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ALGORITHMIC DESIGN OF SINGLE-INPUT SINGLE-OUTPUT SYSTEMS WITH A TWO-INPUT ONE-OUTPUT CONTROLLER

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Abstract

This paper consider the design of linear time-invariant single-input single-output feedback systems with a two-input one-output controller. Three design algorithms for synthesis, computer-aided design, and robust asymptotic tracking are presented.

1. Introduction

This paper presents an easily undstood, straightforward and algorithmic method for designing linear time-invariant single-input single-output feedback system with a two-input one-output controller. It is closely related to the approaches in [Bon.1] and [Åst.1]; it uses the more flexible configuration of Åstrom rather than the unity-feedback structure.

Notations. $\mathbb{C}_+$, ($\mathring{\mathbb{C}}_-$ resp.) := the closed right half plane (open left half plane, resp.). $\mathbb{R}[s]$, ($\mathbb{R}_p(s)$, $\mathbb{R}_{p,o}(s)$, resp.) := the ring of all polynomials (proper rational functions, strictly proper rational functions, resp.) with real coefficients.

2. Problem

Consider the linear time-invariant single-input single-output feedback system as shown in Fig. 1; given a strictly proper plant transfer function p, design a proper controller with two inputs, namely v_1 and e_1, and one output y_1, such that (i) the system is stable, and (ii) prescribed designed goals are achieved.

The controller can be viewed as consisting of a precompensator $\pi : v_1 \mapsto y_1$ and a feedback compensator $f : e_1 \mapsto y_1$. Let $[\pi:f] = [n_\pi:n_f]/d_c$, with $n_\pi, n_f, d_c \in \mathbb{R}[s]$; we realize the controller using the observer canonical form [Kai.1, p. 43, Fig. 2.1.9]. More precisely, $1/d_c$ is first realized by using appropriate constant-gain feedbacks around cascade integrators; the inputs v_1 and e_1 are then fed through appropriate constant gains to the integrator-inputs to obtain n_π and n_f, respectively. Note that $1/d_c$ lies inside the system feedback loop.

3. Analysis

We impose the following assumptions on the system of Fig. 1:

$$\text{(I)} \quad p = \frac{n_p}{d_p} \in \mathbb{R}_{p,o}(s) \tag{3.1}$$

$$\text{(II)} \quad [\pi:f] = [n_\pi:n_f]/d_c \in \mathbb{R}_p(s)^{1\times 2} . \tag{3.2}$$

When (3.1) and (3.2) hold, the system is called the system Σ. Note that (a) p is strictly proper and (n_p,d_p) are assumed coprime; (b) both π and f are proper while the polynomials n_π, n_f and d_c are not necessarily coprime; (c) (3.1) and (3.2) imply $(1+fp)^{-1} \to 1$ as $|s| \to \infty$, hence all the eight closed-loop transfer functions from u_1, u_2, v_1, and d_0 to y_1 and y_2 are all proper.

Clearly, Σ obeys the differential equations:

$$\begin{bmatrix} d_c & n_f n_p \\ -1 & d_p \end{bmatrix} \begin{bmatrix} \xi_1 \\ \xi_2 \end{bmatrix} = \begin{bmatrix} n_f & 0 & n_\pi & -n_f \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ v_1 \\ d_o \end{bmatrix}$$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & n_p \end{bmatrix} \begin{bmatrix} \xi_1 \\ \xi_2 \end{bmatrix}$$

Thus, the closed-loop characteristic polynomial of Σ is

$$\chi := d_c d_p + n_f n_p .$$

and the closed-loop eigenvalues are the zeros of χ.

Let $\mathcal{U} \supset \mathbb{C}_+$ be the closed subset of $\mathbb{C}$, symmetric with respect to the real axis, which includes all "undesirable" locations for poles of transfer functions.

We say that the system Σ is (closed-loop)$\mathcal{U}$-stable iff (i) all the closed-loop eigenvalues are in $\mathbb{C} \backslash \mathcal{U}$, and (ii) all the closed-loop transfer functions are proper.

Since the properness of closed-loop transfer functions is guaranteed, we have

Fact 1. The system Σ is $\mathcal{U}$-stable

$$\Leftrightarrow \quad Z[\chi] \subset \mathbb{C}\backslash\mathcal{U} .$$ ∎

Given f proper, the properness of π is closely related to the system I/O map $h_{y_2v_1} : v_1 \mapsto y_2$; more precisely,

Fact 2. For the system of Fig. 1, let $p \in \mathbb{R}_{p,o}(s)$ and $f \in \mathbb{R}_p(s)$. Then,

$$\pi \in \mathbb{R}_p(s) \Leftrightarrow p^{-1}h_{y_2v_1} \in \mathbb{R}_p(s) .$$ ∎

Proof: By direct calculation, $h_{y_2v_1} = p(1+fp)^{-1}\pi$, or equivalently, $p^{-1}h_{y_2v_1} = (1+fp)^{-1}\pi$. Since $(1+fp)^{-1} \to 1$ as $|s| \to \infty$, the equivalence follows. ∎

4. Synthesis

It is extremely useful for the designer to know the class of I/O maps that are achievable by $\mathcal{U}$-stable closed-loop systems with proper controllers. To exhibit this, we use the device of the following

Algorithm 1. (Synthesis)

Data: (1) $p = \frac{n_{pu}n_{ps}}{d_p} \in \mathbb{R}_{p,o}(s)$ with (i) $(n_{pu}n_{ps}, d_p)$ coprime, (ii) $Z[n_{pu}] \subset \mathcal{U}$, and (iii) $Z[n_{ps}] \subset \mathbb{C}\backslash\mathcal{U}$; (4.1)

(2) $h_{y_2v_1} = \frac{n_{pu}n_{h1}}{d_h} \in \mathbb{R}_p(s)$ with (i) $p^{-1}h_{y_2v_1} \in \mathbb{R}_p(s)$, (ii) (n_{h1}, d_h) coprime, and (iii) $Z[d_h] \subset \mathbb{C}\backslash\mathcal{U}$. (4.2)

Step 1: Choose monic $\chi \in \mathbb{R}[s]$ such that

(1) $Z[\chi] \subset \mathbb{C}\backslash\mathcal{U}$; (4.3)

(2) $\partial\chi \geq 2.\partial d_p - 1$; (4.4)

(3) $n_{ps}d_h | n_{h1}\chi$. (4.5)

Set

$$n_\pi := \frac{n_{h1}\chi}{n_{ps}d_h} . \tag{4.6}$$

Step 2: Choose $n_f \in \mathbb{R}[s]$ such that

(1) $\partial n_f \leq \partial\chi - \partial d_p$; (4.7)

(2) $d_p | (\chi - n_f n_p)$. (4.8)

Set

$$d_c := \frac{(\chi - n_p n_f)}{d_p} . \tag{4.9}$$ ∎

Comments

(a) p strictly proper and condition (4.7) imply that $f := n_f/d_c$ given by Algo. 1 is proper. Indeed, (4.7) gives $\partial n_f + \partial n_p \leq \partial\chi - \partial d_p + \partial n_p$. Now, since p is strictly proper, $\partial n_f n_p < \partial\chi$. So (4.9) implies $\partial d_c = \partial\chi - \partial d_p$. Thus, by (4.7), $f := n_f/d_c$ is proper.

(b) Condition (4.4) guarantees that there will be enough parameters in the polynomial n_f such that (4.8) can be satisfied. Indeed, (4.8) imposes ∂d_p equality constraints on the coefficients of n_f. Consequently, (4.8) can be satisfied if $\partial n_f \geq \partial d_p - 1$, or equivalently, if the polynomial n_f has at least ∂d_p coefficients to be adjusted. Now, with p strictly proper and f proper (see (a) above), we have $\partial\chi = \partial d_p + \partial d_c$; hence, condition (4.4) reads $\partial d_p + \partial d_c \geq 2\partial d_p - 1$, or equivalently, $\partial d_c \geq \partial d_p - 1$. Consequently, condition (4.4) allows us to choose n_f such that $\partial n_f \geq \partial d_p - 1$.

(c) The expression $h_{y_2v_1} = n_\pi \frac{1}{\chi} n_p$ and (4.6) show that the resulting I/O map is actually that required in (4.2).

(d) The polynomials n_f and d_c given by the algo may have common factors. By (4.9), such common factors must be factor of χ, and hence have all their zeros in $\mathbb{C}\backslash\mathcal{U}$: thus, if present, they do not upset the $\mathcal{U}$-stability of the design. Furthermore, the three polynomials n_π, n_f and d_c may have common factors. Such common factors, with zeros necessarily in $\mathbb{C}\backslash\mathcal{U}$, should of course be removed before realizing the required controller. ∎

5. CAD Considerations

The computer - an efficient number cruncher - and nonlinear programming algorithms (see e.g. [Bha.1]) - i.e. algorithms that optimize over a parameter set defined by a finite or infinite number of inequality constraints - suggest a design philosophy very different from the synthesis one. In synthesis, one is given the precisely defined goal and the algorithm delivers a design meeting that goal: often the resulting design is not acceptable because too big or too small parameters are required. To avoid this pitfall, the design procedure should lead to a parameterized family of designs, say, over a parameter set $\Omega \subset \mathbb{R}^m$ such that, $\forall z \in \Omega$, the design obeys the main requirement (e.g. properness of compensators and $\mathcal{U}$-stability). Then the parameter z is determined by optimization over Ω.

The suggested computer-aided design (CAD) methodology can be described as follows:

Algorithm 2. (Computer-Aided Design)

Data: p as in (4.1) with a frequency normalization such that the main poles and zeros are O(1).

Step 1: Let $n_f := \sum_{i=0}^{m} \alpha_{m-i} s^i$, with $m \geq \partial d_p - 1$, and leave the coefficients $(\alpha_i)_0^m$ free.

Step 2: Choose monic $\chi \in \mathbb{R}[s]$, with $\partial\chi \geq \partial d_p + m$, and include in χ a number of real parameters subject to simple inequality constraints such that, for all feasible values of those parameters, $Z[\chi] \subset \mathbb{C}\backslash \mathcal{U}$. (For example, for $\partial d_p = 2$ and $m = 1$, let

$$\chi := (\frac{s}{\beta\omega_0} + 1) [(\frac{s}{\omega_0})^2 + 2\zeta(\frac{s}{\omega_0}) + 1]$$

with three parameters subject to say, $\omega_0 \geq 0.5$, $0.7 \leq \zeta \leq 1.2$ and $\beta \geq 1$).

Step 3: Obtain ∂d_p linear algebraic constraints on $(\alpha_i)_0^m$ by requiring that

$$d_p | (\chi - n_p n_f) .$$

Step 4: Let $n_\pi := \sum_{i=0}^{k} \gamma_{k-i} s^i$, with $k \leq \partial\chi - \partial d_p$, and leave the coefficients $(\gamma_i)_0^k$ free.

Step 5: Obtain the expressions

$$h_{y_2v_1} = \frac{n_p n_\pi}{\chi} ; \; h_{e_2v_1} = \frac{d_p n_\pi}{\chi} ;$$

$$h_{y_2d_0} = 1 - \frac{n_p n_f}{\chi} .$$

Use nonlinear programming algorithm [Bha.1] to adjust the parameters in χ, n_f and n_π so that design goals are achieved. Typically, this is done by (i) requiring "nice" properties of the I/O map $h_{y_2v_1}$ (e.g., "large" bandwidth, "good" step response, ...), and (ii) putting bounds on the output-disturbance sensitivity and on the size of signals say, at the plant input. The bounds can be implemented by imposing the following inequality constraints:

$$\max_{0\leq\omega\leq\omega_1} |h_{y_2d_0}(j\omega)| \leq \ell_1 ; \; \max_{0\leq\omega\leq\omega_2} |h_{e_2v_1}(j\omega)| \leq \ell_2 .$$

■

Comment: This process leads to some "optimal" design or, better, trade-off curves so that the designer may select the trade-off between conflicting design goals. (For examples of such designs see [Gus.1]). ■

6. Tracking

The inputs to be tracked are specified (in terms of Laplace transforms) to belong to the class

$$\Psi := \{\frac{\nu}{\psi} : \nu \in \mathbb{R}[s] \text{ with } \partial\nu < \partial\psi\} \qquad (6.1)$$

where $\psi \in \mathbb{R}[s]$ is a given monic polynomial with

$$Z[\psi] \subset \mathbb{C}_+ \subset \mathcal{U}; \text{ and} \qquad (6.2)$$

$$Z[\psi] \cap Z[n_p] = \phi. \qquad (6.3)$$

We say that the system Σ achieves robust asymptotic tracking over the class Ψ if and only if

(a) Σ is $\mathcal{U}$-stable;

(b) $\forall v_1 \in \Psi$, the tracking error $\check{\eta}(t) = \check{y}_2(t) - \check{v}_1(t) \to 0$ exponetially as $t \to \infty$;

(c) the tracking requirement (b) holds for any perturbed plant $\tilde{p} := \tilde{n}_p/\tilde{d}_p \in \mathbb{R}_{p,o}(s)$ where $\tilde{n}_p$ and $\tilde{d}_p \in \mathbb{R}[s]$ are arbitrary subject to (i) $(\tilde{n}_p, \tilde{d}_p)$ are coprime, (ii) $Z[\psi] \cap Z[\tilde{n}_p] = \phi$, and (ii) $Z[\tilde{\chi}] \subset \mathbb{C}_-$, where $\tilde{\chi} := d_c\tilde{d}_p + n_f\tilde{n}_p$.

Fact 3. The system Σ achieves robust asymptotic tracking over Ψ;

$$\Leftrightarrow \begin{cases} \text{(i)} \quad Z[\chi] \subset \mathbb{C}\backslash\mathcal{U}; & (6.6) \\ \text{(ii)} \quad \psi | d_c; & (6.7) \\ \text{(iii)} \quad \psi | (n_\pi - n_f) . & (6.8) \end{cases}$$

■

Proof:

Note that $\forall v_1 \in \Psi$, the tracking error η is given by

$$\eta := (h_{y_2v_1} - 1)v_1 = \frac{[n_p(n_\pi - n_f) - d_p d_c]}{\chi} \frac{\nu}{\psi} . \qquad (6.9)$$

$\Rightarrow$. By inspection, from (6.6), (6.7), and (6.8), $\forall\nu \in \mathbb{R}[s]$ with $\partial\nu < \partial\psi$, we have $P[\eta] \subset \mathbb{C}\backslash\mathcal{U} \subset \mathbb{C}_-$ and $P[\tilde{\eta}] \subset \mathbb{C}_-$ for the perturbed systems under consideration. Hence robust asymptotic tracking follows.

$\Leftarrow$. For all the perturbed systems under consideration, we have $Z[\tilde{\chi}] \subset \mathbb{C}_-$; so the only way to have $P[\tilde{\eta}] \subset \mathbb{C}_-$ is to have $\psi | d_c$ and $\psi | (n_\pi - n_f)$. ■

Note that (a) (6.6) and (6.7) imply that $Z[\psi] \cap Z[n_p] = \phi$, as expected; (b) (6.9) shows that if ζ is a zero of ψ of order m, then $h_{y_2v_1}(\zeta) = 1$ and $h^{(i)}_{y_2v_1}(\zeta) = 0$, for $i = 1, \cdots, m-1$.

It is easy to verify that the following algo leads to a system that achieves robust asymptotic tracking.

Algorithm 3. (Robust Asymptotic Tracking)

Data: (1) p as in (4.1);
(2) Ψ specified by (6.1), (6.2) and (6.3).

Step 1: Choose monic $\chi \in \mathbb{R}[s]$ such that

(1) $Z[\chi] \subset \mathbb{C} \backslash \mathcal{U}$;

(2) $\partial\chi \geq \partial\psi + 2\,\partial d_p - 1$

Step 2: Choose $n_f \in \mathbb{R}[s]$ such that

(1) $\partial n_f \leq \partial\chi - \partial d_p$;

(2) $(\psi d_p) | (\chi - n_p n_f)$.

Set

$$d_c := \frac{\chi - n_p n_f}{d_p} \quad .$$

Step 3: Choose $n_\pi \in \mathbb{R}[s]$ s.t.

(1) $\partial n_\pi \leq \partial\chi - \partial d_p$;

(2) $\psi | (n_\pi - n_f)$. ∎

7. Conclusion

(a) Three design algorithms for model matching, computer-aided design, and robust asymptotic tracking, respectively, are presented.

(b) The results obtained for continuous time systems extend readily to discrete-time systems by simply replacing s, $\mathbb{C}_-$, and $\mathbb{C}_+$ by z, D(1) ($:= \{z \in \mathbb{C} : |z| < 1\}$), and $\mathbb{C} \backslash D(1)$, respectively. ∎

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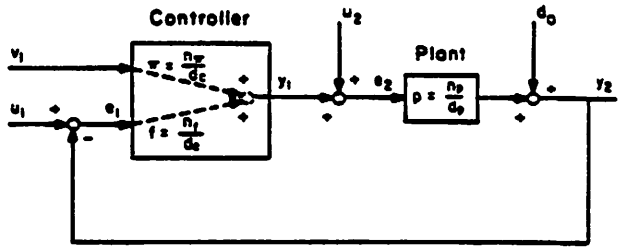

Fig. 1 The system under consideration.

References

[Ast.1] K.J. Åstrom, "Robustness of a design method based on assignment of poles and zeros," IEEE. Trans. Automat. Contr., vol. AC-25, pp. 588-591, June 1980.

[Bha.1] M.A.Bhatti, E. Polak and K.S. Pister, "OPTDYN - A general purpose optimization program for problems with or without dynamic constraints," Report No. UCB/EERC-79/16, Univ. of California, Berkeley, July 1979.

[Bon.1] J.J. Bongiorno, Jr. and D. C. Youla, "On the design of single-loop single-input-output feedback control systems in the complex-frequency domain," IEEE Trans. Automa. Contr., vol. AC-22, pp. 416-423, June 1977.

[Gus.1] C.L. Gustafson and C.A. Desoer, "Controller design for linear multi-variable feedback systems with stable plant using optimization with inequality constraints," Univ. of California, Berkeley, Memo. UCB/ERL M81/51, 1981.

[Kai.1] T. Kailath, Linear Systems, Englewood Cliffs, N.J.: Prentice-Hall, 1980.