COHERENCY IDENTIFICATION FOR POWER SYSTEM DYNAMIC EQUIVALENTS

by

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Memorandum No. UCB/ERL M77/57

26 August 1977

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ABSTRACT

Coherency-based approach to the problem of constructing power system dynamic equivalents has been very successful in applications. The key step in this approach is to identify groups of coherent generators. An analytic study of coherency is conducted. An algebraic characterization of coherency is given. An algorithm, based on the algebraic characterization, to identify coherent groups directly from system data is developed. A physical interpretation of the algebraic characterization of coherency is presented, where the condition for coherency is described in terms of generator inertia constants and their equivalent admittances to the bus of the disturbance.

Research sponsored by the National Science Foundation Grant ENG75-21747.

I. INTRODUCTION

In the analysis of an interconnected power system, normally one is interested only in the responses in a portion of the system, called the study system. The rest of the system is called the external system. Owing to the dimension of the interconnected systems, it is impossible or uneconomical to represent the entire system model in detail. A reduced-order approximate model of the external system is used for stability studies, this is called a dynamic equivalent in power literature. A dynamic equivalent is employed to approximate the effect of the external-system dynamics on the study-system dynamics.

Besides some earlier heuristic methods, there are currently two approaches to the problem of constructing dynamic equivalents. One is the modal appraoch and the other is the coherency-based approach. Modal approach uses a linearized model of the external system and reduces the order of the model by ignoring the contribution to the responses due to slowly-varying modes, as well as the uncontrollable and unobservable modes [1,2]. The modal approach has recently been studied and an efficient algorithm has been developed [3]. This paper addresses itself to the coherency-based approach.

It has been observed that certain generators tend to have the same waveshape for their swing curves (i.e., the response of the rotor angle as a function of time) after a disturbance. These generators are said to be coherent and they are referred to as a coherent group of generators. The concept of coherency has been utilized by Chang and Adibi [4] for dynamic equivalents. This approach has been developed further by Podmore [5,6]. The coherency-based approach to dynamic equivalents involves two stages:

(i) Groups of coherent generators are identified.

(ii) The terminal buses (nodes) for each group of coherent generators are replaced by a single equivalent bus and the models for each coherent group of generating units are combined into one equivalent model.

Coherency is an observed phenomenon. The formation of coherent groups depends on the location and nature of the disturbance. Some previous attempts at the problem of identifying coherency have been heuristically-based. Lee and Schweppe [7] suggested the use of concepts from pattern recognition to identify coherency. Because of the lack of accuracy and consistency in the heuristic methods, the current approach [5] to coherency identification involves numerically solving the (simplified and linearized) system equations and then processing the swing curves by a clustering algorithm to determine the coherent groups.

The objectives of the work reported herein are to understand the phenomenon of coherency and to develop a method for coherency identification. In Section III of this paper, after describing the mathematical model in Section II, an algebraic characterization of coherency is given. It is also shown that the condition for coherency may be expressed in terms of a system matrix which is independent of the disturbance and a vector which describes the disturbance. Consequently a simple characterization for generators to be coherent for a set of disturbances is obtained. An algorithm, based on the algebraic characterization, to identify coherent groups directly from system data is presented in Section IV. The algorithm involves only elementary operations on the matrices of the system equations. A physical interpretation of the algebraic characterization of coherency is given in Section V. It translates the condition for generators to be coherent in terms of their inertia constants and equivalent admittances to the bus of the disturbance.

II. MATHEMATICAL MODEL

We are considering the response of a power system after a disturbance and concerned with the identification of coherency. A simplified and linearized model is used. It has the following characteristics:

(i) The classical model is used to represent the synchronous generators.

(ii) A linearized system model is used.

(iii) The decoupling between real-power-phase-angle and reactive-power-voltage-magnitude is assumed.

The model that we are using is the same as the one used by Podmore [5]. Through a number of computer simulation tests, he has found that this model is adequate for coherency identification.

Detailed derivation of the generator and network models can be found in standard textbooks (for example: 8, Chapter 4; 9, Chapter 5).

1. Generator and network

When the classical model is used for the synchronous generators, the machine dynamics are represented by the so-called swing equation. The linearized swing equation for a generator is given as follows:

$$\Delta\dot{\omega}_i = M_i^{-1}(\Delta PM_i - \Delta PG_i - D_i\Delta\omega_i) \quad (1)$$

$$\Delta\dot{\delta}_i = 2\pi f_0 \Delta\omega_i \quad (2)$$

where

i: subscript for generator i

Δ: deviation

M: inertia constant

ω: speed

δ: rotor angle

D: damping constant

f_0: synchronous frequency

PM: mechanical input power

PG: electrical output power

We assume that during the period of interest $\Delta PM = 0$.

The linearized decoupled load flow equations are used to represent the network. An underlying assumption here is that the effect of changes in voltage magnitude on the real power flows is negligible. This is true for transmission system with high X/R ratios [10,11]. The equations are written below.

$$\begin{bmatrix} \Delta PG \\ \Delta PL \end{bmatrix} = \begin{bmatrix} H_{gg} & H_{g\ell} \\ H_{\ell g} & H_{\ell\ell} \end{bmatrix} \begin{bmatrix} \Delta\delta \\ \Delta\theta \end{bmatrix} \tag{3}$$

where

PG: vector of real power injections at generator internal buses

PL: vector of real power injections at load buses

δ: vector of phase angles at generator internal buses = generator rotor angles

θ: vector of phase angles at load buses

H: matrix of partial derivatives

2. Disturbances

As the result of a fault, a protective measure is usually taken, which may be either load shedding, generator dropping, or line switching. We shall consider the modeling of these changes.

(a) Load shedding

Load shedding at the i^{th} load bus can be modeled as:

$$\Delta PL = (0,...1,0...)^T u(t) \tag{4}$$

for some u(t), where 1 occurs at the i^{th} position. As we shall see later, we do not require the exact waveform of u(t) for coherency identification.

(b) Generator dropping

Let us suppose that the i^{th} generator is to be dropped and that it is connected to the k^{th} load bus (Fig. 1a). The effect of generator dropping can be modeled by leaving the i^{th} generator there and introducing a load $\Delta PG_i(t)$ at the k^{th} load bus to compensate the power generation $\Delta PG_i(t)$ from the i^{th} generator (Fig. 1b). Thus we can model generator dropping the same as change in load.

(c) Line switching

The removal of a line connecting bus i and bus j can be modeled as load changes. Let P_i and P_j be the real power from the line into bus i and bus j respectively before the line removal (Fig. 2a). We assume that $P_i = -P_j$, i.e., the difference in power at two ends of the line due to losses is negligible. First we modify the network by removing the line, hence $H_{\ell\ell}$ is changed into $H'_{\ell\ell}$,

$$H'_{\ell\ell} = H_{\ell\ell} + \mu dd^T \tag{5}$$

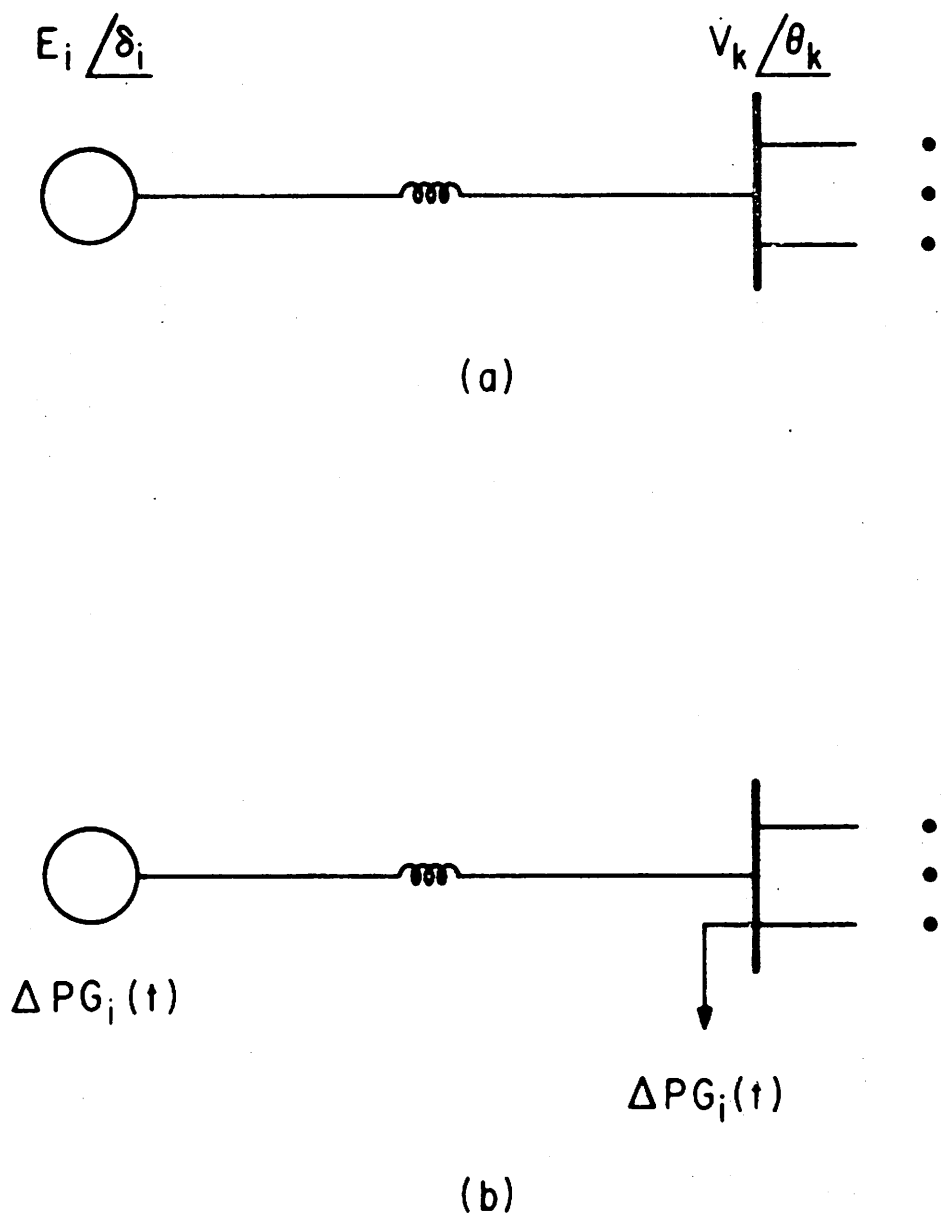

Fig. 1. Modeling of generator dropping as change in power injection at the terminal load bus.

(a) Connection of the generator.

(b) The effect of dropping the generator i can be achieved by simply adding a load at bus k with magnitude the same as the output power of the generator.

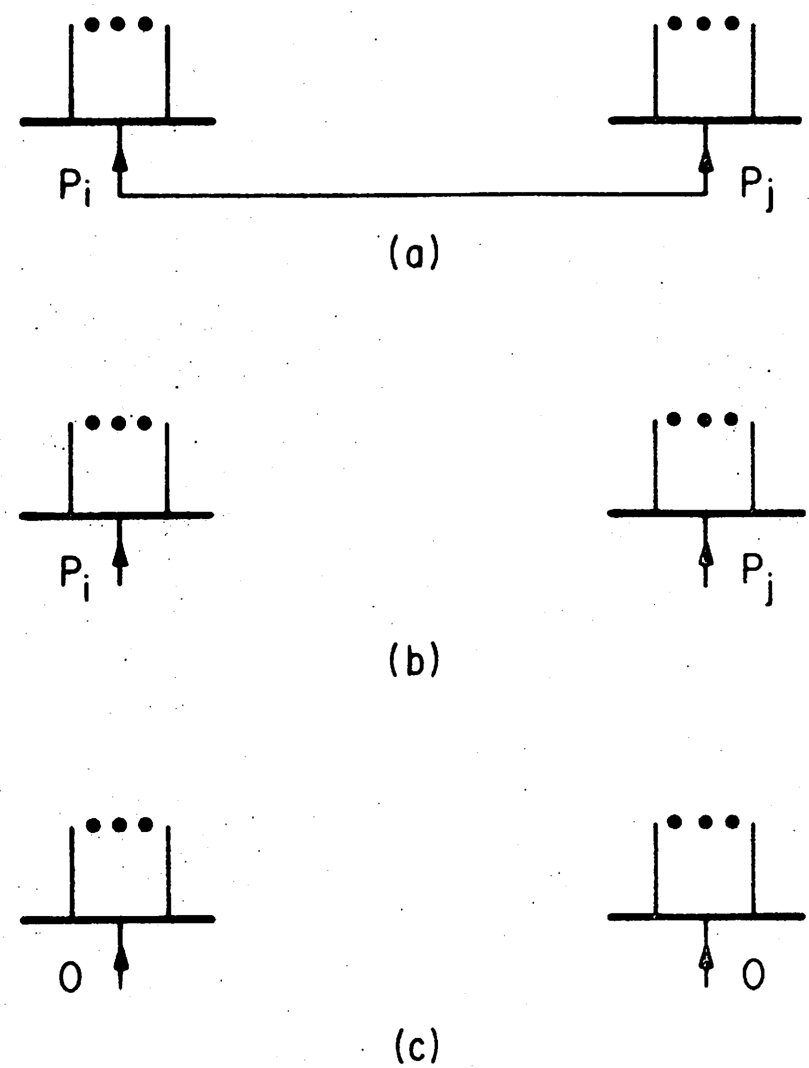

Fig. 2. Modeling of line switching as changes in power injections at the load buses with network modification.

(a) The connection and the power flows in the line before switching.

(b) With power injections P_i and P_j at the terminal buses of the line, we may remove the line and have an equivalent system as in (a).

(c) The removal of the line can now be represented as change in the power injections at buses i and j from P_i and P_j to zero.

for some μ, where $d^T = (0,\ldots+1,\ldots-1,\ldots0)$ with +1 at the i^{th} position and -1 at the j^{th} position. The power flow in the line before its removal can be represented as power injections at buses i and j (Fig. 2b). Now the effect of line removal can be modeled as load changes in buses i and j in this modified network (Fig. 2c), i.e.,

$$PL = d \cdot P_i \tag{6}$$

3. System model

A general model incorporating any of these disturbances may be used to represent the power system after the disturbance. Combining eqs. (1-3) and eq. (4) or eqs. (5-6), we obtain

$$\begin{bmatrix} \dot{\Delta\omega} \\ \dot{\Delta\delta} \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -M^{-1}D & 0 & -M^{-1} & 0 \\ (2\pi f_0)I & 0 & 0 & 0 \\ 0 & H_{gg} & -I & H_{g\ell} \\ 0 & H_{\ell g} & 0 & H'_{\ell\ell} \end{bmatrix} \begin{bmatrix} \Delta\omega \\ \Delta\delta \\ \Delta PG \\ \Delta\theta \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ 0 \\ d \end{bmatrix} u(t) \tag{7}$$

$$H'_{\ell\ell} = H_{\ell\ell} + \mu dd^T \tag{8}$$

where

$D = \text{diag}(D_i)$

$M = \text{diag}(M_i)$

$\Delta\omega$ = vector of $(\Delta\omega_i)$

$\Delta\delta$ = vector of $(\Delta\delta_i)$

For line switching, $\mu \neq 0$ and $d^T = (0,\ldots+1,\ldots-1,\ldots0)$. For load shedding or generator dropping, $\mu = 0$ and $d^T = (0,\ldots1,\ldots0)$.

We further assume that in the case of line switching, it does not split the network into two separable parts. This implies that $H_{\ell\ell}$ and $H'_{\ell\ell}$ are nonsingular.

We shall write eqs. (7) and (8) in a more compact form

$$\begin{bmatrix} \Delta\dot{x} \\ 0 \end{bmatrix} = \begin{bmatrix} A_1 & -A_2 \\ A_3 & A'_4 \end{bmatrix} \begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix} + \begin{bmatrix} 0 \\ e \end{bmatrix} u(t) \tag{9}$$

$$A'_4 = A_4 + \mu e e^T \tag{10}$$

where

$$\Delta x = \begin{bmatrix} \Delta\omega \\ \Delta\delta \end{bmatrix} \qquad \Delta y = \begin{bmatrix} \Delta PG \\ \Delta\theta \end{bmatrix}$$

and A_1, A_2, A_3, A_4, A'_4 and e are the corresponding submatrices from eq. (7). Eliminating Δy in eq. (9) we obtain

$$\Delta\dot{x} = A'\Delta x + b'u(t) \tag{11}$$

where

$$A' \triangleq A_1 + A_2(A'_4)^{-1}A_3 \tag{12}$$

$$b' \triangleq A_2(A'_4)^{-1}e \tag{13}$$

For later use, we shall also define

$$A \triangleq A_1 + A_2(A_4)^{-1}A_3 \tag{14}$$

$$b \triangleq A_2(A_4)^{-1}e \tag{15}$$

III. CHARACTERIZATION OF COHERENCY

Two generators 'i' and 'j' are said to be coherent for a disturbance occurring at time t_0 if $\delta_i(t) - \delta_j(t) = c$, for some constant c, for all $t \geq t_0$. This is equivalent to the condition that $\Delta\delta_i(t) = \Delta\delta_j(t)$ for all $t \geq t_0$, where $\Delta\delta_i(t) = \delta_i(t) - \delta_i(t_0)$ and $\Delta\delta_j(t) = \delta_j(t) - \delta_j(t_0)$. In other words, two generators are coherent for a disturbance if the response curves of the rotor angles have identical waveshape. We will follow the convention of taking $t_0 = 0$.

A group of generators is said to be coherent for a disturbance if the generators are pairwise coherent for that disturbance.

A group of generators is said to be coherent for a set of disturbances if the group is coherent for each of the disturbances in the set.

Let us define a vector $\alpha(i,j)$ in conjunction with the model developed in the previous section.

$$\alpha(i,j)^T \Delta x \triangleq \Delta\delta_i - \Delta\delta_j \tag{16}$$

Clearly, generators 'i' and 'j' are coherent iff $\alpha(i,j)^T \Delta x = 0$.

1. Coherency for a single disturbance

Theorem 1. (Characterization of coherency for a single disturbance)

Consider a system represented by

$$\dot{x} = Ax + bu \tag{17}$$

$$x(0) = 0$$

where

$A \in \mathbb{R}^{nxn}$, $b \in \mathbb{R}^{nx1}$, $u(t) \in \mathbb{R}$, $x(t) \in \mathbb{R}^n$.

Let $\alpha \in \mathbb{R}^n$.

(i) If for some $u(t) \neq 0$, $\alpha^T x(t) = 0$ for all $t \geq 0$ then $\alpha^T A^{k-1} b = 0$ for $k = 1,2,\ldots n$.

(ii) If $\alpha^T A^{k-1} b = 0$ for $k = 1,2,\ldots n$, then $\alpha^T x(t) = 0$ for all $t \geq 0$ for all $u(\cdot)$.

The proof of the theorem is included in Appendix A. For ease of notation we write $\alpha \perp Q[A,b]$ iff $\alpha^T A^{k-1} b = 0$ for $k = 1,2,\ldots.$

<u>Corollary</u> For the system (17) if for some $u \neq 0$, $\alpha^T x(t) = 0$ for all $t \geq 0$ then $\alpha^T x(t) = 0$ for all $t \geq 0$, for all $u(\cdot)$.

The above corollary implies that it is not necessary to know $u(t)$ in order to identify coherency. Theorem 1 provides a necessary and sufficient condition for two generators to be coherent. Using the model (11) it is clear that generators 'i' and 'j' are coherent if and only if

$$\alpha(i,j) \perp Q[A',b'] \tag{18}$$

2. Coherency for a set of disturbances

Normally one is interested not only in coherent behavior of generators for a particular disturbance but also for a set of possible disturbances. In principle, one can always analyze this by considering each disturbance individually. Since the matrices A' and b' change with the location and the nature of the disturbance, the direct application of Theorem 1 to characterize coherency of generators for a set of disturbances will involve a set of independent tests. However it turns out that these tests are actually related. Lemma 2 below shows that the necessary and sufficient condition for coherency (18) can be expressed in terms of the unmodified constant matrix A of eq. (14). This important observation leads us to a simple characterization of coherency for a set of disturbances as stated in Theorem 2.

<u>Lemma 2</u> Under the assumptions made in obtaining the model given by eqs. (11-15), the following are equivalent

(i) generators 'i' and 'j' are coherent.

(ii) $\alpha(i,j) \perp Q[A',b']$

(iii) $\alpha(i,j) \perp Q[A,b']$

(iv) $\alpha(i,j) \perp Q[A,b]$

The proof of Lemma 2 is presented in Appendix C. A direct application of Lemma 2 results in Theorem 2, which provides a simple algebraic characterization for generators to be coherent for a set of disturbances.

<u>Theorem 2</u>. (<u>Characterization of coherency for a set of disturbances</u>)

Let $b_1, b_2, \ldots b_p$ be the 'b' vectors (as defined in eq. (15)) for the p disturbances in the set. Define

$$B \triangleq [b_1, b_2, \ldots b_p] \tag{19}$$

Generators 'i' and 'j' are coherent for <u>each</u> of the disturbances in the set if and only if

$$\alpha(i,j)^T A^{k-1} B = 0 \quad \text{for } k = 1, \ldots n \tag{20}$$

We will write condition (20) simply as $\alpha(i,j) \perp Q[A,B]$.

IV. AN ALGORITHM FOR COHERENCY IDENTIFICATION

The test for coherency (20) can be greatly simplified by the use of a transformation. We shall first state the required transformation and the condition for coherency after this transformation in the following Fact. We then present an efficient algorithm for testing coherency of generators for a set of disturbances.

<u>Fact</u> Consider the system representation

$$\dot{x} = Ax + Bu \tag{21}$$

where $A \in \mathbb{R}^{nxn}$, $B \in \mathbb{R}^{nxp}$, and $u(t) \in \mathbb{R}^{p}$.

Let $q \triangleq \text{rank}[B, AB, \ldots A^{n-1}B]$. (22)

There exists a nonsingular transformation T such that

$$T^{-1}AT = \begin{bmatrix} \bar{A}_{11} & \bar{A}_{12} \\ 0 & \bar{A}_{22} \end{bmatrix} \tag{23}$$

$$T^{-1}B = \begin{bmatrix} \bar{B}_{1} \\ 0 \end{bmatrix} \tag{24}$$

where $\bar{A}_{11} \in \mathbb{R}^{qxq}$ and $\bar{B}_1 \in \mathbb{R}^{qxp}$ and

$$\text{rank}[\bar{B}_1, \bar{A}_{11}\bar{B}_1, \ldots \bar{A}_{11}^{q-1}\bar{B}_1] = q \tag{25}$$

Let $\alpha \in \mathbb{R}^n$ and

$$\alpha^T T = (\bar{\alpha}_1^T \quad \bar{\alpha}_2^T) \tag{26}$$

where $\bar{\alpha}_1 \in \mathbb{R}^q$.

Then $\alpha^T A^{k-1} B = 0$ for $k = 1,2,\ldots n$ if and only if

$$\bar{\alpha}_1 = 0 \tag{27}$$

The proof of the Fact is included in Appendix D for completeness. Now let us apply the Fact to the test for coherency (20). Because of the simple structure of $\alpha(i,j)$ it is extremely easy to check if $\alpha(i,j)^T T$ is of the form $(0\ \bar{\alpha}_2)$. Indeed, let T be partitioned as

$$T = [T_1 \,\vdots\, T_2] \tag{28}$$

where $T_1 \in \mathbb{R}^{nxp}$, then $\alpha(i,j)^T T$ is of the form $(0\ \bar{\alpha}_2)$, i.e., generators 'i' and 'j' are coherent, if and only if the rows corresponding to δ_i and δ_j of T_1 are identical. We may go one step further and say that the rows corresponding to $\delta_{i1}, \delta_{i2}, \ldots \delta_{ik}$ of T_1 are identical if and only if the group of generators i1,i2,...ik are coherent for the set of disturbances under consideration.

The transformation T referred to in the foregoing Fact is not unique. There are several algorithms for obtaining such a transformation. We shall incorporate the one by Rosenbrock-Mayne [12] in our coherency identification algorithm. Rosenbrock-Mayne algorithm utilizes only elementary transformations and is perhaps the most efficient one.

We now summarize our algorithm for coherency identification below.

<u>Algorithm for coherency identification</u>

A. <u>Setting up the equations</u>

1. Construct the matrices A_1, A_2, A_3 and A_4 as in eqs. (7-10) from pre-disturbance system data.
2. For each disturbance in the set construct the 'e' vector (the 'd' vector) as in eq. (9) (in eqs. (4)(6)(7)). Let $e_1, e_2, \ldots e_p$ be the set of 'e' vectors of the p disturbances and define the nxp matrix E as

$$E \triangleq [e_1 \,\vdots\, e_2 \,\vdots\, \ldots\ e_p] \tag{29}$$

3. Obtain

$$A = A_1 + A_2(A_4)^{-1}A_3 \tag{30}$$

$$B = A_2(A_4)^{-1}E. \tag{31}$$

This is carried out by Gaussian elimination as follows:

Set

$$M = \begin{bmatrix} A_4 & A_3 & E \\ -A_2 & A_1 & 0 \end{bmatrix} \tag{32}$$

initially and let A_4 be rxr, A_1 be nxn.

Step 1.1. Set $j = 1$.

Step 1.2. Find $M_{i'j}$, $i' \in \{j,\ldots r\}$ such that $|M_{i'j}| > |M_{ij}|$ for all $i \in \{j,\ldots r\}$.

Step 1.3. If $M_{i'j} = 0$, the process can not continue, stop; else continue.

Step 1.4. Interchange rows i' and j of M.

Step 1.5. For $i = j+1,\ldots r+n$ do the following to make M_{ij} zero:

Subtract $(\frac{M_{ij}}{M_{jj}})$ x (row j) from row i of M.

Step 1.6. If $j = r$, the process is complete, stop; else $j = j+1$ go to Step 1.2.

When the process terminates at Step 1.6, the matrices occupying the middle and right lower blocks are the desired A and B (Fig. 3).

B. <u>Rosenbrock-Mayne algorithm for determining the controllable part</u>

Now we apply the Rosenbrock-Mayne algorithm to obtain the transformation T defined in the Fact. Initially, we set

$N = [B \quad A]$

$T = I$

$$\begin{bmatrix} A_4 & A_3 & E \\ -A_2 & A_1 & O \end{bmatrix} \xrightarrow[\text{row operations}]{\text{elementary}} \begin{bmatrix} U & X & X \\ O & A & B \end{bmatrix}$$

Fig. 3. The system matrices A and B can be easily obtained by Gaussian elimination.

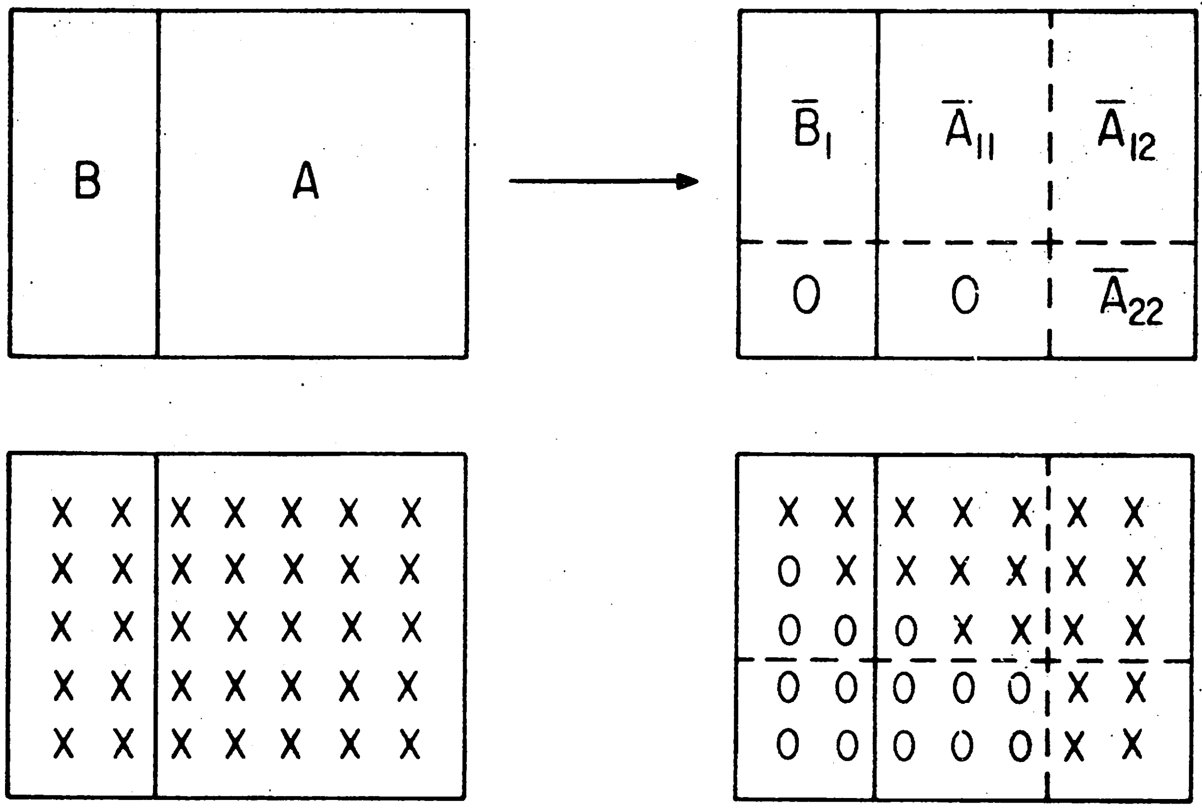

Fig. 4. Rosenbrock-Mayne algorithm provides the desired transformation.

where N is nx(p+n) and T is nxn.

In the following algorithm we shall refer to A as the current content of the last n columns of N.

Step 2.1. Set $i = 1$, $j = 1$.

Step 2.2. Find $N_{k'j}$, $k' \in \{i, i+1, \ldots n\}$ such that $|N_{k'j}| \geq |N_{kj}|$ for all $k \in \{i, i+1, \ldots n\}$.

Step 2.3. If $N_{k'j} = 0$ go to Step 2.7, else continue.

Step 2.4. Interchange rows i and k' of N and columns i and k' of A and T.

Step 2.5. If $i = n$ there is no coherent group, stop; else continue.

Step 2.6. For $k = i+1, \ldots n$ do the following to make N_{kj} zero:

Subtract $(\frac{N_{kj}}{N_{ij}})$ x (row i) from row k of N.

Add $(\frac{N_{kj}}{N_{ij}})$ x (column k) to column i of A and T.

Set $i = i + 1$.

Step 2.7. Set $j = j + 1$, if $j \geq i + p$ the process is complete, set $q = i$ and stop; else go to Step 2.2.

When the above algorithm terminates at Step 2.7, the matrix N can be partitioned as in Fig. 4 and the matrix T is the desired transformation.

C. Identification of coherent groups of generators

Let the first q columns of T be denoted by T_1. We check the rows $(\frac{n}{2})+1, (\frac{n}{2})+2, \ldots n$. (These are the rows that correspond to $\Delta\delta$). Whenever a set of rows $(\frac{n}{2})+i1, (\frac{n}{2})+i2, \ldots (\frac{n}{2})+ik$ of T_1 are identical, the generators $i1, i2, \ldots ik$ are coherent for the set of p disturbances under consideration.

V. A PHYSICAL INTERPRETATION OF THE RESULT

Let us assume that the damping constants of the generators are negligible, i.e., $D = 0$. Suppose the disturbance is modeled as a change in power injection at bus d, ΔP_d. Let subscript e denote the other load buses. We may express eq. (3) as follows

$$\begin{bmatrix} \Delta P_G \\ \Delta P_d \\ 0 \end{bmatrix} = \begin{bmatrix} H_{gg} & H_{gd} & H_{ge} \\ H_{dg} & H_{dd} & H_{de} \\ H_{eg} & H_{ed} & H_{ee} \end{bmatrix} \begin{bmatrix} \Delta\delta \\ \Delta\theta_d \\ \Delta\theta_e \end{bmatrix} \tag{33}$$

Note that the sum of the elements in each row of the matrix in eq. (33) is zero [9, p. 175].

Eliminating $\Delta\theta_e$ in eq. (33) we obtain

$$\begin{bmatrix} \Delta P_G \\ \Delta P_d \end{bmatrix} = \begin{bmatrix} H'_{gg} & H'_{gd} \\ H'_{dg} & H'_{dd} \end{bmatrix} \begin{bmatrix} \Delta\delta \\ \Delta\theta_d \end{bmatrix} \tag{34}$$

The elements of H'_{gg}, etc., may be viewed as the "equivalent admittances" of the network[1] after eliminating all the load buses e, except the one with disturbance. It can be easily verified that the sum of the elements in each row of the matrix in eq. (34) is zero. (See Appendix E.) This can be conveniently expressed by defining a vector $\mathbb{1}$ consisting of 1's.

$$\mathbb{1} \triangleq (1,1,\ldots 1)^T \tag{35}$$

Then we have

$$H'_{gg}\mathbb{1} + H'_{gd} = 0, \quad H'_{dg}\mathbb{1} + H'_{dd} = 0 \tag{36}$$

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[1] Indeed, the elements of H can be approximated by the elements of $-B$, where $Y = G + jB$ is the network admittance matrix [10].

Note that this implies that one of the equations in (34) must be redundant, therefore in what follows we shall use only

$$\Delta P_G = H'_{gg}\Delta\delta + H'_{gd}\Delta\theta_d \tag{37}$$

Using eqs. (37) and (1-2) with D = 0, we obtain the following equation, which corresponds to eq. (11).

$$\begin{bmatrix} \Delta\dot{\omega} \\ \Delta\dot{\delta} \end{bmatrix} = \begin{bmatrix} 0 & M^{-1}H'_{gg} \\ (2\pi f_0)I & 0 \end{bmatrix} \begin{bmatrix} \Delta\omega \\ \Delta\delta \end{bmatrix} + \begin{bmatrix} M^{-1}H_{gd} \\ 0 \end{bmatrix} \Delta\theta_d \tag{38}$$

Now let us apply the coherency test (27) to (38). We shall first pick a transformation T. Let us make one more simplifying assumption that the eigenvalues of the matrix in eq. (38) (which will be referred to as A in what follows) are distinct and lie in the left half plane. Then the matrix consisting of the eigenvectors $x^1, x^2, \ldots x^n$ of A forms a desired transformation T,

$$T = [x^1 \mid x^2 \mid \ldots \; x^n] \tag{39}$$

Let

$$x^1 = \begin{bmatrix} x_1^1 \\ x_2^1 \end{bmatrix} \tag{40}$$

Using the transformation T, we may express the necessary and sufficient condition for all the generators to be coherent as[2]

$$x_2^1 = \mathbb{1} \tag{41}$$

[2]The condition (41) is clearly necessary for all the generators to be coherent. It follows from (42) that there can not be two distinct eigenvectors $\begin{bmatrix} x_1^1 \\ \mathbb{1} \end{bmatrix}$ and $\begin{bmatrix} x_1^2 \\ \mathbb{1} \end{bmatrix}$, hence the condition (41) is also sufficient.

It can be easily shown (see Appendix F) that $\begin{bmatrix} x_1^1 \\ \mathbb{1} \end{bmatrix}$ is an eigenvector if and only if

$$M^{-1}H'_{gd} = \left(\frac{\lambda^2}{2\pi f_0}\right)\mathbb{1} \tag{42}$$

where λ is the corresponding eigenvalue of A.

Let $H'_{gd} = (H'_{1d}, H'_{2d}, \ldots)^T$. We can express condition (42) as follows:

Suppose a disturbance is modeled as a change in power injection at bus d. Let H' be obtained from H, by eliminating all load buses except bus d. Under the stated assumptions, all the generators will be coherent if and only if

$$\frac{H'_{id}}{M_i} = \frac{H'_{jd}}{M_j} \quad \text{for all the generators } i,j \tag{43}$$

As a corollary we have the following:

Suppose a disturbance is modeled as a change in power injection at bus d. Let H' be obtained from H, by eliminating all load buses except bus d. Consider a group of generators i1,i2,...ik. Suppose furthermore that

$$H'_{ij} = 0 \quad \text{for all } i \in \{i1,i2,\ldots ik\} \text{ and } j \notin \{i1,i2,\ldots ik\} \tag{44}$$

Under the state assumptions, if the generators i1,i2,...ik are coherent then

$$\frac{H'_{id}}{M_i} = \frac{H'_{jd}}{M_j} \quad \text{for all } i,j \in \{i1,i2,\ldots ik\} \tag{45}$$

The results above are only for very restrictive cases. We are not suggesting using (45) as a practical criterion for testing coherency. The proposed algorithm in Sec. IV should be used for that purpose. However the foregoing physical interpretation provides some insights to the behavior of coherency.

H'_{id} is a measure of the "electrical distance" from the generator to the disturbance. The result above shows that the ratio $(\frac{H'_{id}}{M_i})$, a combination of electrical distance and inertia constant, is perhaps a quantity more related to the factors that determine coherency. It suggests that a generator electrically far from the disturbance and having small inertia may be coherent with a generator electrically closer to the disturbance but having larger inertia. The result also indicates that identifying coherency based solely on electrical distance is inadequate.

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APPENDIX

A. Proof of Theorem 1

(i) The fact that $\alpha^T x(\cdot) = 0$ (i.e., $\alpha^T x(t)=0$ for all $t \geq 0$) for some $u(\cdot) \neq 0$ implies that $\alpha^T \dot{x}(\cdot) = 0$, or

$$\alpha^T Ax(\cdot) + \alpha^T bu(\cdot) = 0 \tag{A1}$$

We claim that $\alpha^T b = 0$. Suppose not, i.e., $\alpha^T b \neq 0$, then from (A1) we have

$$u(t) = - \frac{\alpha^T Ax(t)}{\alpha^T b} \qquad \text{for all } t \geq 0 \tag{A2}$$

Hence

$$\dot{x} = [A- \frac{1}{\alpha^T b} b\alpha^T A]x \tag{A3}$$

$$x(0) = 0$$

This implies that $x(\cdot) = 0$; from (A2) we have $u(\cdot) = 0$, which is a contradiction. Thus

$$\alpha^T b = 0 \tag{A4}$$

From (A4) and (A1) we obtain

$$\alpha^T Ax = 0 \tag{A5}$$

This in turn implies that $\alpha^T Ab = 0$ and $\alpha^T A^2 x = 0$, etc.

(ii) Define

$$\xi_k = \alpha^T A^{k-1} x \qquad k = 1,2,\ldots n+1 \tag{A6}$$

Since $\alpha^T A^{k-1} b = 0$, we have

$$\dot{\xi}_k = \xi_{k+1} \qquad k = 1,2,\ldots n \tag{A7}$$

From Caley-Hamilton theorem, we have

$$\xi_{n+1} = \sum_{i=1}^{n} \beta_i \xi_i \tag{A8}$$

Let $\xi = (\xi_1, \xi_2, \ldots \xi_n)^T$. It follows from (A7) and (A8)

$$\dot{\xi} = \begin{bmatrix} 0 & 1 & 0 & 0 & \ldots & 0 \\ 0 & 0 & 1 & 0 & \ldots & 0 \\ \cdot & & & & & \\ \beta_1 & \beta_2 & \ldots & & \ldots & \beta_n \end{bmatrix} \xi \tag{A9}$$

$$\xi(0) = 0$$

Hence $\xi(t) = 0$ for all $t \geq 0$, in particular $\xi_1(t) = \alpha^T x(t) = 0$ for all $t \geq 0$. ◻

B. Lemma 1 Let $A \in \mathbb{R}^{nxn}$, $b \in \mathbb{R}^n$, $\gamma \in \mathbb{R}^n$, and

$$A' = A + b\gamma^T \tag{A10}$$

Then $\alpha \perp Q[A',b]$ if and only if $\alpha \perp Q[A,b]$.

Proof ($\Leftarrow$) We shall prove by induction that $\alpha^T A^{k-1} = \alpha^T (A')^{k-1}$, $k = 1,2,\ldots n$. Clearly it is true for $k = 1$. Consider

$$\alpha^T (A')^k = \alpha^T (A')^{k-1} (A + b\gamma^T)$$

$$= \alpha^T A^{k-1} (A + b\gamma^T) \qquad \text{(by induction hypothesis)}$$

$$= \alpha^T A^k + \alpha^T A^{k-1} b\gamma^T$$

$$= \alpha^T A^k \qquad \text{(by assumption)}$$

($\Rightarrow$) Same.

C. Proof of Lemma 2 Lemma 1 will be used in the proof.

(i) $\Leftrightarrow$ (ii) from Theorem 1.

Now consider

$$A - A' = A_2(A_4'^{-1} - A_4^{-1})A_3$$

$$= A_2 A_4'^{-1}(I - A_4' A_4^{-1})A_3$$

$$= A_2 A_4'^{-1}(I - (A_4 + \mu ee^T)A_4^{-1})A_3$$

$$= A_2 A_4'^{-1} e(-\mu e^T A_4^{-1} A_3)$$

$$= b'\gamma^T \tag{A11}$$

where $\gamma^T = -\mu e^T A_4^{-1} A_3$.

It follows from (A11) and Lemma 1 that (ii) $\Leftrightarrow$ (iii).

Next consider

$$b' = A_2 A_4'^{-1} e$$

$$= A_2 (A_4 + \mu ee^T)^{-1} e$$

$$= A_2 A_4^{-1}(I + ee^T A_4^{-1} \mu)^{-1} e$$

$$= A_2 A_4^{-1} e(1 + e^T A_4^{-1} e\mu)^{-1} \tag{A12}$$

The last equality follows from that fact that $(I+PQ)^{-1}P = P(I+QP)^{-1}$ [13, p. 54]. Hence

$$b' = \eta b \tag{A13}$$

where $\eta = (1+e^T A_4^{-1} e\mu)^{-1}$ is a scaler.

It then follows that (iii) $\Leftrightarrow$ (iv). ¤

D. Proof of the Fact The first part of the Fact is a standard theorem in linear system theory [14, p. 172].

Clearly

$$\alpha^T A^{k-1} B = 0, \qquad k = 1,2,\ldots n \tag{A14}$$

or

$$\alpha^T T T^{-1} A^{k-1} T T^{-1} B = 0, \qquad k = 1,2,\ldots n \tag{A15}$$

iff

$$\bar{\alpha}_1^T \bar{A}_{11}^{k-1} \bar{B}_1 = 0, \qquad k = 1,2,\ldots q \tag{A16}$$

Equation (A16) holds if and only if

$$\bar{\alpha}_1 = 0 \tag{A17}$$

because of eq. (25). ¤

E. Verification of (36)

From eqs. (33)(34), we have

$$H'_{gg} = H_{gg} - H_{ge} H_{ee}^{-1} H_{eg} \tag{A18}$$

$$H'_{gd} = H_{gd} - H_{ge} H_{ee}^{-1} H_{ed} \tag{A19}$$

Since $H_{eg}\mathbb{1} + H_{ed} + H_{ee}\mathbb{1} = 0$ and $H_{gg}\mathbb{1} + H_{gd} + H_{ee}\mathbb{1} = 0$ (A20)

it follows from eqs. (A18-A20),

$$H'_{gg}\mathbb{1} + H'_{gd} = 0 \tag{A21}$$

□

F. Verification of (42)

Let λ be the corresponding eigenvector of A associated with x'. With (40) and (41), we have

$$\begin{bmatrix} 0 & M^{-1}H'_{gg} \\ (2\pi f_0)I & 0 \end{bmatrix} \begin{bmatrix} x_1^1 \\ \mathbb{1} \end{bmatrix} = \lambda \begin{bmatrix} x_1^1 \\ \mathbb{1} \end{bmatrix} \tag{A22}$$

Hence

$$M^{-1}H'_{gg}\mathbb{1} = \lambda x_1^1 \tag{A23}$$

$$(2\pi f_0)x_1^1 = \lambda\mathbb{1} \tag{A24}$$

Equation (42) follows from eqs. (A23)(A24) and (36). □