STABILITY ANALYSIS OF MULTIMACHINE POWER NETWORKS WITH LINEAR FREQUENCY DEPENDENT LOADS

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ABSTRACT

This paper presents a complete stability analysis of a new power system model which was presented in [BER2]. The essential feature of the model is the assumption of frequency dependent loads. This facilitates a dynamic representation directly in terms of the network structure. Consequently, concepts and results from circuit theory can play a strong role in the stability analysis of the model. The multivariable Popov criterion is used to obtain general Lure-Postnikov type Lyapunov functions which rigorously allow for the presence of real loads. This has not been possible with the previously used model. Results are given for both local (dynamic) stability and for determination of regions of asymptotic (transient) stability.

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I. Introduction

The direct stability assessment of electric power systems has been largely founded on the use of energy functions to estimate the stability boundaries. The practical usefulness of this approach has been demonstrated in recent years [GUP1, RIB2, ATH1]. For a discussion of the literature on this subject, refer to the survey paper by Fouad [FOU1]. The energy functions are Lyapunov functions of the Lure-Postnikov form. It is well-known that the optimal Lyapunov functions of this form are associated with satisfaction of the Popov stability criterion [AIZ1]. The systemmatic search for general functions for power systems based on this result was developed by Willems [WIL16, WIL10, WIL11]. All of this work uses the so-called classical model for multi-machine power systems [FOU1]. This model is based on several assumptions which limit the practical usefulness of energy function methods. The present paper extends the results given by the authors in [BER2] to remove completely the least reasonable of these assumptions: the transfer conductances are zero.

A longstanding difficulty in power system stability theory has been to allow for nonzero transfer conductances in the derivation of Lyapunov functions of Lure-Postnikov form. The associated power flow equations are nonreciprocal and it has not been possible to find a well-defined energy function. These difficulties have been documented; see for instance [HEN1, FOU1, WIL11]. In [BER2], it is shown that the problem can be avoided by using simple dynamic models for the loads and preserving the network structure. In the usual model, loads are represented by impedances which are absorbed into a reduced network. The real power loads give the transfer conductances in this reduced network. The stability analysis in [BER2] was not fullv develooed: the usefulness of the model was illustracted with a simple energy function.

In this paper, the model in [BER2] is subjected to a complete stability analysis along the lines of that given by Willems for the classical model. For this, it has been found necessary to reformulate the state-space model used in [BER2]. This change corresponds merely to selecting a generator as the reference rather than a load bus. Results are given for local stability (or dynamic stability in power systems terminology) and instability of an equilibrium point. These sharpen considerably a result given in [BER2]. The main contribution is the Lyapunov function which will be derived for checking transient stability. This is the first fully general Lure-Postnikov type Lyapunov function which regorously accounts for real loads.

The structure of the paper is as follows. Section II gives a summary of the structure perserving model which was presented in [BER2]. The state-space model using a generator reference is derived. Section III gives results on local stability and instability. The general Lyapunov function for study of transient stability is derived in Section IV. Section V looks at an example and Section VI gives some conclusions. Some background material and certain calculations have been relegated to the Appendices.

II. Structure Preserving Dynamic Model of a Power System

In this section, the model which was developed in [BER2] will be described. An improved formulation will be used. This is based on using a generator as the reference.

We consider a network of m generators and n_0 buses connected by transmission lines. There are n_0 - m buses which have loads, but no generation. We augment the original network to introduce fictitious buses which represent the internal generation voltages. These are connected to the generator buses via the transient reactances. All the transmission lines are modelled as series reactances. Hence, all the buses in the augmented network model are connected by a network of series reactances. The augmented network has $n = m + n_0$ buses. We number the buses with no generation 1, 2, ..., n_0-m, the remaining buses from the original network n_0-m+1, ..., n_0, and the fictitious generator buses n_0+1, ..., n. (In [BER2], the load buses were numbered last.) The nth bus will be used as a reference.

Let δ_i and P_{D_i} be the bus angle and real power drawn by the load at bus i respectively. The key observation made in [BER2] is that for constant voltage magnitudes and small frequency variations around the operating point P_{D_i}, it is reasonable to assume

$$P_{D_i} = P_{D_i}^0 + D_i \omega_i \qquad i = 1,2,\ldots,n-m = n_0 \tag{1}$$

where $\omega_i = \dot{\delta}_i$ and $D_i > 0$ is the load-frequency coefficient. The generator dynamics are modelled by the swing equation with the usual assumptions of constant voltage (magnitude) behind transient reactance, constant mechanical power and so on [FOU1]. Thus, we are led to

$$M_i \ddot{\delta}_i + D_i \dot{\delta}_i + \sum_{\substack{j=1 \\ j \neq i}}^{n} b_{ij} \sin(\delta_i - \delta_j) = P_{M_i}^0 - P_{D_i}^0 \triangleq P_i^0 \qquad i = 1,\ldots,n \tag{2}$$

where

$M_i = 0$	$i = 1,\ldots,n_0$	
$M_i > 0$	$i = n_0+1,\ldots,n$	(generator inertia constants)
$D_i > 0$	$i = 1,\ldots,n_0$	(load frequency coefficient)
$D_i > 0$	$i = n_0+1,\ldots,n$	(generator damping coefficient)
$P_{M_i}^0 = 0$	$i = 1,\ldots,n_0$	
$P_{D_i}^0 = 0$	$i = n_0+1,\ldots,n$	

It is convenient to assume that $\sum_{i=1}^{n} P_i^0 = 0$. For situations where this is not the case, a shift of speed reference can be chosen to make it true in the new coordinates.

There are various ways to describe the angle variables in the network. We define the internodal angles,

$$\alpha_i \stackrel{\Delta}{=} \delta_i - \delta_n \qquad i = 1,\ldots,n-1;$$

the line angle differences,

$$\sigma_k = \delta_i - \delta_j \qquad k = 1,\ldots,\ell$$

for the kth line joining buses i and j where there are ℓ lines in the augmented network; and the tree branch angles,

$$\theta_p = \delta_i - \delta_r \qquad p = 1,\ldots,n-1$$

for the pth line joining buses i and r in a chosen tree. The angle vectors $\underline{\delta} = [\delta_1, \ldots, \delta_n]^T$, $\underline{\alpha} = [\alpha_1, \ldots, \alpha_{n-1}]^T$, $\underline{\sigma} = [\sigma_1, \ldots, \sigma_\ell]^T$, and $\underline{\theta} = [\theta_1, \ldots, \theta_{n-1}]^T$ are related by various transformations

$$\underline{\alpha} = \underline{T}\,\underline{\delta} \tag{3a}$$

$$\underline{\sigma} = \underline{A}^T\underline{\alpha} \tag{3b}$$

$$\underline{\sigma} = \underline{Q}^T\underline{\theta} \tag{3c}$$

Matrix $\underline{T}$ is given by

$$\underline{T} = [\underline{I}_{n-1} : -\underline{e}_{n-1}]$$

where $\underline{I}_{n-1}$ is the (n-1) identity matrix and $\underline{e}_{n-1}$ is the (n-1) vector with unity entries. Matrices $\underline{A}$ and $\underline{Q}$ are the reduced incidence matrix and fundamental cutset matrix of the augmented network respectively [DES2].

We denote the real power flow in line k by $P_k = g_k(\sigma_k)$. Then

$$g_k(\sigma_k) = b_k \sin \sigma_k$$

where $b_k = b_{ij} > 0$ and it is assumed that branch k connects buses i and j. Now $P_n^0 = -\sum_{i=1}^{n-1} P_i^0$, so there are n-1 independent bus powers. Let $\underline{P}^0 = [P_i^0, \ldots, P_{n-1}^0]^T$. Then from eqn (3b) and nodal analysis, the load flow equations can be written as

$$\underline{P} = \underline{A}\underline{g}(\underline{A}^T\underline{\alpha}) \triangleq \underline{f}(\underline{\alpha}) \qquad (4a)$$

Note that

$$f_i(\underline{\alpha}) = \sum_{\substack{j=1 \\ j\neq i}}^{n-1} b_{ij}\sin(\alpha_i-\alpha_j)+b_{in}\sin \alpha_i \qquad i = 1,\ldots,n-1 \qquad (4b)$$

Now define the diagonal matrices

$$M_g = \text{diag}\{M_{n_0+1}, \ldots, M_n\}$$

$$\underline{D}_\ell = \text{diag}\{D_1, \ldots, D_{n_0}\}$$

$$\underline{D}_g = \text{diag}\{D_{n_0+1}, \ldots, D_n\}$$

It is also convenient to partition $\underline{T}$ according to

$$\underline{T} = \left[\begin{array}{c|cc} \underline{I}_{n_0} & \underline{0} & -\underline{e}_{n_0} \\ \underline{0} & \underline{I}_{m-1} & -\underline{e}_{m-1} \end{array}\right] \triangleq \left[\underline{T}_1 \;\middle|\; \underline{T}_2\right] \qquad (5)$$

From the results of [BER2], with minor changes to accomodate the use of a generator reference, we get that a state-space representation for the power system is

$$\dot{\underline{\omega}}_g = -\underline{M}_g^{-1}\underline{D}_g\underline{\omega}_g - \underline{M}_g^{-1}\underline{T}_2^T[\underline{f}(\alpha)-\underline{P}^0] \tag{6a}$$

$$\dot{\underline{\alpha}} = \underline{T}_2\underline{\omega}_g - \underline{T}_1\underline{D}_\ell^{-1}\underline{T}_1^T[\underline{f}(\underline{\alpha})-\underline{P}^0] \tag{6b}$$

Further, the equilibria are given by $\omega_g = 0$ and the solutions of

$$\underline{f}(\underline{\alpha}^e) = \underline{P}^0 \tag{7}$$

Eqn.(7) typically has numerous solutions. Some indication of what is known about the nature of these solutions is helpful.

We call the function $\underline{f}(\cdot)$ the flow function. Due to the periodic dependence of $\underline{f}(\underline{\alpha})$ on $\underline{\alpha}$, we write $\underline{f}$: $T^{n-1} \to \mathbb{R}^{n-1}$ where T^{n-1} is the n-1 dimensional torus. To study the properties of solutions of eqn.(7), we make extensive use of the Jacobian matrix of the flow function, which is denoted by $\underline{F}(\underline{\alpha})$. We note that $\underline{f} \in C'$. From eqn.(4), it follows that

$$\underline{F}(\underline{\alpha}) \triangleq \frac{\partial \underline{f}}{\partial \underline{\alpha}} = \underline{A}\ \underline{G}(\underline{A}^T\underline{\alpha})\underline{A}^T \tag{8}$$

where

$$\underline{G}(\underline{\sigma}) = \text{diag}\{G_k(\sigma_k)\}$$

and

$$G_k(\sigma_k) = b_k \cos \sigma_k$$

The (i,j)th term of $\underline{F}(\underline{\alpha})$ is given by

$$\frac{\partial f_i}{\partial \alpha_j}(\underline{\alpha}) = \begin{cases} b_{in}\cos \alpha_i + \sum\limits_{\substack{k=1 \\ k\neq i}}^{n-1} b_{ik}\cos(\alpha_i-\alpha_1), & i=j \\ -b_{ij}\cos(\alpha_i-\alpha_j), & i\neq j \end{cases}$$

It is clear that $\underline{F}(\underline{\alpha})$ has full normal rank. Using some terminology from the theory of nonlinear mapping, we call $\underline{P}^0$ regular if det $\underline{F}(\underline{\alpha}) \neq 0$ for all $\underline{\alpha}$ such that (7) is satisfied. (Note that any $\underline{P}^0$ for which (7) has no solution automatically qualifies as being regular.) Sard's Theorem [ORT1] says that almost all $\underline{P}^0 \in \mathbb{R}^{n-1}$ are regular values. Further, on choosing a regular value of $\underline{P}^0$, we are guaranteed that (7) has a finite number of solutions (which includes the possibility of no solutions, of course). In fact, it is easy to show that there must be an even number of solutions [ARA5].

Comments

(1) The important feature of this model is that it explicitly preserves the original network structure. That is, on substituting eqn.(4a) into eqn.(6), the system dynamics can be studied as a function of the parameter values, network topology, and bus powers. Further, the model is still simple enough to undertake direct analytical studies using the tools of system theory. The previously used model did not have structural integrity due to the absorption of loads into an effective admittance matrix. The loads were modelled as impedances; this does not appear suitable for bulk power systems. For further discussion along these lines, refer to [BER2].

(2) The dimension of the state-space in the above model is $m + n - 1 = (2m - 1) + n_0$. Using the classical model, the dimension would be $2m - 1$. Clearly, we have merely added one extra state for each load. It is implicit in the above formulation that damping is nonuniform.

(3) Note the model depends on the assumption that all $D_i > 0$. This is certainly physically reasonable so not very limiting on the scope of the results given here. On setting $\underline{D}_g = \underline{0}$ (or if damping is uniform) the obvious extension of the classical case applies. That is, the state-space dimension is reduced to $m + n$. If $\underline{D}_\ell = \underline{0}$, eqn.(6) is replaced by a set

of algebraic and differential equations. It turns out that a state-space can be defined by setting $\underline{D}_\ell = \varepsilon\underline{I}$ and taking the limit $\varepsilon \to 0+$ [SAS1].

(4) $\underline{F}(\underline{\alpha})$ can be interpreted as the nodal admittance matrix of a linear resistive network. The conductance of branch k connecting nodes i and j is G_k. Later we will have cause to look at the possibility of $G_k = 0$, i.e. $|\sigma_k| = \frac{\pi}{2}$ (mod 2π). Such branches are called zero-valued.

III. Local Stability and Instability

We have seen that eqn.(7) typically has numerous solutions. In this section, we study the local stability properties of the state-space model (6) about each of these as a dynamic equilibrium point.

For the study of local stability, we linearize eqns.(6) about the equilibrium point $(\underline{\alpha}^e, \underline{0})$ to obtain differential equations in variables $\Delta\underline{\alpha} = \underline{\alpha} - \underline{\alpha}^e$ and $\Delta\underline{\omega}_g = \underline{\omega}_g - \underline{\omega}_g^e = \underline{\omega}_g$. This gives

$$\Delta\dot{\underline{\omega}}_g = -\underline{M}_g^{-1}\underline{D}_g\Delta\underline{\omega}_g - \underline{M}_g^{-1}\underline{T}_2^T\underline{F}(\underline{\alpha}^e)\Delta\underline{\alpha} \tag{9a}$$

$$\Delta\dot{\underline{\alpha}} = \underline{T}_2\Delta\underline{\omega}_g - \underline{T}_1\underline{D}_\ell^{-1}\underline{T}_1^T\underline{F}(\underline{\alpha}^e)\Delta\underline{\alpha} \tag{9b}$$

It is convenient to define the polytope

$$\Omega^\ell = \{\underline{\sigma} \in \mathbb{R}^\ell : \ |\sigma_i| \le \tfrac{\pi}{2}(\text{mod } 2\pi), \quad i = 1,\ldots,\ell\}$$

Lemma 1 [TAV1,ARA5]. Consider a solution $\underline{\alpha}^e$ to (7) and suppose that $\underline{\sigma}^e = \underline{A}^T\underline{\alpha}^e \in \Omega^\ell$. Then $\underline{F}(\underline{\alpha}^e) \ge 0$. Further, det $\underline{F}(\underline{\alpha}^e) = 0$ iff there is a cutset of zero-valued lines.

The following result for stable equilibrium points was given in [BER2]. An outline proof is given here since the details are instructive as a lead-up to deriving the instability result.

Theorem 1. Consider an equilibrium point for the power system satisfying (7). Suppose that $\underline{\sigma}^e \in \Omega^\ell$ and the augmented network has no cutsets of zero-valued lines. Then the equilibrium point is asymptotically stable.

Outline Proof: Choose as a Lyapunov function candidate

$$V(\underline{\omega}_g,\Delta\underline{\alpha}) = \frac{1}{2}\underline{\omega}_g^T M\underline{\omega}_g + \frac{1}{2}\Delta\underline{\alpha}^T \underline{F}(\underline{\alpha}^e)\Delta\underline{\alpha} \qquad (10)$$

Differentiating along the solutions of (6) gives

$$\dot{V}(\underline{\omega}_g,\Delta\underline{\alpha}) = -\underline{\omega}_g^T \underline{D}_g \underline{\omega}_g - \Delta\underline{\alpha}^T \underline{F}(\underline{\alpha}^e)\underline{T}_1\underline{D}_\ell^{-1}\underline{T}_1^T\underline{F}(\underline{\alpha}^e)\Delta\underline{\alpha}$$

which is at least negative semi-definite in the ω_g and $\Delta\alpha$ variables. It is straightforward to show $\dot{V}(\underline{\omega}_g,\Delta\underline{\alpha}) \equiv 0$ implies that $\underline{T}^T\ \underline{F}(\underline{\alpha}^e)\ \Delta\underline{\alpha} \equiv \underline{0}$. Since $\underline{T}$ has full rank and, from Lemma 1, $\underline{F}(\underline{\alpha}^e)$ is nonsingular, standard Lyapunov stability theory [HAH1] predicts that the equilibrium point $(0, \underline{\alpha}^e)$ is asymptotically stable.

This result accomodates the intuitive idea that if all lines satisfy $|\sigma_i^e| < \frac{\pi}{2}$, then the equilibrium is stable. Observations of the results of power system analysis also lead us to associate unstable equilibrium points with some branches satisfying $\frac{\pi}{2} < |\sigma_i^e| < \frac{3\pi}{2}$. The instability result clarifies this idea. First, we establish the following.

Lemma 2. Consider a solution $\underline{\alpha}^e$ to (7) and suppose that $\underline{\sigma}^e = \underline{A}^T\ \underline{\alpha}^e$ satisfies $|\sigma_i - \pi| < \frac{\pi}{2}$ on a cutset C_s of branches in the augmented network. Then $\underline{F}(\underline{\alpha}^e)$ has a negative eigenvalue.

Proof: Choose a tree for the network such that the cutset C_s corresponds to a fundamental cutset [DES2]. From eqn. (3c), we have

$$\Delta\underline{\sigma} = \underline{Q}^T\Delta\underline{\theta}$$

Write $\underline{Q}^T = [\underline{q}_1, \underline{q}_2, \ldots, \underline{q}_{n-1}]$; then the designated cutset corresponds to $\underline{q}_s$. Setting $\Delta\theta_i = 0$ for all $i \neq s$ gives

$$\Delta\underline{\sigma}_s = \underline{q}_s\Delta\theta_s$$

The corresponding internodal angles satisfy

$$\Delta\underline{\sigma}_s = \underline{A}^T\Delta\underline{\alpha}_s$$

Now

$$\begin{aligned}\Delta\underline{\alpha}_s^T\underline{F}(\underline{\alpha}^e)\Delta\underline{\alpha}_s &= \Delta\underline{\alpha}_s^T\underline{A}\,\underline{G}(\underline{\sigma}^e)\underline{A}^T\Delta\underline{\alpha}_s \\ &= \Delta\underline{\sigma}_s^T\underline{G}(\underline{\sigma}^e)\Delta\underline{\sigma}_s \\ &= (\Delta\theta_s)^2 q_s^T\underline{G}(\underline{\sigma}^e)q_s \\ &= (\Delta\theta_s)^2\sum_{C_s} G_k(\underline{\sigma}^e)\end{aligned}$$

where the summation is over branches in the cutset C_s. The given conditions on $\underline{\alpha}^e$ imply that $G_k(\underline{\alpha}^e) < 0$ for branches in C_s. It is then clear that $\underline{F}(\underline{\alpha}^e)$ has a negative eigenvalue. ¤

Theorem 2. Consider an equilibrium point for the power system satisfying (7). Suppose that $\underline{\sigma}^e$ satisfies $|\sigma_i - \pi| < \frac{\pi}{2}$ on a cutset of branches in the augmented network and det $\underline{F}(\underline{\alpha}^e) \neq 0$. Then the equilibrium point is unstable.

Proof: As in the stability result, we use standard Lyapunov stability/instability results. Now $V(\underline{\omega}_g, \Delta\underline{\alpha})$ in eqn.(10) takes negative values arbitrarily close to $(\underline{0}, \underline{0})$. Again we have that $\dot{V}(\underline{w}_g, \Delta\underline{\alpha}) \equiv 0$ implies that $(\underline{\omega}_g, \Delta\underline{\alpha}) \equiv (\underline{0}, \underline{0})$. A straightforward application of a standard instability theorem proves the stated result. ¤

Comments

(1) The restriction that det $\underline{F}(\underline{\alpha}^e) \neq 0$ is used in Theorems 1 and 2 to ensure that $\hat{V} \not\equiv 0$ except for the trivial trajectory. Removing it from Theorem 1, no conclusion can be made directly about the stability of $(\underline{0}, \underline{\alpha}^e)$. However, in Theorem 2 the absence of this condition leaves the conclusion that $(\underline{0}, \underline{\alpha}^e)$ is not asymptotically stable.

(2) When all $G_k \geq 0$, Lemma 1 gives a simple characterization of the condition det $\underline{F}(\underline{\alpha}^e) \neq 0$. We should note, of course, that in the usual case of regular $\underline{P}^0$ this condition is automatically satisfied.

(3) Typically, there is one stable solution to (7) and numerous unstable solutions. Precise statements on this issue are only recently beginning to emerge [ARA5]. In [ARA5], geometrical arguments on (7) are used to show that for regular $\underline{P}^0$ there are more unstable solutions than stable ones. There can be multiple stable solutions and, most surprisingly, there can be unstable solutions without stable ones.

IV. Lyapunov Functions for Transient Stability

After establishing that the equilibrium point $(\underline{0}, \underline{\alpha}^s)$ is asymptotically stable, the next step is to estimate the region of stability for large disturbances. In power systems terminology, this is called the transient stability region. A powerful approach is based on using Lyapunov functions for the nonlinear model. Suppose $V(\cdot)$ has been established as a Lyapunov function. A critical value V_ℓ is defined which represents the maximum amount of transient energy which can be injected into the faulted system without causing instability. This is obtained by examining the unstable equilibrium points. The stability region is then defined by the inequality $V(\underline{x}(0)) < V_\ell$.

The most general systemmatic method of producing Lyapunov functions for nonlinear systems is via use of Popov's stability criterion. The application of this technique to the classical model of power systems has been developed by Willems [WIL16,WIL10,WIL11]. In this section, we consider the corresponding results for the model given by eqn. (6). A summary of the results needed on the Popov criterion is given in Appendix A.

Putting eqns.(6) into the form (A.1), we have

$$\underline{F} = \begin{bmatrix} -\underline{M}_g^{-1}\underline{D}_g & \underline{0} \\ \underline{T}_2 & \underline{0} \end{bmatrix} \qquad \underline{G} = \begin{bmatrix} \underline{M}_g^{-1}\underline{T}_2^T \\ \underline{T}_1\underline{D}^{-1}\underline{T}_1^T \end{bmatrix} \qquad \underline{H} = \begin{bmatrix} \underline{0} \\ \underline{I}_{n-1} \end{bmatrix}$$

$$\underline{\psi}(\underline{\alpha}) = \underline{f}(\underline{\alpha}) - \underline{P}^0$$

First, we look at the necessary positive realness condition to insure a solution of eqns.(A.2). The transfer function $\underline{G}_L(s)$ of the linear part is given by

$$\underline{G}_L(s) = \frac{1}{s}[\underline{T}_2(s\underline{I}+\underline{M}_g^{-1}\underline{D}_g)^{-1}\underline{M}_g^{-1}\underline{T}_2^T + \underline{T}_1\underline{D}_\ell^{-1}\underline{T}_1^T]$$

Note that $\lim_{s\to\infty} s\,\underline{G}_L(s) = \underline{T}_1 D_\ell^{-1}\underline{T}_1^T$. This contrasts with the classical case where $s\,\underline{G}_L(s)$ is a strictly proper transfer function matrix. Now consider

$$\underline{Z}(s) = (p+qs)\underline{G}_L(s)$$

where $p \geq 0$, $q > 0$. It is not hard to see that $\underline{Z}(s)$ is positive real if $q \geq p\,\frac{M_i}{D_i}$, $i = n_0+1, \ldots, n$.

To obtain the general Lyapunov function it remains to solve eqns.(A.2). It is useful to introduce the following notation. For r x r diagonal matrix $\underline{J} = \text{diag}\,\{J_1, \ldots, J_r\}$, we define the (r-1) x (r-1) diagonal matrix $\tilde{\underline{J}} = \text{diag}\,\{J_1, \ldots, J_{r-1}\}$.

Let $\underline{P}$, $\underline{L}$ and $\underline{W}$ be partitioned as

$$\underline{P} = \begin{bmatrix} \underline{P}_1 & \underline{P}_2^T \\ \underline{P}_2 & \underline{P}_3 \end{bmatrix} \qquad \underline{L} = \begin{bmatrix} \underline{L}_{11} & \underline{L}_{12} \\ \underline{L}_{21} & \underline{L}_{22} \end{bmatrix} \qquad \underline{W} = \begin{bmatrix} \underline{W}_1 \\ \underline{W}_2 \end{bmatrix}$$

Then eqn. (A.2a) becomes

$$\begin{bmatrix} \underline{P}_1\underline{M}_g^{-1}\underline{D}_g+\underline{D}_g\underline{M}_g^{-1}\underline{P}_1-\underline{P}_2^T\underline{T}_2-\underline{T}_2^T\underline{P}_2 & -\underline{D}_g\underline{M}_g^{-1}\underline{P}_2^T+\underline{T}_2^T\underline{P}_3 \\ -\underline{P}_2\underline{M}_g^{-1}\underline{D}_g+\underline{P}_3\underline{T}_2 & \underline{0} \end{bmatrix} = \begin{bmatrix} \underline{L}_{11}\underline{L}_{11}^T+\underline{L}_{12}\underline{L}_{12}^T & \underline{L}_{11}\underline{L}_{21}^T+\underline{L}_{12}\underline{L}_{22}^T \\ \underline{L}_{21}\underline{L}_{11}^T+\underline{L}_{22}\underline{L}_{12}^T & \underline{L}_{21}\underline{L}_{21}^T+\underline{L}_{22}\underline{L}_{22}^T \end{bmatrix}$$

It follows that

$$\underline{L}_{21} = \underline{0} \qquad \underline{L}_{22} = \underline{0}$$

$$\underline{P}_2\underline{M}_g^{-1}\underline{D}_g = \underline{P}_3\underline{T}_2 \tag{11a}$$

$$\underline{P}_1\underline{M}_g^{-1}\underline{D}_g+\underline{D}_g\underline{M}_g^{-1}\underline{P}_1-\underline{P}_2^T\underline{T}_2-\underline{T}_2^T\underline{P}_2 = \underline{L}_{11}\underline{L}_{11}^T+\underline{L}_{12}\underline{L}_{12}^T \tag{11b}$$

Now note that

$$\underline{H}^T\underline{G} = \underline{T}_1\underline{D}_\ell^{-1}\underline{T}_1^T$$

Eqn.(A.2c) gives

$$\underline{W}_1^T\underline{W}_1+\underline{W}_2^T\underline{W}_2 = 2q\underline{T}_1\underline{D}_\ell^{-1}\underline{T}_1^T$$

Eqn. A2 has an infinite number of solutions in general and thus we may expect some freedom in the choice of $\underline{W}_1$ and $\underline{W}_2$. It is convenient to take

$$\underline{W}_1 = \sqrt{2q}\,\underline{D}_\ell^{\frac{-1}{2}}\underline{T}_1^T$$

$$\underline{W}_2 = \underline{0}$$

Then

$$\underline{LW} = \begin{bmatrix} 2q\underline{L}_{11}\underline{D}_\ell^{\frac{-1}{2}}\underline{T}_1^T \\ \\ \underline{0} \end{bmatrix}$$

We can now express eqn. (A.2b) as

$$\underline{P}_1\underline{M}_g^{-1}\underline{T}_2^T+\underline{P}_2^T\underline{T}_1\underline{D}_\ell^{-1}\underline{T}_1^T = q\underline{T}_2^T-\sqrt{2q}\,\underline{L}_{11}\underline{D}_\ell^{\frac{-1}{2}}\underline{T}_1^T \quad (11c)$$

$$\underline{P}_2\underline{M}_g^{-1}\underline{T}_2^T+\underline{P}_3\underline{T}_1\underline{D}_\ell^{-1}\underline{T}_1^T = p\underline{I}_{n-1} \quad (11d)$$

Thus, the solution of eqns.(A.2) is equivalent to solution of eqn.(11). For the general case, the details are quite cumbersome and have been relegated to Appendix B. A useful result is obtained, and the main calculations are illustrated, for the special case of $p = 0$. For the rest of this section, we assume that $p = 0$.

Eqns.(11a) and (11d) give

$$\underline{P}_3(\underline{T}_2\underline{D}_g^{-1}\underline{T}_2^T+\underline{T}_1\underline{D}_\ell^{-1}\underline{T}_1^T) = \underline{0}$$

Define

$$\underline{K} \triangleq \underline{T}_2\underline{D}_g^{-1}\underline{T}_2^T+\underline{T}_1\underline{D}_\ell^{-1}\underline{T}_1^T$$

$$= \begin{bmatrix} \underline{D}_\ell^{-1} & \underline{0} \\ \underline{0} & \tilde{\underline{D}}_g^{-1} \end{bmatrix} + \frac{1}{D_n}\underline{e}\rangle\langle\underline{e} \quad (12)$$

Now observe that $\underline{K}$ has rank of n-1. Thus $\underline{P}_3 = \underline{0}$, and from (11a), we further conclude that $\underline{P}_2 = \underline{0}$. Eqns.(11b) and (11c) are simplified to

$$\underline{P}_1\underline{M}_g^{-1}\underline{D}_g+\underline{D}_g\underline{M}_g^{-1}\underline{P}_1 = \underline{L}_{11}\underline{L}_{11}^T+\underline{L}_{12}\underline{L}_{12}^T \tag{13a}$$

$$\underline{P}_1\underline{M}_g^{-1}\underline{T}_2^T = q\underline{T}_2^T-\sqrt{2q}\,\underline{L}_{11}\underline{D}_\ell^{\frac{-1}{2}}\underline{T}_1^T \tag{13b}$$

It is convenient to rewrite $\underline{T}_2$ in the form

$$\underline{T}_2 = \begin{bmatrix} \underline{0} & -\underline{e}_{n_0} \\ \hline \multicolumn{2}{c}{\underline{T}_g} \end{bmatrix}$$

where from (5) clearly $\underline{T}_g$ has the same general structure as $\underline{T}$. Then it is easy to see that eqn.(13b) becomes

$$\underline{P}_1\underline{M}_g^{-1}\begin{bmatrix} \underline{0} \\ \\ -\underline{e}_{n_0}^T \end{bmatrix} = -\sqrt{2q}\underline{L}_{11}D_\ell^{\frac{-1}{2}} + q\begin{bmatrix} \underline{0} \\ \\ -\underline{e}_{n_0}^T \end{bmatrix} \tag{14a}$$

$$\underline{P}_1\underline{M}_g^{-1}\underline{T}_g^T = q\underline{T}_g^T \tag{14b}$$

A solution to (14b) is

$$\underline{P}_1 = q\underline{M}_g+\mu\underline{M}_g\underline{1}_{mm}\underline{M}_g \tag{15}$$

where μ is a scalar constant and $\underline{1}_{mm}$ is an m x m matrix with all elements equal to 1. In verifying the result we note that $\underline{1}_{mm}\underline{T}_g^T = \underline{0}$. Substituting into eqn.(14a) gives

$$\underline{L}_{11} = \frac{\mu}{\sqrt{2q}}\underline{M}_g\underline{1}_{mn_0}\underline{D}_\ell^{\frac{1}{2}}$$

where $\underline{1}_{mn_0}$ is the m x n_0 matrix with all elements equal to 1.

Finally, we note that the solution to eqns.(11) is achieved if $\underline{P}_1 > 0$ and eqn.(13a) is satisfied. Regarding this second condition it is to be noted that the required $\underline{L}$ may be found if $\underline{P}_1$ is such that the left hand side of (13a) is positive definite (or positive semi-definite). This then requires that μ is chosen to ensure the following inequalities are satisfied:

$$q\underline{M}_g+\mu\underline{M}_g\underline{1}_{mm}\underline{M}_g > 0 \qquad (16a)$$

$$2q\underline{D}_g + \mu(\underline{M}_g\underline{1}_{mm}\underline{D}_g+\underline{D}_g\underline{1}_{mm}\underline{M}_g) - \frac{\mu^2}{2q}\underline{M}_g'\underline{1}_{mn_0}\underline{D}_\ell\underline{1}_{n_0m}\underline{M}_g \geq 0. \qquad (16b)$$

The Lyapunov function in A-4 reduces to

$$V(\underline{x}) = \frac{q}{2}\underline{\omega}_g^T\underline{M}_g\underline{\omega}_g + \frac{\mu}{2}\underline{\omega}_g^T\underline{M}_g\underline{1}_{mm}\underline{M}_g\underline{\omega}_g + qW(\underline{\alpha},\underline{\alpha}^s) \qquad (17)$$

where

$$W(\underline{\alpha},\underline{\alpha}^s) = \int_{\underline{\alpha}^s}^{\underline{\alpha}}[\underline{f}(\underline{\xi})-\underline{P}^0]^T d\underline{\xi}.$$

Refer to Appendix B for the results in the general case of $p \neq 0$.

Comments

(1) Although the situation treated by Willems [WIL16] is not a special case of the present model, it is reassuring that on putting $\underline{D}_\ell = \underline{0}$, eqns.(16) correspond to those obtained in [WIL16]. It is the extra term in eqn.(17) which effectively solves the so-called transfer conductances problem.

(2) The above Lyapunov function can be applied using techniques which are described elsewhere [GUP1,RIB2,ATH1]. Usually the critical value V_ℓ is evaluated according to

$$V_\ell(\underline{\alpha}^s) = W(\underline{\alpha}^u,\underline{\alpha}^s)$$

where $\underline{\alpha}^u$ represents a selected unstable equilibrium point. Then the stability criterion for large disturbances becomes

$$q\underline{\omega}_g^T \underline{M}_g \underline{\omega}_g + \mu \underline{\omega}_g^T \underline{M}_g \underline{1}_{mm} \underline{M}_g \underline{\omega}_g < 2qW(\underline{\alpha}^u, \underline{\alpha}) \tag{18}$$

That is, all states $(\underline{w}_g, \underline{\alpha})$ satisfying (18) represent stable post fault initial conditions.

(3) The calculation of acceptable values of μ is very similar to that employed for the classical model case [WIL16]. We get that $V(\cdot)$ is a valid Lyapunov function if μ satisfies

$$0 \geq \mu \geq \max(\mu_0, \mu_1)$$

where the value of μ_0 and μ_1 are selected to ensure inequalities (16a) and (16b) are satisfied respectively. We have

$$\mu_0 = \frac{-q}{\sum_{i=n_0+1}^{n} M_i}$$

and μ_1 is a negative root of a polynomial in μ.

(4) The quantity $W(\underline{\alpha}_1, \underline{\alpha}_2)$ can be evaluated as a summation over all the lines in the network, of single variable integrations [BER2].

$$W(\underline{\alpha}_1, \underline{\alpha}_2) = \sum_{k=1}^{\ell} b_k \int_{\sigma_k^1}^{\sigma_k^2} (\sin u - \sin \sigma_k^1) du$$

where $\underline{\sigma}^i = \underline{A}^T \underline{\alpha}_i$, $i = 1, 2$.

(5) In Section III, we used a version of $V(\underline{x})$ in eqn.(17) with $\mu = 0$. It is easy to see that these results on local stability cannot be improved with more elaborate Lyapunov functions derived from setting $\mu \neq 0$.

(6) Note that the Lyapunov function (17) has not precisely followed the recipe given by Theorem A.1. In the above analysis we have $\underline{P} \geq 0$ but not $\underline{P} > 0$. Note that with $p = 0$, $s = 0$ is a pole of $\underline{G}_1(s)$ so the conditions of minimality are violated. However, it is easy to check from eqns. (A.4) and (A.5) that $V(\cdot)$ is still a valid Lyapunov function if μ is chosen so that $\underline{P}_1 > 0$.

V. Example

To illustrate the results of the previous section, we consider the simple power system shown in Figure 1. The buses, including fictitious ones represented by dotted symbols, and lines are numbered according to the convention given in Section II. Note that $m = 2$, $n_0 = 3$ and $n = n_0 + m = 5$. We take bus 5 to be the reference.

The relevant matrices are

$$\underline{D}_\ell = \begin{bmatrix} D_1 & 0 & 0 \\ 0 & D_2 & 0 \\ 0 & 0 & D_3 \end{bmatrix} \qquad \underline{D}_g = \begin{bmatrix} D_4 & 0 \\ 0 & D_5 \end{bmatrix}$$

$$\underline{M}_g = \begin{bmatrix} M_4 & 0 \\ 0 & M_5 \end{bmatrix}$$

First consider the case where $p = 0$, $q = 1$. Then

$$\underline{P}_1 = \begin{bmatrix} M_4+\mu M_4^2 & \mu M_4 M_5 \\ \mu M_4 M_5 & M_5+\mu M_5^2 \end{bmatrix}$$

and it is easy to check that this is positive definite if

$$\mu > \mu_0 = -\frac{1}{M_4+M_5} \tag{19a}$$

Inequality (16b) becomes

$$\begin{bmatrix} 2D_4+2\mu D_4M_4-\frac{\mu^2}{2}D_\ell M_4^2 & \mu(M_4D_5+M_5D_4)-\frac{\mu^2}{2}D_\ell M_4M_5 \\ \mu(M_4D_5+M_5D_4)-\frac{\mu^2}{2}D_\ell M_4M_5 & 2D_5+2\mu D_5M_5-\frac{\mu^2}{2}D_\ell M_5^2 \end{bmatrix} \geq 0$$

where

$$D_\ell \triangleq D_1+D_2+D_3.$$

This is equivalent to the scalar inequalities

$$\mu^2\frac{D_\ell M_4^2}{2} - 2\mu D_4M_4 - 2D_4 \leq 0 \tag{19b}$$

$$\mu^2(D_4M_5^2D_\ell+D_5M_4^2D_\ell+(M_4D_5+M_5D_4)^2-4D_4M_4D_5M_5)$$
$$-4\mu D_4D_5(M_4+M_5) - 4D_4D_5 \leq 0 \tag{19c}$$

Any negative solution of inequalities (19) can be taken as μ. However, by trial and error the one giving the largest estimated stability region would be chosen. The stability of a post fault initial condition $(\underline{\omega}_g(0),\underline{\alpha}(0))$ is determined by (18) which reduces to

$$M_4(1+\mu M_4)\omega_4^2(0) + 2\mu M_4M_5\omega_4(0)\omega_5(0) + M_5(1+\mu M_5)\omega_5^2(0)$$
$$< \sum_{k=1}^{5} b_k \int_{\sigma_k^u}^{\sigma_k(0)} (\sin u - \sin \sigma_k^s)du \tag{20}$$

where

$$\underline{\sigma}(0) = \underline{A}^T\underline{\alpha}(0).$$

In the general case, we obtain from (B.1) and (B.3) that

$$
\underline{P} = \begin{bmatrix}
qM_4+\mu M_4^2 & \mu M_4M_5 & -p\frac{D_1M_4}{D} & -p\frac{D_2M_4}{D} & -p\frac{D_3M_4}{D} & pM_4(1-\frac{D_4}{D}) \\
\mu M_4M_5 & qM_5+\mu M_5^2 & -p\frac{M_5D_1}{D} & -p\frac{M_5D_2}{D} & -p\frac{M_5D_3}{D} & -p\frac{M_5D_4}{D} \\
-p\frac{D_1M_4}{D} & -p\frac{M_5D_1}{D} & pD_1(1-\frac{D_1}{D}) & -p\frac{D_1D_2}{D} & -p\frac{D_1D_3}{D} & -p\frac{D_1D_4}{D} \\
-p\frac{D_2M_4}{D} & -p\frac{M_5D_2}{D} & -p\frac{D_2D_1}{D} & pD_2(1-\frac{D_2}{D}) & -p\frac{D_2D_3}{D} & -p\frac{D_2D_4}{D} \\
-p\frac{D_3M_4}{D} & -p\frac{M_5D_3}{D} & -p\frac{D_3D_1}{D} & -p\frac{D_3D_2}{D} & pD_3(1-\frac{D_3}{D}) & -p\frac{D_3D_4}{D} \\
pM_4(1-\frac{D_4}{D}) & -p\frac{M_5D_4}{D} & -pD_4D_1 & -p\frac{D_4D_2}{D} & -p\frac{D_4D_3}{D} & pD_4(1-\frac{D_4}{D})
\end{bmatrix}
$$

and the stability criterion (20) is modified accordingly. The value of μ is obtained from the inequalities (B.4).

VI. Conclusions

The model presented in [BER2] for a bulk power system is an important new starting point for direct stability analysis techniques. Here a complete theoretical study of the stability of this model has been presented.

A longstanding problem has now been completely overcome: how to allow rigorously for the effect of significant real power loads at load buses. The major step was taken in [BER2] by formulating a better model than the classical one. In this paper, the main contribution is the derivation of the first optimum Lyapunov function (of Lure-Postnikov form) which allows for such loads. As such, this Lyapunov function holds considerable promise for improving the accuracy of transient stability analysis. This aspect will be pursued elsewhere.

Appendix A

We discuss the computation of Lyapunov functions for the system

$$\underline{\dot{x}} = \underline{Fx} - \underline{G}\underline{\psi}(\underline{H}^T\underline{x}) \tag{A.1}$$

The picture of this model representing a linear system with input $\underline{u}$ and output $\underline{y}$ under the influence of nonlinear feedback $\underline{u} = -\underline{\psi}(\underline{y})$ is well-known. The transfer function of the linear system is

$$\underline{G}_L(s) = \underline{H}^T(s\underline{I}-\underline{F})^{-1}\underline{G}$$

The following is adapted from the work of Anderson [AND1,AND6]. We consider the transfer function

$$\underline{Z}(s) = (p+qs)\underline{G}_L(s)$$

where $p \geq 0$ and $q > 0$ and $s = -\frac{p}{q}$ is not a pole of $\underline{G}_L(s)$.

Theorem A.1. Suppose that ($\underline{F}$, $\underline{G}$, $\underline{H}$) is a minimal state-space representation of $\underline{G}_L(s)$. Then if $\underline{Z}(s)$ is positive real there exists real matrices $\underline{P}$, $\underline{L}$ and $\underline{W}$ with $\underline{P}$ positive definite symmetric, such that

$$\underline{PF}+\underline{F}^T\underline{P} = -\underline{LL}^T \tag{A.2a}$$

$$\underline{PG} = p\underline{H}+q\underline{F}^T\underline{H}-\underline{LW} \tag{A.2b}$$

$$\underline{W}^T\underline{W} = q(\underline{H}^T\underline{G}+\underline{G}^T\underline{H}). \tag{A.2c}$$ ¤

Now suppose that the nonlinear feedback satisfies

$$\underline{\psi}(\underline{0}) = \underline{0}$$

$$\underline{\psi}^T(\underline{u})\underline{u} \geq \varepsilon\underline{u}^T\underline{u} \qquad \text{for all } \underline{u} \neq 0 \tag{A.3}$$

where $\varepsilon > 0$, and $\underline{\psi}(\cdot)$ is the gradient of a real-value function.

Theorem A.2. If the transfer function $\underline{Z}(s)$ is positive-real and $\underline{\psi}(\cdot)$ satisfies the properties (A.3), then the null solution of system (A.1) is asymptotically stable in the large. ¤

This result is established using the Lyapunov function.

$$V(\underline{x}) = \frac{1}{2}\underline{x}^T\underline{P}\underline{x} + q\int_{\underline{0}}^{\underline{H}^T\underline{x}} \underline{\psi}(\underline{\xi})^T d\underline{\xi} \tag{A.4}$$

where $\underline{P}$ is the solution of eqns.(A.2). The derivative along solutions of (A.1) is

$$\dot{V}(\underline{x}) = -\frac{1}{2}(\underline{x}^T\underline{L}-\underline{\psi}^T\underline{W})(\underline{L}^T\underline{x}-\underline{W}^T\underline{\psi}) - p\underline{x}^T\underline{H}\underline{\psi}(\underline{H}^T\underline{x}) \tag{A.5}$$

Comments

(1) Under the given conditions, the stability property in Theorem A.2 is global. However, it is more common that property (A.3) is satisfied in some region of Euclidean space. Then $V(\cdot)$ can be used to estimate the boundary of the region of asymptotic stability.

(2) Varying the scalars p and q provides some flexibility in the Lynapunov function. In practice, it is usual to try several combinations until the size of the predicted stability region has been optimized.

(3) It does not appear to be widely recognized that Theorem A.2 is valid when $\underline{\psi}(\cdot)$ is not diagonal in the sense that $\psi_i(\underline{u})$ depends on some u_j, $i \neq j$. The assumption of diagonal $\underline{\psi}(\cdot)$ is usually made. Anderson's paper [AND6] is not precise on this issue and a subsequent paper [MOO1] assumed that the nonlinearity is diagonal.

Appendix B

Some further details on the solution of eqns.(11) are provided for the case of general p and q.

Firstly, note that eqn.(11a) and (11d) now give

$$\underline{P}_3 = p\underline{K}^{-1}$$

From eqn.(12), we have

$$\underline{K} = \tilde{D}^{-1} + \frac{1}{D_n}\underline{e}><\underline{e}$$

The Matrix Inversion Lemma gives

$$\underline{K}^{-1} = \underline{\tilde{D}} - \frac{1}{D}\underline{\tilde{D}}\,\underline{e}\,\underline{e}^T\underline{\tilde{D}}$$

where

$$D = \sum_{i=1}^{n} D_i .$$

Hence

$$\underline{P}_3 = p(\underline{\tilde{D}} - \frac{1}{D}\underline{\tilde{D}}\,\underline{1}_{n-1,n-1}\underline{\tilde{D}}) \qquad \text{(B.1a)}$$

We then get $\underline{P}_2$ from eqns.(11a) and (B.1a).

$$\underline{P}_2 = \underline{P}_3\underline{T}_2\underline{M}_g\underline{D}_g^{-1}$$

A straightforward calculation shows that

$$\underline{P}_2 = \frac{p}{D}\left[\begin{array}{c|c} -\underline{D}_\ell\underline{1}_{n_0,m-1}\underline{M}_g & -M_n\underline{\tilde{D}}\,\underline{e} \\ D\underline{\tilde{M}}_g - \underline{\tilde{D}}_g\underline{1}_{m-1,m-1}\underline{\tilde{M}}_g & \end{array}\right] \qquad \text{(B.1b)}$$

Eqns.(14) are replaced by

$$-\frac{p}{D}\begin{bmatrix}\tilde{\underline{M}}_g\underline{1}_{m-1,n_0} \\ M_n\underline{e}^T{}_{n_0}\end{bmatrix} + \underline{P}_1\underline{M}_g^{-1}\begin{bmatrix}\underline{0} \\ -\underline{e}^T{}_{n_0}\end{bmatrix} = -\sqrt{2q}\,\underline{L}_{11}\underline{D}_\ell^{-\frac{1}{2}} + q\begin{bmatrix}\underline{0} \\ -\underline{e}^T{}_{n_0}\end{bmatrix} \quad \text{(B.2a)}$$

$$\underline{P}_1\underline{M}_g^{-1}\underline{T}_g^T = q\underline{T}_g^T \quad \text{(B.2b)}$$

Hence $\underline{P}_1$ is still given by

$$\underline{P}_1 = q\underline{M}_g + \mu\underline{M}_g\underline{1}_{mm}\underline{M}_g \quad \text{(B.3)}$$

From eqn.(B.2a), a fairly tedious calculation gives

$$\underline{L}_{11} = \frac{1}{\sqrt{2q}}(\mu+\frac{p}{D})\underline{M}_g\underline{1}_{mn_0}\underline{D}_\ell^{\frac{1}{2}}$$

and then the counterpart to inequality (16b) is

$$2q\underline{D}_g + \mu(\underline{M}_g\underline{1}_{mm}\underline{D}_g+\underline{D}_g\underline{1}_{mm}\underline{M}_g) - p(\underline{T}_2^T\tilde{\underline{D}}\,\underline{T}_2\underline{M}_g\underline{D}_g^{-1}+\underline{D}_g^{-1}\underline{M}_g\underline{T}_2^T\tilde{\underline{D}}\,\underline{T}_2)$$

$$+\frac{p}{D}(\underline{T}_2^T\tilde{\underline{D}}\,\underline{1}_{n-1,n-1}\tilde{\underline{D}}\,\underline{T}_2\underline{M}_g\underline{D}_g^{-1}+\underline{D}_g^{-1}\underline{M}_g\underline{T}_2^T\tilde{\underline{D}}\,\underline{1}_{n-1,n-1}\tilde{\underline{D}}\,\underline{T}_2) - \frac{1}{2q}(\mu+\frac{p}{D})^2\underline{M}_g\underline{1}_{mn_0}\underline{D}_\ell\underline{1}_{n_0m}\underline{M}_g$$

$$\geq 0 \quad \text{(B.4)}$$

References

[BER 2] A.R. Bergen and D.J. Hill, "A structure preserving model for power system stability analysis," IEEE Trans. Power Apparatus and System, Vol. PAS-to appear.

[GUP 1] C.L. Gupta and A.H. El-Abiad, "Determination of the closest unstable equilibrium state for Liapunov methods in transient stability studies," IEEE Trans. Power Apparatus and Systems, Vol. PAS-94, pp. 1699-1712, September/October, 1976.

[RIB 2] M. Ribbens-Pavella, et al., "Transient stability analysis by scalar Liapunov functions: Recent improvements and practical results," Coll. des Public de la Faculté des Sc. Appliquées, Univ. of Liege, No. 67, 1977.

[ATH 1] T. Athay, R. Podmore and S. Virmani, "A practical method for the direct analysis of transient stability," IEEE Trans. Power Apparatus and Systems, Vol. PAS-98, pp. 573-584, March/April 1979.

[FOU 1] A.A. Fouad, "Stability theory - Criteria for transient stability," Proc. Eng. Foundation Conf. on Systems Eng. for Power: Status and Prospects, Henniker, New Hampshire, pp. 421-450, 1975.

[AIZ 1] M.A. Aizerman and F.R. Giantmacher, Absolute Stability of Regulator Systems Holden-Day, San Francisco, 1964 (Russian ed. 1963).

[WIL16] J.L. Willems, "Optimum Lyapunov functions and stability regions for multimachine power systems," Proc. IEE Vol. 117, No. 3, pp. 573-577, March 1970.

[WIL10] J.L. Willems, "Direct methods for transient stability studies in power system analysis," IEEE Trans. Auto. Control, Vol. AC-16, No. 4, August 1971, pp. 332-341.

[WIL11] J.L. Willems, "A partial stability approach to the problem of transient power system stability," Int. J. Control, Vol. 19, No. 1, pp. 1-14, 1974.

[HEN 1] V.E. Henner, "Comments on 'On Lyapunov functions for power systems with transfer conductances,'" IEEE Trans. Automatic Control, Vol. AC-19, pp. 621-622, October 1974.

[WIL17] J.L. Willems, "Comments on 'A General Liapunov function for multimachine power systems with transfer conductances,'" Int. J. Control, Vol. 23, No. 1, pp. 147-148, 1976.

[DES 2] C.A. Desoer and E.S. Kuh, Basic Circuit Theory, McGraw-Hill, New York, 1969.

[ORT 1] J.M. Ortega and W.C. Rheinboldt, Iterative Solution of Non-linear Equations in Several Variables, Academic Press, New York, 1970.

[ARA 5] A. Arapostathis, S. Sastry and P. Varaiya, "Analysis of the power flow equation," Memorandum No. UCB/ERL M80/35, College of Engineering, University of California, Berkeley, August 1980.

[SAS 1] S. Sastry and P. Varaiya, "Hierarchical stability and alert state steering control of interconnected power systems," Memorandum No. UCB/ERL M79/81, College of Engineering, University of California, Berkeley, December 1979.

[TAV 1] C.J. Tavora and O.J.M. Smith, "Equilibrium analysis of power systems," IEEE Trans. Power Apparatus and Systems, Vol. PAS-91, pp. 1131-1137, May/June 1971.

[HAH 1] W. Hahn, Stability of Motion, Springer Verlog, Berlin, 1967.

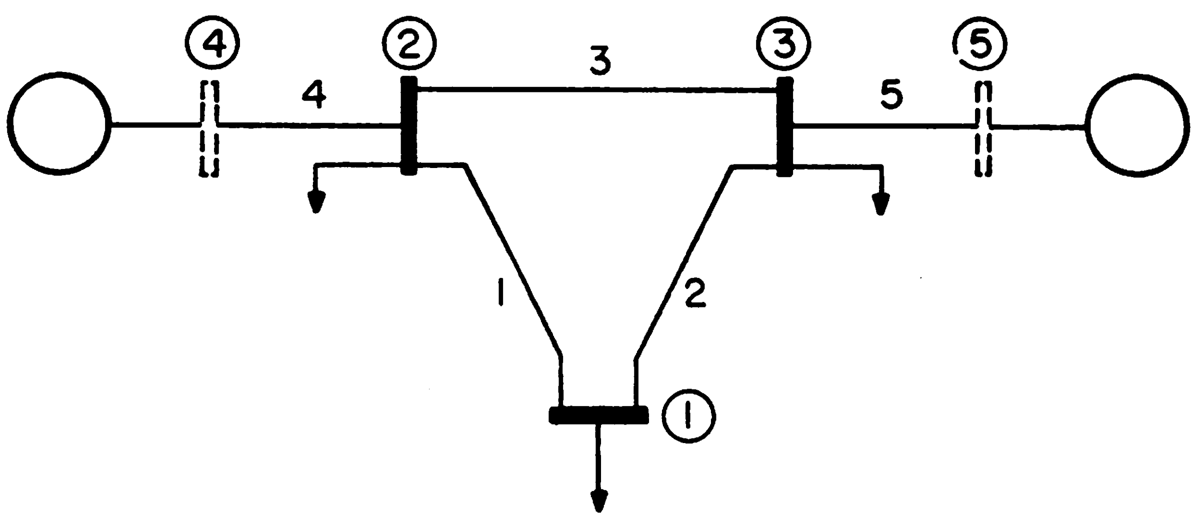
[AND 1] B.D.O. Anderson, "A system theory criterion for positive real matrices," SIAM Control. Vol. 5, pp. 171-182, May 1967.

[AND 6] B.D.O. Anderson, "Stability of control systems with multiple nonlinearities," J. of the Franklin Institute, Vol. 282, No. 3, September 1966, pp. 155-160.

[MOO 1] J.B. Moore and B.D.O. Anderson, "A generalization of the Popov criterion," J. Franklin Institute, Vol. 285, No. 6, June 1968, pp. 488-492.

Figure Caption

Fig. 1. Power system.



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