TWO WAY DETERMINISTIC FINITE AUTOMATA ARE EXPONENTIALLY MORE SUCCINCT THAN SWEEPING AUTOMATA

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KEYWORDS.

One way deterministic finite automaton (1dfa).

Two way deterministic finite automaton (2dfa).

One way non deterministic finite automaton (1nfa).

DEFINITION

Sweeping automaton (sa): a 2dfa which can halt or change the direction of its head motion only at the ends of the input tape.

PROBLEM SET UP AND MAIN RESULT.

This note is a refinement of a work of M.Sipser [1]. His main result is:

(*) For all n there is a language Bn which is accepted by an n-state 1nfa but not by any sa with less than $2^n - 1$ states.

i.e. 1nfa are exponentially more succinct than sa. We add the following contribution:

(**) 2dfa are are exponentially more succinct than sa.

The following lemma is fundamental in our proof.
LEMMA: Let L be a language on a finite alphabet Σ such that

1) If w belongs to L then all substrings of w belong to L.

2) In L there is at least one word x such that for all words u and v in L uxv belongs to L.

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3) There exists a string d over such that

i) d has length 2^n

ii) d does not belong to L

iii) after the removal of any of its non empty substrings d belongs to L.

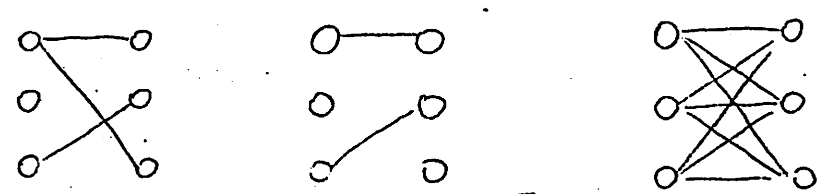
Then L cannot be accepted by an sa with less than 2^n-1 states.

PROOF: The proof can be carried out by following, step by step, the demonstration given in [1] that Bn cannot be accepted by an sa with less than 2^n-1 states. In fact properties (1),(2) and (3) hold for Bn and they are the only features of Bn involved in that proof.

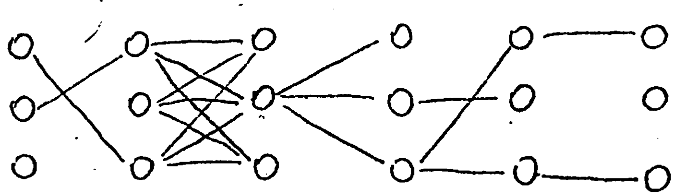
In order to prove (**) we define the language An. Say that a bipartite graph is of type 1 if no two edges meet at a right node and of type 2 if it is a complete bipartite graph. the alphabet Σ_n of An consists of all bipartite graphs satisfying these two properties:

i) The graphs have n left nodes and n right nodes

ii) The graphs are either of type 1 or of type 2. A sequence of such symbols constitutes a string by identifying right and left nodes of adjacent bipartite graphs.



some members of Σ_3



a string over Σ_3

A string s on Σ_n is a word of An iff

a) there is a path leading from a leftmost node of s to a rightmost one.

Definition: a chain of a string s is a maximal substring of s consisting of symbols of type 1.

Say a chain is good iff there is a path from one of its leftmost nodes to one of its rightmost nodes. Then s belongs to An iff

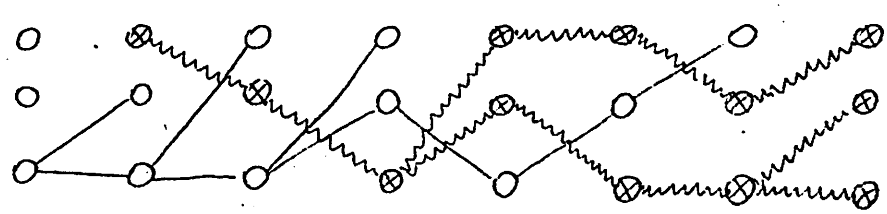
b) all chains of s are good.

Note that a chain is circuit free and thus (b) can be checked using depth first search by a 2dfa with $O(n^2)$ states. We summarize these observations in the following theorem 1.

THEOREM 1: An is accepted by an $O(n^2)$-state 2dfa.

THEOREM 2: An cannot be accepted by an sa with less than 2^n-1 states.

PROOF: We show that An satisfies the conditions of the lemma. Let w belong to An and s be a substring of w. As there is a path from a left most node of w to a rightmost one, such a path must also connect a leftmost node of s with a rightmost node of s. Thus property (1) holds for An. The word consisting of a single complete bipartite graph is a valid x for property (2). Let's now construct d_n (a valid d for An): write down 2^n columns of n nodes numbered 1 through n (top-down). Order the subsets of In={1,...,n} first by cardinality and then lexicographically. In the ith column from the left, mark the nodes that correspond to the ith subset of In. For i=1 to 2^n-1, connect the 1st marked node of column i with the 1st marked node of column i+1, 2nd with 2nd and so on; let last marked node connect to all remaining marked nodes (at most one) of column i+1. For unmarked nodes, connect the last of them in column i with all unmarked nodes of column i+1. d for n=3 looks as follows



d_n has length 2^n. In d_n no path runs from a leftmost node to a rightmost one but the removal of any non empty substring will create one. Thus d_n has all the properties required in (3) ; this completes the proof.

Acknowledgements

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References

[1] M.Sipser, Lower bounds on the size of sweeping automata,

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