A DERIVATION OF THE TIME TO FAILURE DISTRIBUTION IN CONTINUOUS - TIME MARKOV CHAINS

by

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Memorandum No. UCB/ERL M79/14

December 1978

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A DERIVATION OF THE TIME TO FAILURE DISTRIBUTION IN CONTINUOUS-TIME MARKOV CHAINS

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ABSTRACT

In this paper the expression for the 'time to failure distribution' for systems modelled as continuous-time finite state Markov Chains is derived using only elementary concepts of probability. This is used to obtain the expressions for expected time to failure and expected cycle time of the system. It is shown that under steady state assumptions the system can be modeled as if it were a two state Markov Chain for the computations of commonly used reliability indices.

Research sponsored by the Department of Energy Grant EC-77-S-01-5105.

1. Introduction

We consider a finite-state continuous-time Markov Chain. The state space is partitioned into two sets: U, the set of 'up-states' and D, the set of 'down-states.' Suppose that initially the system is in U with a given probability distribution. In this paper we derive the expression for the distribution of the time at which the system leaves U. Based on this general expression for the time to failure distribution, we present a simple derivation of the expected cycle time.

This problem arises in reliability studies [1]. Consider, for example, a complex repairable system with many independent components where each component has an exponentially distribured failure and rapair time. The system can then be modeled as a 2^n-state continuous-time Markov Chain, where n is the number of components in the system. Of these states, some correspond to the system being up (working) while the others correspond to system failure. In such models it is of great interest to find 'time to failure' distribution. Such models have been applied to large-scale power system studies [2] in the so called frequency and duration method.

Brown [3] has derived the time to failure distribution for a parallel system (system for which the set D consists of only one state, viz, where all components have failed). Kielson [4] has considered the structure of various failure time distributions of a general system and their inter-relationships. Ross [5] and Barlow and Proschan [6] have considered the case where initially all the components are up. They have derived some important properties of the time to failure distributions.

The derivation presented here makes use of only elementary concepts of probability. We first note that the time to failure distribution function satisfies a linear first order differential equation with time varying coefficient. This coefficient is seen to depend on the vector of conditional probabilities. This vector satisfies a first order non-linear differential equation. We obtain the solution of this differential equation to derive the expression for the mean time to failure distribution. This expression is used to obtain the expression for expected cycle time and the expected time to failure of the system.

2. Preliminaries

To make the analysis manageable let us number the states 1 to N with the states in U numbered 1 to M. Let $\underline{x}(t)$ denote the state at time t. The assumptions inherent in the system model are given by:

Basic Assumptions.

1. For each time t, for each i,j such that $i \neq j$ there exists a $\lambda_{i,j}$ such that

$$\text{Prob}\{\underline{x}(t+\Delta t)=j \,|\, \underline{x}(t)=i\} = \lambda_{i,j}\Delta t + o(\Delta t);\ \Delta t \geq 0 \tag{1}$$

2. The Markovian assumption that:

$$\text{Prob}\{\underline{x}(t_1)=j \,|\, \underline{x}(t)=i;S\} = \text{Prob}\{\underline{x}(t_1)=j \,|\, \underline{x}(t)=i\}$$

where $t_1 > t$ and S is any condition prior to time t.

3. $\lambda_{i,j}$ is independent of time t.

4. $\text{Prob}\{\underline{x}(t+\Delta t)=i;\underline{x}(t+\theta\Delta t)=j \text{ for some } \theta \in (0,1) \,|\, \underline{x}(t)=k\} = o(\Delta t)$ whenever $i \neq j$ and $j \neq k$.

In view of the above assumptions the system is time-invariant and without loss of generality let us assume that the initial time of interest is 0 (the result that we obtain can be easily translated to any other starting time).

Problem Formulation.

We are given that at time 0 the system is in the set of up-states and we are also given the initial probability distribution $\underline{p}_o$ where the i-th component of $\underline{p}_o$ is the Prob{$\underline{x}(0)=i$}. Let T represent the time at which the system leaves the set U. We are interested in obtaining the distribution of T viz.

$$F(t) = \text{Prob}\{T>t|\underline{x}(0) \in U;\underline{p}_o\}$$

$$= \text{Prob}\{\underline{x}([0,t]) \subset U|\underline{x}(0) \in U;\underline{p}\}$$

In order to obtain F(t) we shall obtain the differential equation governing the function F with $\Delta t > 0$

$$F(t+\Delta t) = \text{Prob}\{\underline{x}([0,t+\Delta t]) \subset U|\underline{x}(0) \in U;\underline{p}_o\}$$

$$= \text{Prob}\{\underline{x}([t,t+\Delta t]) \subset U|\underline{x}([0,t]) \subset U;\underline{p}_o\}$$

$$\text{Prob}\{\underline{x}([0,t]) \subset U|\underline{x}(0) \in U;\underline{p}_o\}$$

$$= (1-\text{Prob}\{\underline{x}(t') \text{ is not in U for some } t' \text{ in}$$

$$[t,t+\Delta t]|\underline{x}([0,t]) \subset U;\underline{p}_o\})\ F(t)$$

$$= [1-(\lambda(t)\ \Delta t + o(\Delta t))]F(t)$$

$$\text{where } \lambda(t) = \lim_{\substack{\Delta t \geq 0 \\ \Delta t \geq 0}} \frac{\text{Prob}\{\underline{x}(t+\Delta t)\in D|\underline{x}([0,t])\subset U;\underline{p}_o\}}{\Delta t}$$

From this it follows that

$$\frac{dF}{dt}(t) = -\lambda(t)F(t) \tag{2}$$

Remarks. We can regard $\lambda(t)$ as the rate of departure from the set U. This would depend on the state of the system at time t. Thus in order to obtain $\lambda(t)$ we require the probability distribution of the state at time t. This as we shall see presently depends on t and satisfies a nonlinear differential equation. We are however given the initial probability distribution viz. $\underline{p}_o$ and the initial value problem has a closed form solution. Using this solution we obtain the expression for F(t).

3. Expression for F(t)

Notation: When we use $t_0, t_1, \ldots,$ it is assumed that $t_0 < t_1, \ldots,$.

Definitions.

$$\lambda_{i,i} = -\sum_{\substack{j=1 \\ j \neq i}}^{N} \lambda_{i,j} \text{ and } \Lambda = \{\lambda_{i,j}\}_{i,j=1}^{N} = \begin{bmatrix} \Lambda_{UU} & \Lambda_{UD} \\ \Lambda_{DU} & \Lambda_{DD} \end{bmatrix}$$

$\underline{p}(t)$ a row vector of dimension M such that

$$\underline{p}^i(t) = \text{Prob}\{\underline{x}(t)=i \,|\, \underline{x}([0,t]) \subset U; \underline{p}_o\},$$

$\underline{s}(t)$ a row vector of dimension N such that

$$\underline{s}^i(t_2,t_1) = \text{Prob } \underline{x}(t_2)=i \,|\, \underline{x}([0,t_1]) \subset U; \underline{p}_o\};$$

clearly $\underline{s}^i(t,t) = \underline{p}^i(t) \quad i = 1,2,\ldots,M.$

Also we define $\underline{1}$ as a column vector of proper dimension with each component equal to 1. This vector is useful for summation of components of row vectors.

3.1. The Differential Equation for $\underline{p}(t)$

$$\underline{s}^i(t+\Delta t,t) = \sum_{j=1}^{M} \text{Prob}\{\underline{x}(t+\Delta t)=i|\underline{x}(t)=j\}$$

$$\text{Prob}\{\underline{x}(t)=j|\underline{x}([0,t]) \subset U;\underline{p}_o\}$$

$$= \sum_{\substack{j=1\\j\neq i}}^{M} \underline{s}^j(t,t)(\lambda_{j,i}\Delta t+o(\Delta t))+\underline{s}^i(t,t)(1+\lambda_{i,i}\Delta t+o(\Delta t)) \qquad (3)$$

Now

$$\text{Prob}\{\underline{x}(t+\Delta t) \in U|\underline{x}([0,t]) \subset U;\underline{p}_o\} = \sum_{i=1}^{M} \underline{s}^i(t+\Delta t,t)$$

$$= \sum_{i=1}^{M} \{\underline{s}^i(t,t) + \sum_{j=1}^{M} \underline{s}^j(t,t)\{\lambda_{j,i}\Delta t+o(\Delta t)\}\}$$

$$= 1 + \sum_{i=1}^{M} \sum_{j=1}^{M} \underline{s}^j(t,t)\{\lambda_{j,i}\Delta t+o(\Delta t)\}$$

$$= 1 + \underline{p}(t)\Lambda_{UU}\underline{1}\Delta t+o(\Delta t)$$

Hence for i = 1,2,... M

$$\underline{p}^i(t+\Delta t) = \frac{\underline{s}^i(t+\Delta t,t)}{1+\underline{p}(t)\Lambda_{UU}\underline{1}\ t+o(\Delta t)}$$

Using (3) we get

$$\underline{p}(t+\Delta t) = \frac{\underline{p}(t)+\underline{p}(t)\Lambda_{UU}\Delta t+o(\Delta t)}{1+\underline{p}(t)\Lambda_{UU}\underline{1}\Delta t+o(\Delta t)}$$

and

$$\underline{p}(t+\Delta t)-\underline{p}(t) = \frac{\underline{p}(t)\Lambda_{UU}\Delta t-\underline{p}(t)\Lambda_{UU}\underline{1}\underline{p}(t)\Delta t+o(\Delta t)}{1+\underline{p}(t)\Lambda_{UU}\underline{1}\Delta t+o(\Delta t)}$$

where

$$\frac{d\underline{p}}{dt} = \underline{p}\Lambda_{UU} - \underline{p}\Lambda_{UU}\underline{1}\underline{p}; \qquad \underline{p}(0) = \underline{p}_o \tag{4}$$

3.2. The Expression for F(t)

Also

$$\lambda(t) = \lim_{\substack{\Delta t\to 0\\ \Delta t\geq 0}} \frac{\text{prob}\{\underline{x}(t+\Delta t) \in D|\underline{x}([0,t]) \subset U;\underline{p}_o\}}{\Delta t}$$

$$= \lim_{\substack{\Delta t\to 0\\ t\geq 0}} \frac{\sum_{j=M+1}^{N} \text{prob}\{\underline{x}(t+\Delta t)=j|\underline{x}([0,t]) \subset U;\ \underline{p}_o\}}{\Delta t}$$

$$= \lim_{\substack{\Delta t\to 0\\ \Delta t\geq 0}} \sum_{j=M+1}^{N} \sum_{i=1}^{M} [\frac{\text{Prob}\{\underline{x}(t+\Delta t)=j|\underline{x}(t)=i\}}{\Delta t}$$

$$\text{Prob}\{\underline{x}(t)=i|\underline{x}([0,t]) \subset U;\underline{p}_o\}]$$

$$= \lim_{\substack{\Delta t\to 0\\ t\geq 0}} \frac{\sum_{j=m+1}^{N} \sum_{i=i}^{M} \underline{p}^i(t)\lambda_{i,j}\Delta t+o(\Delta t)}{\Delta t}$$

$$= \sum_{j=m+1}^{N} \sum_{i=1}^{M} \underline{p}^i(t)\lambda_{i,j} = \sum_{i=1}^{M} \sum_{j=m+1}^{N} \underline{p}^i(t)\lambda_{i,j}$$

$$= \sum_{i=1}^{M} \underline{p}^{i}(t) \sum_{j=m+1}^{N} \lambda_{i,j} = - \sum_{i=1}^{M} \underline{p}^{i}(t) \sum_{j=1}^{M} \lambda_{i,j}$$

$$= -\sum_{i=1}^{M} \sum_{j=1}^{M} \underline{p}^{i}(t)\lambda_{i,j}$$

$$= - \underline{p}(t)\Lambda_{UU}\underline{1} \tag{5}$$

It is easily verified that the solution for (4) is given by

$$\underline{p}(t) = \frac{\underline{p}_o e^{\Lambda_{UU}t}}{\underline{p}_o e^{\Lambda_{UU}t}\underline{1}} \tag{6}$$

Substituting (6) in (5) we get

$$\lambda(t) = - \frac{\underline{p}_o e^{\Lambda_{UU}} \Lambda_{UU}\underline{1}}{\underline{p}_o e^{\Lambda_{UU}t}\underline{1}} \tag{7}$$

Substituting (7) in (2) and noting that the numerator in (7) is the defivative of the denominator we get

$$F(t) = \underline{p}_o e^{\Lambda_{UU}t}\underline{1} \tag{8}$$

<u>Remarks</u>. Suppose that the following holds:

Sum of the elements in each row of Λ_{UD} is the same (say λ_{out}). (9)

Then $\Lambda_{UU}\underline{1} = - \lambda_{out}\underline{1} \Rightarrow e^{\Lambda_{UU}t} = \underline{1}e^{-\lambda_{out}t}$. This with the initial condition that $\underline{p}_o\underline{1} = 1$ yields

$$F(t) = e^{-\lambda_{out} t} \tag{10}$$

Also it easily verified that F(t) is of the form (10) for any $\underline{p}_o$ (with $\underline{p}_o \underline{1} = 1$) if and only if (9) holds.

Professor R. E. Barlow, after seeing the priliminary version of this report, has suggested an alternative derivation based on the theory of Markov Chains. His derivationis based on considering a new Markov Chain with M+2 states with transition rate matrix Λ^* given by

$$\begin{aligned}
\lambda^*_{i,j} &= \lambda_{i,j} \text{ for } 1 \leq i,\ j \leq M \\
&= -\sum_{k=1}^{M} \lambda_{i,k} \text{ for } 1 \leq i \leq M \text{ and } j = M+1 \\
&= -\lambda \quad \text{for } i = j \text{ and } i > M \\
&= \lambda \quad \text{for } i \neq j \text{ and } i,j > M \\
&= 0 \quad \text{for all other cases}
\end{aligned}$$

With this {M+1, M+2} becomes an absorbing set and F(t) is the same as the probability of finding the system in one of the first M states at time t. Using the standard expression for this probability one obtains Eq. (8).

4. Application to Reliability Evaluation

4.1. Preliminaries

In the previous section we derived an expression for the time spent in a group of states, given the initial probability vector. In this section we shall apply the result to obtain the distribution of certain important random variables used in reliability analysis. Since the expression (8) requires that the initial probability vector be given we proceed under the assumption that the system is in steady state. This idea will be made precise after we define the random variables of interest.

Definition

We say that the time spent in the down (up) states at time t_o is T_D (T_U) if the following event takes place

$$\underline{x}(0^-) \in U; \quad \underline{x}([t_o, t_o+T_d)) \subset D; \quad \underline{x}(t_o+T_d) \in U.$$

Also we define the cycle time T_c at t_o as follows

$$\underline{x}(t_o^-) \in D; \quad \underline{x}([t_o, t_o+t_1)) \subset U; \quad \underline{x}(t_o+t_1, t_o+t_c)) \subset D; \quad \underline{x}(t_p+t_c) \in U.$$

$$\text{with } 0 < t_1 < t_c.$$

Also it is useful to define another random variable $y = t_c - t_1$. Throughout this analysis we take $t_o = 0$.

Underlying Assumptions

1. $\forall t \; \pi^i(t) = \text{Prob}\{x(t)=i\}$ exists.

   Lemma: $\pi(t) = (\pi^1(t), \pi^2(t) \ldots \pi^N(t))$ is a constant and satisfies $\pi(t)\Lambda = 0$.

   Proof: Using (5) we have

$$\underline{\pi}(t) = \frac{\underline{\pi}(t_o)e^{\Lambda(t-t_o)}}{\underline{\pi}(t_o)e^{\Lambda(t-t_o)}\underline{1}}$$

$$= \underline{\pi}(t_o)e^{\Lambda(t-t_o)}$$

since $\Lambda\underline{1} = 0$, $\underline{\pi}(t_o)\underline{1} = 1$

Hence $\underline{\pi}(\cdot)$ is bounded

$\Longleftrightarrow \underline{\pi}(\cdot)$ is a constant

$\Longleftrightarrow \underline{\pi}\Lambda = 0$

2. In view of the above lemma let $\underline{\pi} = \underline{\pi}(t)$.

We assume that $\forall i\ \underline{\pi}^i > 0$

4.2. Distributions

Statement of the problem:

Given the above two assumptions we require the following distributions

1. $F_U(t) = \text{Prob}\{T_U>t\}$ and similarly F_D
2. $F_c(t) = \text{Prob}\{T_c>t\}$

Since $T_c = T_U + y$ we have an equivalent formulation of 2 as

2'. $F_y(t,x) = \text{Prob}\{T_y>t|T_U=x\}$

Remark. If we have $F_y(t,x)$ then clearly

$$F_c(t) = -\int F_y(t-x,x)dF_U(x)$$

In order to use (8) we require the initial probability vector $\underline{p}_o$. Since the following problem comes up often in our work we solve it first.

Problem. Given a set of states A, given that at 0^-, $x(0^-) \in \bar{A}$, the complement of A, given $\underline{p}(0^-)$ the probability vector at time 0^-, find

$\underline{p}(0)$ the probability vector at time 0, given that $x(0) \in A$.

Clearly

$$\text{Prob}\{\underline{x}(0)=i|\underline{x}(-\delta)=j\} = \lambda_{j,i}\delta + o(\delta)$$

hence

$$\text{Prob}\{\underline{x}(0)=i\} = \sum_j \underline{p}_j(-\delta)\lambda_{j,i}\delta + o(\delta)$$

$$\Rightarrow \text{Prob}\{\underline{x}(0)=i|\underline{x}(0) \in A\}$$

$$= \frac{\sum_j \underline{p}_j(-\delta)\lambda_{j,i}\delta+o(\delta)}{\sum_{i\in A}\sum_j \underline{p}_j(-\delta)\lambda_{j,i}\delta+o(\delta)}$$

$$= \frac{\sum_j \underline{p}_j(0^-)\lambda_{j,i}}{\sum_{i\in A}\sum_j \underline{p}_j(0^-)\lambda_{j,i}}$$

Hence $\underline{p}_A(0)$ is a multiple of $\underline{p}_{\bar{A}}(0^-)\Lambda_{A\bar{A}}$

Let $\underline{p}_U(0)$ be a row vector such that

$$\underline{p}_u^i(0) = \text{Prob}\{\underline{x}(0)=i|\underline{x}(0^-) \in D;\underline{x}(0) \in U\}$$

Also let $\underline{p}_D(t)$ be a row vector such that

$$\underline{p}_D^i = \text{Prob}\{\underline{x}(t)=i+M|\underline{x}(t) \in D;\underline{x}[0,t) \subset U;\underline{x}(0^-) \in D\}$$

In view of the result just stated we have

$\underline{p}_U(0)$ is a multiple of $\underline{\pi}_D\Lambda_{DU} = -\underline{\pi}_U\Lambda_{UU}$

and

$\underline{p}_D(t)$ is a multiple of $\underline{p}_U(0)e^{\Lambda t}\Lambda_{UD}$

Direct application of (8) yields

$$F_U(t) = \underline{p}_U(0)e^{\Lambda_{UU}t}\underline{1}$$

$$= \frac{\underline{\pi}_U\Lambda_{UU}e^{\Lambda_{UU}t}\underline{1}}{\underline{\pi}_U\Lambda_{UU}\underline{1}}$$

$$= \frac{1}{\underline{\pi}_U\Lambda_{UU}\underline{1}}\ \frac{d}{dt}\underline{\pi}_U e^{\Lambda_{UU}t}\underline{1}$$

and

$$F_y(t,x) = \underline{p}_D(x)e^{\Lambda_{DD}t}\underline{1}$$

$$= \frac{\underline{\pi}_U\Lambda_{UU}e^{\Lambda_{UU}x}\Lambda_{UD}e^{\Lambda_{DD}t}\underline{1}}{\underline{\pi}_U\Lambda_{UU}e^{\Lambda_{UU}x}\Lambda_{UD}\underline{1}}$$

where

$$F_c(t) = -\int F_y(t-x,x)dF_U(x)$$

$$= -\int_0^t F_y(t-x,x)dF_U(x) - \int_t^\infty dF_U(x)$$

$$= F_U(t) - \int_0^t F_y(t-x,x)dF_U(x)$$

$$= F_U(t) + \int_0^t \frac{\underline{\pi}_U\Lambda_{UU}e^{\Lambda_{UU}x}\Lambda_{UD}e^{\Lambda_{DD}(t-x)}\underline{1}dx}{\underline{\pi}_U\Lambda_{UU}\underline{1}}$$

where we have made use of the fact that

$$\frac{de^{\Lambda_{UU}x}\underline{1}}{dx} = e^{\Lambda_{UU}x}\Lambda_{UU}\underline{1}$$

$$= - e^{\Lambda_{UU}x}\Lambda_{UD}\underline{1}$$

Remark: The integral is a standard convolution integral of the form $\int_0^t H(x)G(t-x)dx$. While a closed form for the integral is not possible we have the following easily verifiable result.

$$\int_0^\infty \int_0^t H(x)G(t-x)dxdt = \int_0^\infty H(x)dx \int_0^\infty G(t)dt$$

4.3. Expectations

While the distributions we obtained in Section 4.2 are of importance, in reliability studies one is more interested in the expected values of these random variables. These values are sometimes used for reliability indices. In this section we shall compute these reliability indices. While these computations are straightforward, the results we obtain can be given extremely simple physical interpretations.

1. $E(T_U)$ = Expected time spent in up-states

$$= \int_0^\infty F_u(t)dt$$

$$= \frac{1}{\underline{\pi}_U \Lambda_{UU}\underline{1}} - \underline{\pi}_U\underline{1} = \frac{\underline{\pi}_U\underline{1}}{\underline{\pi}_U\Lambda_{UD}\underline{1}}$$

Similarly

2. $E(T_D)$ = Expected time spent in down states

$$= \frac{\underline{\pi}_D \underline{1}}{\underline{\pi}_D \Lambda_{DU} \underline{1}}$$

Remark: The vector $\underline{\pi}_{UD} \underline{1}$ represents the vector of rates out of U, i.e.

$$(\underline{\pi}_{UD}\underline{1})^i = \lim_{\substack{\Delta t > 0 \\ \Delta t \to 0}} \frac{\text{prob}\{\underline{x}(\Delta t) \in D | \underline{x}(0)=i\}}{\Delta t} = \lambda_{i,D}$$

and $\frac{\underline{\pi}_U}{\underline{\pi}_U \underline{1}}$ represents the vector of conditional steady state probability, given that the system is up. If we define the steady state failure rate of the system (λ) as

$$\lambda = \sum_{i \in U} \text{Prob}\{\underline{x}=\underline{i} | x \in U\} \lambda_{i,D} = \lim_{\substack{\Delta t \to 0 \\ \Delta t > 0}} \frac{\text{Prob}\{\underline{x}(\Delta t) \in D | \underline{x}(0) \in U\}}{\Delta t}$$

and similarly steady state repair rate of the system (μ) we have the following:

$$E(T_U) = \frac{1}{\mu} \quad \text{and} \quad E(T_D) = \frac{1}{\mu}$$

Also noting that $\underline{\pi}_D \Lambda_{DD} \underline{1} = \underline{\pi}_U \Lambda_{UU} \underline{1}$ we have

$$\text{Prob}\{x \in U\} = \frac{\underline{\pi}_U \underline{1}}{\underline{\pi}_U \underline{1} + \underline{\pi}_D \underline{1}} = \frac{\mu}{\lambda+\mu} \quad \text{and} \quad \text{Prob}\{x \in D\} = \frac{\lambda}{\lambda+\mu}$$

3. $E(T_c)$ = Expected cycle time

$$= \int_0^\infty F_c(t)dt$$

$$= E(T_U) + \frac{\underline{\pi}_U \Lambda_{UD} \Lambda_{DD}^{-1} \underline{1}}{\underline{\pi}_U \Lambda_{UU} \underline{1}}$$

where we have made use of the fact following the remark on convolution integrals. Making use of the fact that

$$\underline{\pi}_U \Lambda_{UD} = -\underline{\pi}_D \Lambda_{DD} \quad \text{and} \quad \underline{\pi}_U \Lambda_{UU} = -\underline{\pi}_D \lambda_{DU}$$

we obtain

$$E(T_c) = \frac{1}{\lambda} + \frac{1}{\mu}$$

5. Conclusion

We collect the results in the previous section as the following two key observations.

1. Even under steady state assumptions the transitions between two groups of states in a time finite-state Markov chain' will not be a 'two-state Markov chain' and hence the residence time in any group of states will in general not be exponentially distributed.

2. However, for the calculations of the expected time spent in the up-states, the expected time spent in the set of down-states, the expected cycle time (or the mean time between failures), the probability of finding the system up or down in a system modelled as 'continuous-time finite-state Markov chain' we can model the system as if it were a 'two state Markov chain' the two states being the 'up-state' and the'down-state.'

6. Acknowledgements

I would like to thank Ms. Claudia Grief for all the discussions we had, and for her thought provoking counter examples to the various false starts I made prior to this work. Also I wish to thank Professor R. E. Barlow for his comments on this work.

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