TWO-STEP COMPENSATION OF NONLINEAR SYSTEMS

by

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Abstract

We study the problem of stabilization of nonlinear plants. We show that given a nonlinear plant P, if there exists a (nonlinear) compensator F, possibly unstable, which stabilizes P, then, with $P_1 := P(I-F(-P))^{-1}$, any C defined by $C := F + Q(I-P_1Q)^{-1}$ for some finite-gain stable Q will stabilize P.

Keywords: Nonlinear feedback, nonlinear stability, compensator design, multiple-loop systems.

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I. Introduction

The Q-parametrization method, used by Newton et al. [New. 1] for the conventional linear single-input single-output systems, was correctly established by Zames [Zam. 1] for a very general class of linear multiple-input multiple-output systems; this method has proven to be useful in controller design [Des. 1], [Gus. 1]. The method has been extended to the nonlinear case by Desoer and Liu [Des. 2] who used input/output representations. In all the cases above, the method requires that the plant be stable. Since unstable plants do occur in practice, it is of interest to find out whether this method extends to unstable plants. For the linear case, Zames obtained a "decomposition principle" [Zam. 2, p. 307]: he described a "two-stage feedback scheme" for a class of linear plants which are stabilizable by stable linear compensators. Namely, given an unstable linear plant, Zames proposed to stabilize it by a stable linear compensator first, then use the Q-parametrization to obtain the class of all linear stabilizing compensators. For the nonlinear case, Anantharam and Desoer [Ana. 1] showed that given an unstable nonlinear plant (specified by its input-output map P) which can be stabilized by an incrementally stable compensator F, then, for any finite-gain stable map Q, with $P_1 := P(I-F(-P))^{-1}$, the compensator $C := F + Q(I-P_1Q)$ will also stabilize P.

In this paper, we show that the assumption that the nonlinear plant P be stabilizable by an incrementally stable compensator is not necessary. Roughly speaking, we show that given a nonlinear plant P, if there exists a (nonlinear) compensator F, possibly unstable, which stabilizes P, then, with $P_1 := P(I-F(-P))^{-1}$, any C defined by

$C := F + Q(I-P_1Q)^{-1}$, for some finite-gain stable Q will also stabilize P.

II. Definitions and Notations

Let $(\mathcal{L}, \|\cdot\|)$ be a normed space of 'time functions': $T \to V$ where T is the time-set (typically $\mathbb{R}_+$ or $\mathbb{N}$), V is a normed space (typically $\mathbb{R}, \mathbb{R}^n, \mathbb{C}^n, \ldots$) and $\|\cdot\|$ is the chosen norm on $\mathcal{L}$. Let $\mathcal{L}_e$ be the corresponding extended space [Wil. 1], [Des. 3], [Vid. 1]. A nonlinear causal map $P : \prod_{j=1}^{m} \mathcal{L}_e^{n_j} \to \prod_{k=1}^{\ell} \mathcal{L}_e^{m_k}$ is said to be finite-gain (f.g.) stable iff $\exists\ \gamma(P) < \infty$, s.t. $\forall\ T > 0$, $\forall\ (u_1,u_2,\ldots,u_m) \in \prod_{j=1}^{m} \mathcal{L}_e^{n_j}$

$$\|P(u_1,u_2,\ldots,u_m)\|_T \le \gamma(P)(\|u_1\|_T + \|u_2\|_T + \ldots + \|u_m\|_T).$$

P is said to be incrementally (inc.) stable iff a) P is f.g. stable, b) $\exists\ \tilde{\gamma}(P) < \infty$, s.t. $\forall\ T > 0$, $\forall\ (u_1,u_2,\ldots,u_m)$, $(\bar{u}_1,\bar{u}_2,\ldots,\bar{u}_m) \in \prod_{j=1}^{m} \mathcal{L}_e^{n_j}$,

$$\|P(u_1,u_2,\ldots,u_m) - P(\bar{u}_1,\bar{u}_2,\ldots,\bar{u}_m)\|_T \le \tilde{\gamma}(P)(\|u_1-\bar{u}_1\|_T + \|u_2-\bar{u}_2\|_T$$

$$+ \ldots + \|u_m-\bar{u}_m\|_T).$$

A nonlinear system N with input $(u_1,u_2,\ldots,u_m) \in \prod_{j=1}^{m} \mathcal{L}_e^{n_j}$ and output $(z_1,\ldots,z_\ell) \in \prod_{k=1}^{\ell} \mathcal{L}_e^{m_k}$ is said to be f.g. stable iff $\exists\ \gamma(N) < \infty$ s.t. $\forall\ T > 0$, $\forall (u_1,u_2,\ldots,u_m) \in \prod_{j=1}^{m} \mathcal{L}_e^{n_j}$, for any corresponding output $(z_1,z_2,\ldots,z_\ell)$, $\|z_1\|_T+\|z_2\|_T+\ldots+\|z_\ell\|_T \le \gamma(N)(\|u_1\|_T+\|u_2\|_T+\ldots+\|u_m\|_T)$. In case the system N is specified by its input-output map, then the f.g. stability (inc. stability, resp.) of the system N is equivalent to the f.g. stability (inc. stability, resp.) of its input-output map. ${}^{1}S(P,F)$ denotes the system shown in Fig. 1, with (e_2'', u_3) as input and (y_2, y_3) as output. ${}^{1}S(P,C)$ and ${}^{1}S(P_1, C-F)$ are defined similarly (see Fig. 2 and Fig. 3, resp.).

III. Main Result

Theorem:

Let $P : \mathcal{L}_e^{n_i} \to \mathcal{L}_e^{n_o}$ and $F : \mathcal{L}_e^{n_o} \to \mathcal{L}_e^{n_i}$ be nonlinear causal maps such that

(A.1) (i) The I/O map of the system ${}^{1}S(P,F)$, $\psi := (\psi_2, \psi_3) : (e_2'', u_3) \to (y_2, y_3)$ is a well-defined causal map from $\mathcal{L}_e^{n_i} \times \mathcal{L}_e^{n_o} \to \mathcal{L}_e^{n_o} \times \mathcal{L}_e^{n_i}$; and

(ii) ${}^{1}S(P,F)$ is inc. stable.

Let $P_1 := P(I-F(-P))^{-1}$ and let $Q : \mathcal{L}_e^{n_o} \to \mathcal{L}_e^{n_i}$ be f.g. stable such that $(I-P_1Q)^{-1}$ is a well-defined causal map from $\mathcal{L}_e^{n_o} \to \mathcal{L}_e^{n_o}$. Let

$$C := F + Q(I-P_1Q)^{-1} \tag{1}$$

satisfy the following condition:

(A.2) The map $H : (u_1, u_2, u_3) \to (y_1, y_2, y_3, e_2'')$ associated with the system ${}^{3}S(P,F,C-F)$ shown in Fig. 4 is a well-defined causal map from $\mathcal{L}_e^{n_o} \times \mathcal{L}_e^{n_i} \times \mathcal{L}_e^{n_o} \to \mathcal{L}_e^{n_i} \times \mathcal{L}_e^{n_o} \times \mathcal{L}_e^{n_i} \times \mathcal{L}_e^{n_i}$.

Under these conditions

(i) ${}^{3}S(P,F,C-F)$ is f.g. stable, and

(ii) ${}^{1}S(P,C)$ is f.g. stable.

Comments

(a) ${}^{3}S(P,F,C-F)$ is f.g. stable iff the map $(u_1, u_2, u_3) \mapsto (y_1, y_2, y_3)$ is f.g. stable.

(b) Note that by (A.1), $P_1(\cdot) = \psi_2(\cdot,0)$ is a well-defined incrementally stable causal map from $\mathcal{L}_e^{n_i} \to \mathcal{L}_e^{n_o}$.

(c) Note that none of the maps P, F, C-F, C are required to be stable.

<u>Proof</u>:

We first prove (i).

By (A.1), $P_1 := P(I-F(-P))^{-1}$ is inc. stable. Since Q is f.g. stable, it follows that, with $C-F := Q(I-P_1Q)^{-1}$, the system ${}^1S(P_1,C-F)$, shown in Fig. 3, is f.g. stable [Theorem N, Des. 2].

Fig. 4 shows the system ${}^3S(P,F,C-F)$ with input (u_1,u_2,u_3) and output (y_1,y_2,y_3,e_2''). When $u_3 = 0$, y_2 is given in terms of e_2'' by $y_2 = P(I-F(-P))^{-1}e_2''$. Hence, if we set $u_3 = 0$ and consider only the output (y_1,y_2,e_2''), then the system ${}^3S(P,F,C-F)$ reduces to ${}^1S(P_1,C-F)$ of Fig. 3. Now ${}^1S(P_1,C-F)$ has (u_1,u_2) as input and (y_1,y_2,e_2'') as output, and is f.g. stable. By (A.2) it follows that, for the system ${}^3S(P,F,C-F)$, the <u>partial</u> map $(u_1,u_2,0) \mapsto (y_1,y_2,e_2'')$ is f.g. stable.

Next consider the system ${}^3S(P,F,C-F)$ with (u_1,u_2,u_3) as input and call $(\bar{y}_1,\bar{y}_2,\bar{y}_3,\bar{e}_2'')$ the corresponding output. Define

$$\Delta\tilde{y}_2 := \psi_2(\bar{e}_2'',u_3) - \psi_2(\bar{e}_2'',0) \tag{2}$$

To ${}^3S(P,F,C-F)$, apply $(u_1-\Delta\tilde{y}_2,u_2,0)$. Call $(\tilde{y}_1,\tilde{y}_2,\tilde{y}_3,\tilde{e}_2'')$ the corresponding output. We claim that $\bar{y}_1 = \tilde{y}_1$, $\bar{y}_2 = \tilde{y}_2 + \Delta\tilde{y}_2$ and $\bar{e}_2'' = \tilde{e}_2''$.

To prove this, consider the equations defining $\bar{y}_1$, $\bar{y}_2$, and $\bar{e}_2''$:

$$\bar{y}_1 = (C-F)(u_1-\bar{y}_2)$$

$$\bar{e}_2'' = \bar{y}_1 + u_2$$

$$\bar{y}_2 = \psi_2(\bar{e}_2'', u_3)$$

By (A.2), these equations have a unique solution. Writing the equations defining $\tilde{y}_1$, $\tilde{y}_2$, and $\tilde{e}_2''$, using (2), and invoking the uniqueness, we easily conclude that $\bar{y}_1 = \tilde{y}_1$, $\bar{y}_2 = \tilde{y}_2 + \Delta\tilde{y}_2$, $\bar{e}_2'' = \tilde{e}_2''$. Since, by (A.1), ${}^1S(P,F)$ is inc. stable, $\exists\ \tilde{\gamma}_2 < \infty$ s.t. $\forall\ T > 0$, $\forall\ (u_1,u_2,u_3) \in \mathcal{L}_e^{n_o} \times \mathcal{L}_e^{n_i} \times \mathcal{L}_e^{n_o}$

$$\|\Delta\tilde{y}_2\|_T = \|\psi_2(\bar{e}_2'',u_3) - \psi_2(\bar{e}_2'',0)\|_T \leq \tilde{\gamma}_2\|u_3\|_T \leq \tilde{\gamma}_2(\|u_1\|_T + \|u_2\|_T + \|u_3\|_T) \tag{3}$$

Thus, for ${}^3S(P,F,C\text{-}F)$, the map $(u_1,u_2,u_3) \mapsto \Delta\tilde{y}_2$ is f.g. stable, consequently, so is the map $(u_1,u_2,u_3) \mapsto (u_1 - \Delta\tilde{y}_2, u_2, 0)$; finally, using the f.g. stability of the partial map $(u_1,u_2,0) \mapsto (y_1,y_2,e_2'')$ proved earlier, we see that the composed map $(u_1,u_2,u_3) \mapsto (\tilde{y}_1,\tilde{y}_2,\tilde{e}_2'')$ is f.g. stable; and hence by (3), $(u_1,u_2,u_3) \mapsto (\bar{y}_1,\bar{y}_2,\bar{e}_2'')$ is f.g. stable. Now, by (A.1), $\psi_3 : (\bar{e}_2'',u_3) \mapsto \bar{y}_3$ is f.g. stable, consequently, for the system ${}^3S(P,F,C\text{-}F)$, the map $(u_1,u_2,u_3) \mapsto (\bar{y}_1,\bar{y}_2,\bar{y}_3,\bar{e}_2'')$ is f.g. stable. Thus, (i) is proved.

Consider the equations of ${}^3S(P,F,C\text{-}F)$ written in terms of the $\bar{e}_i$'s:

$$\bar{e}_1 = u_1 - P\bar{e}_2 \tag{4}$$

$$\bar{e}_2'' = u_2 + (C-F)\bar{e}_1 \tag{5}$$

$$\bar{e}_2 = \bar{e}_2'' + F\bar{e}_3 \tag{6}$$

$$\bar{e}_3 = u_3 - P\bar{e}_2 \tag{7}$$

If we set $u_3 = u_1$, then (4) and (7) show that $\bar{e}_3 = \bar{e}_1$, and the equations (4)-(6) reduce to

$$\bar{e}_1 = u_1 - P\bar{e}_2$$

$$\bar{e}_2 = u_2 + C\bar{e}_1$$

The last two equations describe ${}^1S(P,C)$. Hence the proof of (i) implies that ${}^1S(P,C)$ is f.g. stable. □

Corollary

Let $P : \mathcal{L}_e^{n_i} \to \mathcal{L}_e^{n_o}$, $F : \mathcal{L}_e^{n_o} \to \mathcal{L}_e^{n_i}$, and $N : \mathcal{L}_e^{n_o} \times \mathcal{L}_e^{n_i'} \to \mathcal{L}_e^{n_i} \times \mathcal{L}_e^{n_o'}$ be nonlinear causal maps such that

(A.1) (i) for the system ${}^1S(P,F)$ shown in Fig. 1, $\psi : (e_2'', u_3) \mapsto (y_2, y_3)$ is a well-defined causal map from $\mathcal{L}_e^{n_i} \times \mathcal{L}_e^{n_o} \to \mathcal{L}_e^{n_o} \times \mathcal{L}_e^{n_i}$,

(ii) ${}^1S(P,F)$ is inc. stable;

(A.3) For the nonlinear feedback system S shown in Fig. 5, $\underline{S} : (v_1, u_1, u_2, u_3) \mapsto (y_1, y_2, y_3, y_4)$ is a well-defined causal map from $\mathcal{L}_e^{n_i'} \times \mathcal{L}_e^{n_o} \times \mathcal{L}_e^{n_i} \times \mathcal{L}_e^{n_o} \to \mathcal{L}_e^{n_i} \times \mathcal{L}_e^{n_o} \times \mathcal{L}_e^{n_i} \times \mathcal{L}_e^{n_o'}$.

Under these conditions,

S is f.g. stable if and only if

S_o is f.g. stable (see Fig. 6).

Comments

(a) The corollary yields a simplification in the stability analysis of a class of nonlinear feedback systems containing a minor-loop: if the minor-loop ${}^1S(P,F)$ is inc. stable, then to check the stability of S of Fig. 5, it is enough to check the stability of S_0 of Fig. 6, where the minor-loop ${}^1S(P,F)$ is replaced by the map $P(I-F(-P))^{-1}$.

(b) From a synthesis point of view, the corollary may be useful in designing two-input compensators for a class of nonlinear plants, namely, the class of all nonlinear plants P which are stabilizable under the configuration ${}^1S(P,F)$ of Fig. 1, for some nonlinear compensator F.

(c) In actual design, a minor-loop is not necessary. Consider the system S of Fig. 5, instead of putting F as a feedback around P, we may put F in parallel with N such that F takes e_1 as input and such that F feeds its output into the summing node associated with y_1 and u_2. The resulting system $\tilde{S}$ with input (v_1, u_1, u_2) is "equivalent" to the system S with input (v_1, u_1, u_2, u_1). Hence $\tilde{S}$ is f.g. stable if S is f.g. stable.

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Figure Captions

Fig. 1 shows the system ${}^1S(P,F)$ in which F stabilizes P.

Fig. 2 shows the system ${}^1S(P,C)$.

Fig. 3 shows the system ${}^1S(P_1,C)$ in which $P_1 = P(I-F(-P)^{-1}$.

Fig. 4 shows the system ${}^3S(P,F,C-F)$ in which the structure of P_1 is shown in detail.

Fig. 5 shows the system S with the two-input compensator N.

Fig. 6 shows the system S_0: it differs from S in that the internal structure of P_1 is ignored and the input u_3 is absent.

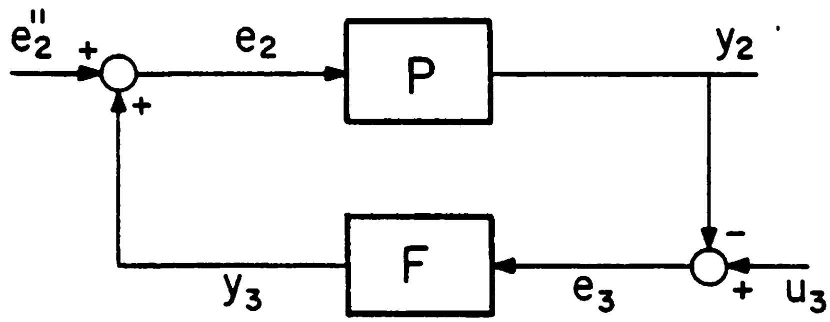

Figure 1

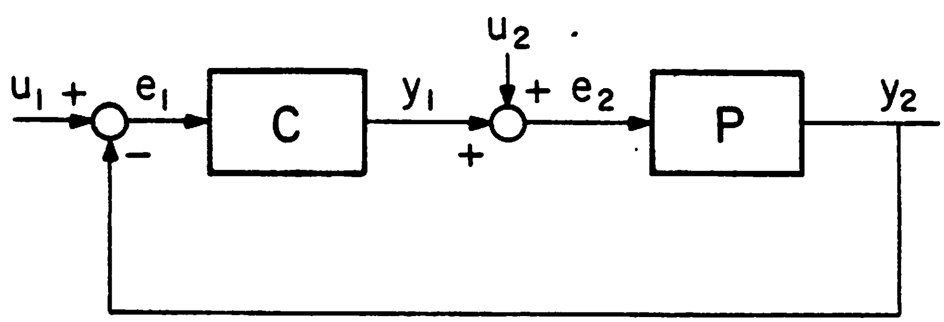

Figure 2

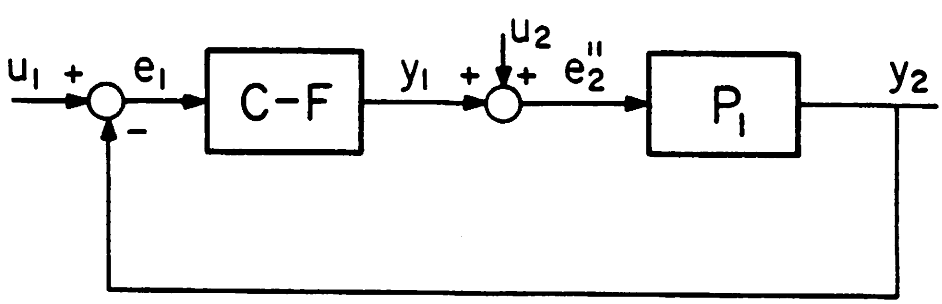

Figure 3

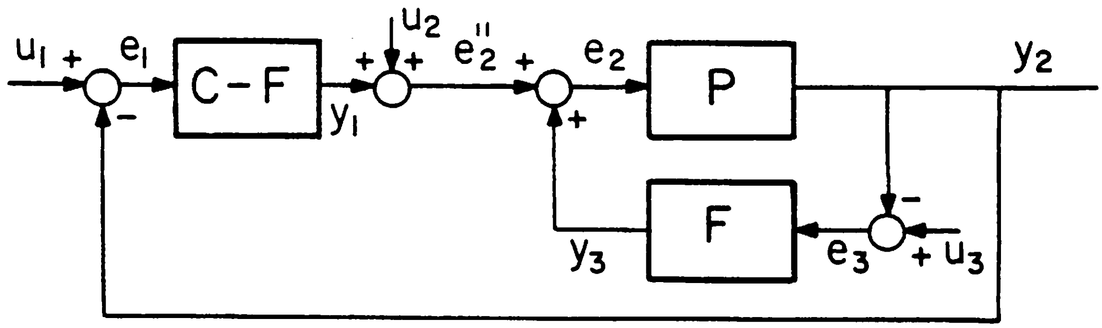

Figure 4

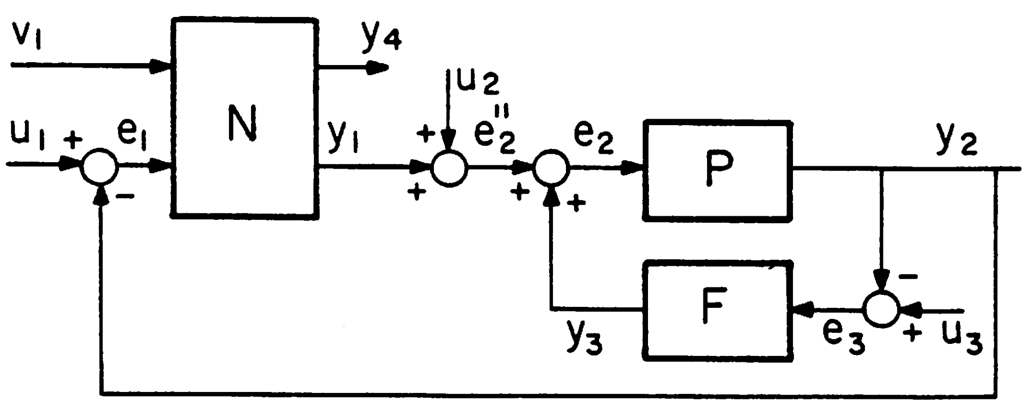

Figure 5

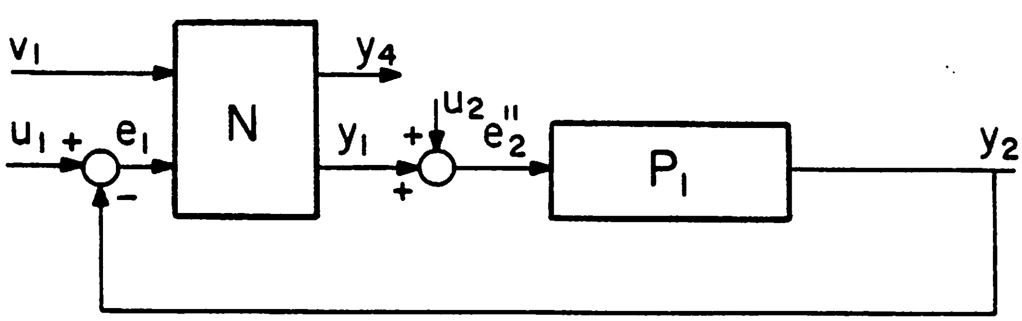

Figure 6