POSSIBILITY DISTRIBUTIONS, FUZZY INTERVALS AND POSSIBILITY MEASURES IN A LINGUISTIC APPROACH TO PATTERN CLASSIFICATION IN MEDICINE

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Memorandum No. UCB/ERL M82/13

5 March 1982

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POSSIBILITY DISTRIBUTIONS, FUZZY INTERVALS AND POSSIBILITY MEASURES IN A LINGUISTIC APPROACH TO PATTERN CLASSIFICATION IN MEDICINE*

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ABSTRACT

The methodology that is described in this tutorial paper refers to a linguistic approach to pattern classification in medicine. A reference pattern of "medical knowledge" here consists of a tableau with linguistic entries or of a set of propositions and, in both cases, relationships between attributes (plasma lipids, serum proteins, cardiac indexes, etc.) and types, or groups, syndromes, etc., are not expressed by numbers but by verbal denotations of fuzzy sets. Possibility distributions and fuzzy intervals are simply introduced to give a meaning to the fuzzy propositions issued from the pattern. Finally, it is shown how possibility measures allow an automatic classification of patients, with observed or measured attributes, yielding a grade of membership in each type of the pattern.

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*This paper was the support of one of the lectures the author gave in China during summer 1981.

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Research sponsored by the National Science Foundation Grants MCS79-06543 and IST-8018196.

1. INTRODUCTION

This paper was written with tutorial aims for presentation to practitioners or biologists who may not be familiar with fuzzy set theory and derived concepts (note also a short appendix at the end of the paper) or to a wider family of researchers who can be simply shown how fuzzy set techniques may actually apply to a class of practical problems in medicine.

As stressed by many practitioners, the definition of "limits of normal" for biological analyses shows some arbitrariness. In fact, most experts agree on a central zone of normality and on some zones of more or less decreased or increased quantities (of course, with respect to "normal" positions), in relation to given classes of disorders. Symbols obtained by repeating "↓" or "↑" are frequently used to denote variations. For example, ↓↓, ↓, → (or N) ↑↑↑, respectively stand for *strongly* (*or very*) *decreased*, *slightly* (*or moderately*) *decreased*, *normal*, *very strongly* (*or markedly*) *increased*. Generally, a set of cut-off points helps to define limits of values that characterize abnormalities, hence numerical intervals are mostly used to describe standards in variations. The problem that naturally arises stems from the ill-definition of the boundaries of such intervals, so that to cope with borderline cases, fuzzy set theory appears to provide efficient tools. We here postulate that the imprecision in the description of abnormalities is of a fuzzy nature and we shall assume that terms like *slightly decreased*, *very increased*, etc., are labels of fuzzy subsets of appropriate universes of discourse of variables to which they apply. Such fuzzy sets will consist of linguistic intervals, so that around cut-off points, very close points will not be accepted or rejected in automatic procedures, depending on their position with respect to the frontier. Moreover, the usual coding with ascending or descending arrows is sometimes too restrictive, it is not always possible for one to make up one's mind between ↑ and ↑↑ for example, in some patterns one may encounter "↑ to ↑↑" or even "↓ to ↑". A graduate membership extending from 0 to 1 is very convenient. Note that one does not need to set up precise values in the unit interval; usually, one has a rough idea of the compatibility curve expressing the concept, derived from "normality", that he/she is going to use and manipulate in interpreting patterns.

Note that there exist notions of "statistical normality" derived from frequency distributions, not always confined to Gaussian distributions. But depending on the measurement procedures of a given laboratory, on the epidemiologist, the biologist, the geneticist or the clinician who manipulates and interprets measures, on the different populations under study, on conditions of "physiological (biological) normality", etc., one is usually forced to rely on fiducial limits, see [1,4] for discussions on "normality".

What is here proposed is an automatic and reproducible methodology, of special interest in the processing of borderline cases, and which offers the physician practical assistance in obtaining always the same results in the same abnormal profiles.

2. LINGUISTIC PATTERN

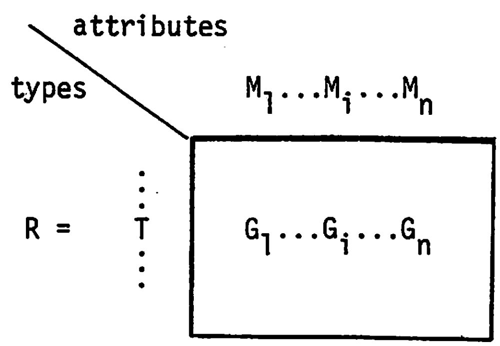
Let a reference pattern be described in two possible ways:

(i) a tableau with linguistic entries,

(ii) a set of propositions.

In both cases, we assume that "medical knowledge" is interpreted by fuzzy propositions of the form "M is G" where M is a variable taking values in a universe of discourse U, and G is a fuzzy subset of U.

In the case of a tableau expressing relationships between attributes (M), such as temperature, plasma lipids, arterial pressure, serum proteins, etc., and groups, syndromes, diseases, diagnoses, or types, the linguistic entries are assumed to be labels of fuzzy sets (G), or more precisely, fuzzy intervals. We assume that a given type (T) is represented on a row of a tableau R, with linguistic entries:



where for i = 1,...,n, M_i is the name of a variable (attribute) taking values in a universe of discourse U_i and G_i is a fuzzy subset of U_i. The "medical knowledge" involved in T is then interpreted by the conjunction (AND's) of elementary propositions:

$$T \rightarrow P = P_1 \text{ AND...AND } P_i \text{ AND...AND } P_n$$

where for i = 1,...,n, P_i has the standard form:

M_i is G_i.

For example, in the WHO-Fredrikson classification of hyperlipoproteinemias, Type IIb (T) may be interpreted as follows [9]:

Lipids (M_1) are *increased* (G_1)

AND...AND

Triglycerides (M_i) are *slightly increased* (G_i)

AND...AND

Pre-β lipoproteins (M_n) are *increased* (G_n).

Finally, in the two possible dscriptions (tableau with linguistic entries or set of propositions), we have at our disposal, conjunctions of fuzzy propositions, and each of them (P_i) induces a possibility distribution [16] defined by the possibility assignment equation:

$$M_i \text{ is } G_i \rightarrow \Pi_{M_i} = G_i$$

For example,

Triglycerides are *slightly increased* $\rightarrow \Pi_{\text{Triglycerides}}$ = SLIGHTLY INCREASED, where SLIGHTLY INCREASED (G_i), the denotation of *slightly increased,* is a fuzzy subset of $U_i = [0,45]$, with triglycerides expressed in mmol/l, and with G_i depicted in figure 1.

Note that the information contained in "Triglycerides are *slightly increased*" does not provide a precise characterization of the numerical values to be assigned to a variable named "Triglycerides", but it indicates a soft constraint on its possible values. This constraint is here translated by a possibility distribution, associated with the variable, and it is expressed by the fuzzy set SLIGHTLY INCREASED. This fuzzy set is viewed as a fuzzy interval that extends the definition of an ordinary (crisp) interval g_i in U_i (see figure 2).

Just like its elementary constituent propositions, a reference proposition P may be written in the standard form

$$M \text{ is } G,$$

where $M = (M_1,\ldots,M_n)$ is a n-ary variable and, $G = G_1 \times\ldots\times G_n$ is a fuzzy relation in $U = U_1 \times\ldots\times U_n$, defined by

$$\mu_G(u) = \min(\mu_{G_1}(u_1),\ldots,\mu_{G_n}(u)), \text{ for } u = (u_1,\ldots,u_n) \text{ in } U.$$

Hence, the following translation rule of P yields the possibility distribution associated with M.

$$M \text{ is } G \rightarrow \Pi_{(M_1,\ldots,M_n)} = G_1 \times\ldots\times G_n.$$

For example, the information conveyed by the fuzzy proposition P characterizing Type IIb is expressed by

$$\Pi_{(\text{Lipids},\ldots,\text{Triglycerides},\ldots,\text{Pre-}\beta\text{ lipoproteins})}$$
$$= \text{INCREASED} \times\ldots\times \text{SLIGHTLY INCREASED} \times\ldots\times \text{INCREASED}.$$

Let us now see how the G_i's are practically defined.

3. FUZZY INTERVALS AND FUZZY NUMBERS

Fuzzy intervals on the real line are here defined as simple combinations of the S-membership function, which is a piecewise quadratic function defined by [16]:

$$
\begin{aligned}
S(u;\alpha,\gamma) &= 0 && \text{for } u \leq \alpha \\
&= 2(u-\alpha)^2/(\gamma-\alpha)^2 && \text{for } \alpha \leq u \leq \beta \\
&= 1 - 2(u-\gamma)^2/(\gamma-\alpha)^2 && \text{for } \beta \leq u \leq \gamma \\
&= 1 && \text{for } u \geq \gamma
\end{aligned}
$$

where $\beta = (\alpha+\gamma)/2$ is the crossover point, i.e., $S(\beta;\alpha,\gamma) = 0.5$. The S-membership function is depicted in figure 3.

A fuzzy set with an S-membership function may be viewed as a fuzzy interval of the form $\underset{\sim}{\geq} \beta$, whereas a fuzzy interval of the form $\underset{\sim}{\leq} \beta$ is characterized by an S'-membership function, where $S'(u;\alpha,\gamma) = 1 - S(u;\alpha,\gamma)$ for all u in the domain of S'. A fuzzy interval of the form $\underset{\sim}{[\beta_1,\beta_2]}$ is characterized by an S_1 and an $(1-S_2)$-membership functions:

$$\underset{\sim}{[\beta_1,\beta_2]} = (\underset{\sim}{\geq} \beta_1) \cap (\underset{\sim}{\leq} \beta_2)$$

so that $\underset{\sim}{[\beta_1,\beta_2]}$ identifies a possibility distribution of points that are *greater than* β_1 AND *smaller than* β_2 (the notation of intersection ($\cap$)

replaces the notation of cartesian product (X) for we now consider fuzzy subsets of a same universe of discourse), as illustrated in figure 4. When $\gamma_1 = \alpha_2$ (we denote by "a" this common value) and when $\gamma_1 - \alpha_1 = \gamma_2 - \alpha_2$, the fuzzy interval $\underset{\sim}{[\beta_1,\beta_2]}$ reduces to a fuzzy number $\underset{\sim}{a}$ expressing *around* or *close to* "a", as shown in figure 5, where b, the distance corresponding to $\beta_2 - \beta_1$ is the bandwidth (separation between the crossover points β_1 and β_2), it is the fuzziness indicator of $\underset{\sim}{a}$. As it is defined, a fuzzy number $\underset{\sim}{a}$ implicitly depends on the only parameter b and, in fact, it should be written $\underset{\sim}{a(b)}$.

Note that in practical problems, fuzziness attached to limits of the intervals may be different on both sides, so that a fuzzy interval is not necessarily the union of fuzzy numbers with equal bandwidths.

Let us now return to the reference pattern of "medical knowledge" in which the fuzzy sets G_i are assumed to be fuzzy intervals issued from one of the three classes:

Class 1: Fuzzy intervals of the form $\underset{\sim}{\leq} \beta$

Class 2: Fuzzy intervals of the form $\underset{\sim}{\geq} \beta$

Class 3: Fuzzy intervals of the form $\underset{\sim}{[\beta_1,\beta_2]}$.

A class 1 or a class 2 fuzzy interval depends on two parameters (α,γ), whereas a class 3 fuzzy interval depends on four parameters, say $(\alpha_1,\gamma_1,\alpha_2,\gamma_2)$. In a previous example, the fuzzy set SLIGHTLY INCREASED associated with triglycerides could be written $\underset{\sim}{[2.2,5.5]}$ and defined by $\alpha_1 = 1.7$, $\gamma_1 = 2.7$, $\alpha_2 = 4.4$ and $\gamma_2 = 6.6$. Very often a fuzzy set expressing a "normal concept" is a class 3 fuzzy interval that fuzzifies usual intervals given by experts; the setting of such intervals is the first step in the design of the reference pattern. Usually, for each laboratory analysis, the biologist defines, modifies for a specific purpose or, more generally, is given a variation range in which should fall the "normal" quantitative measurements and he has a rough idea of the variation ranges for abnormalities, so that he has in mind more or less well-defined intervals (with respect to the boundaries). In order to determine a patient's condition it is then sufficient to check in which interval the measured value falls. This process will now be extended to our linguistic pattern analysis in which we postulated that imprecision in the definition of the intervals is of a fuzzy nature.

If we consider a non fuzzy proposition of the form "M_i is a number in the interval [2.2,5.5]", we mean that any number in the interval [2.2,5.5] is a possible value for the variable M_i and it is not possible for a number outside the interval to be assigned to M_i; the given proposition induces a possibility distribution Π_{M_i} defined by the interval, implying that

$$\begin{aligned} \text{Poss}\{M_i = u_i\} &= 1 \quad \text{for } 2.2 \leq u_i \leq 5.5 \\ &= 0 \quad \text{for } u_i < 2.2 \text{ or } u_i > 5.5. \end{aligned}$$

Returning now to the fuzzy case, let us consider the proposition "Triglycerides (M_i) are *slightly increased*" which translates into

$$\Pi_{\text{Triglycerides}} = \underset{\smile}{[2.2,5.5]} \; (=G_i),$$

and implies that

$$\text{Poss}\{\text{Triglycerides} = u_i\} = \mu_{\text{SLIGHTLY INCREASED}}(u_i)$$

or

$$\text{Poss}\{M_i = u_i\} = u_{G_i}(u_i),$$

where the second member may be viewed [16] as the possibility distribution function π_{M_i} which is defined by being numerically equal to the membership function of G_i (π_{M_i} corresponds the possibility distribution Π_{M_i}):

$\pi_{M_i}(u_i) = \mu_{G_i}(u_i)$.

In the case of the reference proposition "M is G", we have $\mathrm{Poss}\{M = u\} = \pi_M(u)$, where $\pi_M(u) = \mu_G(u)$, M being a n-ary variable.

4. POSSIBILITY MEASURES AND PATTERN CLASSIFICATION

In order to compare a patient's condition with a reference proposition, say P, issued from a type, say T, in the pattern of "medical knowledge", the attributes $M_1,\ldots,M_n$ involved in P, have to be measured in the patient yielding the values

$M_1(\text{patient}) = a_1$ in U_1
....
$M_n(\text{patient}) = a_n$ in U_n .

Hence, given the proposition P: "M is G" where $M = (M_1,\ldots,M_n)$ and $G = G_1 \times\ldots\times G_n$, we derive:

$$\mathrm{Poss}\{M(\text{patient}) = a\} = \mu_G(a),\ a = (a_1,\ldots,a_n) \in U$$

or

$$\mathrm{Poss}\{M_1(\text{patient}) = a_1 \text{ AND}\ldots\text{AND } M_n(\text{patient}) = a_n\}$$
$$= \mu_{G_1 \times\ldots\times G_n}(a_1,\ldots,a_n)$$
$$= \min(\mu_{G_1}(a_1),\ldots,\mu_{G_n}(a_n)),$$

this value yields a grade of compatibility of the patient's condition as regards T.

In fact, usually, the measured data are of a fuzzy nature in at least two aspects:

i) imprecision in the measurement procedures.

ii) interpretation of the measured values, so that it is natural to transform each measured value into a fuzzy number, e.g. "M_i(patient) = a_i" is transformed into "M_i(patient) is $\underset{\sim}{a_i}$", with a bandwidth b_i that only depends on M_i and not on the patient, so that a possibility distribution is induced:

$$M_i(\text{patient}) \text{ is } \underset{\sim}{a_i} \rightarrow \Pi_{M_i(\text{patient})} = \underset{\sim}{a_i}$$

For example, "John's triglycerides are 6.3 mmol/l" is transformed into "Triglycerides (John) = $\underset{\sim}{6.3}$", inducing the possibility distribution shown in figure 6.

The patient's condition is now expressed by a conjunction of fuzzy propositions involving fuzzy numbers: M_1(patient) is $\underset{\sim}{a_1}$ AND...AND M_n(patient) is $\underset{\sim}{a_n}$, which is equivalent to

$$M(\text{patient}) \text{ is } \underset{\sim}{a},$$

where M(patient) = (M_1(patient),...,M_n(patient)), and $\underset{\sim}{a} = \underset{\sim}{a_1} \times \ldots \times \underset{\sim}{a_n}$ is a fuzzy relation in $U_1 \times \ldots \times U_n$. Hence, the patient's condition can now be compared to a pattern reference proposition, say P : "M is G", by means of a possibility measure [16]:

<u>If</u>, M is G $\rightarrow \Pi_M = G$

<u>Then</u>, Poss{M(patient) is $\underset{\sim}{a}$ <u>given</u> Π_M} = $\Pi(\underset{\sim}{a})$, the possibility measure of $\underset{\sim}{a}$ with respect to the possibility distribution Π_M, is defined by

$$\sup_{u \in U} \min(\mu_{\underset{\sim}{a}}(u), \pi_M(u))$$

or, shortly,

$$\Pi(\underset{\sim}{a}) = \sup(\underset{\sim}{a} \cap \Pi_M).$$

Note that Π now stands for a function defined on fuzzy sets and taking values in the unit interval. Moreover, we shall use the notation $\Pi^G(\underset{\sim}{a})$ to remember that the possibility measure is evaluated with respect to $\Pi_M = G$. Recalling now that $G = G_1 \times \ldots \times G_n$ and that $\underset{\sim}{a} = \underset{\sim}{a}_1 \times \ldots \times \underset{\sim}{a}_n$, one verifies that

$$\Pi^G(\underset{\sim}{a}) = \min(\Pi^{G_1}(\underset{\sim}{a}_1), \ldots, \Pi^{G_n}(\underset{\sim}{a}_n)),$$

where, for $i = 1, \ldots, n$, M_i is $G_i \to \Pi_{M_i} = G_i$, and $\Pi^{G_i}(a_i)$ $= \text{Poss}\{M_i(\text{patient}) \text{ is } \underset{\sim}{a}_i$ <u>given</u> $\Pi_{M_i}\}$. For example, Poss{John's triglycerides are *close* to 6.3 mmol/l <u>given</u> $\Pi_{\text{triglycerides}}$ = SLIGHTLY INCREASED (= G_i in Type IIb)} = $\Pi^{G_i}(\underset{\sim}{6.3}) = \text{Sup}(\underset{\sim}{6.3} \cap \Pi_{M_i}) = 0.2$. See figure 7 for illustration. This value (0.2) indicates a weak compatibility of "*close to* 6.3 mmol/l" with the fuzzy interval SLIGHTLY INCREASED, whereas a possibility measure with respect to $\Pi_{\text{triglycerides}}$ = MARKEDLY INCREASED would have yielded the value "1".

When possibility measures are computed over all attributes, the minimum of these values yields $\Pi^G(\underset{\sim}{a})$, which stands for a grade of membership (or a matching degree) of the patient in a reference type T. Doing so for all reference types, a patient is finally assigned a grade of membership, lying between 0 and 1, in all possible profiles of a pattern. In well defined cases only one "1" is assigned for one type to the patient, the other grades being equal to 0. Fuzziness is, of course, of great interest for borderline cases.

Note that when $\underset{\sim}{a}$ is nonfuzzy, it reduces to $(a_1, \ldots, a_n)$ and, $\Pi^G(\underset{\sim}{a})$, as expected, becomes equal to $\min(\mu_{G_1}(a_1), \ldots, \mu_{G_n}(a_n))$. In this case, for $i = 1, \ldots, n$, $\mu_{\underset{\sim}{a}_i} = \delta_{a_i}$, i.e., $\delta_{a_i}(a_i) = 1$ and $\delta_{a_i}(u_i) = 0$ for $u_i \neq a_i$, so that $\underset{\sim}{a}_i$ is identified with a_i.

5. CONCLUDING REMARKS

It is natural to assert that a reference proposition is constituted by conjunctions (ANDs) of elementary propositions, but what may be discussed is the correspondence between a logical AND a "min" operator.

It seems that something between "min" and "max" operators should bring satisfactory answers, as will be discussed in a forthcoming paper. The methodology we proposed in this paper works pretty well and it is being improved in several directions. Note that if one, and only one, observed attribute in a patient does not match the corresponding fuzzy interval in the reference pattern (i.e., we get one low value for the possibility measure), the whole type is rejected ("min" operator and one low value), but a high grade of membership in a given type makes us entirely confident in the result. In fact, we here assumed that all attributes are "independent" and of equal importance in relation to all types, but an interesting study has been carried out [12-14] taking into account the two notions of "evocation power" and "rejection power" of an attribute with respect to a type. Such powers were introduced to better fit the expert's perception of relationship between attributes and types, so that in most cases, results were improved.

In some of our practical applications, starting with a small number of attributes and a small number of types, some results (low grades of membership, for patients, in all types) suggested the need, from a technical point of view, for introducing new fuzzy intervals expressing evolutionary or secondary variations [5,9]. Experts, of course agreed with such considerations for they were very often aware of the meaning of these minor types, but in a simple model discussed at the beginning of a study, evolutionary types did not need to be defined. Note that in some cases, definition of types does not necessarily correspond to diseases or diagnostic entities; it is only a formalization of standard variations, so that types may be somehow modified, bearing in mind that their number should be optimized.

The proposed methodology has satisfactorily been applied to thyroid pathology [11], to cardiac insufficiency problems [5], to the Fredrickson-WHO classification of hyperlipoproteinemia [9], to the classification of some icteria [10], and we are thinking of epidemiologic studies of abnormal patterns in large population samples.

APPENDIX ON FUZZY SETS

A universe of discourse U is a specific set or an arbitrary collection of objects in the traditional sense of set theory. For example, U may denote the set of all real numbers, the set of all diagnoses in a pathology, the reference set of values in a biological test, the set of all attendees at a conference, etc.

Given a subset X of U, any element u in U is assumed to be, or not, a member of X; there are only two possibilities; membership or nonmembership in X. In real world problems, one encounters ill-defined classes in which imprecision is located at the boundaries; for some elements u in U, it is not natural to have to decide whether or not they are members of one of these classes. Many problems of misclassification arise from the introduction of subjective, or more or less arbitrary, thresholds to define strict classes. Among the very common examples of such classes are those of *young* ladies, *ripe* fruits, *big* men, *old* animals, *narrow* roads, etc.

Zadeh's fuzzy set theory [18] provides an appropriate conceptual framework to deal with classes of objects in which the transition from membership to non-membership is gradual; an object in U may have a grade of membership, in a fuzzy subset A of U, intermediate between unity and zero. A fuzzy subset A of U is characterized by a membership function $\mu_A : U \to [0,1]$ which associates, which each element u in U, a number $\mu_A(u)$ in the real interval [0,1], with $\mu_A(u)$ representing the grade of membership of u in A($\mu_A(u)$ may also be viewed as the degree of compatibility of u with the concept represented by A). For example, if U expresses the possible values (in Celsius degree) resulting from the observation of a temperature, $\mu_{high}(39°) = 0.8$, indicates that the assignment of 39° to temperature is compatible, to the grade 0.8, with the fuzzy set labelled *high* or that the membership of 39° to the fuzzy set "*high temperature*" is equal to 0.8.

Note that fuzziness is not a form of randomness, it results from the subjective evaluation of the perception of an imprecise, approximate, phenomenon.

Let us finally recall some basic definitions in fuzzy set theory. Let U denote a specific universe of discourse and let A and B denote two fuzzy subsets of U characterized by their respective membership functions μ_A and μ_B.

<u>Equality</u>: $A = B \Leftrightarrow \mu_A(u) = \mu_B(u),\ u \in U$

<u>Containment</u>: $A \subset B \Leftrightarrow \mu_A(u) \leq \mu_B(u),\ u \in U$

<u>Union</u>: $A \cup B \Leftrightarrow \mu_{A \cup B}(u) = \max(\mu_A(u), \mu_B(u)),\ u \in U$

<u>Intersection</u>: $A \cap B \Leftrightarrow \min(\mu_A(u), \mu_B(u)),\ u \in U$

If U is the cartesian product of n universes of discourse $U_1, \ldots, U_n$, then a n-ary fuzzy relation R, in U, is a fuzzy subset of U. R is characterized by its membership function $\mu_R : U_1 \times \ldots \times U_n \rightarrow [0,1]$. Some examples of 2-ary fuzzy relations may be given by *approximately equal, much larger than, close to, coherent with*, etc.

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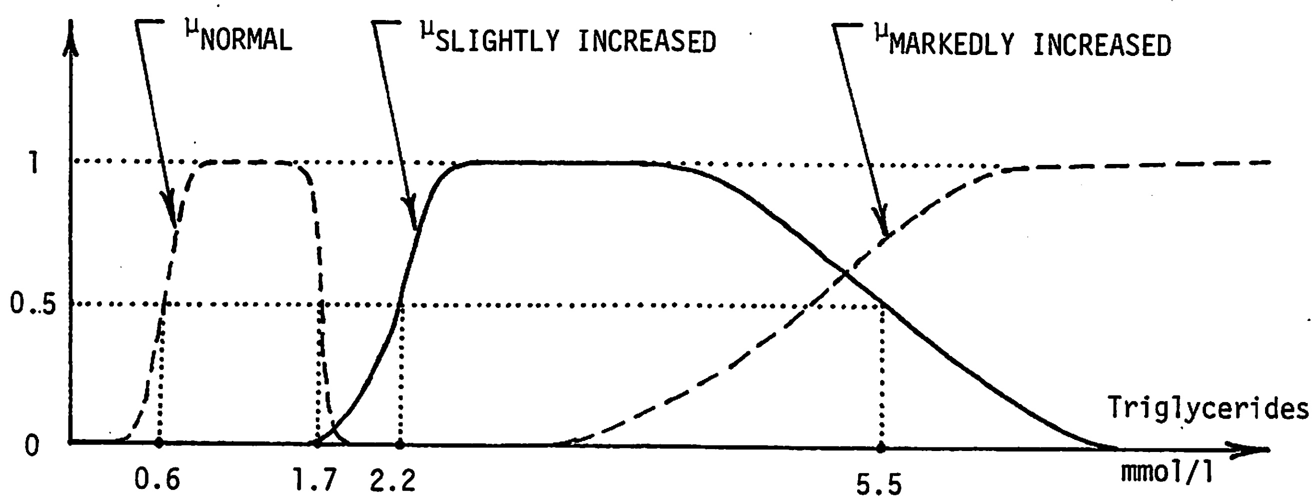

Fig. 1. Membership function for (triglycerides) slightly increased.

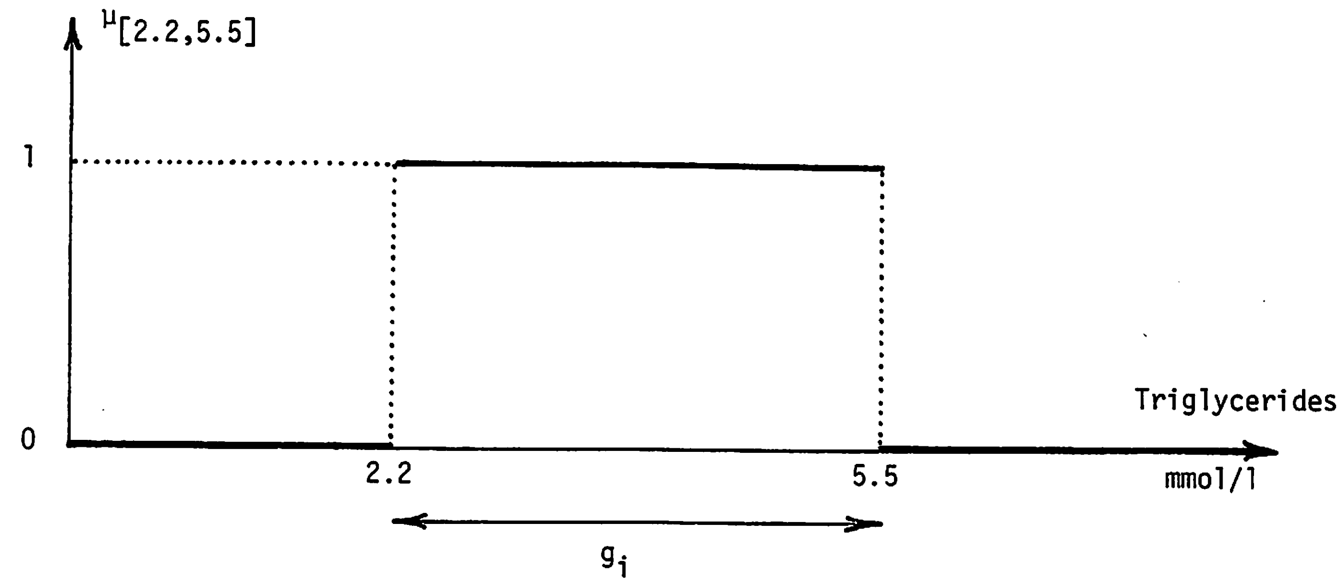

Fig. 2. Non-fuzzy interval for (triglycerides) slightly increased.

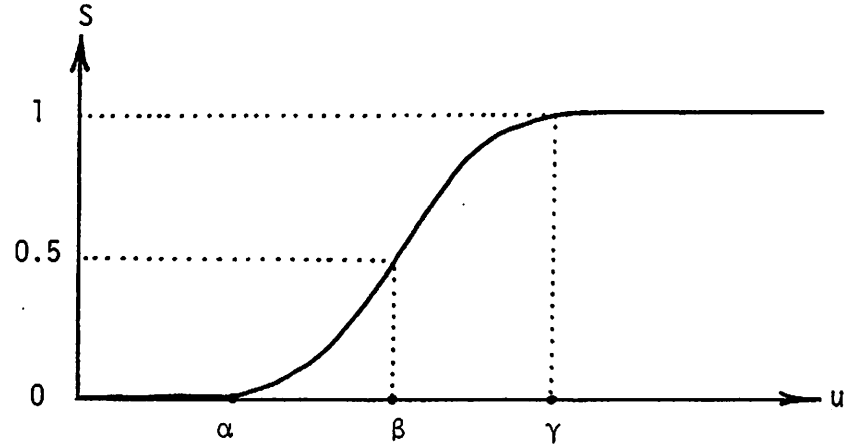

Fig. 3. The S-membership function.

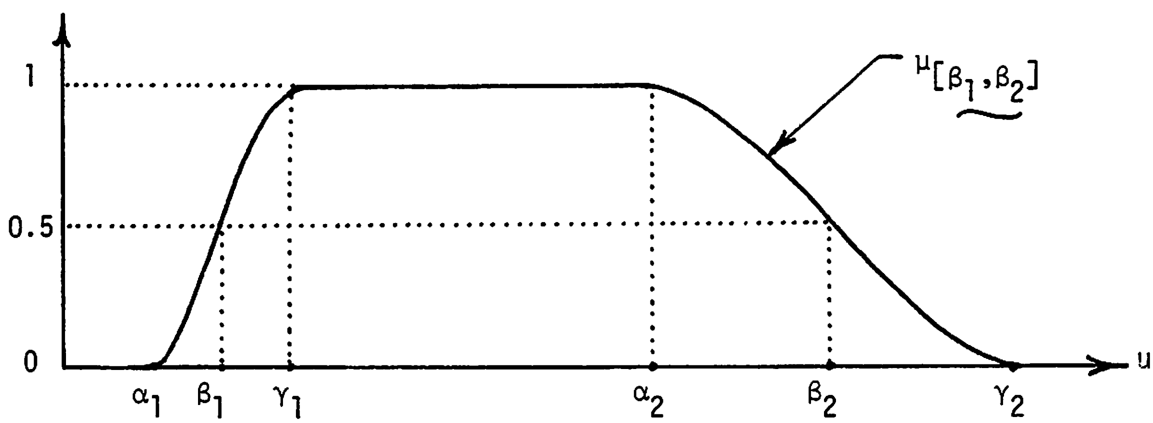

Fig. 4. A fuzzy interval $\underset{\sim}{[\beta_1,\beta_2]}$.

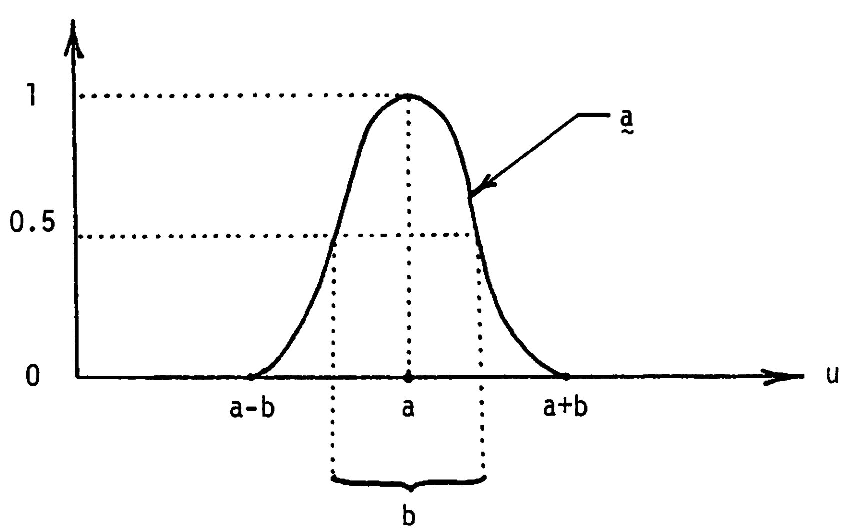

Fig. 5. 'Around a', as a fuzzy number $\underset{\sim}{a}$.

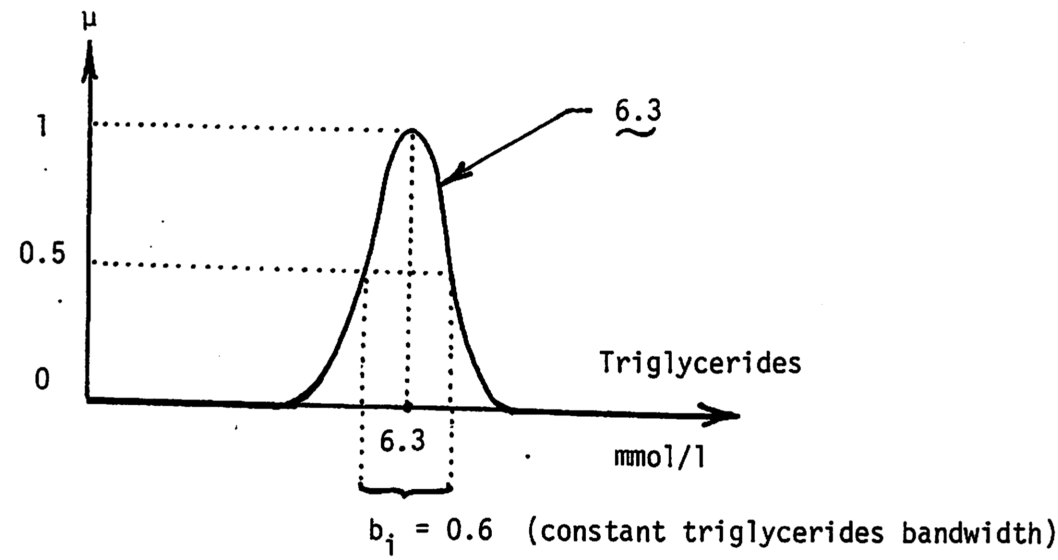

Fig. 6. Possibility distribution for 'triglycerides around 6.3'.

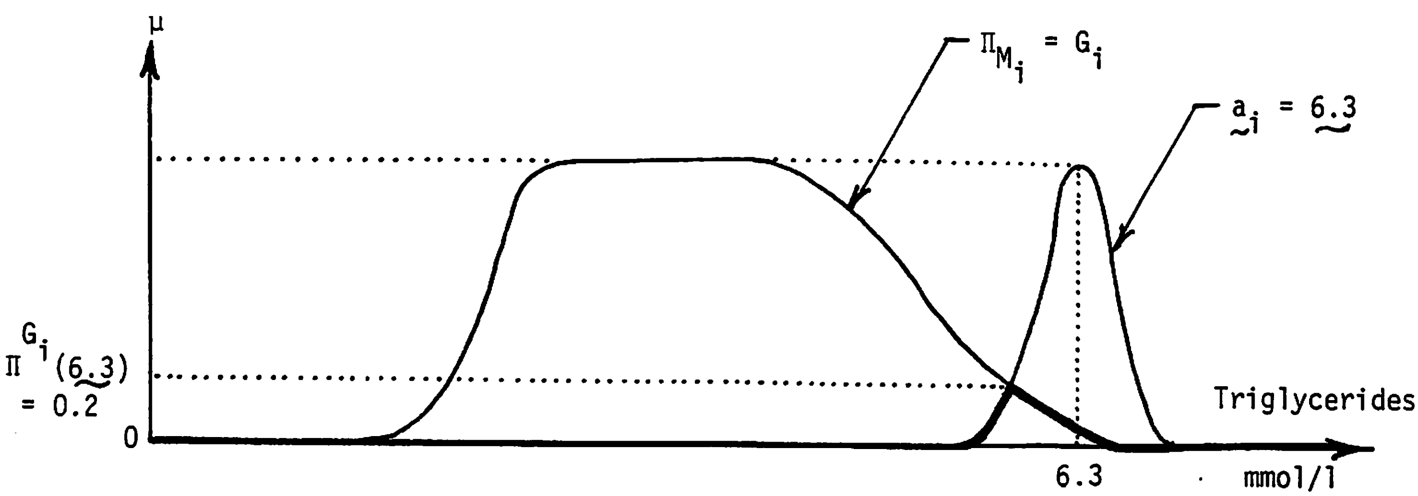

Fig. 7. Possibility measure of $\underset{\sim}{a}_i$ with respect to Π_{M_i}.