultural wages and "modern" urban sector wages. (For a particularly interesting example, see [7].) But there are possibilities to introduce property income into such a framework.

On the output side, extensions to more than two sectors suggest themselves. The principal difficulty in such extension lies in the rapid increase in the number of parameters needed to specify the demand structure. For instance, an extension of the framework from two to three sectors means that the number of such parameters increases from six to twelve. However, if a transportation sector were added to Simon's "agricultural" and "industrial" sectors, the analytical difficulty could be eased by the introduction of quite plausible constraints on the parameters concerned. These various possibilities will be explored in further work.

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