SIMULTANEOUS STABILIZATION OF NONLINEAR SYSTEMS

by

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Memorandum No. UCB/ERL M82/79

2 November 1982

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Abstract - We study the problem of simultaneous stabilization of a given set of nonlinear plants by one nonlinear, not necessarily stable compensator. We obtain a necessary and sufficient condition under which there is a single compensator which stabilizes a given set of n nonlinear plants.

Research sponsored by the National Science Foundation Grant ECS-8119763 and the Joint Services Electronics Program Contract F49620-79-C-0178.

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I. INTRODUCTION

The problem of simultaneous stabilization of a given set of plants by one compensator arises frequently in practice, due to plant uncertainty, plant variation (failure modes, etc.) or plants with several modes of operation. Therefore, it is of interest to know the conditions under which there exists a solution to this problem.

For the linear case, Saeks and Murray [Sae. 1] obtained a necessary and sufficient condition which guarantees simultaneous stabilization of a given set of linear plants by one linear compensator. Vidyasagar and Viswanadham [Vid. 1] showed that the problem of simultaneously stabilizing n linear plants by a linear compensator is equivalent to the problem of simultaneously stabilizing n-1 linear plants by a stable linear compensator.

In this paper, we study the problem of simultaneous stabilization of a given set of nonlinear plants by one nonlinear, not necessarily stable compensator. We obtain a necessary and sufficient condition under which there is a single (nonlinear) compensator which stabilizes a given set of n nonlinear plants.

II. DEFINITIONS AND NOTATIONS

Let $(\mathcal{L},\|\cdot\|)$ be a normed space of "time functions": $\mathcal{T} \to V$ where $\mathcal{T}$ is the time-set (typically $\mathbb{R}_+$ or $\mathbb{N}$), V is a normed space (typically $\mathbb{R}$, $\mathbb{R}^n$, $\mathbb{C}^n,\cdots$) and $\|\cdot\|$ is the chosen norm on $\mathcal{L}$. Let $\mathcal{L}_e$ be the corresponding extended space [Wil. 1], [Des. 1], [Vid. 2]. A nonlinear causal map $P : \prod_{j=1}^{m} \mathcal{L}_e^{n_j} \to \prod_{k=1}^{\ell} \mathcal{L}_e^{m_k}$ is said to be finite-gain (f.g.) stable iff $\exists\, \gamma(P) < \infty$ s.t. $\forall T > 0$, $\forall\, (u_1,u_2,\cdots,u_m) \in \prod_{j=1}^{m} \mathcal{L}_e^{n_j}$,

$$\|P(u_1,u_2,\cdots,u_m)\|_T \le \gamma(P)(\|u_1\|_T+\|u_2\|_T+\cdots+\|u_m\|_T) .$$

We shall use repeatedly the fact that the sum and the composition of f.g. stable maps are f.g. stable.

P is said to be <u>incrementally (inc.) stable</u> iff

a) P is f.g. stable, b) $\exists\tilde{\gamma}(P) < \infty$ s.t. $\forall T > 0$
$\forall\ (u_1,u_2,\cdots,u_m),\ (\bar{u}_1,\bar{u}_2,\cdots,\bar{u}_m) \in \prod_{j=1}^{m} \mathcal{L}_e^{n_j}$,
$\|P(u_1,u_2,\cdots,u_m) - P(\bar{u}_1,\bar{u}_2,\cdots,\bar{u}_m)\|_T \le \tilde{\gamma}(P)(\|u_1-\bar{u}_1\|_T + \|u_2-\bar{u}_2\|_T+\cdots+\|u_m-\bar{u}_m\|_T)$.
A nonlinear <u>system</u> N with input $(u_1,\cdots,u_m) \in \prod_{j=1}^{m} \mathcal{L}_e^{n_j}$ and output $(z_1,z_2,\cdots,z_\ell) \in \prod_{k=1}^{\ell} \mathcal{L}_e^{m_k}$ is said to be f.g. stable iff $\exists\gamma(N) < \infty$ s.t. $\forall T > 0$, $\forall\ (u_1,u_2,\cdots,u_m) \in \prod_{j=1}^{m} \mathcal{L}_e^{n_j}$, for any corresponding output $(z_1,z_2,\cdots,z_\ell) \in \prod_{k=1}^{\ell} \mathcal{L}_e^{m_k}$,

$$\|z_1\|_T + \|z_2\|_T + \cdots + \|z_\ell\|_T \le \gamma(N)(\|u_1\|_T+\|u_2\|_T + \cdots + \|u_m\|_T)\ .$$

For the purpose of this paper, we say that a feedback system is <u>well-formed</u> iff the relation between the inputs of interest and the outputs of interest is a well-defined causal map between suitable extended spaces. More precisely, the system ${}^1S(\bar{P}_i,C+F)$ of Fig. 1, with $\bar{P}_i : \mathcal{L}_e^{n_i} \to \mathcal{L}_e^{n_o}$ and $C + F : \mathcal{L}_e^{n_o} \to \mathcal{L}_e^{n_i}$ causal maps, is said to be well-formed iff $H : (u_1,u_2) \mapsto (e_1,e_2,y_1,y_2)$ is a well-defined causal map from $\mathcal{L}_e^{n_o} \times \mathcal{L}_e^{n_i} \to \mathcal{L}_e^{n_o} \times \mathcal{L}_e^{n_i} \times \mathcal{L}_e^{n_i} \times \mathcal{L}_e^{n_o}$. The f.g. stability of a well-formed feedback system is equivalent to the f.g. stability of its input-output map.

We <u>assume</u> throughout that each system under consideration is well-formed.

III. MAIN RESULTS

The main result of this paper is a theorem. A simplified version of the theorem can be described as follows: Consider two nonlinear plants described by nonlinear causal input-output maps, P_1 and P_2, where <u>P_1 is inc. stable</u>, the theorem shows that there exists a compensator C which stabilizes both P_1 and P_2, (i.e., the systems ${}^1S(P_1,C)$ and ${}^1S(P_2,C)$

shown in Fig. 2 are f.g. stable) if and only if there exists a f.g. stable Q such that the system ${}^1S(P_2-P_1,Q)$ of Fig. 3 is f.g. stable. To ease the restriction that P_1 be inc. stable, the theorem is preceded by a reduction lemma which is used to replace the condition that P_1 be inc. stable by the condition that P_1 be stabilizable by an inc. stable compensator.

<u>Lemma 1</u>: Let $\bar{P}_i : \mathcal{L}_e^{n_i} \to \mathcal{L}_e^{n_o}$ and $C, F : \mathcal{L}_e^{n_o} \to \mathcal{L}_e^{n_i}$ be nonlinear causal maps. Let[1] $P_i := \bar{P}_i(I-F(-\bar{P}_i))^{-1}$. Under these conditions, assuming that F is inc. stable,

$${}^1S(\bar{P}_i,C+F) \text{ is f.g. stable}$$

$$\Leftrightarrow$$

$${}^1S(P_i,C) \text{ is f.g. stable.}$$

Comments. a) None of the maps $\bar{P}_i$, P_i, and C are required to be stable.
b) Contrary to some popular arguments based on block diagram manipulations, it is a fact that F must be inc. stable. Consider the following example: Let $\bar{P}_i = (s-1)/(s+3) =: \bar{n}/\bar{d}$, $F = 3/(s-1)$, and $C = 3/1 =: n_c/d_c$. By calculation, $C+F = 3s/(s-1) =: n_{c+f}/d_{c+f}$ and $P_i = \bar{P}_i(1-F(-\bar{P}_i))^{-1}$ $= (s-1)/(s+6) =: n/d$. The system ${}^1S(P_i,C)$ is stable, since its characteristic polynomial is $nn_c + dd_c = 4s+3$. However, the system ${}^1S(\bar{P}_i,C+F)$ is unstable since its characteristic polynomial is $\bar{n}n_{c+f} + \bar{d}d_{c+f} = (s-1)(4s+3)$.

Proof:

($\Rightarrow$)

Consider the system ${}^1S(P_i,C)$ shown in Fig. 4, write the equations defining $\tilde{e}_1$ and $\tilde{e}_2$:

$$\tilde{e}_1 = u_1 - \bar{P}_i\tilde{e}_2 \tag{1}$$

$$\tilde{e}_2 = u_2 + C\tilde{e}_1 + F(-\bar{P}_i\tilde{e}_2)$$

$$= u_2 + C\tilde{e}_1 + F(\tilde{e}_1 - u_1) \tag{2}$$

By adding and subtracting $F\tilde{e}_1$ to (2) and rearranging terms, we have

$$\tilde{e}_2 = u_2 + F(\tilde{e}_1 - u_1) - F(\tilde{e}_1) + (C+F)\tilde{e}_1 \tag{3}$$

Define

$$\tilde{u}_1 := u_1 \tag{4}$$

$$\tilde{u}_2 := u_2 + F(\tilde{e}_1 - u_1) - F(\tilde{e}_1) \tag{5}$$

By using (4) and (5) in (1) and (3), we obtain

$$\tilde{e}_1 = \tilde{u}_1 - \bar{P}_i\tilde{e}_2 \tag{6}$$

$$\tilde{e}_2 = \tilde{u}_2 + (C+F)\tilde{e}_1 \tag{7}$$

Note that (6) and (7) describe the system ${}^1S(\bar{P}_i, C+F)$ of Fig. 1 with input $(\tilde{u}_1,\tilde{u}_2)$; hence by assumption, the map $(\tilde{u}_1,\tilde{u}_2) \mapsto (\tilde{e}_1,\tilde{e}_2)$ is f.g. stable.

Since F is inc. stable, $\exists \tilde{\gamma}(F) < \infty$ s.t. $\forall t > 0$, $\forall\ (u_1,u_2) \in \mathcal{L}_e^{n_o} \times \mathcal{L}_e^{n_i}$,

$$\|F(\tilde{e}_1 - u_1) - F(\tilde{e}_1)\|_T \le \tilde{\gamma}(F)\|u_1\|_T \tag{8}$$

Inequality (8), equations (4) and (5) show that the map $(u_1,u_2) \mapsto (\tilde{u}_1,\tilde{u}_2)$ is f.g. stable. Since the composition of f.g. stable maps is f.g. stable, we have that, for ${}^1S(P_i, C)$, the map $(u_1,u_2) \mapsto (\tilde{e}_1,\tilde{e}_2)$ is f.g. stable. Now, we see from Fig. 4 that

$$\tilde{y}_2 = u_1 - \tilde{e}_1 \tag{9a}$$

$$\tilde{e}_2'' = \tilde{e}_2 - F(-\tilde{y}_2) \qquad (9b)$$

$$\tilde{y}_1 = \tilde{e}_2'' - u_2 \qquad (9c)$$

Since F is f.g. stable, successive examination of equations (9a)-(9c) shows that, for ${}^1S(P_i,C)$, the map $(u_1,u_2) \mapsto (\tilde{y}_1,\tilde{y}_2)$ is f.g. stable. This completes the proof that the system ${}^1S(P_i,C)$ is f.g. stable.

($\Leftarrow$)

By noting that[2] $\bar{P}_i = P_i(I+F(-P_i))^{-1}$, we see that the system shown in Fig. 5 is ${}^1S(\bar{P}_i,C+F)$. Refer to Fig. 5, write the equations defining e_1 and e_2'':

$$e_1 = u_1 - P_i e_2'' \qquad (11)$$

$$e_2'' = u_2 + (C+F)e_1 - F(-P_i e_2'')$$

$$= u_2 + Fe_1 - F(e_1-u_1) + Ce_1 \qquad (12)$$

Define

$$\bar{u}_1 := u_1 \qquad (13)$$

$$\bar{u}_2 := u_2 + Fe_1 - F(e_1-u_1) \qquad (14)$$

By using (13) and (14) in (11) and (12), we obtain

$$e_1 = \bar{u}_1 - P_i e_2'' \qquad (15)$$

$$e_2'' = \bar{u}_2 + Ce_1 \qquad (16)$$

Note that (15) and (16) describe the system ${}^1S(P_i,C)$ with input $(\bar{u}_1,\bar{u}_2)$; since by assumption that ${}^1S(P_i,C)$ is f.g. stable, the map $(\bar{u}_1,\bar{u}_2) \mapsto (e_1,e_2'')$ is f.g. stable.

Since F is inc. stable, $\exists\tilde{\gamma}(F) < \infty$, s.t. $\forall T > 0$, $\forall\ (u_1,u_2) \in \mathcal{L}_e^{n_o} \times \mathcal{L}_e^{n_i}$,

$$\|F(e_1) - F(e_1-u_1)\|_T \leq \tilde{\gamma}(F)\|u_1\|_T \tag{17}$$

By (17) and equations (13) and (14), the map $(u_1,u_2) \mapsto (\bar{u}_1,\bar{u}_2)$ is f.g. stable. By noting that the composition of f.g. stable maps is f.g. stable, we conclude that, for the system ${}^1S(\bar{P}_i,C+F)$ as shown in Fig. 5, the map $(u_1,u_2) \mapsto (e_1,e_2'')$ is f.g. stable. Now, from Fig. 5 we see that

$$y_2 = u_1 - e_1 \tag{18a}$$

$$e_2 = F(-y_2) + e_2'' \tag{18b}$$

$$y_1 = e_2 - u_2 \tag{18c}$$

The successive examination of equations (18a)-(18c) and the finite gain of F show that, for ${}^1S(\bar{P}_i,C+F)$, the map $(u_1,u_2) \mapsto (y_1,y_2)$ is f.g. stable. This completes the proof.

<u>Theorem</u> (Simultaneous Stabilization): Let $\bar{P}_1$, $\bar{P}_2 : \mathcal{L}_e^{n_i} \to \mathcal{L}_e^{n_o}$ be nonlinear causal maps. Let $F : \mathcal{L}_e^{n_o} \to \mathcal{L}_e^{n_i}$ be an <u>inc. stable</u> causal map such that $P_1 := \bar{P}_1(I-F(-\bar{P}_1))^{-1}$ is <u>inc. stable</u>. Let $P_2 := \bar{P}_2(I-F(-\bar{P}_2))^{-1}$.

Under these conditions,

(a) if there exists a <u>f.g. stable</u> $Q : \mathcal{L}_e^{n_o} \to \mathcal{L}_e^{n_i}$ such that ${}^1S(P_2-P_1,Q)$ is f.g. stable, then, with

$$C := Q(I-P_1Q)^{-1}, \tag{20}$$

${}^1S(\bar{P}_i,C+F)$ is <u>f.g. stable</u>, for $i = 1, 2$;

(b) if there exists a $C : \mathcal{L}_e^{n_o} \to \mathcal{L}_e^{n_i}$ such that ${}^1S(\bar{P}_i,C+F)$ is f.g. stable for $i = 1,2$, then, with

$$Q := C(I+P_1C)^{-1}, \tag{21}$$

Q is <u>f.g. stable</u> and ${}^1S(P_2-P_1,Q)$ is <u>f.g. stable.</u>

Comments. a) None of the maps $\bar{P}_1$, $\bar{P}_2$, P_2, and C are required to be stable.
b) Roughly speaking, the conclusion says that ${}^1S(\bar{P}_i, C+F)$ is f.g. stable, for i = 1, 2, if and only if ${}^1S(P_2-P_1,Q)$ is f.g. stable for some f.g. stable Q.
c) Suppose that we have n plants $\bar{P}_1$, $\bar{P}_2$, ⋯, $\bar{P}_n$, then we may apply successively the theorem to the pairs $(\bar{P}_i,\bar{P}_1)$, i = 2,3,⋯,n and reach the conclusion that ${}^1S(\bar{P}_i,C+F)$ is f.g. stable for i = 1,2,⋯,n if and only if ${}^1S(P_i-P_1,Q)$ is f.g. stable for i = 2,3,⋯,n for some f.g. stable Q.
d) To the best of the authors' knowledge, there are no known general conditions under which a general nonlinear plant is stabilizable by an inc. stable compensator.

Proof of (a):

(i) We show that the system ${}^1S(\bar{P}_1,C+F)$ is f.g. stable.

Since P_1 is inc. stable, and by assumption Q is f.g. stable, it follows that, with $C := Q(I-P_1Q)^{-1}$, ${}^1S(P_1,C)$ is f.g. stable (see Fig. 2) [Des. 2]. Thus, by Lemma 1, ${}^1S(\bar{P}_1,C+F)$ is f.g. stable.

(ii) Figure 6 shows the system ${}^2S(P_2-P_1,C,P_1)$ with input (u_1,u_2,u_3) and output (y_1,y_2,y_3). When $u_3 = 0$, y_1 is given in terms of e_1' by $y_1 = Qe_1' = C(I+P_1C)^{-1}e_1'$.[3] Hence, if we set $u_3 = 0$, and consider only the output (y_1,y_2), then the system ${}^2S(P_2-P_1,C,P_1)$ reduces to ${}^1S(P_2-P_1,Q)$ of Fig. 3. Now the system ${}^1S(P_2-P_1,Q)$ has (u_1,u_2) as input and (y_1,y_2) as output and, by assumption, is f.g. stable. Hence, for the system ${}^2S(P_2-P_1,C,P_1)$, the partial map $(u_1,u_2,0) \mapsto (y_1,y_2)$ is f.g. stable.

(iii) Next consider the system ${}^2S(P_2-P_1,C,P_1)$ with (u_1,u_2,u_3) as input and call $(\bar{y}_1,\bar{y}_2,\bar{y}_3,\bar{e}_1')$ the corresponding output. Define

$$\Delta\tilde{y}_3 := P_1(u_3+\bar{y}_1) - P_1(\bar{y}_1) \tag{22}$$

To ${}^2S(P_2-P_1,C,P_1)$, apply $(u_1-\Delta\tilde{y}_3,u_2,0)$, call $(\tilde{y}_1,\tilde{y}_2,\tilde{y}_3;\tilde{e}_1')$ the corresponding output. We claim that $\bar{y}_1 = \tilde{y}_1$, and $\bar{y}_2 = \tilde{y}_2$.

To prove this, we obtain from Fig. 6 the equations relating $(\bar{y}_1,\bar{y}_2,\bar{e}_1')$ to (u_1,u_2,u_3)

$$\bar{e}_1' = u_1 - \bar{y}_2$$

$$\bar{y}_1 = C(\bar{e}_1'-P_1(u_3+\bar{y}_1))$$

$$\bar{y}_2 = (P_2-P_1)(u_2+\bar{y}_1)$$

By the well-formedness assumption, these equations have a unique solution. Writing the equations defining $\tilde{y}_1$, $\tilde{y}_2$ and $\tilde{e}_1'$, using (22), and invoking the uniqueness, we easily conclude that $\bar{y}_1 = \tilde{y}_1$, $\bar{y}_2 = \tilde{y}_2$ and $\bar{e}_1' = \tilde{e}_1' + \Delta\tilde{y}_3$. Since P_1 is inc. stable, $\exists\tilde{\gamma}_3 < \infty$, s.t. $\forall T > 0$, $\forall\ (u_1,u_2,u_3) \in \mathcal{L}_e^{n_o} \times \mathcal{L}_e^{n_i} \times \mathcal{L}_e^{n_i}$,

$$\|\Delta\tilde{y}_3\|_T = \|P_1(u_3+\bar{y}_1) - P_1(\bar{y}_1)\|_T \le \tilde{\gamma}_3\|u_3\|_T \le \tilde{\gamma}_3(\|u_1\|_T+\|u_2\|_T+\|u_3\|_T) \quad (23)$$

Thus, for ${}^2S(P_2-P_1,C,P_1)$, the map $(u_1,u_2,u_3) \mapsto \Delta\tilde{y}_3$ is f.g. stable, consequently, so is the map $(u_1,u_2,u_3) \mapsto (u_1-\Delta\tilde{y}_3,u_2,0)$. Finally, using the f.g. stability of the partial map $(u_1,u_2,0) \mapsto (y_1,y_2)$ proved earlier, we see that the composed map $(u_1,u_2,u_3) \mapsto (\tilde{y}_1,\tilde{y}_2)$ is f.g. stable; and hence, for ${}^2S(P_2-P_1,C,P_1)$, the map $(u_1,u_2,u_3) \mapsto (\bar{y}_1,\bar{y}_2)$ is f.g. stable. Since $\bar{y}_3 = P_1(u_3+\bar{y}_1)$ and by assumption P_1 is f.g. stable, we conclude that for ${}^2S(P_2-P_1,C,P_1)$, the map $(u_1,u_2,u_3) \mapsto (\bar{y}_1,\bar{y}_2,\bar{y}_3)$ is f.g. stable. Thus, ${}^2S(P_2-P_1,C,P_1)$ is f.g. stable.

(iv) We show that the f.g. stability of ${}^2S(P_2-P_1,C,P_1)$ implies that of ${}^1S(P_2,C)$. Consider the equations of ${}^2S(P_2-P_1,C,P_1)$ written in terms $\bar{e}_i$'s:

$$\bar{e}_1' = u_1 - (P_2-P_1)\bar{e}_2 \quad (24)$$

$$\bar{e}_1 = \bar{e}_1' - P_1\bar{e}_3 \quad (25)$$

$$\bar{e}_2 = u_2 + C\bar{e}_1 \quad (26)$$

$$\bar{e}_3 = u_3 + C\bar{e}_1 \quad (27)$$

If we set $u_2 = u_3$, then (26) and (27) shows that $\bar{e}_2 = \bar{e}_3$, and the equations (24)-(26) reduce to

$$\bar{e}_1 = u_1 - P_2\bar{e}_2$$

$$\bar{e}_2 = u_2 + C\bar{e}_1$$

The last two equations describe ${}^1S(P_2,C)$. Hence, ${}^2S(P_2-P_1,C,P_1)$ is f.g. stable implies that ${}^1S(P_2,C)$ is f.g. stable.

(v) From (iii) and (v), we have that ${}^1S(P_2,C)$ is f.g. stable. Then by Lemma 1, ${}^1S(\bar{P}_2,C+F)$ is f.g. stable. This together with (i) completes the proof.

Proof of (b): By assumption, ${}^1S(\bar{P}_i,C+F)$ is f.g. stable, for i = 1,2, and F is inc. stable. Hence by Lemma 1, ${}^1S(P_i,C)$ is f.g. stable for i = 1,2. Since ${}^1S(P_1,C)$ is f.g. stable, it follows that $Q := C(I+P_1C)^{-1}$ is f.g. stable, since $Q : (u_1,0) \mapsto y_1$.

Consider the system ${}^2S(P_2,Q,-P_1)$ shown in Fig. 7, with input (u_1,u_2,u_3) and output (y_1,y_2,y_3), note that when $u_3 = 0$, y_1 is given in terms of e_1 by $y_1 = Ce_1 = Q(I-P_1Q)^{-1}e_1$. Hence if we set $u_3 = 0$ and consider only the output (y_1,y_2), the system reduces to ${}^1S(P_2,C)$ of Fig. 2. Now ${}^1S(P_2,C)$ has input (u_1,u_2) and output (y_1,y_2) and is f.g. stable. Consequently, for the system ${}^2S(P_2,Q,-P_1)$, the partial map $(u_1,u_2,0) \mapsto (y_1,y_2)$ is f.g. stable. Since P_1 is inc. stable, it follows, by similar arguments as those in (iii) and (iv) of the proof of (a), that ${}^2S(P_2,Q,-P_1)$ is f.g. stable and hence ${}^2S(P_2-P_1,Q)$ is f.g. stable. The assertion is established.

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List of footnotes

[1] $(I-F(-\bar{P}_i))^{-1}$ is a well-defined map by the well-formedness assumption.

[2] By definition $P_i = \bar{P}_i(I-F(-\bar{P}_i))^{-1}$, hence $I+F(-P_i) = I+F(-\bar{P}_i)(I-F(-\bar{P}_i))^{-1}$ by taking the inverse and operating on the left by P_i, we obtain $\bar{P}_i = P_i(I+F(-P_i))^{-1}$.

[3] From (20) we have $I+P_1C = I + P_1Q(I-P_1Q)^{-1} = (I-P_1Q)^{-1}$, taking the inverse, operating on the left by C and using (20) we obtain $C(I+P_1C)^{-1} = Q$.

Figure Captions

Fig. 1. Shows the system ${}^{1}S(\bar{P}_i, C+F)$.

Fig. 2. Shows the system ${}^{1}S(P_i, C)$ where $P_i = \bar{P}_i(I-F(-\bar{P}_i))^{-1}$.

Fig. 3. Shows the system ${}^{1}S(P_2-P_1, Q)$ where $Q = C(I+P_1C)^{-1}$.

Fig. 4. Shows the system ${}^{1}S(P_i, C)$ in which the structure of P_i is shown in detail.

Fig. 5. Shows the system ${}^{1}S(\bar{P}_i, C+F)$ in which $\bar{P}_i$ is represented as a feedback connection of P_i and F, and $\bar{P}_i = P_i(I+F(-P_i))^{-1}$.

Fig. 6. Shows the system ${}^{2}S(P_2-P_1, C, P_1)$ in which the structure of Q is shown in detail. If $u_3 = 0$, and if y_3, e_3 are ignored, the system reduces to ${}^{1}S(P_2-P_1, Q)$ of Fig. 3.

Fig. 7. Shows the system ${}^{2}S(P_2, Q, -P_1)$.

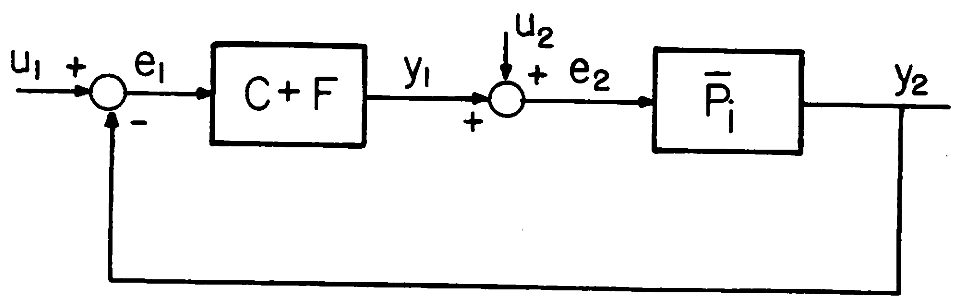

Fig. 1

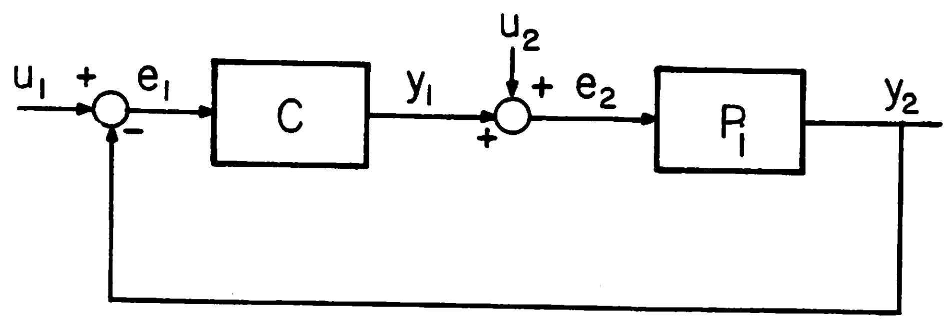

Fig. 2

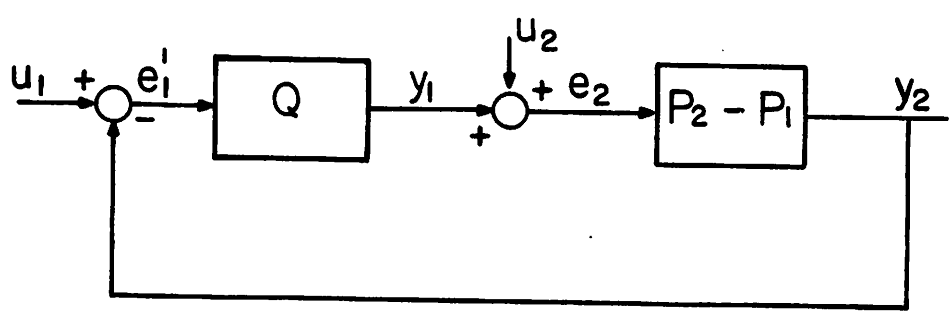

Fig. 3

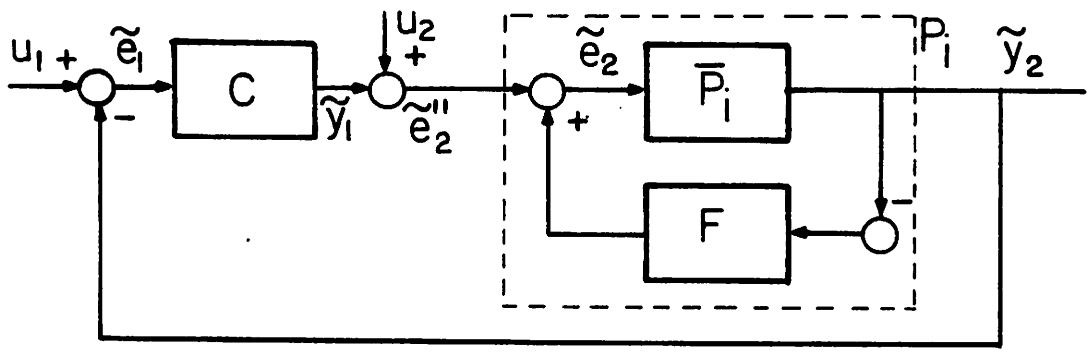

Fig. 4

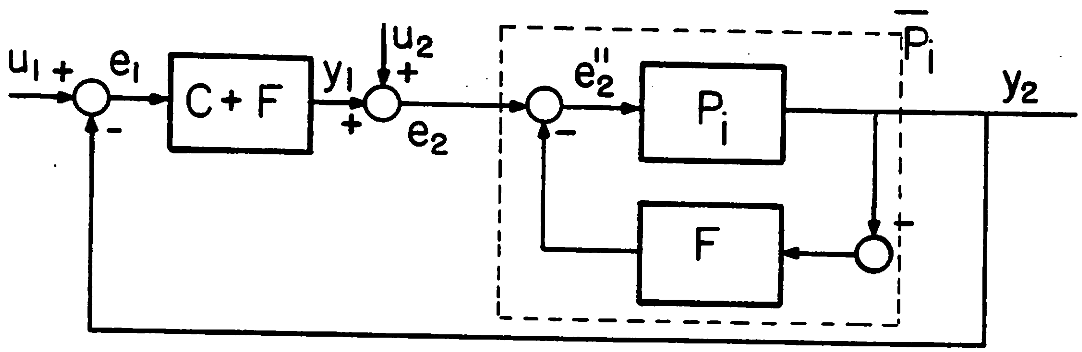

Fig. 5

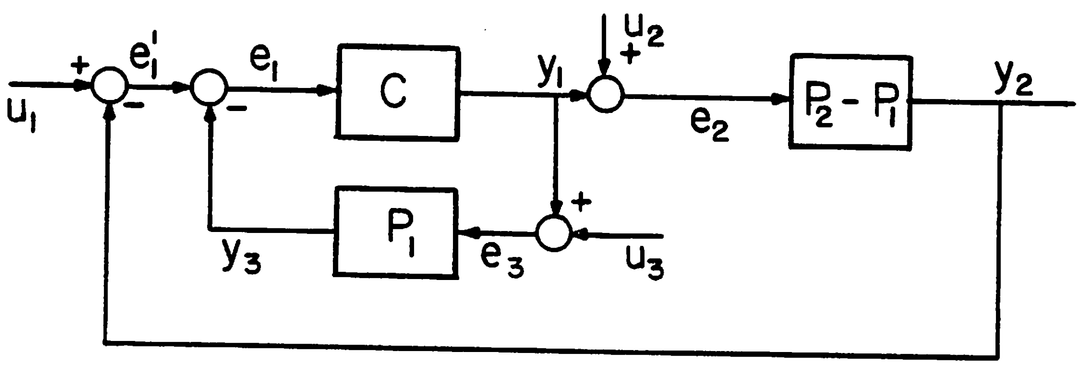

Fig. 6

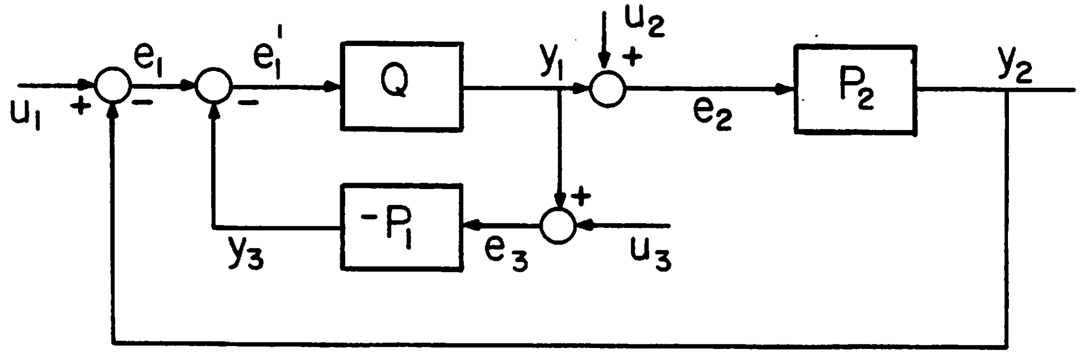

Fig. 7