STRUCTURE-PRESERVING MODEL REDUCTION WITH APPLICATIONS TO POWER SYSTEM DYNAMIC EQUIVALENTS

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STRUCTURE-PRESERVING MODEL REDUCTION WITH APPLICATIONS TO POWER SYSTEM DYNAMIC EQUIVALENTS

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ABSTRACT

Consider systems that have an intrinsic mathematical representation of the form: $A_n\underline{x}^{(n)} + \cdots + A_0\underline{x} = B\underline{u}$, where $\underline{x}^{(i)}$ is the ith derivative of $\underline{x}$. The matrices A_i may possess certain properties. The problem is to construct a reduced model having the same form with corresponding matrices $\bar{A}_i$ of smaller dimension and possessing the same properties. An algorithm for this structure-preserving model reduction is presented. An algorithm for constructing an approximate reduced model, called ε-structure preserving reduced model is also presented, together with the error bounds. The application of the method to power system dynamic equivalents is described.

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I. INTRODUCTION

Model reduction, i.e., constructing a reduced model of a given system, is essential in the analysis and simulation of many practical large systems. Frequently for systems such as interconnected power systems [1], or space craft [2,3], the modeling leads naturally to a mathematical representation of the form:

$$A_n \underline{x}^{(n)} + A_{n-1} \underline{x}^{(n-1)} + \cdots + A_0 \underline{x} = B\underline{u}$$

where $\underline{x} \in \mathbb{R}^m$, $\underline{x}^{(i)}$ denotes the i-th derivative of $\underline{x}$, and A_i, $i = 0,1,\cdots,n$, are mxm real matrices. The matrices A may possess certain properties, for example, symmetry, diagonal, or positive-definiteness. It is desired to construct a reduced model which preserves the structure of the physical system. That is, the mathematical representation of the reduced system still has the form:

$$\overline{A}_n \underline{x}_r^{(n)} + \overline{A}_{n-1} \underline{x}_r^{(n-1)} + \cdots + \overline{A}_0 \underline{x}_r = \overline{B}\underline{u}$$

where $\underline{x}_r$ is of lower dimension than m and the matrices $\overline{A}_i$ possess the same properties as A_i. We shall refer to this problem as the <u>structure-preserving (SP) minimal realization</u>. Moreover sometimes it is desired to preserve the identity of certain variables in $\underline{x}$ corresponding to the subsystem under study and to have a <u>structural-preserving model reduction of the external system</u>, or simply <u>SP model reduction</u>. The SP minimal realization is a special case of the SP model reduction.

Preserving the structure of the physical system in the reduced model is necessary for the reduced model to be analyzed by the same large computer simulation program. For example transient stability simulation programs are available for the analysis of power networks consisting of hundreds of generators and interconnecting lines. One would like to use the same simulation program to study a reduced model of an even larger system. Conventional model reduction methods, either time-domain approach [4,5] or frequency-domain approach [6,7], lead to a set of reduced-order equations which can no longer be interpreted as a representation of the same physical system.

In this paper a structure-preserving model reduction scheme is presented. The development of the method utilized concepts of reachability grammian and its singular value decomposition, which are collected for easy reference in Sec. 2. The structure-preserving minimal realization is discussed in Sec. 3. An exact SP model reduction scheme is presented in Sec. 4.1. A practical algorithm to construct an approximate reduced model, called ε-structure preserving reduced model, is presented next. Bounds on the resulting errors in system variables are given. The application of the method to power systems dynamic equivalent is presented in Sec. 5. This is a generalization of our previous work on ε-coherency dynamic equivalent of power systems [8].

2. PRELIMINARIES

2.1. Grammian and Reachability Set

Consider the linear time-invariant system

$$\underline{\dot{x}} = A\underline{x} + B\underline{u} \quad \underline{x}(0) = \underline{0} \tag{1}$$

where $A \in \mathbb{R}^{nxn}$ and the admissible input $\underline{u}$ satisfying the constraint

$$\int_0^\tau \|\underline{u}(t)\|^2 dt \leq a^2 \tag{2}$$

The reachability grammian at τ is defined to be the matrix W_τ^2.

$$W_\tau^2 := \int_0^\tau e^{At} BB^T e^{A^T t} \, dt \tag{3}$$

Note that W_τ^2 is real, symmetric and positive semidefinite. Therefore we have

$$W_\tau^2 = U\,\Sigma^2\,U^T \tag{4}$$

where $\Sigma^2 = \mathrm{diag}(\sigma_1^2,\sigma_2^2,\cdots,\sigma_r^2,0,\cdots,0)$, $\sigma_1^2 \geq \sigma_2^2,\cdots \geq \sigma_r^2 > 0$, are the real eigenvalues of W_τ^2 and columns of U are the corresponding orthonormal eigenvectors. The positive root of W_τ^2, W_τ, is defined to be

$$W_\tau := U\Sigma U^T \tag{4a}$$

where $\Sigma = \mathrm{diag}(\sigma_1,\sigma_2,\cdots,\sigma_r,\ 0,\cdots 0)$. $\sigma_1 \geq \sigma_2 \cdots \geq \sigma_r > 0$.

The <u>set of reachable states at τ</u> is given by

$$S_\tau := \{\underline{x} \mid \underline{x} = \int_0^\tau e^{A(\tau-t)}B\underline{u}(t)dt,\ \int_0^\tau \|\underline{u}(t)\|^2 dt \leq a^2\} \tag{5}$$

and the <u>set of reachable states in $[0,\tau]$</u> is given by

$$S_{[0,\tau]} := \{\underline{x} \mid \exists t \in [0,\tau] \ \text{ s.t. } \ \underline{x} = \int_0^t e^{A(t-t')}B\underline{u}(t')dt' \text{ , and } \int_0^\tau \|\underline{u}(t)\|^2 dt \leq a^2\} \tag{6}$$

also, the image S under the map W_τ of the ball with radius a.

$$S := \{\underline{x} \mid \underline{x} = W_\tau \underline{p},\ \ \|\underline{p}\| \leq a\} \tag{7}$$

<u>Fact 1</u>. $S_{[0,\tau]} = S_\tau = S$ [8-9]

2.2. <u>Singular Value Decomposition of the Grammian</u>

Consider the set

$$S = \{\underline{x} \mid \underline{x} = W_\tau \underline{p},\ \ \|\underline{p}\| \leq a\}$$

where $W_\tau = U\Sigma U^T$ and $\Sigma = \mathrm{diag}(\sigma_1,\cdots\sigma_r,\ 0,\cdots 0)$

If we change coordinates to a basis formed by the orthonormal columns of U, i.e., $\underline{x} = U\underline{x}'$, $\underline{p} = U\underline{p}'$, then the set S can be described as follows:

$$S = \{\underline{x}' \,|\, (\frac{x_1'}{\sigma_1})^2 + (\frac{x_2'}{\sigma_2})^2 + \cdots + (\frac{x_r'}{\sigma_r})^2 \leq a^2, \quad x_{r+1}' = \cdots = x_n' = 0\} \tag{8}$$

Thus S is an r-dimensional ellipsoid in $\mathbb{R}^n$.

2.3. Differential Equation of the Grammian

The reachability grammian W_τ^2 can be obtained from the solution of a linear matrix differential equation as stated in the following fact, whose proof is immediate by the definition of W_τ^2.

Fact 2. [10, p. 84] The reachability grammian W_t^2 as a function of t, satisfies the following linear matrix differential equation

$$\dot{X} = AX + XA^T + BB^T, \quad X(0) = 0 \tag{9}$$

3. STRUCTURE-PRESERVING MINIMAL REALIZATION

3.1. System Representation

Consider the first-order representation of the zero-state response of a linear time-invariant differential dynamical system,

$$\dot{\underline{z}} = A\underline{z} + Bu, \quad \underline{y} = C\underline{z}, \quad \underline{z}(0) = \underline{0} \tag{10}$$

We shall use F[A,B,C], or $F(\underline{z})$, or simply F to denote the representation (10).

Consider next the n-th order representation of a linear time-invariant differential dynamical system,

$$A_n\underline{x}^{(n)} + A_{n-1}\underline{x}^{(n-1)} + A_{n-2}\underline{x}^{(n-2)} + \cdots + A_0\underline{x} = B\underline{u}, \quad \underline{y} = \underline{x},$$
$$A_n \text{ nonsingular}, \quad \underline{x}^{(n-1)}(0) = \underline{x}^{(n-2)}(0) = \cdots = \underline{x}(0) = \underline{0}, \quad \underline{x} \in \mathbb{R}^m \tag{11}$$

where $\underline{x}^{(i)}$ is the i-th derivative of $\underline{x}$ with respect to t, and all A_i's are real mxm matrices. We shall use $N[A_n,\cdots A_0,B,I]$, or $N(\underline{x})$, or simply N to denote the representation (11). We shall call n the order of the representation, and m the dimension of the representation.

Now if we consider the class of all first order representation R^1 with the form of (10),

$$R^1 := \{F(z)\,|\,F \text{ is represented as (10) for some matrices A, B, C}\} \quad (12)$$

and the class of all n-th order representation R^n with form (11),

$$R^n := \{N(\underline{x})\,|\,N \text{ is represented as (11), for some matrices } A_n, A_{n-1}, A_{n-2}, \cdots A_0, B\} \quad (13)$$

Clearly, if $N(\underline{x}) \in R^n$, choose

$$z := \begin{pmatrix} \underline{x} \\ \underline{x}^{(1)} \\ \vdots \\ \underline{x}^{(n-1)} \end{pmatrix} \quad (14)$$

as state variable, then it is obvious that $N(\underline{x})$ can be represented by a first-order representation $F(\underline{z})$. Hence $R^n \subset R^1$. $F(\underline{z})$ is called the embedded first-order representation of $N(\underline{x})$. Let us define the embedding map Φ,

$\Phi : R^n \to R^1$, such that for any $N(\underline{x}) \in R^n$, $\Phi(N(\underline{x})) = F(\underline{z})$, $F(\underline{z})$ is the embedded first-order representation of $N(\underline{x})$.

3.2. Structure-Preserving Equivalent

For a dynamical system which is modeled as an n-th order system (11), we are interested in finding a minimal realization of (11) within the same class R^n. Roughly speaking, we are not only looking for a system representation with least complexity, but also preserving the physical structure of the model.

Two representation $N(\underline{x})$, $N'(\underline{x}')$, are said to be SP equivalent iff $N(\underline{x}) \in R^n$, $N'(\underline{x}') \in R^n$, and $\underline{y}'(\cdot) = \underline{y}(\cdot)$ for $\forall\ \underline{u}(\cdot)$. Note that the SP equivalent is defined only for zero state equivalent. The SP reachable space is defined for $N(\underline{x}) \in R^n$ as,

$$S_N := \{\underline{x} \mid \exists \underline{u}(\cdot), \text{ for some } t \geq 0,\ \underline{x}(t) = \underline{x}, \text{ and } x(\cdot) \text{ satisfies (11)}\} \tag{15}$$

We say that $N(\underline{x})$ is SP completely reachable iff $\dim S_N = \dim(\underline{x})$. And $N'(\underline{x}')$ is said to be a Structural-preserving minimal realization, or SP minimal realization of $N(\underline{x})$ iff $N'(\underline{x}')$ is SP equivalent to $N(\underline{x})$ and $N'(\underline{x}')$ is SP completely reachable.

3.3. Structure-Preserving Minimal Realization v.s. Conventional Minimal Realization

A minimal realization on R^n is not necessary a minimal realization on R^1. In this section we study the implications when an SP minimal realization will also be a minimal realization on R^1.

Let the reachability grammian at τ, W_τ^2 for $N(\underline{x}) \in R^n$ be defined as the reachability grammian at τ of its embedded first-order representation F[A,B,C]

$$W_\tau^2 := \int_0^\tau e^{At} B B^T e^{A^T t} dt \tag{16}$$

The first m rows of W_τ is denoted, by H, $m = \dim(\underline{x})$. Thus H is a real mxmn matrix.

Theorem 1: For any $\tau > 0$, minimal realization of $\Phi(N(\underline{x}))$ is algebraically equivalent to Φ (SP minimal realization of $N(\underline{x})$) iff

$$\dim R(W_\tau) = n \dim R(H)$$

where $R(W_\tau)$, $R(H)$ denote the range space of W_τ, H respectively.

Proof

For a linear time invariant system, the range space of W_τ for any $\tau > 0$ equals to the reachability space (controllability space). That is, $(\underline{x}^T, \dot{\underline{x}}^T, \cdots \underline{x}^{(n-1)^T})^T \in R(W_\tau)$, and $\underline{x} \in R(H)$. Besides $\Phi(N)$ is completely observable. Thus the dimension of minimal realization of $\Phi(N(\underline{x}))$ equals dim $R(W_\tau)$, and dim S_N = dim R(H). But the dimension of Φ (SP minimal realization of $N(\underline{x})$) equals to n x dim S_N. Therefore,

dim $R(W_\tau)$ = n dim R(H) for any $\tau > 0$

iff dim of minimal realization of $\Phi(N(\underline{x}))$

= dim of Φ (SP minimal realization of $N(\underline{x})$)

iff Φ(SP minimal realization of $N(\underline{x})$) is a minimal realization

iff minimal realization of $\Phi(N(\underline{x}))$ is algebraically equivalent to

to Φ (SP minimal realization of $N(\underline{x})$) [11]. ¤

4. STRUCTURE-PRESERVING MODEL REDUCTION

Sometimes in model reduction it is desired to preserve the identity of a subsystem, called the study system. The rest of the system will be called the external system. A reduced model (i) which retains the interconnection of the study system and (ii) whose reduced external system is an SP minimal realization of the external system is called an SP reduced model. The SP model reduction to be presented below includes the special case where the study system is empty, i.e., constructing an SP minimal realization of a system.

4.1. Structure-Preserving Model Reduction

Let us consider an n-th order matrix differential system $N(\underline{x})$ as described by (11). Without loss of generality, we renumber the components of the vector $\underline{x}$ such that the first q components are associated with the study system, the last m-q components are associated with the external system. The reachability grammian at $\tau > 0$ of $N(\underline{x})$, W_τ^2, and its positive root, W_τ, are defined in (16) and (4a) respectively. Let us partition W_τ into n blocks.

$$W_\tau := \begin{bmatrix} W_1 \\ W_2 \\ \vdots \\ W_n \end{bmatrix} \tag{17a}$$

where all W_i's $\in \mathbb{R}^{mxmn}$.

Let us further partition W_1 into

$$W_1 := \begin{bmatrix} W_{S1} \\ W_{E1} \end{bmatrix} \tag{17}$$

where $W_{S1} \in \mathbb{R}^{qxmn}$, $W_{E1} \in \mathbb{R}^{(m-q)xmn}$

Let the singular values of W_{E1} be ordered as $\sigma_1 \geq \sigma_2 \cdots \geq \sigma_r > \sigma_{r+1} = \cdots = \sigma_{m-q} = 0$, and $\Sigma_1 = [\Sigma_{11}, 0] \in \mathbb{R}^{(m-q)xmn}$, where $\Sigma_{11} = \text{diag}(\sigma_1, \cdots \sigma_r, \cdots \sigma_{m-q})$. Let U_1 denote the (m-q)x(m-q) matrix whose columns are the left singular vectors of W_{E1}. Partition $U_1 = [U_{11}, U_{12}]$, where $U_{11} \in \mathbb{R}^{(m-q)xr}$ and $U_{12} \in \mathbb{R}^{(m-q)x(m-q-r)}$. Let V_1 denote the (mn)x(mn) matrix whose columns are the right singular vectors of W_{E1}.

The singular value decomposition (S.V.D.) of W_{E1} can be written as

$$W_{E1} = U_1 \Sigma_1 V_1^T \tag{18}$$

Let us define a mx(q+r) matrix

$$K := \begin{bmatrix} I & 0 \\ 0 & U_{11} \end{bmatrix} \tag{19}$$

Theorem 2.

$$N_r(\underline{x}_r): \quad \bar{A}_n\underline{x}_r^{(n)} + \bar{A}_{n-1}\underline{x}_r^{(n-1)} + \cdots + \bar{A}_0\underline{x}_r = \bar{B}\underline{u}, \quad \underline{y}_r = \bar{C}\underline{x}_r$$

$$\bar{A}_n \text{ nonsingular}, \ \underline{x}_r^{(n-1)}(0) = \cdots = \underline{x}_r(0) = \underline{0} \tag{20}$$

is a SP reduced model of $N(\underline{x})$ defined by (11).

where $\bar{A}_i := K^T A_i K$

$\bar{B} := K^T B$

$\bar{C} := K$

Furthermore, if A_i, $i = 0,1,\cdots n$, are symmetry (skew symmetry, positive definite, negative definite, resp.), $\bar{A}_i$, $i = 0,1,\cdots n$, are symmetry (skew symmetry, positive definite, negative definite, resp.)

Proof

It is clear that K retains the interconnection of study sytem. By Fact 1 and the proof of Theorem 1, $\exists$ $\underline{g}(t)$ such that

$$\underline{x}(t) = W_1\underline{g}(t) \tag{21}$$

Therefore

$$\bar{U}_1^T\underline{x}(t) = \underline{0} \qquad \text{for } \forall t \geq 0 \tag{22}$$

where

$$\bar{U}_1^T := [0, U_{12}^T], \quad \bar{U}_1 \in R^{mx(m-q-r)}$$

Equation (22) also implies $\bar{U}_1^T\underline{x}^{(i)}(t) = \underline{0}$, $\forall t \geq 0$, $\forall i = 0,1,2,\cdots$

Hence

$$\begin{aligned} A_i\underline{x}^{(i)} &= A_i[K,\bar{U}_1][K,\bar{U}_1]^T \underline{x}^{(i)} \\ &= A_iKK^T\underline{x}^{(i)} \end{aligned} \tag{23}$$

Substitute (23) into (11) and premultiply the resulting equation by K_1^T. It is immediately that $K^T\underline{x}$ satisfies (20). By the uniqueness of differential equations, we have

$$K^T\underline{x} = \underline{x}_r \tag{24}$$

Because of (22) and the fact that $[K,\bar{U}_1]$ is unitary, we have

$$\begin{aligned} \underline{x} &= [K,\bar{U}_1][K,\bar{U}_1]^T\underline{x} \\ &= KK^T\underline{x} + \bar{U}_1\bar{U}_1^T\underline{x} \\ &= K\underline{x}_r \end{aligned} \tag{25}$$

Equation (25) shows that $\underline{y} = \underline{y}_r$, therefore $N_r(\underline{x}_r)$ is SP equivalent to $N(\underline{x})$. Besides by (21) and (24) the reachable space of the external system of $N_r(\underline{x}_r)$ is SP completely reachable. Therefore the external system of $N_r(\underline{x}_r)$ is a SP minimal realization of that of $N(\underline{x})$. The nonsingularity of $\bar{A}_n$ follows immediately from the fact that A_n is nonsingular and K is of full rank.

Clearly $\bar{A}_i$ preserves symmetry, etc. ¤

Remark. If we let q = 0, i.e., there is no subsystem to be retained, the above SP model reduction gives a SP minimal realization of the system.

4.2. ε-Structure Preserving Model Reduction

In practical implementation some singular values of W_{E1} are very small but not exactly zero. For all practical purposes they should be treated as zero. In this section we present an SP model reduction which takes into account of this fact. We also present in Theorem 3 the error bounds for this approximation.

In this section we assume that

$$\sigma_1 \geq \sigma_2 \geq \cdots\cdot \geq \sigma_r >> \varepsilon = \sigma_{r+1} \geq \cdots\cdot \geq \sigma_{m-q} \geq 0 \tag{26}$$

For ease of presentation let us consider second order matrix differential equations only. The extension to higher order system is obvious.

The matrix W_2 in (17a) is partitioned into

$$W_2 = \begin{bmatrix} W_{S2} \\ W_{E2} \end{bmatrix} \tag{27}$$

where $W_{S2} \in \mathbb{R}^{qxmn}$, $W_{E2} \in \mathbb{R}^{(m-q)xmn}$.

Let the singular values of $U_{12}^T W_{E2}$ be ordered as

$$\bar{\sigma}_1 \geq \bar{\sigma}_2 \cdots \geq \bar{\sigma}_j > \varepsilon \geq \bar{\sigma}_{j+1} \cdots \geq \bar{\sigma}_{m-q-r} \geq 0\ . \tag{28}$$

and $\Sigma_2 = [\Sigma_{22}, 0] \in \mathbb{R}^{(m-q-r)x(mn)}$ where $\Sigma_{22} = \text{diag}(\bar{\sigma}_1, \cdots \bar{\sigma}_{m-q-r})$. Let U_2 be the (m-q-r) x (m-q-r) matrix whose columns are the left singular vectors of $U_{12}^T W_{E2}$, and $U_2 = [U_{21}, U_{22}]$ where $U_{21} \in \mathbb{R}^{(m-q-r)xj}$ and $U_{22} \in \mathbb{R}^{(m-q-r)x(m-q-r-j)}$. Let V_2 be the (mn)x(mn) matrix whose columns are the right singular vectors of $U_{12}^T W_{E2}$. The S.V.D. of $U_{12}^T W_{E2}$ can be written as

$$U_{12}^T W_{E2} = U_2 \Sigma_2 V_2^T$$

For the case where n = 2, (11) becomes

$$N(\underline{x}):\quad A_2\underline{\ddot{x}} + A_1\underline{\dot{x}} + A_0\underline{x} = B\underline{u},\quad \underline{y} = \underline{x}\ ,$$

$$A_2 \text{ nonsingular } \underline{\dot{x}}(0) = \underline{x}(0) = \underline{0} \tag{29}$$

Let

$$\bar{U}_{22} := U_{12}U_{21} \tag{30a}$$

and

$$K_1 := \begin{bmatrix} I & 0 & 0 \\ 0 & U_{11} & \bar{U}_{22} \end{bmatrix} \in \mathbb{R}^{m\times(q+r+j)} \tag{30b}$$

We define the ε-SP reduced model to be the system:

$$N_r(\underline{x}_r): \quad K_1^T A_2 K_1 \ddot{\underline{x}}_r + K_1^T A_1 K_1 \dot{\underline{x}}_r + K_1^T A_0 K_1 \underline{x}_r = K_1^T B \underline{u}, \quad \underline{y}_r = K_1 \underline{x}_r$$

$$\dot{\underline{x}}_r(0) = \underline{x}_r(0) = \underline{0} \tag{30}$$

Next we will present error bounds on the approximate ε-SP reduced model and the original system. We will separate the errors in the study system response and the external system response. The error vector $\underline{e}(t)$ between (29), (30) is defined to be

$$\underline{e}(t) := \begin{bmatrix} \underline{e}_S(t) \\ \underline{e}_E(t) \end{bmatrix} := \underline{y}(t) - \underline{y}_r(t)$$

where $\underline{e}_S(t) \in \mathbb{R}^q$ denotes the error vector of the study system and $\underline{e}_E \in \mathbb{R}^{m-q}$ denotes the error vector of the external system.

Let us consider the set of inputs $\underline{u}(\cdot)$, such that

(i) $\underline{u}(t)$ is continuous on $(0,\tau)$

(ii) $\underline{u}(t) = \underline{u}_1(t) + \underline{v}$

where $\underline{v}$ is the discontinuity of $\underline{u}(t)$ at $t = 0$, $\underline{u}_1(0) = \underline{0}$,

(iii) $\underline{u}_1(t)$ is piecewise differentiable on $[0,\tau)$.

(iv) $\int_0^\tau \|\underline{u}(t)\|^2 dt \le a^2$

(v) $\int_0^\tau \|\dot{\underline{u}}_1(t)\|^2 dt \le h^2$

Let us denote the set by

$$L(a,h) := \{\underline{u}(\cdot) | (i)-(v) \text{ are satisfied}\}. \tag{31}$$

Let us define the following quantities

$$\ell_1 := \|A_2\|h + \|A_1\|a + \|A_0\|a \tag{32}$$

where $\|\cdot\|$ is the Euclidean norm.

$$K_2^T := [0,(U_{12}U_{22})^T]$$

$$\ell_2^2 := \int_0^\tau \underline{v}^T B^T e^{A^T t} C^T K_2 K_2^T A_2^T A_2 K_2 K_2^T C e^{At} B \underline{v} dt, \quad F[A,B,C] = \Phi(N(\underline{x})) \ . \tag{33}$$

Let $(\sigma_1')^2$ be the largest singular value of the reachability grammian at τ of $N(\bar{A}_2,\bar{A}_1,\bar{A}_0,K_1^T,I)$

<u>Theorem 3</u>. For $\forall\ \underline{u}(\cdot) \in L(a,h)$

1. $$\max_{t\in[0,\tau]} \|\underline{e}_S(t)\| \le \sigma_1'(\tau\ell_1^2\varepsilon^2+\ell_2^2+2\tau^{1/2}\ell_1\ell_2\varepsilon)^{1/2} \tag{34}$$

2. $$\max_{t\in[0,\tau]} \|\underline{e}_E(t)\| \le \varepsilon a + \sigma_1'(\tau\ell_1^2\varepsilon^2+\ell_2^2+2\tau^{1/2}\ell_1\ell_2\varepsilon)^{1/2} \tag{35}$$

<u>Remark</u>: $W_0^2 := \int_0^\tau B^T e^{A^T t} C^T K_2 K_2^T A_2^T A_2 K_2 K_2^T e^{At} B\ dt$ can be interpreted as the observability grammian at τ of $\Phi(N(A_2,A_1,A_0,B,A_2K_2K_2^T))$. Thus $\ell_2^2 = \underline{v}^T W_0^2 \underline{v}$.

<u>Proof of Theorem 3</u>.

We shall express $\underline{e}(t)$ as the sum of $\underline{x}-K_1K_1^T\underline{x}$ and $K_1K_1^T\underline{x}-K_1\underline{x}_r$.

1. The matrix $[K_1,K_2]$ is unitary, where $K_2^T := [0,(U_{12}U_{22})^T] \in \mathbb{R}^{(m-q-r-j)xm}$

By Fact 1, $\exists\ \underline{p}(t)$, $\|\underline{p}(t)\| \le a$, $0 \le t \le \tau$ such that

$$\underline{x} - K_1K_1^T\underline{x} = K_2K_2^T\underline{x}$$
$$= \begin{bmatrix} 0 & 0 \\ 0 & U_{12}U_{22}U_{22}^TU_{12}^T \end{bmatrix} W_1\underline{p}(t) \tag{36}$$

Substitute (18), into (36), it can be easily shown that:

$$(\underline{x}-K_1K_1^T\underline{x})_S = \underline{0} \tag{37}$$

and

$$\|(\underline{x}-K_1K_1^T\underline{x})_E\| \leq \varepsilon a \tag{38}$$

where $(\underline{x})_S$ and $(\underline{x})_E$ denote the first q and last m-q components of $\underline{x}$ respectively.

2. Let $\underline{z}$ denote the state vector of embedded first order system of $N(\underline{x})$. The Laplace transform of $\underline{z}(t)$ is $\underline{Z}(s)$,

$$\underline{Z}(s) = (sI-A)^{-1}\, B(\underline{u}_1(s) + \frac{1}{s}\,\underline{v}) \tag{39}$$

where $\underline{u}_1(s)$ is the Laplace transform of $\underline{u}_1(t)$.

Therefore $s\underline{Z}(s) = (sI-A)^{-1}\, B(s\underline{u}_1(s)+\underline{v})$, this implies

$$\dot{\underline{z}}(t) = \underline{z}_1(t) + e^{At}B\underline{v} \tag{40}$$

where

$$\underline{z}_1 = A\underline{z}_1 + B\dot{\underline{u}}_1,\ \underline{z}_1(0) = \underline{0},\ \int_0^\tau \|\dot{\underline{u}}_1(t)\|^2 dt \leq h^2 \tag{41}$$

Note $\ddot{\underline{x}}$ is the last m components of $\dot{\underline{z}}(t)$. By (40) we have

$$K_2^T\ddot{\underline{x}} = K_2^T\underline{z}_{12}(t) + K_2^T[0,I]e^{At}B\underline{v} \tag{42}$$

where $\underline{z}_{12}$ is the last m components of $\underline{z}_1$.

Again by Fact 1,

$$\max_{t\in[0,\tau]} \|K_2^T\underline{z}_{12}(t)\| \leq \|K_2^T W_2\|h \leq \bar{\sigma}_{j+1}h \tag{43}$$

3. Now if we let $\underline{y}_1 := \underline{x}_r - K_1^T\underline{x}$, and use the fact $K_1K_1^T+K_2K_2^T = I$, it is clear $\underline{y}_1$ satisfies

$$\bar{A}_2\ddot{\underline{y}}_1 + \bar{A}_1\dot{\underline{y}}_1 + \bar{A}_0\underline{y}_1 = K_1^T(A_2K_2K_2^T\ddot{\underline{x}} + A_1K_2K_2^T\dot{\underline{x}} + A_0K_2K_2^T\underline{x}) \tag{44}$$

Thus by Fact 1,

$$\max_{t\in[0,\tau]} \|\underline{y}_1(t)\| \leq \sigma_1' a_1 \tag{45}$$

where

$$\int_0^\tau \|A_2K_2K_2^T\ddot{\underline{x}} + A_1K_2K_2^T\dot{\underline{x}} + A_0K_2K_2^T\underline{x}\|^2 dt = a_1^2 \tag{46}$$

In order to get a bound for a_1^2, let's denote,

$$\underline{f}_1(t) := A_2K_2K_2^T\underline{z}_{12} + A_1K_2K_2^T\dot{\underline{x}} + A_0K_2K_2^T\underline{x} \tag{47}$$

$$\underline{f}_2(t) := A_2K_2K_2^T[0,I]e^{At}B\underline{v} \tag{48}$$

Using (43) and the fact that $\|K_2^T\dot{\underline{x}}\| \leq \bar{\sigma}_{j+1}a$, $\|K_2^T\underline{x}\| \leq \varepsilon a$, we have

$$\max_{t\in[0,\tau]} \|\underline{f}_1(t)\| \leq \|A_2\|\bar{\sigma}_{j+1}h + \|A_1\|\bar{\sigma}_{j+1}a + \|A_0\|\varepsilon a$$

$$\leq \varepsilon\cdot\ell_1$$

By Schwarz's inequality

$$a_1^2 \leq \tau\varepsilon^2\ell_1^2 + \ell_2^2 + 2\tau^{1/2}\ell_1\ell_2\varepsilon \tag{49}$$

(45), (38) together with (49) implies:

$$\max_{t\in[0,\tau]} \|\underline{e}_S(t)\| \leq \sigma_1'(\tau\varepsilon^2\ell_1^2+\ell_2^2+2\tau^{1/2}\ell_1\ell_2\varepsilon)^{1/2}$$

$$\max_{t\in[0,\tau]} \|\underline{e}_E(t)\| \leq \varepsilon a + \sigma_1'(\tau\varepsilon^2\ell_1^2+\ell_2^2+2\tau^{1/2}\ell_1\ell_2\varepsilon)^{1/2}$$

¤

4.3. Procedure for ε-SP Model Reduction

The following procedure may be used for ε-SP model reduction of $N(\underline{x})$ defined in (11).

1. Determine ε,τ.

2. Solve

$$\dot{x} = AX + XA^T + BB^T, \; X(0) = 0$$

and set

$$W_\tau^2 = x(\tau)$$

where A, B are system matrices of the embedded first order system of $N(\underline{x})$.

3. Perform singular value decomposition of W_τ^2 and obtain W_τ.

4. Perform singular value decomposition of W_{E1} and obtain U_{11}, U_{12}. U_{11}, U_{12} are the matrices whose column vectors are the left singular vectors of W_{E1} with corresponding singular values greater than and less than ε respectively.

5. For $i = 2,\cdots,n$, perform the singular value decomposition of

$\bar{U}_i^T W_{Ei}$ and obtain U_{i1}, U_{i2}.

$\bar{U}_i := U_{12}U_{22} \cdots U_{(i-1)2}$ and U_{i1}, U_{i2} are the matrices whose column vectors are the left singular vectors of $\bar{U}_i^T W_{Ei}$ with singular values greater than and less than ε respectively.

6. Set

$$\bar{U}_{i1} = \bar{U}_i U_{i1} \; , \; \bar{U}_{i2} = \bar{U}_i U_{i2}$$

and

$$K_1 = \begin{bmatrix} I & 0 & 0 & \cdots\cdots\cdots\cdots & 0 \\ 0 & U_{11} & \bar{U}_{21} & \cdots \; \bar{U}_{i1} \; \cdots & \bar{U}_{n1} \end{bmatrix}$$

$$K_2 = \begin{bmatrix} 0 \\ \bar{U}_{n2} \end{bmatrix}$$

7. The reduced system

$$N_r(\underline{x}_r): \quad \bar{A}_n\underline{x}_r^{(n)} + \bar{A}_{n-1}\underline{x}_r^{(n-1)} + \cdots + \bar{A}_0\underline{x}_r = \bar{B}\underline{u}, \quad \underline{y}_r = \bar{C}\underline{x}_r$$

$$\underline{x}_r^{(n-1)}(0) = \cdots = \underline{x}_r(0) = \underline{0}$$

where $\bar{A}_i = K_1^T A_i K \quad \bar{B} = K_1^T B \quad \bar{C} = K_1 C$ is the ε-SP reduced system of (11).

Corollary 1. For $\forall\ \underline{u}(\cdot) \in L(a,h)$ defined in (31)

1. $\max\limits_{t\in[0,\tau]} \|\underline{e}_S(t)\| \le \sigma_1'(\tau\ell_1^2\varepsilon^2+\ell_2^2+2\tau^{1/2}\ell_1\ell_2\varepsilon)^{1/2}$

2. $\max\limits_{t\in[0,\tau]} \|\underline{e}_E(t)\| \le \varepsilon a + \sigma_1'(\tau\ell_1^2\varepsilon^2+\ell_2^2+2\tau^{1/2}\ell_1\ell_2\varepsilon)^{1/2}$

where

$$\ell_1 := \|A_n\|h + (\|A_{n-1}\| + \cdots + \|A_0\|)a$$

$$\ell_2 := \int_0^\tau \underline{v}^T B^T e^{A^T t} C^T K_2 K_2^T A_n^T A_n K_2 K_2^T C e^{At} B \underline{v}\, dt$$

$(\sigma_1')^2$ is the largest singular value of the reachability grammian at τ of $N(\bar{A}_n, \bar{A}_{n-1}, \cdots \bar{A}_0, K_1^T, I)$.

Corollary 2. For $\forall\ \underline{u}(\cdot) \in \{\underline{u}(\cdot)\,|\,\underline{u}(t)$ is piecewise differentiable on $[0,\tau)$, and $\int_0^\tau \|\underline{u}(t)\|^2 dt \le a^2$, $\int_0^\tau \|\dot{\underline{u}}(t)\|^2 dt \le h^2\}$

1. $\max\limits_{t\in[0,\tau]} \|\underline{e}_S(t)\| \le \varepsilon\sigma_1'\ell_1\tau^{1/2}$

2. $\max\limits_{t\in[0,\tau]} \|\underline{e}_E(t)\| \le \varepsilon a + \varepsilon\sigma_1'\ell_1\tau^{1/2}$

5. POWER SYSTEM DYNAMIC EQUIVALENTS

In transient stability study of power systems one is interested in the detail response in one's own service area (the study system). The effects of the rest of the interconnection (the external system) have to be modeled. A reduced model of the system is called a dynamic equivalent in power system literature. It is desired to preserve the identity of the study system. Moreover it is desired that the reduced external system be identifiable to a power system, i.e., interconnection of generators and lines.

In Sec. 5.1 the ε-SP model reduction is applied to power systems. In Sec. 5.2 an external power network is synthesized from the result. For convenience, we will use r' for (r+j) as the dimension of the reduced external system.

5.1. ε-SP Model Reduction of Power System

We will use the model [8] which consists of the linearized classical swing equations for the generators and the linearized decoupled real power flow equations for the network. The disturbances are modeled as inputs. The system is represented by the following second order matrix differential equation.

$$M\Delta\ddot{\underline{\delta}} + D\Delta\dot{\underline{\delta}} + H\Delta\underline{\delta} = B\underline{u},\ \underline{y} = \Delta\underline{\delta},\ \Delta\dot{\underline{\delta}}(0) = \Delta\underline{\delta}(0) = \underline{0} \tag{50}$$

where $\Delta\underline{\delta} = (\Delta\delta_1,\cdots\Delta\delta_m)$ vector of deviation of generator rotor angles

$M = \mathrm{diag}(M_1,\cdots,M_m)$ of machine inertia constants

$D = \mathrm{diag}(D_1,\cdots,D_m)$ of machine damping constants

H is obtained from node reduction on the linearized network.

B is obtained from the data on the location and type of disturbance.

Following the scheme proposed in Sec. 4.2, we construct an ε-SP reduced model for (50),

$$K_1^TMK_1\Delta\ddot{\underline{\delta}}_r + K_1^TDK_1\Delta\dot{\underline{\delta}}_r + K_1^THK_1\Delta\underline{\delta}_r = K_1^TB\underline{u},\ \underline{y}_r = K_1\underline{x}_r,\ \Delta\dot{\underline{\delta}}_r(0) = \Delta\underline{\delta}_r(0) = \underline{0} \tag{51}$$

where

$$K_1 = \begin{bmatrix} I & 0 \\ 0 & U_r \end{bmatrix} . \tag{52}$$

$U_r := [U_{11}, \bar{U}_{22}]$, U_{11}, $\bar{U}_{22}$ are defined in (30a), (30b).

Now let us denote the matrix of machine inertias of study and external system by M_s, M_e respectively, and that of damping constant by D_s, D_e respectively.

Fact 3. If $M_e > 0$, $D_e \geq 0$, (& diagonal) then there exists a nonsingular real matrix K_3 such that both $K_3^T U_r^T M_e U_r K_3 > 0, K_3^T U_r^T D_e U_r K_3 \geq 0$ are diagonal. $K3 = V_r \Lambda^{-1} \bar{V}$.

where

V_r is the matrix whose column vectors are the orthonormal eigenvectors of $(U_r^T M_e U_r)$ and Λ^2 is the diagonal matrix whose diagonal elements are the eigenvalues of $(U_r^T M_e U_r)$.

$\bar{V}$ is the matrix whose column vectors are the orthonormal eigenvectors of $(\Lambda^{-1})^T \; V_r^T (U_r^T D_e U_r) V_r \Lambda^{-1}$.

Fact 3 is proved in [12, p. 106].

By Fact 3 and the structure of K_1, the reduced model (54) below represents a power system which retains the study system and has a reduced external power system structure

$$M_r \Delta\ddot{\underline{\delta}}_r' + D_r \Delta\dot{\underline{\delta}}_r' + H_r \Delta\underline{\delta}_r' = B_r \underline{u}, \; \underline{y}_r' = K_4 \Delta\underline{\delta}_r', \; \Delta\dot{\underline{\delta}}_r'(0) = \Delta\underline{\delta}_r'(0) = \underline{0} \tag{54}$$

where

$$M_r := K_4^T M K_4 = \text{diag}(M_1, \cdots M_q, \bar{M}_{q+1}, \cdots \bar{M}_{q+r'}) \tag{55}$$

$$D_r := K_4^T D K_4 = \text{diag}(D_1, \cdots D_q, \bar{D}_{q+1}, \cdots \bar{D}_{q+r'}) \tag{56}$$

$$H_r := K_4^T H K_4 \tag{57}$$

$$B_r := K_4^T B \tag{58}$$

$$K_4 := K_1 \begin{bmatrix} I & 0 \\ 0 & K_3 \end{bmatrix} \tag{59}$$

Remarks: 1) Since (51) is algebraic equivalent to (54), $\underline{y}_r(\cdot) = \underline{y}_r'(\cdot)$ for $\forall \underline{u}$, therefore the reduction error of (54) is equal to that of (51) and can be estimated by Theorem 3.

2) Suppose U_r is a column vector with all components equal, then by Eq. (38), the phase angles of external generators satisfy:

$$|(\underline{x}(t)-U_r U_r^T \underline{x}(t))_i| \leq \varepsilon a, \quad i = q+1,\cdots m, \ \forall t \in [0,\tau]$$

Now $U_r^T = \frac{1}{\sqrt{m-q}} (1 \ \cdots \ 1)$ implies

$$|x_i(t) - \frac{1}{m-q} [\sum_{j=q+1}^{m} x_j(t)]| \leq \varepsilon a, \quad \forall i = q+1 \ \cdots \ m, \forall t \in [0,\tau]$$

It follows that $\forall i,j,\ q+1 \leq i,j \leq m$

$$\max_{t \in [0,\tau]} |x_i(t)-x_j(t)| \leq 2\varepsilon a$$

Hence this is the case where all external generators are ε-coherent [8]. Clearly the ε-coherent dynamic equivalent of our previous work is a special case of ε-SP model reduction.

5.2. Power Network Synthesis

The matrices H and B in Eq. (50) of the original system are derived from a power network. In this section we are going to synthesize a reduced power network for which the corresponding relations give the matrices H_r and B_r in Eq. (54) of the reduced system.

The matrices H and B are derived from the linearized real power flow equations [8] of the power network (Fig. 1)

$$\begin{bmatrix} \Delta PG \\ \Delta PL \end{bmatrix} = \begin{bmatrix} H_{gg} & H_{g\ell} \\ H_{\ell g} & H_{\ell\ell} \end{bmatrix} \begin{bmatrix} \Delta\underline{\delta} \\ \Delta\underline{\theta} \end{bmatrix} \quad . \tag{60}$$

where ΔPG: vector of incremental real power injections at generator internal buses

ΔPL: vector of incremental real power injections at load buses

$\Delta\theta$: vector of deviation of voltage phase angles at load buses

The subscripts g and ℓ in the matrix refer to generator and load buses. The ij-th element of the matrix in Eq. (60) is

$$h_{ij} = -\frac{V_i^0 V_j^0 \cos(\theta_i^0 - \theta_j^0)}{x_{ij}} \qquad i \neq j \tag{61}$$

$$h_{ii} = \sum_{j \text{ connect to } i} \frac{V_i^0 V_j^0 \cos(\theta_i^0 - \theta_j^0)}{x_{ij}} \tag{61a}$$

where

$V_i^0 \angle \theta_i^0$: voltage phasor at bus i at the operating point.

x_{ij}: the reactance between buses i and j.

Substituting the linearized real power flow equations (60) into the linearized swing equations of the generators

$$M\Delta\ddot{\underline{\delta}} + D\Delta\dot{\underline{\delta}} + \Delta PG = 0 \tag{62}$$

we get

$$H = H_{gg} - H_{g\ell} H_{\ell\ell}^{-1} H_{\ell g} \tag{63}$$

$$B = -H_{g\ell} H_{\ell\ell}^{-1} \tag{64}$$

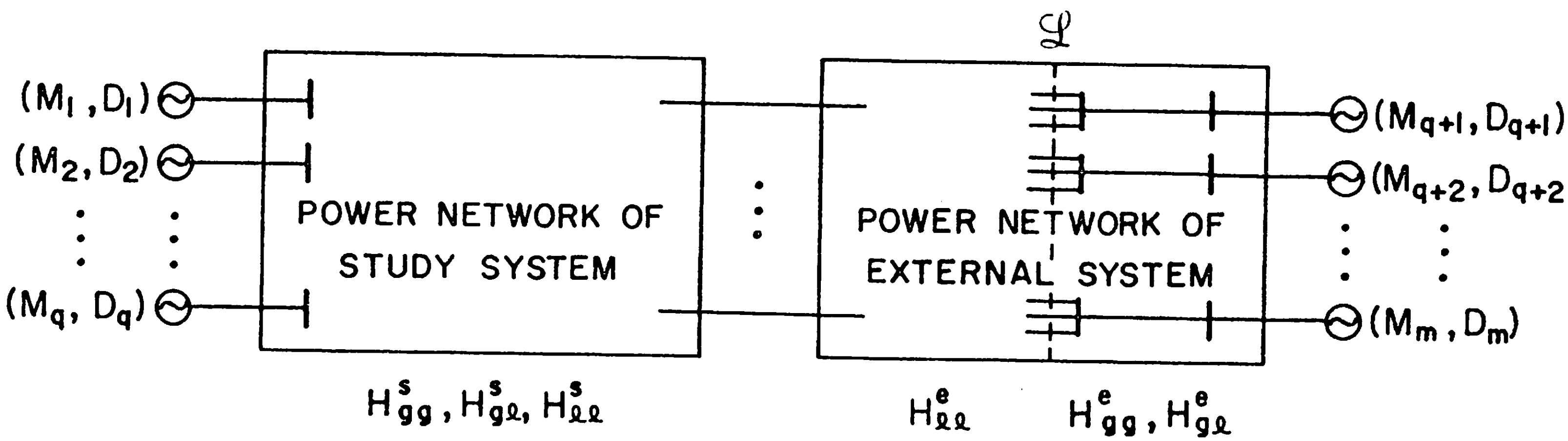

Figure 1

Original power system.

Note that the swing equations (62) concerns with real power balance at generators whereas the network equations (60) involve the voltage phasors at the operating point (61).

If we further partition the system into the study system and the external system the matrix in Eq. (60) takes the form

$$\left[\begin{array}{c|c} H_{gg} & H_{g\ell} \\ \hline H_{\ell g} & H_{\ell\ell} \end{array}\right] = \left[\begin{array}{cc|cc} H^s_{gg} & 0 & H^s_{g\ell} & 0 \\ 0 & H^e_{gg} & 0 & H^e_{g\ell} \\ \hline H^s_{\ell g} & 0 & \multicolumn{2}{c}{\multirow{2}{*}{$H_{\ell\ell}$}} \\ 0 & H^e_{\ell g} & & \end{array}\right] \tag{65}$$

where superscripts s and e refer to study system and external system respectively. Because in the model each generator internal bus is connected through the synchronous reactance of the generator to a bus designated as a load bus, the matrices H^s_{gg} and H^e_{gg} are diagonal and $H^s_{g\ell} = [-H^s_{gg}, 0]$, $H^e_{g\ell} = [-H^e_{gg}, 0]$ if we order the load buses so that those connected to generators come first. Let us denote the set of load buses of the external system that are connected to generators $\mathcal{L}$.

For the reduced system (54) we have (57) (58)

$$H_r = K_4^T H K_4 \tag{66}$$

$$B_r = K_4^T B \tag{67}$$

We are going to synthesize a power network whose linearized real power flow equations are

$$\begin{bmatrix} \Delta\overline{PG} \\ \Delta\overline{PL} \end{bmatrix} = \begin{bmatrix} \overline{H}_{gg} & \overline{H}_{g\ell} \\ \overline{H}_{\ell g} & \overline{H}_{\ell\ell} \end{bmatrix} \begin{bmatrix} \Delta\underline{\delta}'_r \\ \Delta\underline{\theta} \end{bmatrix} \tag{68}$$

where

$$\bar{h}_{ij} = \frac{-\bar{V}_i^0 \bar{V}_j^0 \cos(\bar{\theta}_i^0 - \bar{\theta}_j^0)}{\bar{x}_{ij}} \quad i \neq j \tag{69}$$

$$\bar{h}_{ii} = \sum_{j \text{ connected to } i} \frac{\bar{V}_i^0 \bar{V}_j^0 \cos(\bar{\theta}_i^0 - \bar{\theta}_j^0)}{\bar{x}_{ij}} \tag{69a}$$

Upon substituting (68) into the linearized swing equations of the generators in the reduced system

$$M_r \Delta \ddot{\underline{\delta}}'_r + D_r \Delta \dot{\underline{\delta}}'_r + \Delta \overline{PG} = 0 \tag{70}$$

we should get Eq. (54), i.e.,

$$H_r = \bar{H}_{gg} - \bar{H}_{g\ell} \bar{H}_{\ell\ell}^{-1} \bar{H}_{\ell g} \tag{71}$$

$$B_r = -\bar{H}_{g\ell} \bar{H}_{\ell\ell}^{-1}$$

Similarly if we further partition the system into the study system and the reduced external system, the matrix in (68) takes the form

$$\left[\begin{array}{c|c} \bar{H}_{gg} & \bar{H}_{g\ell} \\ \hline \bar{H}_{\ell g} & \bar{H}_{\ell\ell} \end{array}\right] = \left[\begin{array}{cc|cc} \bar{H}_{gg}^s & 0 & \bar{H}_{g\ell}^s & 0 \\ 0 & \bar{H}_{gg}^e & 0 & \bar{H}_{g\ell}^e \\ \hline \bar{H}_{\ell g}^s & 0 & & \\ 0 & \bar{H}_{\ell g}^e & \multicolumn{2}{c}{\bar{H}_{\ell\ell}} \end{array}\right] \tag{73}$$

We are going to take three steps to synthesize a reduced power network.

Step 1. Matching $\bar{h}_{ij}$

(i) Let us first define

$$K_5 = U_r K_3 \tag{74}$$

By comparing Eqs. (63)(64), (71)(72), (66)(67) and (74), it clearly indicates that it is sufficient to have

$$\bar{H}^e_{gg} = K_5^T H^e_{gg} K_5 \tag{75}$$

$$\bar{H}^e_{g\ell} = K_5^T H^e_{g\ell} \tag{76}$$

$$H^e_{\ell g} = H^e_{\ell g} K_5$$

$$\bar{H}_{\ell\ell} = H_{\ell\ell} \tag{77}$$

and $\bar{H}^s_{gg} = H^s_{gg}$, $\bar{H}^s_{g\ell} = H^s_{g\ell}$, $\bar{H}^s_{\ell g} = H^s_{\ell g}$ for maintaining the identity of the study system.

(ii) It follows from (77) that we should retain the network connecting load buses to load buses (Fig. 2). The matrix $\bar{H}^e_{gg}$ is no longer diagonal (Fig. 2), $\bar{H}^e_{g\ell} = [-K_5^T H^e_{gg}, 0] \neq [-\bar{H}^e_{gg}, 0]$, and $K_5^T H^e_{gg}$ is no longer diagonal (Fig 2).

(iii) In the original network $h_{ii} = -\sum_{j\neq i} h_{ij}$. But $\bar{h}_{ii} \neq -\sum_{j\neq i} \bar{h}_{ij}$. In order to have the form of (61) for $\bar{h}_{ii}$ we create an "infinite bus," which is a node whose voltage is kept constant at $1\underline{/0}$, and connect it with each generator bus and load buses in $\mathcal{L}$ of the reduced external system (Fig. 3).

Step 2. Selection of Voltage

(i) The voltage at each load bus of the external load bus is chosen to be the same as the original one, i.e.,

$$\bar{V}^0_i \underline{/\bar{\theta}^0_i} = V^0_i \underline{/\theta^0_i} \tag{80}$$

The voltages at the generator buses of the reduced system are chosen to be $1\ \underline{/0}$.

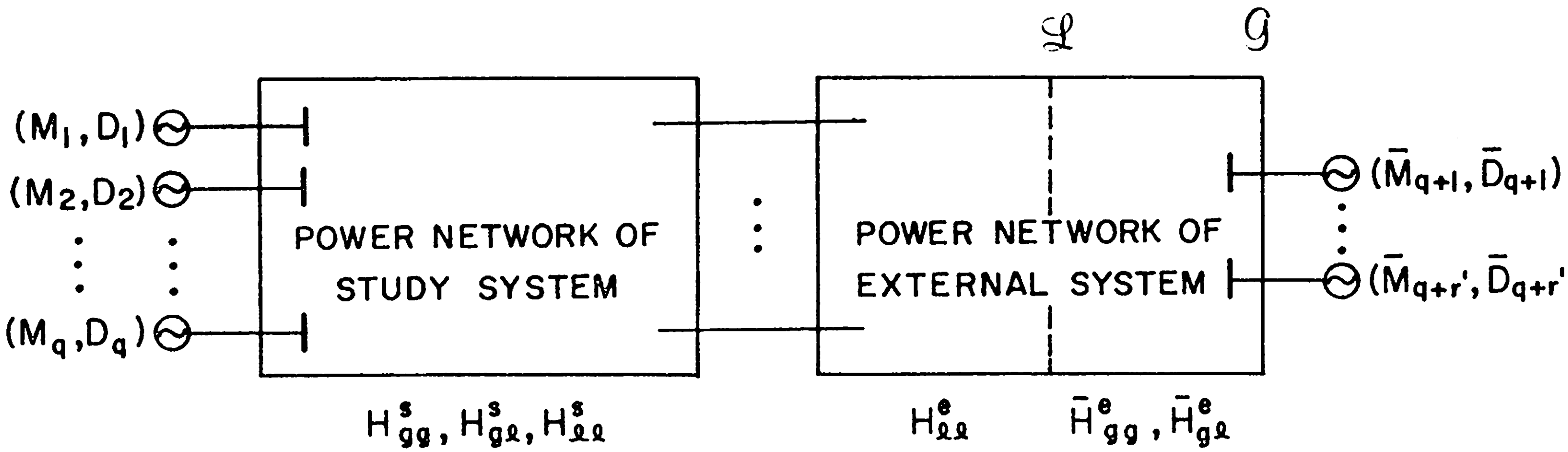

Figure 2

Reduced power system.

(ii) We want power flows in the retained portion of the reduced network exactly the same as before. This is done by adjusting power flow from the load buses in the reduced external network to the infinite bus. For this purpose, we use transformers (Fig. 3).

Step 3. Selection of Impedances

The reactances $\bar{x}_{ij}$ in the retained portion of the reduced external network are chosen the same as the original network. The reactances $\bar{x}_{ij}$ in the altered portion of the reduced external network are selected to match $\bar{h}_{ij}$, voltages, and the power injections as described in Steps 1 and 2.

(i) generator to generator

Let the i-th diagonal of H^e_{gg} be denoted by h_i and the ij-th element of the matrix K_5 be denoted by k_{ij}. The reactances $\bar{x}_{ij}$ between generators of the reduced external system

$$\frac{1}{\bar{x}_{ij}} = -\sum_{\lambda=1}^{m-q} k_{\lambda i} h_{\lambda\lambda} k_{\lambda j}$$

(ii) generator to infinite bus

Let $\mathcal{G}$ denote the set of indices correspond to the generators in the reduced external system. The reactances $\bar{x}_{i\infty}$ between a generator and the infinite bus of the reduced external system are:

$$\frac{1}{\bar{x}_{i\infty}} = \sum_{\lambda=1}^{m-q} k_{\lambda i} h_{\lambda\lambda} \left(\sum_{\mu \in \mathcal{G}} k_{\lambda\mu} - 1 \right)$$

(iii) generator to load

The reactance $\bar{x}_{ij}$ between generator bus i and load bus j is:

$$\frac{1}{\bar{x}_{ij}} = \frac{h_{jj} k_{ji}}{V_j^0 \cos \theta_j^0}$$

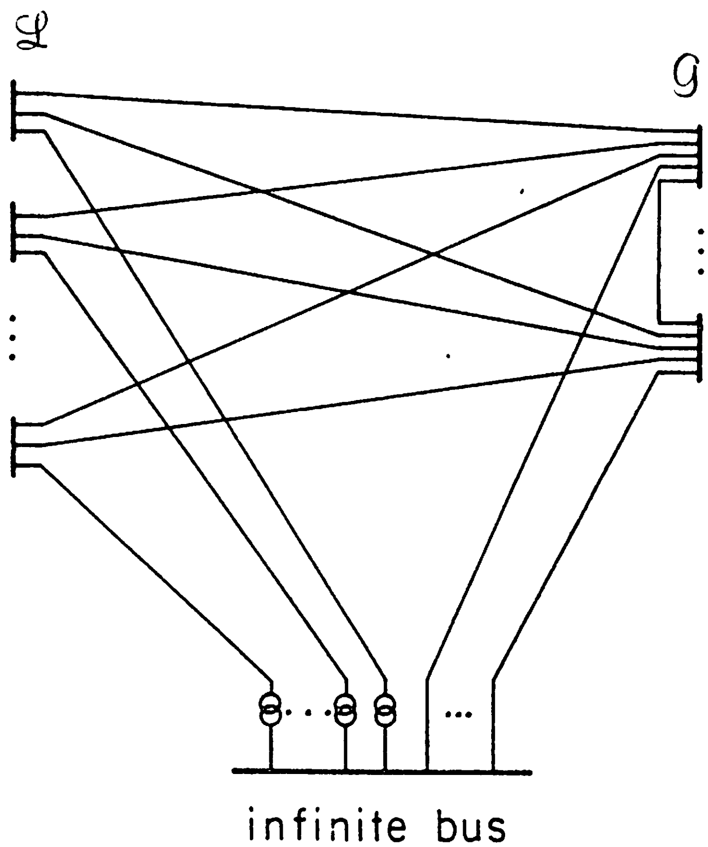

Figure 3

Altered portion of external system.

(iv) transformer ratio

Let S_i^0 be the original complex power injection into the load bus i of $\mathcal{L}$ from the external generator at the operating point, we define

$$\Delta P_i + j\Delta Q_i = -S_i^0 - j \sum_{\lambda \in \mathcal{G}} \frac{1}{\bar{x}_{i\lambda}} V_i^0 (V_i^0 - e^{j\theta_i^0})$$

$$f_i = h_{ii}(1 - \sum_{\lambda \in \mathcal{G}} k_{i\lambda})$$

$$\theta_{\infty i} = \theta_i^0 - \tan^{-1} (\frac{\Delta P_i}{f_i})$$

$$V_{\infty i} = \frac{V_i^0 \, \Delta P_i}{(\Delta Q_i + f_i)\sin(\theta_i^0 - \theta_{\infty i})}$$

The transformer ratio β_i is set to be

$$\beta_i = V_{\infty i} e^{j\theta_{\infty i}}$$

(v) load to infinite bus

The reactance $\bar{x}_{i\infty}$ between load i and the infinite bus is

$$\frac{1}{\bar{x}_{i\infty}} = \frac{\Delta Q_i + f_i}{(V_i^0)^2}$$

5.3. Dynamic Equivalent of External System

Consider the external system in Fig. 2. The number of generators has been reduced but not the load buses. Next we convert all load demands in the external load buses into impedance loads. We then apply Gaussian elimination on the corresponding admittance matrix to eliminate those buses, we obtain the reduced system as shown in Fig. 4.

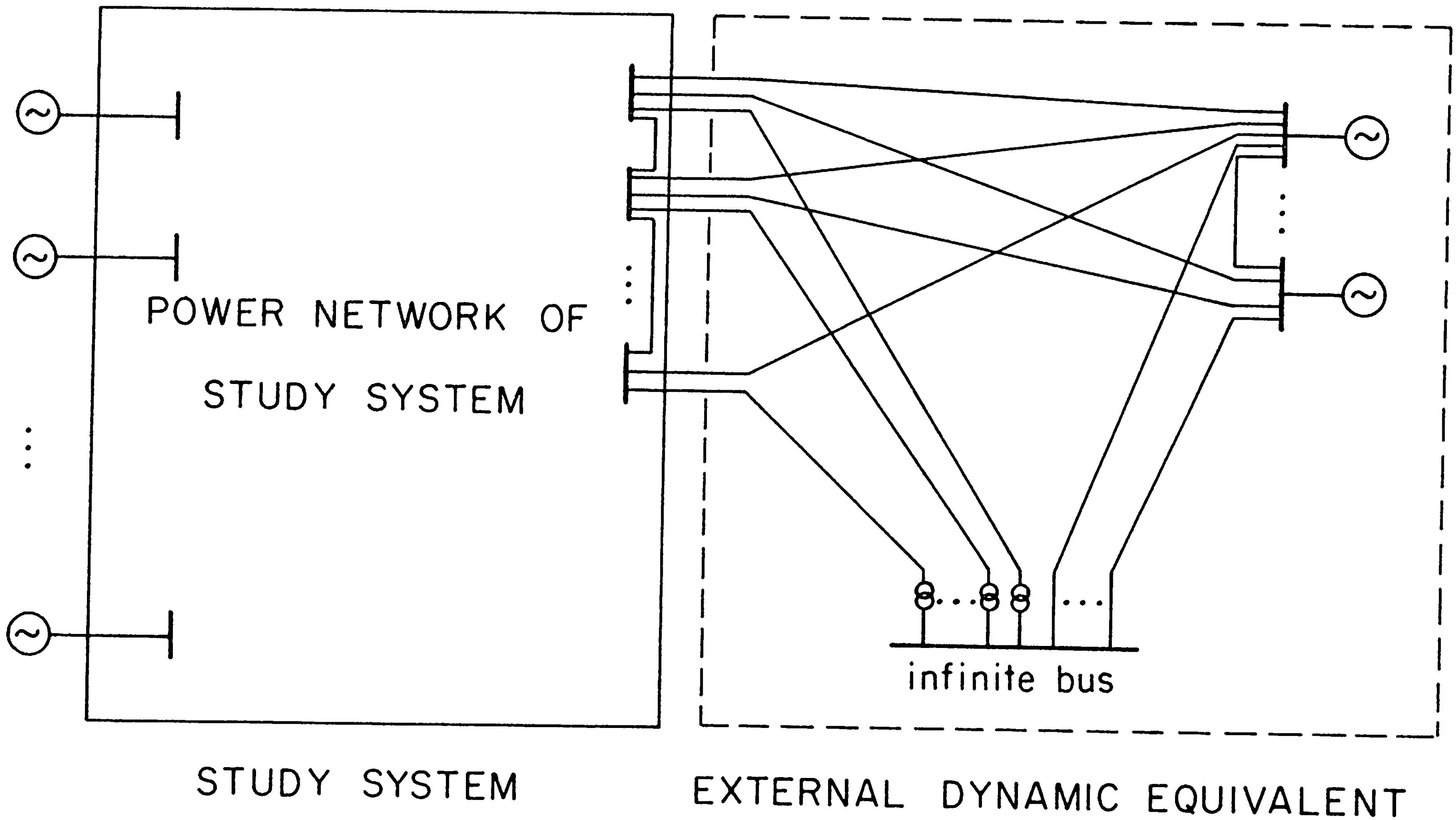

Figure 4

External dynamic equivalent.

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