AN ALGORITHM FOR SINGLE-ROW ROUTING WITH PRESCRIBED STREET CONGESTIONS

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AN ALGORITHM FOR SINGLE-ROW ROUTING WITH PRESCRIBED STREET CONGESTIONS*

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ABSTRACT

The single-row routing approach for layout has attracted a great deal of interest and is in a position to become one of the fundamental routing methods for high density multilayer printed wiring boards (PWB's). A specific development has recently been accomplished on this approach [12], namely: Necessary and sufficient conditions for optimum routing have been obtained. Nonetheless, there still remains a fundamental problem to be overcome, that is, to develop an algorithm to find the optimum solution.

The present paper derives an alternate set of necessary and sufficient conditions for the same problem. These are easy to check and are tailored for algorithm development. An efficient algorithm in the special cases of upper and lower street congestions up to two has been proposed. These special cases are particularly of interest in the design of practical PWB's.

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1. Introduction

In the assembly of digital systems, multilayer printed wiring boards (PWB's) are used very often to provide the necessary interconnection among circuit modules. Recent advances in the technology of micro-electronics have changed the design rule for these PWB's. The number of etch paths between two consecutive pins of an ordinary dual in line package (DIP) is allowed to be three or more. In such a case a set of sophisticated routing schemes to realize 100 percent wiring would be one of the most essential factors to reduce the cost and time incurred in laying out wire patterns. Thus, development is continuing on further sophisticated routing approaches for PWB's.

As is pointed out by [1], the deficiency common to conventional routers, such as maze-running routers [2-4] and line-search routers [5,6], is that they lack in "topological fluidity," that is, the capability to defer detailed wire patterns until connections have been considered. Especially for line-search routers, which are used to be executed track by track, this deficiency is fatal when several etch paths are allowed between two consecutive pins in a DIP. Thus, in the case of high density wiring, of practical use are other routing approaches, which exhibit more topological fluidity, such as the channel router [7] and single-row routers [8-10].

The single-row routing approach, first introduced by So [8], has the promise to become one of the fundamental routing methods and is attracting a great deal of interest in terms of its applications not only to large-scale backboard wiring [8,9], but also to circuit card wiring [1,10,11].

A specific development has been accomplished on this approach recently. Necessary and sufficient conditions for optimum single-row routing have been obtained [12]. However, there remains the key problem of deriving an efficient algorithm for finding a realization with a prescribed set of upper and lower street congestions (or track numbers). If this can be obtained, optimum single-row routing can be employed in conjunction with the line-search router [5] in such a way that the search for possible line segments is implemented channel by channel, with all connections within each channel completed later by single-row routing. In this case, of primary importance is the problem as to what is necessary and sufficient for realizability of single-row routing with some specific numbers imposed on upper and lower street congestions.

The present paper first considers necessary and sufficient conditions for the single-row routing problem, and then describes an efficient routing algorithm for the special cases of upper and lower street congestions up to two. These have enormous potentialities to be extensively used in the practice of PWB wiring.

The conditions obtained are alternate to that of in [12], but they are particularly suited for algorithm development as illustrated in the cases where the number of street congestions is small. For purpose of clarity, necessary terminology and concepts are given in Sections 2 and 3. Necessary and sufficient conditions for realizability are given in Section 4. Theorems and algorithms for the special cases with street congestions 2 or less are derived in Sections 5 and 6.

2. Preliminaries

Consider a set of r nodes evenly spaced on the real line R as shown in Fig. 1, where each node corresponds to a pin (drilled-through hole to reach all layers) or a via (plated-through hole to be used for interconnection between layers). A set of nodes on R to be interconnected is referred to as a <u>net</u>, and a set of nets is designated as a <u>net list</u>.

Given a net list $L = \{N_1, N_2, \ldots, N_n\}$ on R, the interconnection for each net N_i is to be realized by means of a set of paths, which consist of horizontal and vertical line segments according to specification. For example, given a net list

$$
\begin{aligned}
&L = \{N_1, N_2, N_3, N_4\}; \\
&N_1 = \{v_1, v_4, v_7\},\ N_2 = \{v_2, v_9\},\ N_3 = \{v_3, v_5\},\ N_4 = \{v_6, v_8\},
\end{aligned}
\tag{1}
$$

the interconnection for each net is realized as shown in Fig. 2. This way of realization for a given net list on R is called <u>single-row</u> (<u>single-layer</u>) routing [9], where upward and downward zigzagging is allowed, but not forward and backward zigzagging. In this realization, the space above the real line R is designated as the <u>upper street</u>, and the space below R as the <u>lower street</u>. Given a realization, the number of horizontal tracks necessary in the upper (lower) street is called the <u>upper</u> (<u>lower</u>) <u>street congestion</u>. For example, in the realization of Fig. 2, the upper and lower street congestions are one and two, respectively.

Using these terms, the problem to be considered in this paper is formulated as follows: Given a net list L defined for r nodes on real line R, and integers K_u and K_w, find a realization with the upper and lower street congestions equal to or less than K_u and K_w, respectively.

Henceforth, without loss of generality, this problem is discussed under the following assumptions;

I. every net of a given net list contains at least two nodes,

II. every node belongs to a net, and

III. any net does not contain a pair of consecutive nodes v_i and v_{i+1}.

With respect to III, some explanation is necessary: If any net N_k contains a pair of consecutive nodes v_i and v_{i+1}, then delete node v_{i+1} from N_k and seek a realization. After that we can complete the realization by connecting v_i and v_{i+1} by means of a straight line segment on R, without changing the upper and lower street congestions.

3. Interval Graphical Representation

Associated with an ordered sequence $s = (N_{i_1}, N_{i_2}, \ldots, N_{i_n})$ of a given net list $L = \{N_1, N_2, \ldots, N_n\}$, the interval graphical representation is defined [12]. For example, given a net list L of Eq.(1), consider a sequence $s = (N_2, N_1, N_3, N_4)$, then the interval graphical representation associated with s is depicted as in Fig. 3, where each horizontal line segment represents the interval covered by a net, and they are arranged according to the order in s. Nodes which pertain to a net are marked as shown. Obviously, there are n! ordered sequences for a net list of n nets, and hence there are a total of n! interval graphical representations. In an interval graphical representation, let us define the reference line [12] as the continuous line segments which connect the nodes in different nets in succession from left to right. For example, the reference line for the interval graphical representation of Fig. 3 is shown by the broken lines in Fig. 4(a). Now, let us stretch out the reference line and map it into the real line R. Associated with this topological mapping, let each interval line be transformed into a path composed of horizontal and vertical line segments, as shown in Fig. 4(b), where the portions above and below the reference line are mapped into paths in the upper and lower streets, respectively. As readily seen, this topological mapping yields a realization of a given net list. Thus, for each interval graphical representation, there corresponds a unique realization.

Given an interval graphical representation, let us draw a vertical line at each node v_i, and let us define the cut number $c(v_i)$ as the number of interval lines cut by the vertical line at v_i, ignoring the one to which v_i belongs [9,12]. Let us also define the upper and lower cut numbers for a given ordered sequence s, $c_u^s(v_i)$ and $c_w^s(v_i)$ as the numbers of interval lines cut by the vertical line above and below v_i, respectively [12]. Obviously, $c(v_i) = c_u^s(v_i) + c_w^s(v_i)$ for all v_i. Let

$$c_u^s \triangleq \max_i [c_u^s(v_i)] \text{ and } c_w^s \triangleq \max_i [c_w^s(v_i)], \tag{2}$$

be referred to as the maximum upper and lower cut numbers, respectively [12]. Then, c_u^s and c_w^s represent the upper and lower street congestions, respectively, in the realization corresponding to this interval graphical representation.

Thus, given a net list L and integers K_u and K_w, there exists a realization with the upper and lower street congestions not greater than K_u and K_w, respectively, if and only if there exists a sequence s such that $c_u^s \le K_u$ and $c_w^s \le K_w$. Hence we can formulate the following decision problem on single-row routing.

Problem $P(L,K_u,K_w)$:

INPUT: Net list L and integers K_u and K_w.

PROPERTY: There exists a sequence s such that $c_u^s \le K_u$ and $c_w^s \le K_w$.

In what follows, we consider necessary and sufficient conditions that Problem $P(L,K_u,K_w)$ be true.

4. Necessary and Sufficient Conditions

Let $\{v_1,v_2,\ldots,v_r\}$ be a set of r nodes evenly spaced on R, as shown in Fig. 1, and denote by $I = [v_i,v_j]$ $(i \leq j)$ a closed interval on R between v_i and v_j. Given a net list L defined for these nodes on R, let an interval $I = [v_i,v_j]$ such that $c(v_k) \geq h$ for all v_k on I and $c(v_{i-1}) = c(v_{j+1}) = h-1$, be referred to as an h-interval. For any interval $I = [v_i,v_j]$ on R, let $\overline{L}(I)$ denote a set of nets which have no node on I, but have two nodes v_a and v_b such that $a < i$ and $j < b$; and let $L(I)$ represent the union of $\overline{L}(I)$ and a set of nets having nodes on I. The definition and properties of an h-interval I together with the sets of nets $L(I)$ and $\overline{L}(I)$ are illustrated in Fig. 5(a)

Given a sequence s of nets of the form $s = (\ldots,N_i,\ldots,N_j,\ldots)$, let N_i (or N_j) be said to precede (or follow) N_j (or N_i) in s. With the use of these notations, we can obtain the following necessary conditions for Problem $P(L,K_u,K_w)$ to be true.

LEMMA 1: If Problem $P(L,K_u,K_w)$ is true, then

(i) for any h-interval I with $h > \min\,[K_u,K_w]$, $|\overline{L}(I)| \geq 2h - (K_u+K_w)$, and

(ii) for any sequence s such that $P(L,K_u,K_w)$ is true, there are at least $h - K_w$ nets in $\overline{L}(I)$ which precede the other nets of $L(I)$ in s, and at least $h - K_u$ nets in $\overline{L}(I)$ which follow the other nets of $L(I)$ in s.

Proof: Let s be any sequence for which $P(L,K_u,K_w)$ is true, and consider an h-interval I. Let $N_u \in L(I) - \overline{L}(I)$ be the one which precedes all the other nets of $L(I) - \bar{L}(I)$ in s, and let $N_w \in L(I) - \bar{L}(I)$ be the one which follows all the other nets of $L(I) - \bar{L}(I)$ in s. Assume that there are x nets in L(I) which precede N_u in s and y nets in L(I) which follow N_w in s. Then, as readily seen, these x + y nets belong to $\bar{L}(I)$. Since N_u and $N_w \in L(I) - \bar{L}(I)$, N_u has a node on I, say v_u, and N_w has a node on I, say v_w (see Fig. 5(b)). Then, by definition we have

$$c(v_u) \geq h \text{ and } c(v_w) \geq h, \tag{3}$$

$$\begin{aligned} K_w &\geq c_w^s \geq c_w^s(v_u) = c(v_u) - x, \\ K_u &\geq c_u^s \geq c_u^s(v_w) = c(v_w) - y. \end{aligned} \tag{4}$$

Thus, we have

$$\begin{aligned} K_w &\geq h - x, \text{ or } x \geq h - K_w, \\ K_u &\geq h - y, \text{ or } y \geq h - K_u, \end{aligned} \tag{5}$$

and

$$x + y \geq 2h - (K_u + K_w). \tag{6}$$

Hence the lemma. ¤

It should be noted that this lemma is a generalization of the necessary condition stated in [9] which was considered under the assumption that the upper street congestion is equal to the lower street congestion.

As in [9], let us define the cut <u>number</u> $q(N_i)$ <u>of net</u> N_i as the maximum of the cut numbers of nodes belonging to N_i, i.e.,

$$q(N_i) \triangleq \max_{v_j \in N_i} [c(v_j)] . \tag{7}$$

Let

$$q_M \triangleq \max_k [q(N_i)]. \tag{8}$$

Then, we have the following lemma without proof.

LEMMA 2 [12]: Assume that $P(L,K_u,K_w)$ is true, then there is at least one net N_i such that $q(N_i) \leq K_u$, and at least one net N_j such that $q(N_j) \leq K_w$, and for all $N_k \in L$, $q(N_k) \leq K_u + K_w - 1$.

Thus, the necessary conditions so far stated for $P(L,K_u,K_w)$ to be true are summarized as follows.

A1: $q_M \leq K_u + K_w - 1,$ (9)

A2: For any h-interval I ($q_M \geq h > \min[K_u,K_w]$),

$|\bar{L}(I)| \geq 2h - (K_u+K_w),$ (10)

A3: There exists a sequence s for a given net list L such that for any h-interval I ($q_M \geq h > \min [K_u,K_w]$), there are at least $h - K_w$ nets in $\bar{L}(I)$ which precede all the other nets of L(I) in s, and at least $h - K_u$ nets in $\bar{L}(I)$ which follow all the other nets of L(I) in s.

Note here that the condition stated in Lemma 2 that there is at least one net N_i (N_j) such that $q(N_i) \leq K_u$ ($q(N_j) \leq K_w$) can be readily derived from A3, and hence it is not explicitly described above.

Given a net list L satisfying conditions A1 and A2, let us assume that there is a sequence s which satisfies condition A3, and consider the interval graphical representation associated with s. Since any node v_j for which $c(v_j) = h > \min [K_u,K_w]$, is on an h-interval I, for such v_j we have

$$c_u^s(v_j) \geq h - K_w \quad \text{and} \quad c_w^s(v_j) \geq h - K_u, \tag{11}$$

and hence

$$c_u^s(v_j) = h - c_w^s(v_j) \leq K_u,$$
$$c_w^s(v_j) = h - c_u^s(v_j) \leq K_w. \tag{12}$$

On the other hand, for any node v_j for which $c(v_j) \leq \min [K_u, K_w]$, it can be readily seen that

$$c_u^s(v_j) \leq K_u \quad \text{and} \quad c_w^s(v_j) \leq K_w. \tag{13}$$

Hence, in the realization corresponding to the interval graphical representation with respect to s, the upper and lower street congestions are equal to or less than K_u and K_w, respectively; and therefore $P(L,K_u,K_w)$ is true.

Thus, these conditions A1, A2, and A3 are also sufficient conditions for $P(L,K_u,K_w)$ to be true; and hence we have the following theorem.

THEOREM 1: Problem $P(L,K_u K_w)$ is true if and only if A1, A2, and A3 are satisfied.

Hence the main problem here is how to construct an efficient algorithm to generate a sequence s so as to satisfy A3 for a given net list L satisfying A1 and A2.

In the following we restrict ourselves to the special cases of (i) $K_u = 2$ and $K_w = 1$ and (ii) $K_u = K_w = 2$, which are of practical use in high density wiring for PWB's.

5. Special Case of $K_u = 2$ and $K_w = 1$

Given a net list L such that $q_M \leq 1$, then P(L,2,1) is obviously true, and therefore we consider only the case of $q_M = 2$. We first prove the following lemma.

LEMMA 3: Given a net list L such that $q_M = 2$, $|\overline{L}(I)| \leq 1$ for any

2-interval I.

Proof: Since $q_M = 2$, $|\bar{L}(I)| \le 2$ for any 2-interval I. Assume that there is a 2-interval $I' = [v_p, v_q]$ for which $|L(\bar{I}')| = 2$. Since $c(v_{p-1}) = 1$, the net N_i to which v_p belongs has a node v_k with $k > p + 1$. Thus, N_i and at least one of $\bar{L}(I')$ pass over node v_{p+1}, and hence $c(v_{p+1}) \ge 2$, which implies that v_{p+1} is on I'. However, any net in $\bar{L}(I')$ has no node on I', and hence $c(v_{p+1}) = 3$, which contradicts to the assumption of $q_M = 2$. Hence the lemma. ¤

Thus, for this special case of $K_u = 2$ and $K_w = 1$, the necessary and sufficient conditions A1, A2, and A3 can be rewritten as follows.

B1: $q_M \le 2$.

B2: For any 2-interval I, $|\bar{L}(I)| = 1$. (Equality sign due to Lemma 3 and Eq. (10)).

B3: There exists a sequence s such that for any 2-interval I, the single net of $\bar{L}(I)$ precedes the other nets of L(I) in s.

Among these three conditions, B3 has much room to be simplified. In the following we consider this simplification.

LEMMA 4: Given a net list L such that $q_M = 2$, let $I^1 = [v_a, v_b]$ and $I^2 = [v_c, v_d]$ (b<c) be two consecutive 2-interval, such that there is no other 2-interval on $[v_{b+1}, v_{c-1}]$, then we have

1° $|L(I^1) \cap L(I^2)| \le 2$, and

2° if $|L(I^1) \cap L(I^2)| = 2$, then for the interval $I^{12} \triangleq [v_{b+1}, v_{c-1}]$,

$$\bar{L}(I^1) \cup \bar{L}(I^2) \subset L(I^{12}) = L(I^1) \cap L(I^2). \tag{14}$$

__Proof__: Suppose that $|L(I^1) \cap L(I^2)| \geq 3$, then for a node v_j on I^{12}, $c(v_j) \geq 2$, contrary to the assumption. Thus, 1° holds. Now, consider the case of $|L(I^1) \cap L(I^2)| = 2$. We can easily see that $L(I^1) \cap L(I^2) \subset L(I^{12})$. Suppose that $L(I^1) \cap L(I^2)$ is a proper subset of $L(I^{12})$, then there is a node v_j on I^{12} which belongs to a net not in $L(I^1) \cap L(I^2)$, which implies that $c(v_j) \geq 2$, contrary to the assumption. Hence, $L(I^{12}) = L(I^1) \cap L(I^2)$. Furthermore, we have $\overline{L}(I^1) \subset L(I^{12})$ and $\overline{L}(I^2) \subset L(I^{12})$. Thus, 2° holds. ¤

__LEMMA 5__: Suppose that $q_M = 2$ and for any 2-interval I, $|\overline{L}(I)| = 1$. If there are two consecutive 2-intervals I^1 and I^2 such that $|L(I^1) \cap L(I^2)| = 2$ and $\overline{L}(I^1) \neq \overline{L}(I^2)$, then Problem P(L,2,1) is false.

__Proof__: Let $\{N_a, N_b\} \triangleq L(I^1) \cap L(I^2)$, then we may assume from 2° of Lemma 4 that $\overline{L}(I^1) = \{N_a\}$ and $\overline{L}(I^2) = \{N_b\}$. Then, in order that __B3__ must be satisfied, N_a should precede N_b in s and at the same time N_b should precede N_a in s, which is impossible. Hence the lemma. ¤

Using Lemmas 4 and 5, we can prove the following theorem.

__THEOREM 2__: Problem P(L,2,1) is true if and only if

__C1__ (=__B1__): $q_M \leq 2$,

__C2__ (=__B2__): $|\overline{L}(L)| = 1$ for any 2-interval I, and

__C3__: there do not exist any two consecutive 2-intervals I^1 and I^2 such that $|L(I^1) \cap L(I^2)| = 2$ and $\overline{L}(I^1) \neq \overline{L}(I^2)$.

The necessity is evident from lemmas stated above. To prove the sufficiency we have only to show that given a net list L satisfying __C1__, __C2__, and __C3__, we can always find an ordered sequence s of nets as stated in __B3__. To this end, we can construct an algorithm to generate such a sequence s correctly, as follows.

ALGORITHM I

1°: Given a net list L, check whether or not the following conditions hold,

(i) $q_M \leq 2$,

(ii) there is at least one net N_i such that $q(N_i) \leq 1$, and

(iii) for each 2-interval I, $|\bar{L}(I)| = 1$.

If L satisfies all these conditions, then go to 2°. Otherwise halt; at this stage P(L,2,1) is proved to be false.

2°: If there is no 2-interval, then halt; at this stage P(L,2,1) is true and an arbitrary sequence s is a solution to our problem. Otherwise, let G = [V,E] be a null graph such that a set of V of vertices and a set of E of edges are both empty.

3°: If for all 2-intervals the following processes are conducted, then go to 6°. Otherwise, let I* be the left most 2-interval for which the following processes have not been conducted, and associated with it, let us construct a directed graph G* = [V*,E*] such that

(1) $V^* = L(I^*)$, that is, each $N_i \in V^*$ corresponds to $N_i \in L(I^*)$, and

(2) $E^* = \{(N_a,N_k) \mid \{N_a\} = \bar{L}(I^*),\ N_k \in L(I^*) - \bar{L}(I^*)\}$, that is, there is an edge (N_a,N_k) incident from N_a into N_k if and only if $\{N_a\} = \bar{L}(I^*)$ and $N_k \in L(I^*) - \bar{L}(I^*)$.

Then, (i) if $|V \cap V^*| \leq 1$, then go to 4°, and (ii) if $|V \cap V^*| = 2$, then go to 5°.

4°: Let $G \leftarrow G \cup G^*$, where for two graphs $G_1 = [V_1,E_1]$ and $G_2 = [V_2,E_2]$, $G_1 \cup G_2 \triangleq [V_1 \cup V_2, E_1 \cup E_2]$. Then return to 3°.

5°: Let $G \leftarrow G \cup G^*$. If G is an acyclic graph, then return to 3°. Otherwise halt; at this stage P(L,2,1) is proved to be false.

6°: Halt: At this stage P(L,2,1) is proved to be true. Let L_0 be a set of nets such that the corresponding vertices are contained in G. Then let $V_0 \triangleq L-L_0$, and add V_0 to G, where V_0 constitutes a set of isolated vertices in G. Then, as seen from the construction rule of G, G is acyclic, and hence we can construct a sequence s of the nodes in G such that for any edge $(N_i,N_j) \in E$, N_i precedes N_j in s, which forms a solution to our problem.

Noting that if at the stage of 5°, G is found to contain a cycle, then C3 and therefore B3 are proved to be violated, it can be seen that ALGORITHM I always seeks a desired sequence if C1, C2, and C3 are satisfied.

6. Special Case of $K_u = K_w = 2$

Given a net list, if $q_M \leq 2$, then P(L,2,2) is obviously true. Henceforth, we consider the case of $q_M = 3$. First, we can prove the following lemma similarly as Lemma 3.

LEMMA 6: If $q_M = 3$ for a given net list, then for every 3-interval I, $|\overline{L}(I)| \leq 2$.

Thus, for this special case of $K_u = K_w = 2$, the necessary and sufficient conditions A1, A2, and A3 can be rewritten as follows.

D1: $q_M \leq 3$

D2: $|\overline{L}(I)| = 2$ for each 3-interval I. (Equality sign due to Lemma 6 and Eq.(10)).

D3: There exists a sequence s such that for any 3-interval I, one of $\overline{L}(I)$ precedes all the other nets of L(I) in s, and another one of $\overline{L}(I)$ follows all other nets of L(I) in s.

The following lemmas can be readily verified similarly as Lemmas 4 and 5.

LEMMA 7: Given a net list such that $q_M = 3$, let $I^1 = [v_a,v_b]$ and $I^2 = [v_c,v_d]$ $(b<c)$ be two consecutive 3-intervals, then we have

1° $|L(I^1) \cap L(I^2)| \leq 3$, and

2° if $|L(I^1) \cap L(I^2)| = 3$, then for the interval $I^{12} \triangleq [v_{b+1},v_{c-1}]$

$$\bar{L}(I^1) \cup \bar{L}(I^2) \subset L(I^{12}) = L(I^1) \cap L(I^2). \tag{15}$$

LEMMA 8: Suppose that $q_M = 3$ and $|\bar{L}(I)| = 2$ for any 3-interval I. If there exist any two consecutive 3-intervals I^1 and I^2 such that $|L(I^1) \cap L(I^2)| = 3$ and $\bar{L}(I^1) \neq \bar{L}(I^2)$, then P(L,2,2) is false.

We can also prove the following theorem corresponding to Theorem 2.

THEOREM 3: Problem P(L,2,2) is true if and only if

E1 (=D1): $q_M \leq 3$,

E2 (=D2): $|\bar{L}(I)| = 2$ for each 3-interval I, and

E3: There do not exist any two consecutive 3-intervals I^1 and I^2 such that $|L(I^1) \cap L(I^2)| = 3$ and $\bar{L}(I^1) \neq \bar{L}(I^2)$.

The sufficiency of the theorem is proved by showing an algorithm to generate a sequence as stated in D3 for a given net list satisfying E1, E2, and E3.

ALGORITHM II

1°: Given a net list L, check whether or not the following conditions hold,

(i) $q_M \leq 3$,

(ii) there are at least two nets of cut number less than or equal to two, and

(iii) for every 3-interval I, $|\bar{L}(I)| = 2$.

If all these conditions are satisfied, then go to 2°. Otherwise halt; at this stage $P(L,2,2)$ is proved to be false.

<u>2°</u>: If there is no 3-interval, then halt; at this stage $P(L,2,2)$ is true and any sequence s constitutes a solution to our problem. Otherwise, let $G = [V,E]$ be a null graph such that $V = \phi$ and $E = \phi$.

<u>3°</u>: If for all 3-intervals the following processes are conducted, then go to 7°. Otherwise, choose the left-most interval I^* for which the following processes have not been conducted. Let us define a set V^* of vertices as $V^* = L(I^*)$, that is, each $N_i \in V^*$ corresponds to $N_i \in L(I^*)$. Then, (i) if $|V \cap V^*| \leq 1$, then go to 4°, (ii) $|V \cap V^*| = 2$, then go to 5°, and (iii) if $|V \cap V^*| = 3$, then go to 6°.

<u>4°</u>: Construct a graph $G^* = [V^*,E^*]$ such that for $\{N_a,N_b\} \wedge \bar{L}(I^*)$

$$E^* \triangleq \{(N_a,N_k),(N_k,N_b) \ |N_k \in L(I^*) - \bar{L}(I^*)\}, \tag{16}$$

where we note that N_a is a source and N_b a sink in G^*. Then, let $G \leftarrow G \cup G^*$, and return to 3°.

<u>5°</u>: Let $\{N_s,N_t\} \triangleq V \cap V^*$.

(i) If there is no directed path in G from N_s to N_t or from N_t to N_s, then construct G^* by Eq. (16). Put $G \leftarrow G \cup G^*$, and return to 3°.

(ii) If there is a directed path between N_s and N_t, then let us suppose without loss of generality that its direction is from N_s to N_t. If $N_s \in \bar{L}(I^*)$, then for $N_a \triangleq N_s$ and $N_b \in \bar{L}(I^*) - \{N_s\}$, construct G^* by Eq. (16). If $N_t \in \bar{L}(I^*)$, then for $N_b \triangleq N_t$ and $N_a \in \bar{L}(I^*) - \{N_t\}$, construct G^* by Eq. (16). On the other hand, if N_s, $N_t \notin \bar{L}(I^*)$, construct G^* for $\{N_a,N_b\} \wedge \bar{L}(I^*)$

by Eq.(16). Put $G \leftarrow G \cup G^*$, and return to 3°.

6°: Let $\{N_s, N_w, N_t\} \triangleq V \cap V^*$, then there must be a directed path passing these three vertices, as seen from the construction rule for G. Let us suppose without loss of generality that there is a directed path in G passing N_s, N_w, and N_t in this order. If $\{N_s, N_t\} = \bar{L}(I^*)$, then put $N_a \triangleq N_s$ and $N_b \triangleq N_t$, and construct G^* by Eq.(16). Then, let $G \leftarrow G \cup G^*$, and return to 3°. Otherwise, halt; at this stage P(L,2,2) is proved to be false.

7°: Let L_0 be a set of nets such that the corresponding vertices are contained in G. Let $V_0 \triangleq L - L_0$, and add this V_0 to G. Then, as seen from the construction rule for G, G is an acyclic graph, and hence we can always find a sequence s of the nodes in G such that for any edge $(N_i, N_j) \in E$, N_i precedes N_j in s, which constitutes a solution to our problem.

Here, it should be remarked that if $\{N_s, N_t\} \neq \bar{L}(I^*)$ in 6°, then E3 and therefore D3 turn out to be violated, and hence we can see that ALGORITHM II always finds a desired sequence if D1, D2, and D3 are satisfied.

[Example] Let us consider a net list $L = \{N_1, N_2, \ldots, N_{10}\}$ with four 3-intervals $I^1 = [v_4, v_5]$, $I^2 = [v_{10}, v_{11}]$, $I^3 = [v_{14}, v_{16}]$, and $I^4 = [v_{18}, v_{19}]$, as shown in Fig. 6(a). Apply ALGORITHM II to this L, then the acyclic graph G^* is constructed for each 3-interval and combined into the graph G. Consequently, the graph G of Fig. 6(b) is generated, from which we can obtain a desired sequence, for example, $s = (N_8, N_5, N_1, N_7, N_9, N_{10}, N_6, N_3, N_4, N_2)$. Associated with this s, L is realized as shown in Fig. 6(c). Here, it should be noted that for this L the algorithm proposed in [9] can not find any realization with $K_u = K_w = 2$.

Finally, suppose that a data structure is appropriately provided, then ALGORITHMS I and II can be easily implemented in $O(n \cdot r)$ time, where n and r are the numbers of nets and nodes, respectively. In the following, we show this time complexity for the case of ALGORITHM II. For the case of ALGORITHM I, we can show it in a similar way.

Let $G_i = [V_i, E_i]$ be the graph G* constructed at Step 4° in the ith iteration of loop 3°-6°, and let

$$G = [V,E] \triangleq \bigcup_{i=1}^{\ell} G_i,$$

where ℓ is the number of 3-intervals. Then, we can easily see that the following is true:

$$|E_i| = 2(|V_i|-2),$$

$$|V| = |\bigcup_{i=1}^{\ell} V_i| \leq n, \text{ and}$$

$$|V_i \cap (\bigcup_{j=1}^{i-1} V_j)| \leq 3.$$

Therefore, we have

$$|V_1| + |V_2| + \cdots + |V_\ell| \leq n + 3(\ell-1), \text{ and}$$

$$|E| = |E_1| + |E_2| + \cdots + |E_\ell| \leq 2n + 2(\ell-3).$$

Noting that Steps 4°, 5°, and 6° are easily implemented in $O(|V|+|E|)$ time, we see that the total time required by loop 3°-6° is $O(\ell \cdot (|V|+|E|))$, therefore $O(\ell \cdot (n+\ell))$.

On the other hand, Steps 1° and 7° are easily implemented in $O(n \cdot r)$ time and $O(n)$ time, respectively.

Thus, ALGORITHM II is implemented in $O(n \cdot r)$ time, since $\ell < n < r$.

7. Conclusion

In this paper we have first derived necessary and sufficient conditions for realizability of single-row routing with some specific numbers imposed on upper and lower street congestions. Then, based on them, we have considered the special cases of upper and lower street congestions up to two, and proposed two algorithms. These have good possiblities to be extensively used in the practice of PWB wiring, especially when the wiring density increases to such an extent that several etch paths are allowed between two consecutive pins in an ordinary DIP.

The extension of the algorithms to larger street congestions still needs to be worked out.

Finally, it should be pointed out that between-nodes congestion, i.e., the maximum number of connecting paths between two consecutive nodes on R, is often of practical interest. Although we have not dealt with this, it can be readily seen, however, that the between-nodes congestions is equal to or less than $\min[K_u, K_w]$.

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FIGURE CAPTIONS

Fig. 1 A set of r nodes on R.

Fig. 2. A realization of net list L of (1).

Fig. 3. An interval graphical representation.

Fig. 4(a). The reference line for the interval graphical representation of Fig. 3.

Fig. 4(b). A realization associated with the interval graphical representation of (a).

Fig. 5(a). An interval $I = [v_i, v_j]$ with $h = 3$, $c(v_{i-1}) = c(v_{j+1}) = 2$; $\bar{L}(I) = \{N_1, N_3\}$ and $L(I) = \{N_1, N_2, N_3, N_4, N_5, N_6\}$

Fig. 5(b). Nets N_u and N_w in interval I.

Fig. 6(a). Given net list L.

Fig. 6(b). Graph G.

Fig. 6(c). A realization.

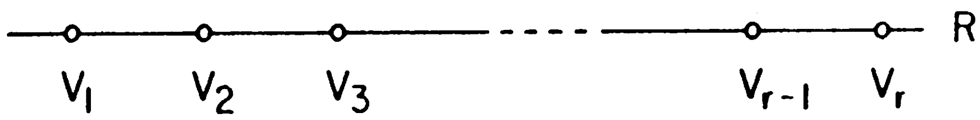


Fig. 1. A set of r nodes on R.

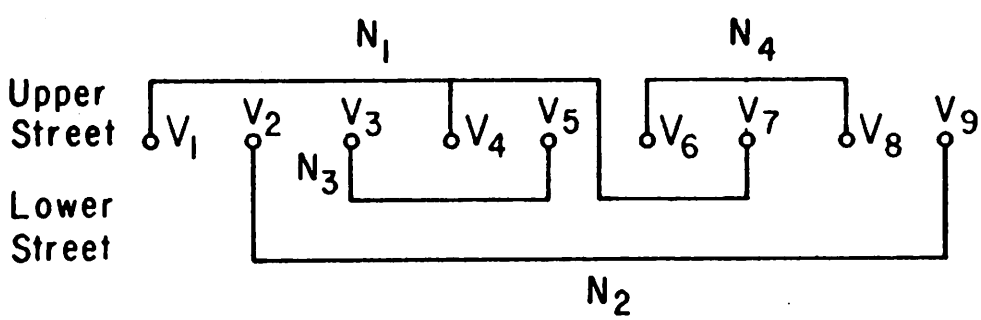


Fig. 2. A realization of net list L of (1).

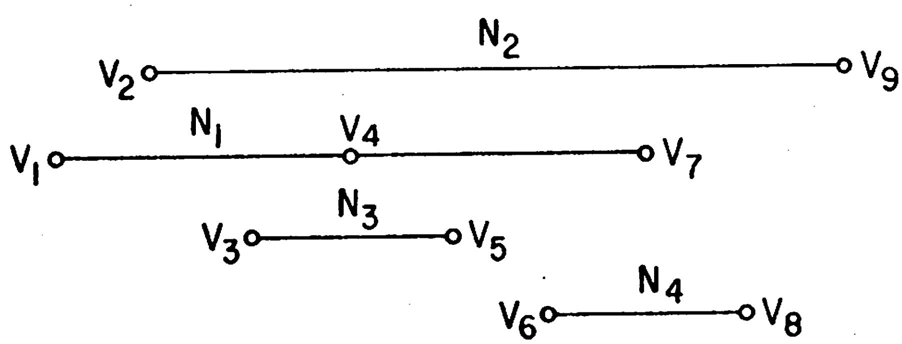
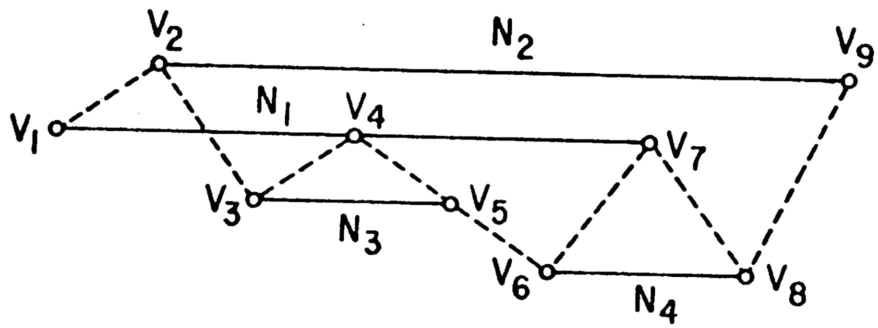
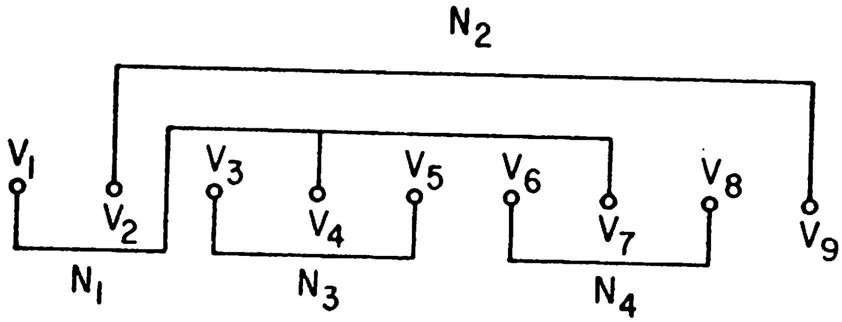


Fig. 3. An interval graphical representation.



(a)

Fig. 4(a). The reference line for the interval graphical representation of Fig. 3.



(b)

Fig. 4(b). A realization associated with the interval graphical representation of (a).

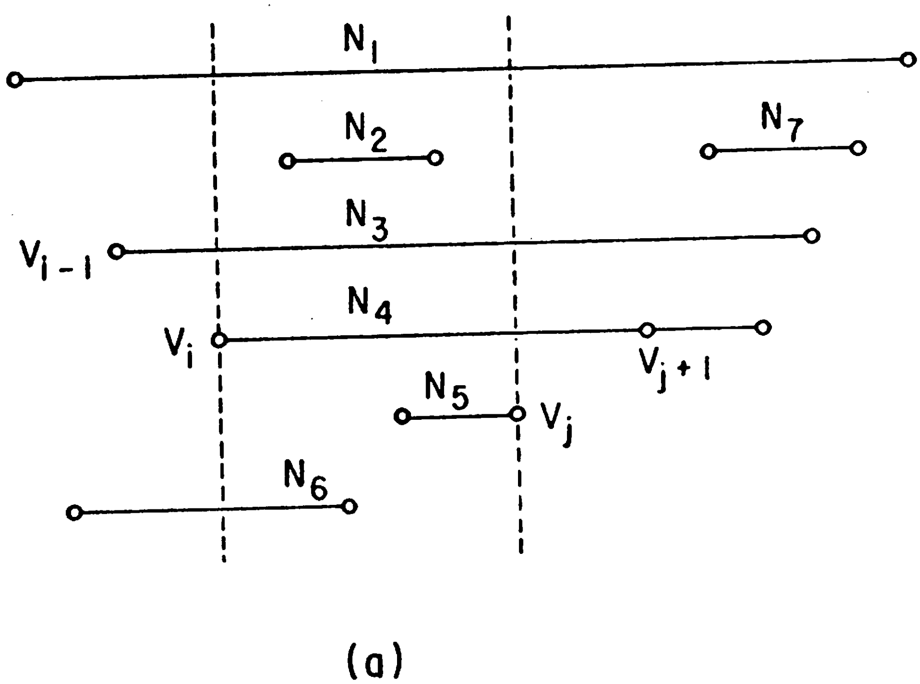


Fig. 5(a). An interval $I = [v_i, v_j]$ with $h = 3$, $c(v_{i-1}) = c(v_{j+1}) = 2$; $\overline{L}(I) = \{N_1, N_3\}$ and $L(I) = \{N_1, N_2, N_3, N_4, N_5, N_6\}$.

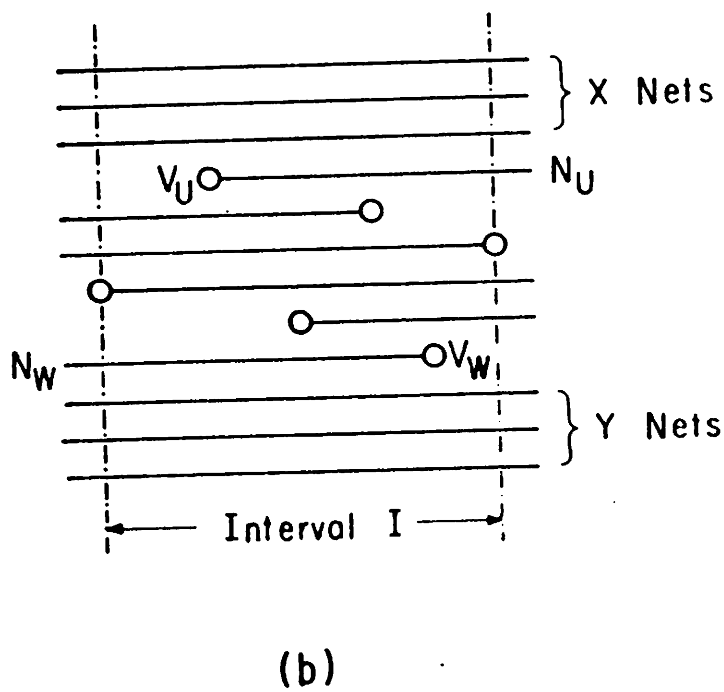


Fig. 5(b). Nets N_u and N_w on the interval I.

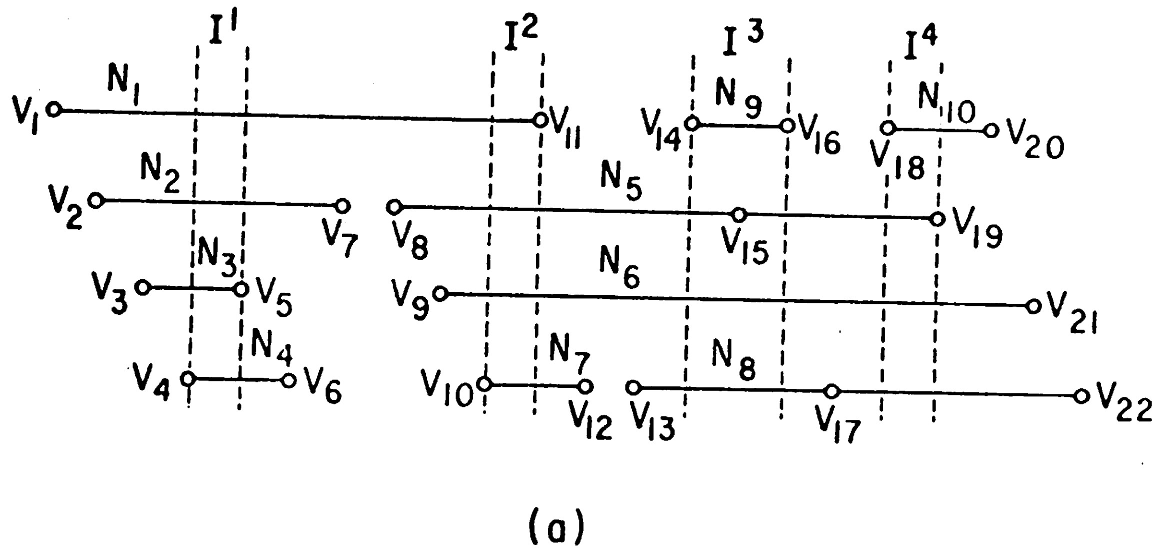


Fig. 6(a). Given net list L.

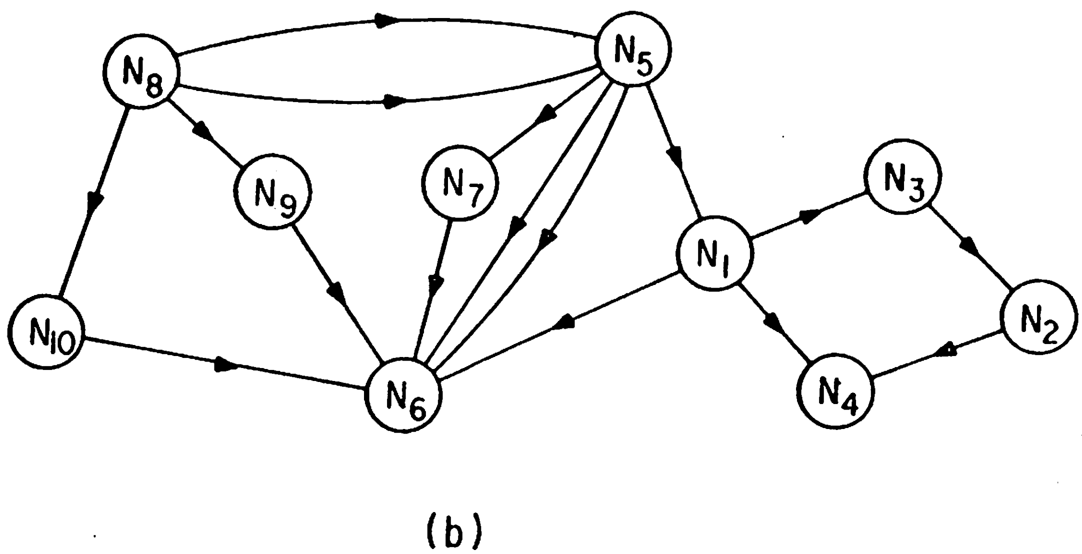


Fig. 6(b). Graph G

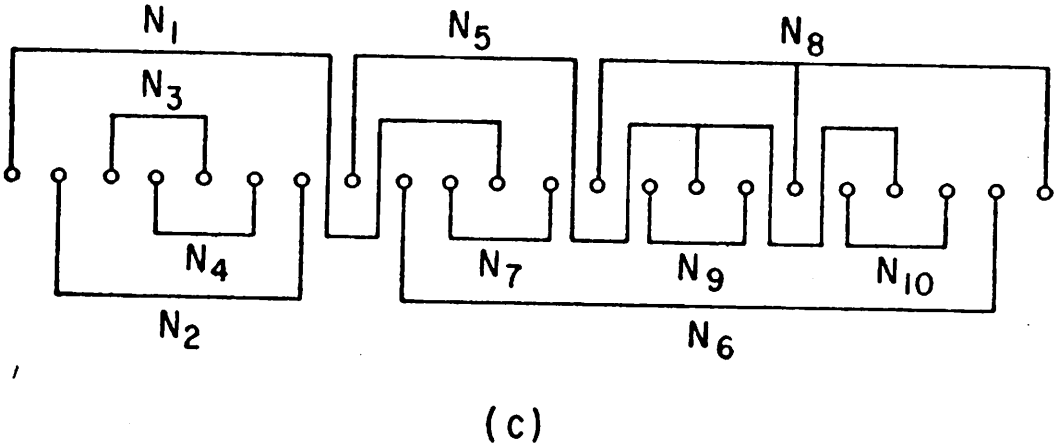


Fig 6(c). A realization.