STEADY-STATE SECURITY REGIONS OF POWER SYSTEMS

by

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Memorandum No. UCB/ERL M82/7

13 January 1982

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ABSTRACT

A steady-state security region is a set of real and reactive power injections (load demands and power generations) for which the power flow equations and the security constraints imposed by equipment operating limits are satisfied. The problem of determining steady-state security regions is formulated as one of finding sufficient conditions for the existence of solutions to the power flow map within the security constraint set. Explicit limits on real and reactive power injections at each bus are obtained, such that if each injection lies within the corresponding limits, the system is guaranteed to operate with security constraints satisfied.

Research sponsored by the Dept. of Energy, Electric Energy Systems Division, under contract DE-AC01-79-ET29364 and the National Science Foundation Grant ENG-75-21747.

1. INTRODUCTION

The steady-state operation of an electric power supply system requires that the power supply and the load demand must be balanced. This is described by a set of nonlinear equations known as the power flow, or load flow, equations [1,2]. Furthermore, the system has to be operated within the designed limits of the equipments. This is described by a set of inequality constraints, sometimes referred to as the security constraints. The fundamental problem in the steady-state analysis of power systems is to determine, for a given set of load demand and generation pattern, whether the system can be operated in such a way that all the equipments are loaded within their security constraints. We define a steady-state security region as a set of power injections (load demands and power generations) for which the power flow equations and the security constraints are satisfied. The concepts of steady state security regions and dynamic security regions are used in our proposed framework for probabilistic dynamic security assessment [3]. A method for deriving dynamic security regions is presented in [4]. In this paper an analytic approach for deriving steady-state security regions is proposed and the resulting regions are presented.

The conventional approach to the steady-state analysis of power systems is to solve the power flow equations numerically and then check whether the security constraints are satisfied [5,6]. Recently there have been some attempts using analytic approach to tackle the problem. Hnylicza et al. [7] used the linearized (DC) power flow equations to derive steady-state security regions with respect to a set of given contingencies. Fischl and De Maio [8] have suggested a scheme for identifying these regions. Dersin and Levis [9] have derived, using mathematical programming duality, some characterizations of these regions. All these works are based on the DC load flow model. On the other hand, Galiana and Banakar [10] have used the rectangular coordinates of the power flow equations to derive linear and quadratic approximation formulas for the characterizations of the steady-state security regions. The work reported in this paper is based on the full-fledged power flow model. The proposed approach is to formulate the problem as one of determining the conditions for the existence of solutions to the power flow equations in the set defined by the security constraints.

The formulation of the problem is presented in Sec. 2. The proposed approach to solve the problem is outlined in Sec. 3. The results are presented in Sec. 4. A simple example is given in Sec. 5. The proposed approach has two steps. At the first step, two simpler existence-of-solution problems using the approximate formulation of the decoupled power flow equations are considered. Leray-Schauder fixed point theorem and concepts from Circuit Theory are used in the derivation. The results we obtain for the two simpler problems are utilized to derive results for the original existence-of-solution problem using the full-fledged power flow equations at the second step. The analytic tool used here is the theory of the degree of mapping. The theory of degree has previously been applied successfully to the investigation of the existence-of-solution problems in nonlinear circuit theory [11-13]. It should be noted that in our approach the decoupled power flow equations are used merely as a stepping stone and the resulting steady-state security regions are <u>exact</u>, without approximations.

Standard notation is used in this paper. Q_k denotes the k-th component of a vector $\underline{Q}$, Y_{ki} denotes the ki-th element of the matrix Y, $\underline{V} \leq \underline{V}^M$ means $V_i \leq V_i^M$ for all i, and x := E means that x is defined by the expression E.

2. FORMULATION

2.1. Power Flow Equations

The branches of a power network represent transmission lines, transformers, etc., which are modeled as linear time-invariant RLC elements. The nodes of the network other than the ground node are called <u>buses</u>. They correspond to generation stations and load-center substations. For steady-state analysis the network is considered as in sinusoidal steady state.

Consider a power network with N+1 buses. Let [Y] denote the $(N+1) \times (N+1)$ node (bus) admittance matrix of the network and $Y_{ki} = G_{ki} + jB_{ki}$ be its ki-th element. Using the standard models of transmission lines and transformers [14, p. 189 and p. 122], we have†

<u>Fact 1</u> $G_{kk} > 0$, $B_{kk} < 0$; $G_{ki} \leq 0$ and $B_{ki} \geq 0$ for $i \neq k$.

$$|B_{kk}| \geq \sum_{\substack{i=0 \\ i \neq k}}^{N} B_{ki} \text{ and } G_{kk} \geq \sum_{\substack{i=0 \\ i \neq k}}^{N} |G_{ki}|.$$

We assume that:

(A1) The network is connected

(A2) The matrix [Y] is symmetric; in particular, $B_{ki} = B_{ik}$.††

Let E_k denote the bus voltage phasor of bus k and $S_k = P_k + jQ_k$ denote the injected complex power at bus k. Let $\underline{E}$ and $\underline{P}$ be the vectors of complex voltages and complex power injections, respectively. For

†In our model, loads are represented by real and reactive power demands, rather than shunt impedances.

††This is true if there is no phase-shifting transformer in the system.

convenience, we introduce a diagonal matrix $[E] = \text{diag}\{E_1, E_2, \cdots, E_N\}$. Then we have

$$\underline{S}^* = [E^*][Y]\underline{E} \quad (1)$$

where superscript * denotes complex conjugate. There are three types of buses:

(i) Slack bus: a bus whose voltage magntidue and phase angle are specified.

(ii) PQ bus: a bus where the injected real and reactive power are specified.

(iii) PV bus: a bus where the injected real power and the voltage magnitude are specified.

Normally PQ buses are load buses and PV buses and the slack bus are generator buses. We let subscript 0 correspond to the slack bus, subscripts $\{1,2,\cdots N_Q\}$ correspond to PQ buses, and subscripts $\{N_Q+1,\cdots,N\}$ correspond to PV buses. Let $E_k = V_k e^{j\theta_k}$ and $\theta_{ki} = \theta_k - \theta_i$. We may express (1) as

$$\sum_{i=0}^{N} V_k V_i \,(G_{ki} \sin\theta_{ki} - B_{ki}\cos\theta_{ki}) = Q_k \qquad k = 1,2,\cdots N_Q \quad (2)$$

$$\sum_{i=0}^{N} V_k V_i \,(G_{ki} \cos\theta_{ki} + B_{ki}\sin\theta_{ki}) = P_k \qquad k = 1,2,\cdots N \quad (3)$$

where $\underline{V} = (V_1, V_2, \cdots V_{N_Q})^T$ and $\underline{\theta} = (\theta_1, \theta_2, \cdots \theta_N)^T$ are the unknown variables or the state variables, and $\underline{Q} = (Q_1, \cdots Q_{N_Q})^T$ and $\underline{P} = (P_1, \cdots P_N)^T$ are the power injections. Equations (2) and (3) are known as the power flow equations [1,2,14]. For ease of later reference we represent Eqs. (2) and (3) in the form

$$f(\underline{x}) = \underline{y} \quad (4)$$

where† $\underline{x} = (\underline{V},\underline{\theta})$ is the set of state variables and $\underline{y} = (\underline{Q},\underline{P})$ is the set of power injections.

2.2. Decoupled Power Flow Equations

Suppose that we make the following simplifying assumptions:

(SA1) The line resistances are negligible, i.e., $G_{ki} = 0$.

(SA2) The phase angles across the branches $\theta_{ki} = \theta_k - \theta_i$ are small so that the second and higher order terms in the series expansions of $\sin \theta_{ki}$ and $\cos \theta_{ki}$ are negligible, i.e., $\cos \theta_{ki} \simeq 1$, $\sin \theta_{ki} \simeq \theta_{ki}$.

Then the power flow equations (2)(3) become

$$Q_k = \tilde{Q}_k(\underline{V}) := -V_k \sum_{i=0}^{N} B_{ki} V_i \qquad k = 1,2,\cdots N_Q \tag{5}$$

$$P_k = \tilde{P}_k(\underline{V}, \underline{\theta}) := V_k \sum_{i=0}^{N} B_{ki} V_i (\theta_k - \theta_i) \qquad k = 1,2,\cdots N \tag{6}$$

Equations (5) and (6) may be written in a compact matrix form

$$-[V] \{[B] \underline{V} + [B^\circ] \underline{V}^\circ\} = \underline{Q} \tag{7}$$

$$[B'(\underline{V})] \underline{\theta} = \underline{P} \tag{8}$$

where $\underline{V}^\circ = (V_0, V_{N_Q+1},\cdots V_N)^T$, $\underline{Q} = (Q_1,\cdots Q_{N_Q})^T$, $\underline{P} = (P_1,P_2\cdots P_N)^T$, the ki-th element of [B] is B_{ki}, $k, i \in \{1,2,\cdots N_Q\}$ and the elements of $[B^\circ]$ are B_{ki}, $k \in \{1,\cdots N_Q\}$, $i \in \{0,N_Q+1,\cdots N\}$. $[B'(\underline{V})]$ is an NxN matrix whose elements are functions of $\underline{V}$. The diagonal and off-diagonal elements of $[B'(\underline{V})]$ are

†It is understood that $\underline{x} = (\underline{V},\underline{\theta})$ means $\underline{x} = (V_1,\cdots V_{N_Q},\theta_1,\cdots \theta_N)^T$.

$$B'_{kk}(\underline{V}) = V_k(\sum_{\substack{i=0\\i\neq k}}^{N} B_{ki}V_i) \qquad k = 1,2,\cdots N \qquad (9)$$

$$B'_{ki}(\underline{V}) = -V_kV_iB_{ki} \qquad \begin{array}{l} k \neq i \\ k,i = 1,2,\cdots N \end{array} \qquad (10)$$

Equations (7) and (8) are called the decoupled power flow equations [15,16].

2.3. Security Constraints

2.3.1. Voltage Magnitude Constraints

Operating limits are imposed on the voltage magnitudes of PQ buses, i.e.,

$$\underline{V}^m \leq \underline{V} \leq \underline{V}^M \qquad (11)$$

Let us denote the region inside the limits by R_V, i.e.,

$$R_V := \{\underline{V} \mid \underline{V}^m \leq \underline{V} \leq \underline{V}^M \qquad (12)$$

2.3.2. Line Current Constraints

Thermal considerations limit the amount of current flowing through transmission lines and transformers. The current I_j through branch j connecting bus k and bus i may be approximated as follows:

$$\begin{aligned} I_j &= -Y_{ki}(E_k - E_i) \\ &= -jB_{ki}(V_k e^{j\theta_k} - V_i e^{j\theta_i}) \\ &= -jB_{ki}V_k E^{j\theta_i} (\cos \theta_{ki} + j \sin \theta_{ki} - \frac{V_i}{V_k}) \\ &\approx -jB_{ki}V_k e^{j\theta_i} (j\theta_{ki}) \end{aligned}$$

where we used the approximations $\cos \Theta_{ki} \simeq 1$ $\sin \theta_{ki} \approx \theta_{ki}$ and $\frac{V_i}{V_k} \simeq 1.$†

†This is true when per-unit system is used.

Hence

$$|I_j| \approx |B_{ki}||V_k||\theta_k - \theta_i| \tag{13}$$

Therefore the line flow constraints $|I_j| \leq I_j^{max}$ may be expressed approximately in terms of the phase angle difference

$$|\theta_k - \theta_i| \leq \delta_j \qquad j \text{ is a branch connecting buses } k \text{ and } i \tag{14}$$

where $\delta_j = \dfrac{I_j^{max}}{|B_{ki}||V_k^M|}$. We assume that:

(A3) $\delta_j \leq \frac{\pi}{2}$.

We may use the incidence matrix A of the network and write (14) for all the branches in the network in a vector inequality

$$-\underline{\delta} \leq A^T\underline{\theta} \leq \underline{\delta} \tag{14}$$

Let us denote the corresponding region in $\underline{\theta}$ by R_θ:

$$R_\theta := \{\underline{\theta} \mid -\underline{\delta} \leq A^T \underline{\theta} \leq \underline{\delta}\} \tag{15}$$

We are going to use the approximate expression (15) for line flow constraints even when the full-fledged power flow equations are used. The justification for this is that unlike the "hard" constraints on power generation due to equipment limitation that wiill be introduced shortly, the line flow constraints are "soft" constraints for which approximation is usually adequate.

2.3.3. Real Power Generation Constraints

Physical limitation imposes constraints on the amount of real and reactive power that can be generated at PV buses, as well as the slack bus. The real power constraints on PV buses are

$$p_k^m \leq P_k \leq p_k^M \qquad k = N_Q + 1, \cdots N \tag{17}$$

where p_k^m and p_k^M are, respectively the minimum and maximum real power generation at bus k. The real power generation from the slack bus is a function of $(\underline{V}, \underline{\theta})$ so the constraints are of the form

$$p_0^m \leq P_0(\underline{V},\underline{\theta}) \leq p_0^M \tag{18}$$

where p_0^m and p_0^M are, respectively, the minimum and maximum real power generation at the slack bus. Let us denote the region in which (18) is satisfied to be

$$R_p := \{(\underline{V},\underline{\theta}) \mid p_0^m \leq P_0(\underline{V},\underline{\theta}) \leq p_0^M\} \tag{19}$$

2.3.4. Reactive Power Generation Constraints

The reactive power generation at the slack bus or a PV bus k is

$$Q_k(\underline{V},\underline{\theta}) := \sum_{i=0}^{N} V_k V_i \; (G_{ki} \sin \theta_{ki} - B_{ki} \cos \theta_{ki}) \tag{20}$$

$$k = 0, N_Q+1, \cdots N$$

The reactive power constraints may be expressed as

$$q_k^m \leq Q_k \; (\underline{V},\underline{\theta}) \leq q_k^M \qquad K = 0, N_Q+1, \cdots N \tag{21}$$

where q_k^m and q_k^M are, respectively, the minimum and maximum reactive power generation at bus k. Let us denote the region in which (21) is satisfied as R_q

$$R_q := \{(\underline{V},\underline{\theta}) \mid q_k^m \leq Q_k(\underline{V},\underline{\theta}) \leq q_k^M, \; k = 0, N_Q+1, \cdots N\} \tag{22}$$

2.3.5. Security Constraint Set

We call the constraints (11) (14) (18) and (21) the _security constraints_, and the set R in $(\underline{V},\underline{\theta})$-space

$$R := (R_V \times R_\theta) \cap R_p \cap R_q \tag{23}$$

the _security constraint set_.

Sufficient conditions that guarantee $R = R_V \times R_\theta$ are given in Fact 2 below.

Fact 2. If conditions (C1) and (C2) below hold,

(C1) The reactive power generation limits q_k^m and q_k^M at the slack bus and any PV bus k satisfy

$$q_k^m \leq \tilde{Q}_k(\underline{V}^M) - \alpha_k^m \text{ and } q_k^M \geq \tilde{Q}_k(\underline{V}^m) + \alpha_k^M \tag{24}$$

$$k = 0, N_Q+1, \cdots N$$

where

$$\alpha_k^M := -\sum_{\substack{i=0 \\ i\neq k}}^{N} V_k^M V_i^M \{G_{ki} \sin \delta_j + B_{ki}(\cos \delta_j - 1)\} \tag{25}$$

$$\alpha_k^m := -\sum_{\substack{i=0 \\ i\neq k}}^{N} V_k^M V_i^M G_{ki} \sin \delta_j \tag{26}$$

and we set $V_i^M = V_i$ for $i = 0, N_Q+1, \cdots N$ in Eqs. (25) (26).

(C2) The real power generation limits p_0^m and p_0^M at the slack bus satisfy

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$^\dagger \tilde{Q}_k(\underline{V})$ for $k = 0, N_Q+1, \cdots N$ is similarly defined as in Eq. (5).

$$p_0^m \leq (-\sum_{i=1}^{N} P_i) + L^m \text{ and } p_0^M \geq (-\sum_{i=1}^{N} P_i) + L^M \tag{27}$$

where

$$L^M := \sum_{k=0}^{N} (V_k^M)^2 G_{kk} + \sum_{k=0}^{N} \sum_{\substack{i=0 \\ i\neq k}}^{N} V_k^m V_i^m G_{ki} \cos \delta_j \tag{28}$$

$$L^m := \max\{0, \sum_{k=0}^{N} (V_k^m)^2 G_{kk} + \sum_{k=0}^{N} \sum_{\substack{i=0 \\ i\neq k}}^{N} V_k^M V_i^M G_{ki}\} \tag{29}$$

then $R = R_V \times R_\theta$.

<u>Proof</u>: Comparing Eq. (5) (for $k = 0, N_Q+1, \cdots N$) and Eq. (20), we have

$$Q_k(\underline{V},\underline{\theta}) = \tilde{Q}_k(\underline{V}) + \alpha_k(\underline{V},\underline{\theta}), \quad k = 0, N_Q+1, \cdots N \tag{30}$$

where

$$\alpha_k(\underline{V},\underline{\theta}) = \sum_{\substack{i=0 \\ i\neq k}}^{N} V_k V_i G_{ki} \sin \theta_{ki} - \sum_{\substack{i=0 \\ i\neq k}}^{N} V_k V_i B_{ki} (\cos \theta_{ki} - 1) \tag{31}$$

It follows from Fact 1 that for any $\underline{V} \in R_V$ and $\underline{\theta} \in R_\theta$,

$$\tilde{Q}_k(\underline{V}^M) \leq \tilde{Q}_k(\underline{V}) \leq \tilde{Q}_k(\underline{V}^m) \tag{32}$$

Fact 1 and assumption (A3) imply that

$$-\alpha_k^m \leq \alpha_k(\underline{V},\underline{\theta}) \leq \alpha_k^M, \quad k = 0, N_Q+1, \cdots N \tag{33}$$

Hence

$$\tilde{Q}_k(\underline{V}^M) - \alpha_k^m \leq Q_k(\underline{V},\underline{\theta}) \leq \tilde{Q}_k(\underline{V}^m) + \alpha_k^M, \quad k = 0, N_Q+1, \cdots N \tag{34}$$

Condition (C1) thus implies $(R_V \times R_\theta) \subseteq R_q$

From the power flow equations (3), we obtain

$$P_0 + \sum_{i=1}^{N} P_i = L(\underline{V},\underline{\theta}) \tag{35}$$

where

$$L(\underline{V},\underline{\theta}) := \sum_{k=0}^{N} \sum_{i=0}^{N} V_k V_i \; G_{ki} \cos\,\theta_{ki}$$

$$= \sum_{k=0}^{N} V_k^2 \; G_{kk} + \sum_{k=0}^{N} \sum_{\substack{i=0 \\ i\neq k}}^{N} V_k V_i \; G_{ki} \cos\,\theta_{ki} \tag{36}$$

It follows from Fact 1 and assumption (A3) that for any $\underline{V} \in R_V$ and $\underline{\theta} \in R_\theta$, we have

$$L^m \leq L(\underline{V},\underline{\theta}) \leq L^M \tag{37}$$

Hence condition (C2) implies $(R_V \times R_\theta) \subseteq R_p$. ¤

Throughout this paper, we consider cases where conditions (C1) and (C2) are satisfied, hence

$$R = R_V \times R_\theta \tag{38}$$

2.4. Steady-State Security Regions

A steady-state security region Ω_{ss} is defined to be a set of power injections for which the power flow equations and the security constraints are satisfied, i.e.,

$$\Omega_{ss} := \{\underline{y} : \exists \underline{x} \in R \ni f(\underline{x}) = \underline{y}\} \tag{39}$$

Note that the power flow map f depends on the system configuration. Hence Ω_{ss} is defined for a fixed system configuration [3].

3. OUTLINE OF THE PROPOSED APPROACH

The problem of finding a steady-state security region can be expressed as a mathematical problem of determining the conditions for the existence of a solution to the power flow equations, i.e.,

(P) Determine a set of power injections $(\underline{Q},\underline{P})$ for which there exists a solution to the power flow equations (2)-(3) that lies in the security constraint set R.

The proposed approach to solve this problem has two steps: (i) Two simpler problems are solved first, (ii) The results are then used to solve the original problem (P). We first consider the simpler model of the decoupled power flow equations. Because of the decoupling of Q-V and P-θ in the decoupled power flow equations (7)-(8), the problem (P) is split into two, namely,

(P1) Determine a set of reactive power injections $\underline{Q}$ for which there exists a solution $\underline{V}$ to the decoupled reactive power flow equations (7) that lies in the voltage magnitude constraint set R_V.

(P2) Determine, for any $\underline{V}$ in R_V, a set of real power injections $\underline{P}$ for which there exists a solution $\underline{\theta}$ to the decoupled real power flow equations (8) that lies in R_θ.

The problem (P1) is solved in Sec. 4.1. The mapping $\underline{V} \mapsto \underline{Q}$ is nonlinear, Leray-Schauder fixed point theorem [17, p. 162] is used in the derivation. The results are presented in Theorem 1. The problem (P2) is solved in Sec. 4.2. The mapping $\underline{\theta} \mapsto \underline{P}$ is linear and has a circuit-theoretic interpretation, hence concepts from Circuit Theory [18] are used in the derivation. The results are presented in Theorems 2 and 3. These results (Theorem 1-3) are utilized to solve the problem (P) in Sec. 4.3. A simple homotopy is constructed from the decoupled power flow map to the power flow map and concepts from the degree theory [17, pp. 147-164] are used in the derivation. The results are presented in Theorems 4 and 5.

4. THE RESULTS

4.1. Existence of Secure Solution to Decoupled Reactive Power Flow Equations

The problem (P1) is solved in this subsection. Lemma 1 below is used to prove the results in Theorem 1.

Lemma 1. Let C be an open and bounded set in $\mathbb{R}^n$ containing the origin and $\underline{g}: C \to \mathbb{R}^n$ be a continuous map. If for all $\underline{x}$ on the boundary of C, denoted by ∂C, there is a k such that $x_k \neq 0$ and $x_k\, g_k(\underline{x}) \leq 0$, then there exists a $\underline{x}^*$ in the closure of C, denoted by $\bar{C}$, that satisfies $\underline{g}(\underline{x}^*) = 0$.

Proof: Let $\underline{h}(\underline{x}) = \underline{g}(\underline{x}) + \underline{x}$. We will show that the conditions in Lemma 1 imply that $\underline{h}(\underline{x})$ has a fixed point.

The conditions in Lemma 1 can be restated as follows: $\forall \underline{x} \in \partial C$ $\exists k \ni x_k \neq 0$ and $x_k\{x_k - h_k(\underline{x})\} \geq 0$. This implies that $\forall \underline{x} \in \partial C$, $\{\underline{x} - \underline{h}(\underline{x})\} + (\lambda - 1)\underline{x} \neq 0$ for $\lambda > 1$, i.e., $\forall \underline{x} \in \partial C$, $\underline{h}(\underline{x}) \neq \lambda \underline{x}$ for $\lambda > 1$. By Leray-Schauder theorem [17, p. 162], $\underline{h}(\underline{x})$ has a fixed point $\underline{x}^*$ in $\bar{C}$. ¤

Theorem 1.

If the reactive power injections Q_k at the PQ buses satisfy condition (C3) below,

$$\text{(C3)} \quad \tilde{Q}_k(\underline{V}^m) \leq Q_k \leq \tilde{Q}_k(\underline{V}^M), \quad k = 1,2,\cdots N_Q \tag{42}$$

then the decoupled reactive power flow equation (40) has a solution $\underline{V}$ in the set $R_V = \{\underline{V} \mid \underline{V}^m \leq \underline{V} \leq \underline{V}^M\}$.

Proof: Let $x_k := V_k - \frac{V_k^m + V_k^M}{2}$, and $g_k(\underline{x}) := Q_k + V_k\{\sum_{i=0}^{N_Q} V_i B_{ki} + \sum_{i=N_Q+1}^{N} V_i B_{ki}\}$.

The constraint set R_V becomes $C = \{\underline{x} \mid \frac{\underline{V}^m - \underline{V}^M}{2} \leq \underline{x} \leq \frac{\underline{V}^M - \underline{V}^m}{2}\}$, which contains the origin.

The boundary of C is the union of the boundary defined by each x_k. Let ∂C_k^+ and ∂C_k^- be the boundaries defined by $x_k = \frac{V_k^M - V_k^m}{2}$ and $x_k = \frac{V_k^m - V_k^M}{2}$, respectively.

Consider any point $\underline{x} \in \partial C_k^+$. We have $x_k > 0$ and

$$g_k(\underline{x}) = Q_k + V_k^M \{ \sum_{\substack{i=0 \\ i \neq k}}^{N_Q} V_i B_{ki} + V_k^M B_{kk} + \sum_{i=N_Q+1}^{N} V_i B_{ki} \}$$

Because $B_{ki} \geq 0$ for $i \neq k$ (Fact 1), it follows that

$$\begin{aligned} g_k(\underline{x}) &\leq Q_k + V_k^M \{ \sum_{\substack{i=0 \\ i \neq k}}^{N_Q} V_i^M B_{ki} + V_k^M B_{kk} + \sum_{i=N_Q+1}^{N} V_i B_{ki} \} \\ &= Q_k - \tilde{Q}_k(\underline{v}^M) \end{aligned} \tag{43}$$

Condition (C3) and Eq. (43) imply that $g_k(\underline{x}) \leq 0$ on ∂C_k^+. Similarly for any point $\underline{x} \in \partial C_k^-$, $x_k < 0$ and $g_k(\underline{x}) \geq 0$. Hence the conditions in Lemma 1 are satisfied and Theorem 1 is thus proved. ¤

4.2. Existence of Secure Solution to Decoupled Real Power Flow Equations

Consider Problem (P2): Determine, for any $\underline{V}$ in R_V, a set of real power injections $\underline{P} = (P_1, P_2, \ldots P_N)^T$ for which there exists a solution $\underline{\theta} = (\theta_1, \theta_2, \ldots \theta_N)^T$ to the decoupled real power flow equation

$$[B'(\underline{v})]\underline{\theta} = \underline{P} \tag{44}$$

in the constraint set

$$R_\theta = \{\underline{\theta} : -\underline{\delta} \leq A^T \underline{\theta} \leq \underline{\delta}\} \tag{45}$$

Lemmas 2 and 3 below provide some properties of the inverse of the matrix $[B'(\underline{V})]$ that are used in the proof of Theorems 2 and 3. First we give a circuit-theoretic interpretation of $[B'(\underline{V})]$.

Fact 3. Suppose that assumptions (A1) and (A2) hold. Consider a linear resistive network of two-terminal elements having the same topology as the power network and wish the conductance of branch ki being $V_k V_i B_{ki}$, then the node conductance matrix of this resistive network with the slack bus as the datum is precisely $[B'(\underline{V})]$. Consequently, $[B'(\underline{V})]$ is nonsingular, $\det[B'(\underline{V})] > 0$ and the elements of the inverse

$$[X(\underline{V})] := [B'(\underline{V})]^{-1} \tag{46}$$

are nonnegative.

Lemma 2. $X_{ki}(\underline{V}) \leq X_{kk}(\underline{V})$, $i = 1,2,\ldots N$

Proof: We use the resistive circuit interpretation of $[B'(\underline{V})]$ for the proof. Let us connect a current source across node k and the datum with 1 unit of current flowing into node k. $X_{kk}(\underline{V})$ is then the voltage magnitude across this current source and $X_{ki}(\underline{V}) = X_{ik}(\underline{V})$ is the voltage magnitude at node i. By the no gain property [18, pp. 777-778] $X_{ki}(\underline{V}) \leq X_{kk}(\underline{V})$. ¤

Lemma 3. $\underline{V}^* \geq \underline{V}$ implies $X_{kk}(\underline{V}^*) \leq X_{kk}(\underline{V})$.

Proof. $X_{kk}(\underline{V})$ is the driving-point resistance of a network consisting of two-terminal resistors with positive resistance. To increase $\underline{V}$ to $\underline{V}^*$ amounts to increasing the conductances of the branches from $V_i V_j B_{ij}$ to $V_i^* V_j^* B_{ij}$. Let us increase the conductances of these branches one at a time, the resulting driving-point resistance X_{kk} never increases. Hence at the end we have $X_{kk}(\underline{V}^*) \leq X_{kk}(\underline{V})$. ¤

We will first consider Problem (P2) for a solution in a hypercube R_θ^0 inscribed inside R_θ and then enlarge the set of interest to R_θ^1 by by moving the hypercube along a given direction.

Let

$$\phi := \min_j \frac{1}{2} \delta_j \tag{47}$$

and define a hypercube R_θ^0 in R_θ as follows:

$$R_\theta^0 := \{\underline{\theta} : -\phi\underline{1} \leq \underline{\theta} \leq \phi\underline{1}\} \tag{48}$$

where $\underline{1} = (1,1,\ldots1)^T$. We then define R_θ^1 by moving R_θ^0 along the direction of $\underline{1}$, i.e.,

$$R_\theta^1 := \{\underline{\theta} : \underline{\theta} = \underline{\theta}^0 + t\mu\underline{1},\ \underline{\theta}^0 \in R_\theta^0,\ -1 \leq t \leq 1\} \tag{49}$$

where

$$\mu := \min_{j \in S} \delta_j - \phi \tag{50}$$

$$S := \left\{ \begin{array}{l} \text{index set of branches that} \\ \text{are incident with the slack bus} \end{array} \right\} \tag{51}$$

Clearly $R_\theta^0 \subseteq R_\theta^1 \subseteq R_\theta$ (See Fig. 1).

Theorem 2 below provides the limits on real power injections in order to guarantee the existence of a solution to the decoupled real power flow equations (44) in the hypercube R_θ^0. Theorem 3 provides the limits on real power injections for Eq. (44) to have solutions in the enlarged region R_θ^1.

Theorem 2.

For any $\underline{V}$ satisfying $\underline{V}^m \leq \underline{V} \leq \underline{V}^M$, if the real power injections P_k at the PQ and PV buses satisfy condition (C4) below

(C4) $|P_k| \leq \frac{\phi}{N} \frac{1}{X_{kk}(\underline{V}^m)}$ (52)

then the decoupled real power flow equation (44) has a solution $\underline{\theta}$ in R_θ^0

<u>Proof</u>: Since $\underline{\theta} = [X(\underline{V})]\underline{P}$, we want to show

$$\left|\sum_{k=1}^{N} X_{ik}(\underline{V})P_k\right| \leq \phi, \; i = 1,2,\ldots N$$

Consider

$$\left|\sum_{k=1}^{N} X_{ik}(\underline{V})P_k\right| \leq \sum_{k=1}^{N} X_{ik}(\underline{V})|P_k| \tag{53}$$

$$\leq \sum_{k=1}^{N} X_{kk}(\underline{V}^m)|P_k| \tag{54}$$

$$\leq \phi \tag{55}$$

From (53) to (54) it is due to Lemmas 2 and 3. From (54) to (55) it is due to condition (C4). ¤

<u>Theorem 3.</u>

For any $\underline{V}$ satisfying $\underline{V}^m \leq \underline{V} \leq \underline{V}^M$, if for some $t \in [-1, 1]$ the real power injections P_k, $k = 1,2,\ldots N$, satisfy condition (C5) below

(C5) $\left(-\frac{\phi}{N}\frac{1}{X_{kk}(\underline{V}^m)} + t\mu V_k^M V_0 B_{k0}\right) \leq P_k \leq \left(\frac{\phi}{N}\frac{1}{X_{kk}(\underline{V}^m)} + t\mu V_k^m V_0 B_{k0}\right)$ (56)

then the decoupled real power flow equation (44) has a solution $\underline{\theta}$ in R_θ^1.

<u>Proof</u>: Let $\underline{P}' := \underline{P} - t\mu[B'(\underline{V})]\underline{1}$. We claim that $\underline{P}'$ satisfies (C4). Indeed,

$$\min_{\underline{V}\in R_V} \sum_{j=1}^{N} B'_{kj}(\underline{V}) = V_k^m V_0 B_{k0} \tag{57}$$

$$\max_{\underline{V}\in R_V} \sum_{j=1}^{N} B'_{kj}(\underline{V}) = V_k^M V_0 B_{k0} \tag{58}$$

Hence there exists a $\underline{\theta}^0$ in R_θ^0 such that

$$[B'(V)]\underline{\theta}^0 = \underline{P}' = \underline{P} - t\mu[B'(V)]\underline{1} \tag{59}$$

or

$$[B'(\underline{V})](\underline{\theta}^0 + t\mu\underline{1}) = \underline{P} \tag{60}$$

That is, $(\underline{\theta}^0 + t\mu\underline{1}) \in R_\theta^1$. ¤

4.3. Existence of Secure Solution to Power Flow Equations

We now tackle Problem (P) using the results (Theorems 1-3) we have obtained for the decoupled power flow equations. Theorem 4 below, which utilizes the results of Theorems 1 and 2, provides the limits on real and reactive power injections in order to guarantee the existence of a solution to the power flow equations (2)(3) in the constraint set $R_V \times R_\theta^0$, whereas Theorem 5, which utilizes the results of Theorems 1 and 3, provides the limits for a solution in $R_V \times R_\theta^1$. Each of them defines a steady-state security region.

Theorems 4 and 5 are derived with the construction of a homotopy [17, p. 135] from the approimate decoupled power flow map to the power flow map and applying the homotopy invariance property of the degree [17, pp. 156] to obtain conditions for the power flow equations to have secure solutions. Fact 4 below is used for the evaluation of the degree of the decoupled power flow map.

Fact 4. If assumptions (A4) and (A5) below, as well as assumption (A1), are satisfied,

$$\text{(A4)} \quad 2V_k > V_i \quad k,i = 1,2,\ldots N. \tag{61}$$

$$\text{(A5)} \quad \tilde{Q}_k\,(V_k = V_k^m;\ V_i = V_i^M \text{ for } i = 1,\ldots,N_Q \text{ and } i \neq j)$$

$$\geq -\Big(\sum_{i=0,N_Q+1}^{N} B_{ki}\Big)(V_k^m)^2 \qquad k = 1,2,\ldots N_Q. \tag{62}$$

and the strict inequality holds for at least one k,† then the Jacobian $D\underline{\tilde{Q}}(\underline{V})$ of the decoupled reactive power flow map $\underline{\tilde{Q}}(\cdot)$ is nonsingular for all $\underline{V}$ in R_V. Furthermore, det $D\underline{\tilde{Q}}(\underline{V}) > 0\ \forall \underline{V} \in R_V$.

Proof: Using the definition of $\tilde{Q}_k(\underline{V})$ in eq (5), the elements of the Jacobian can be computed.

$$D\tilde{Q}_{kk}(\underline{V}) = -2B_{kk}V_k - \sum_{\substack{i=0\\ i\neq k}}^{N} B_{ki}V_i,\ k = 1,2,\ldots,N_Q \tag{63}$$

$$D\tilde{Q}_{ki}(\underline{V}) = -B_{ki}V_k,\quad i = 1,2,\ldots,N_Q,\quad i \neq k \tag{64}$$

Assumption (A1) implies that $D\underline{\tilde{Q}}$ is irreducible [17, pp. 46-47]. Fact 1 and assumption (A4) implies that $D\tilde{Q}_{kk}(\underline{V}) > 0$ and $D\tilde{Q}_{ki}(\underline{V}) \leq 0$. We claim that the Jacobian is diagonally dominant. Consider

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† $\sum_{i=0,N_Q+1}^{N} B_{ki}$ denotes $B_{k0} + \sum_{i=N_Q+1}^{N} B_{ki}$.

$$|D\tilde{Q}_{kk}(\underline{V})| - \sum_{i=1}^{N_Q} |D\tilde{Q}_{ki}(\underline{V})| \tag{65}$$

$$= -2B_{kk}V_k - \sum_{\substack{i=0\\ i\neq k}}^{N} B_{ki}V_i - \sum_{i=1}^{N_Q} B_{ki}V_k \tag{66}$$

$$\geq -B_{kk}V_k - \sum_{\substack{i=0\\ i\neq k}}^{N} B_{ki}V_i + \sum_{i=0,N_Q+1}^{N} B_{ki}V_k \tag{67}$$

$$\geq -B_{kk}V_k^m - \sum_{\substack{i=0\\ i\neq k}}^{N} B_{ki}V_i^M + \sum_{i=0,N_Q+1}^{N} B_{ki}V_k^m \tag{68}$$

$$= \frac{1}{V_k^m} \{\tilde{Q}_k(V_k = V_k^m;\ V_i = V_i^M \text{ for } i = 1,\ldots N_Q \text{ and } i \neq k)$$

$$+ \sum_{i=0,N_Q+1}^{N} B_{ki}(V_k^m)^2\} \tag{69}$$

$$\geq 0 \tag{70}$$

From (66) to (67) we used the fact $-B_{kk} \geq \sum_{\substack{i=0\\ i\neq k}}^{N} B_{ki}$. The inequality from (67) to (68) is true because $\underline{V} \in R_V$. The last inequality from (69) to (70) is due to assumption (A5). Hence $D\tilde{\underline{Q}}(\underline{V})$ is diagonally dominant for all $\underline{v}$ in R_V, and by assumption (A5) the strict inequality (70) holds for at least one k. Therefore $D\tilde{\underline{Q}}(\underline{V})$ is nonsingular [17, pp. 48-49]. In fact $D\tilde{\underline{Q}}(\underline{V})$ is an M-matrix [17, p. 55] and $D\tilde{\underline{Q}}(\underline{V}) > 0$. ¤

The difference between the power flow equations (2)-(3) and the decoupled power flow equations (5)-(6) are given below:

$$\alpha_k(\underline{V},\underline{\theta}) := +\sum_{i=0}^{N} V_kV_iG_{ki}\sin\theta_{ki} - \sum_{i=0}^{N} V_kV_iB_{ki}(\cos\theta_{ki}-1)$$

$$k = 1,2,\ldots N_Q$$

$$\beta_k(\underline{V},\underline{\theta}) := +\sum_{i=0}^{N} V_kV_iG_{ki}\cos\theta_{ki} + \sum_{i=0}^{N} V_kV_iB_{ki}(\sin\theta_{ki}-\theta_{ki})$$

$$k = 1,2,\ldots N$$

We now introduce the error bounds of the approximation. Fact 5 below can easily be checked by Fact 1 and assumption (A3).

<u>Fact 5</u>. For $(\underline{V},\underline{\theta}) \in R_V \times R_\theta$,

$$-\alpha_k^m \le \alpha_k(\underline{V},\underline{\theta}) \le \alpha_k^M, \quad k = 1,2,\ldots N_Q$$

$$-\beta_k^m \le \beta_k(\underline{V},\underline{\theta}) \le \beta_k^M, \quad k = 1,2,\ldots N$$

where

$$\alpha_k^M := -\sum_{\substack{i=0 \\ i\neq k}}^{N} V_k^M V_i^M \{G_{ki} \sin \delta_j + B_{ki}(\cos \delta_j - 1)\} \qquad k = 1,2,\ldots N_Q \tag{71}$$

$$\alpha_k^m := -\sum_{\substack{i=0 \\ i\neq k}}^{N} V_k^M V_i^M G_{ki} \sin \delta_j \qquad k = 1,2,\ldots N_Q \tag{72}$$

$$\beta_k^M := \sum_{\substack{i=0 \\ i\neq k}}^{N} \{V_k^m V_i^m G_{ki} \cos \delta_j - V_k^M V_i^M B_{ki}(\sin \delta_j - \delta_j)\} + (V_k^M)^2 G_{kk} \qquad k = 1,2,\ldots N \tag{73}$$

$$\beta_k^m := -\sum_{\substack{i=0 \\ i\neq k}}^{N} \{V_k^M V_i^M G_{ki} + V_k^M V_i^M B_{ki}(\sin \delta_j - \delta_j)\} - (V_k^m)^2 G_{kk} \qquad k = 1,2,\ldots N \tag{74}$$

<u>Theorem 4</u>.

Let assumptions (A1-5), as well as conditions (C1) and (C2), hold. If the reactive power injections Q_k at the PQ buses satisfy condition (C6) below

$$\text{(C6)} \quad \tilde{Q}_k(\underline{V}^m) + \alpha_k^M < Q_k < \tilde{Q}_k(\underline{V}^M) - \alpha_k^m \qquad k = 1,2,\ldots N_Q \tag{75}$$

and the real power injections P_k at the PQ and PV buses satisfy condition (C7) below

$$\text{(C7)} \quad -\frac{\phi}{N}\frac{1}{X_{kk}(\underline{V}^m)} + \beta_k^M < P_k < \frac{\phi}{N}\frac{1}{X_{kk}(\underline{V}^m)} - \beta_k^m \quad k = 1,2,\ldots N \tag{76}$$

then the power flow equations (2)-(3) have a solution $(\underline{V},\underline{\theta})$ in the security constraint set R defined in Eq. (23). In other words, Eqs. (75) and (76) define a steady-state security region Ω_{ss}.

<u>Proof</u>. Consider the decoupled power flow map

$$F : (\underline{V},\underline{\theta}) \to (\underline{\tilde{Q}}(\underline{V},\underline{\theta}),\ \underline{\tilde{P}}(\underline{V}\ \underline{\theta})) \tag{77}$$

We will construct a simple homotopy from the decoupled power flow map F to the power flow map and apply the homotopy invariance property of the degree [17, pp. 156] to prove the theorem.

First we claim that $\deg(F, R_V x R_\theta^0, \underline{y}) \neq 0$ for any

$$\underline{y} \in \Omega_{ss} := \{(\underline{Q},\underline{P})\,|\,\underline{Q} \text{ satisfies (C5) and } \underline{P} \text{ satisfies (C6)}\} \tag{78}$$

The Jacobian $DF(\underline{v},\underline{\theta})$ of F is given by

$$DF(\underline{V},\underline{\theta}) = \begin{bmatrix} D\tilde{Q}(\underline{V}) & 0 \\ * & [B'(\underline{V})] \end{bmatrix} \tag{79}$$

It follows from Facts 3 and 4 that $\det DF(\underline{V},\underline{\theta}) = \det D\tilde{Q}(\underline{V}) \cdot \det[B'(\underline{V})] > 0$. This, together with Theorems 1 and 2, and the definition of the degree [17, pp. 147-160] establishes the claim.

We introduce the following homotopy $H : ((\underline{V},\underline{\theta}),t) \to (H^Q((\underline{V},\underline{\theta}),t),\ H^P((\underline{V},\underline{\theta}),t))$ from F to the power flow map (Eqs. (2) and (3)):

$$H((\underline{V},\underline{\theta}),t) := F(\underline{V},\underline{\theta}) + t\, G(\underline{V},\underline{\theta}) \qquad 0 \le t \le 1 \tag{80}$$

where $G : (\underline{V},\underline{\theta}) \to (\underline{\alpha}(\underline{V},\underline{\theta}),\ \underline{\beta}(\underline{V},\underline{\theta}))$.

We claim that for any $\underline{y} \in \Omega_{ss}$,

$$H((\underline{V},\underline{\theta}),t) \neq y, \quad t \in [0,1],\ (\underline{V},\underline{\theta}) \in \partial(R_V x R_\theta^0) \tag{83}$$

Consider the following two cases.

Case 1). $(\underline{V},\underline{\theta}) \in \partial(R_V x R_\theta^0)$ and $\underline{V} \in \partial R_V$.

Suppose the boundary is defined by $V_k = V_k^M$. We have on this boundary

$$\tilde{Q}_k(\underline{V}) \geq \tilde{Q}_k(\underline{V}^M) \tag{84}$$

On the other hand from Fact 5, for $(\underline{V},\underline{\theta}) \in \overline{R_V x R_\theta^0}$,

$$-\alpha_k^m \le \alpha_k(\underline{V},\underline{\theta}) \le \alpha_k^M \tag{85}$$

Consequently

$$H_k^Q((\underline{V},\underline{\theta}),t) \geq \tilde{Q}_k(\underline{V}^M) - \alpha_k^m \tag{86}$$

Condition (C6) and Eq, (86) establish our claim (83) for this case. Similarly for $V_k = V_k^m$.

Case 2). $(\underline{V},\underline{\theta}) \in \partial(R_V x R_\theta^0)$ and $\underline{\theta} \in \partial R_\theta^0$.

$[X(\underline{V})]$ is a nonsingular linear map, therefore, it maps the (relative) interior of a convex set onto the (relative) interior of a convex set [19, p. 44]. The convex set $\{\underline{P} : |P_k| \le \frac{\phi}{N} \frac{1}{X_{kk}(\underline{V}^m)}\}$ is mapped by $[X(\underline{V})]$ into a subset of R_θ^0 by Theorem 2. Consequently the points on the boundary of R_θ^0 are necessarily mapped to the boundary or the exterior of the set $\{\underline{P} : |P_k| \le \frac{\phi}{N} \frac{1}{X_{kk}(\underline{V}^m)}\}$, i.e., for $\underline{\theta} \in \partial R_\theta^0$ and $\underline{V} \in R_V$,

$$|\tilde{P}_k(\underline{V},\underline{\theta})| \geq \frac{\phi}{N} \frac{1}{X_{kk}(\underline{V}^m)} \tag{87}$$

On the other hand from Fact 5, for $(\underline{V},\underline{\theta}) \in \overline{R_V x R_\theta^0}$,

$$-\beta_k^m \leq \beta_k(\underline{V},\underline{\theta}) \leq \beta_k^M \tag{88}$$

Hence

$$H_k^P((\underline{V},\underline{\theta}),t) \geq \frac{\phi}{N} \frac{1}{X_{kk}(\underline{V}^m)} - \beta_k^m \tag{89}$$

or

$$H_k^P((\underline{V},\underline{\theta}),t) \leq -\frac{\phi}{N} \frac{1}{X_{kk}(\underline{V}^m)} + \beta_k^M \tag{90}$$

Condition (C7) and Eqs. (89) (90) establish our claim (83) for this case.

By homotopy invariance property of the degree [17, pp. 156], we have $\deg(F+G, R_V x R_\theta^0, \underline{y}) \neq 0$ for any $\underline{y} \in \Omega_{ss}$. Theorem 4 then follows from the Kronecker theorem [17, pp. 161]. ¤

If we use the results from Theorem 3 instead of Theorem 2, we obtain the following characterization of a steady-state security region.

<u>Theorem 5</u>.

Let assumptions (A1-5), as well as conditions (C1) and (C2), hold. If the reactive power injections Q_k at the PQ buses satisfy conditions (C6) below

$$\text{(C6)} \quad \tilde{Q}_k(\underline{V}^m) + \alpha_k^M < Q_k < \tilde{Q}_k(\underline{V}^M) + \alpha_k^m \qquad k = 1,2,\ldots N_Q \tag{91}$$

and the real power injections P_k at the PQ and PV buses satisfy condition (C8) below

(C8) $-\frac{\phi}{N}\frac{1}{X_{kk}(\underline{V}^m)} + t\mu V_k^M V_0 B_{k0} + \beta_k^M < P_k < \frac{\phi}{N}\frac{1}{X_{kk}(\underline{V}^m)} + t\mu V_k^m V_0 B_{k0} + \beta_k^m$

$$\text{for some} \quad t \in [-1, 1], \qquad k = 1,2,\ldots N \tag{92}$$

then the power flow equations (2)-(3) have a solution $(\underline{V},\underline{\theta})$ in the security constraint set R defined in Eq. (23). In other words, Eqs. (91) and (92) define a steady-state security region.

5. EXAMPLE

The following simple example illustrates the fact that the numbers one obtains by applying the results of this paper are reasonable.

Consider a power system consisting of a generator connected to a load through a transmission line with impedance $z = 0.001 + j0.1$ (Fig. 2).

Let $R_V = \{V_1 | 0.95 \le V_1 \le 1.05\}$. $R_\theta = \{\theta_1 | -0.1745 \le \theta_1 \le 0.1745\}$, $R_q = \{(V_1,\theta_1) | -0.7 \le Q_0(V_1,\theta_1) \le 0.7\}$, $R_p = \{(V_1,\theta_1) | 0 \le P_0(V_1,\theta_1) \le 1.0\}$, and $V_0 = 1$.

For this example, we have $\tilde{Q}_1(V_1^M) = 0.525$, $\tilde{Q}_1(V_1^m) = -0.475$, $\tilde{Q}_0(V_1^M) = -0.5$, $\tilde{Q}_0(V_1^m) = 0.5$, $\phi = 0.0873$, $\alpha_0^M = 0.1778$, $\alpha_0^m = 0.0182$, $\alpha_1^M = 0.1778$, $\alpha_1^m = 0.0182$, $\phi/X_{11}(V_1^m) = 0.8293$, $\beta_1^M = 0.262$, $\beta_1^m = 0.0243$, $L^M = 0.0232$, and $L^m = 0$.

Clearly assumption (A5) is satisfied, because $\tilde{Q}_1(V_1^m) = -0.475 \ge -B_{10}(V_1^m)^2 = -9.02$. The condition (C1) is satisfied:

$$q_0^M = 0.7 \ge \tilde{Q}_0(V_1^M) + \alpha_0^M = -0.3222$$

$$q_0^m = -0.7 \le \tilde{Q}_0(V_1^m) - \alpha_0^m = 0.4818.$$

The condition (C6) requires that the reactive power injection Q_1 lies within the limits:

$$-0.2972 < Q_1 < 0.5068$$

The condition (C7) requires that the real power injection P_1 lies with in the limits:

$$-0.8031 < P_1 < 0.8050$$

In order to satisfy condition (C2), the following inequalities must hold,

$$P_1 \leq 0$$

$$P_1 \geq -0.9768$$

Therefore a steady state security region is defined by

$$\Omega_{ss} = \{(P_1,Q_1) | -0.8031 < P_1 \leq 0, -0.2972 < Q_1 < 0.5068\}.$$

6. CONCLUSION

In this paper we have posed the problem of determining the steady-state security regions as one of finding sufficient conditions for the existence of solutions to the power flow map within the security constraint set. We look for conditions that are easy to check. The results which we obtain are expressed in terms of limits on real and reactive power injections at each bus. In other words, the steady-state security regions that we have obtained are hyperboxes in $\mathbb{R}^n$. The assumptions (A1-5) we impose on the system are not restrictive at all. The conditions that define the regions may be conservative, however, we believe that the proposed approach has the potential to yield improved results.

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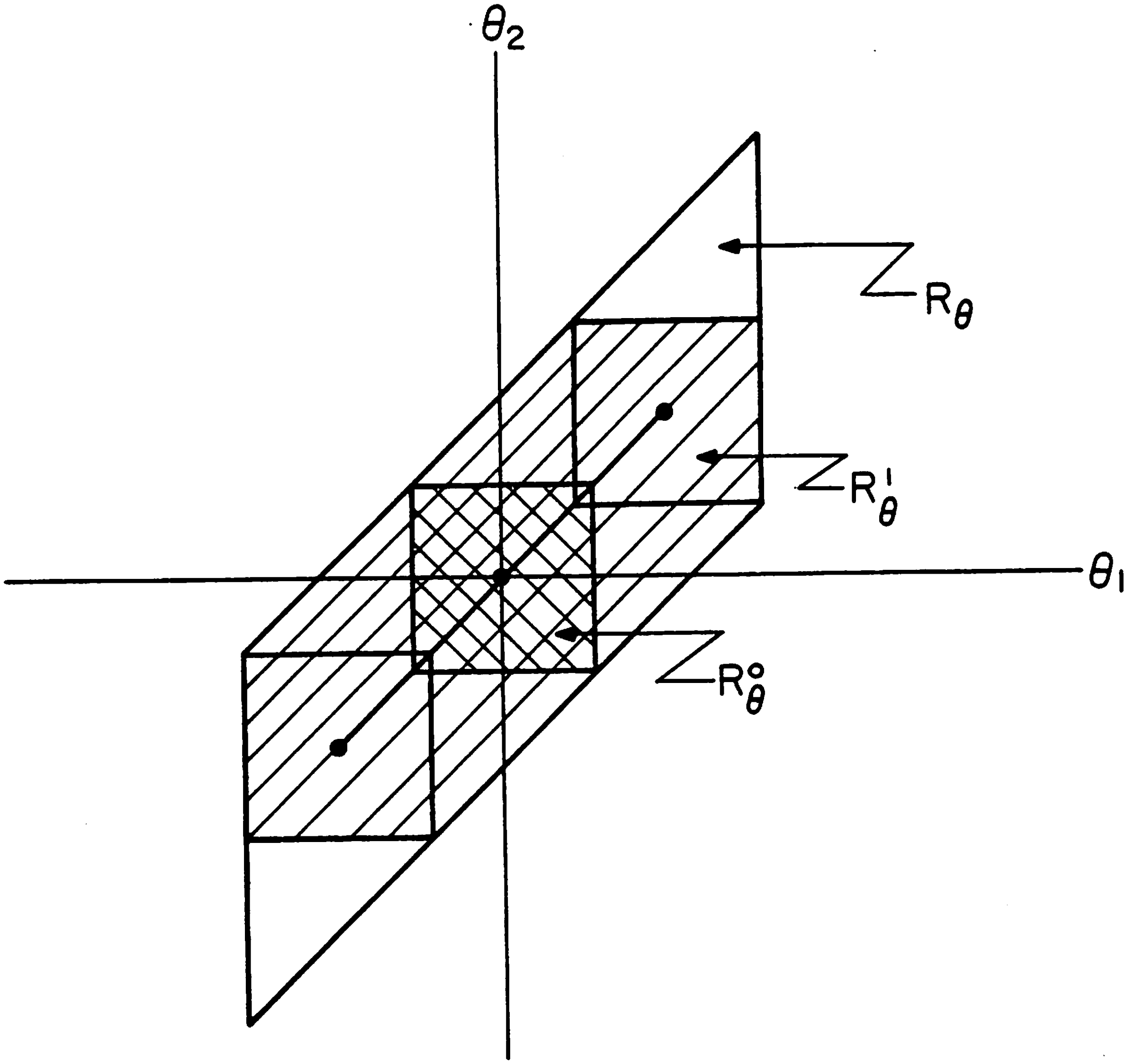
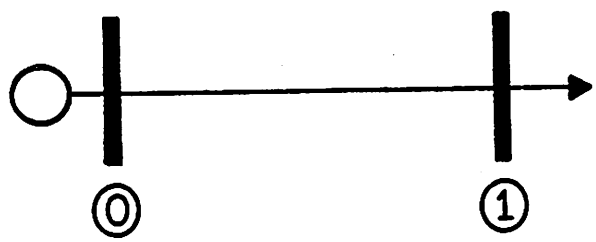
Figure Captions

Fig. 1.. A two-dimensional example to illustrate the regions $R_\theta^0 \subseteq R_\theta^1 \subseteq R_\theta$.

Fig. 2. Example of a two-bus system.



θ_2
θ_1
R_θ
R_θ^1
R_θ^0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