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ABSTRACT

The classical swing equation for a power generator is shown to undergo a Hopf bifurcation to periodic solutions if it is augumented to include any of the following effects: variable net damping, frequency dependence of the electrical torque, lossy transmission line, or excitation control. Oscillations are seen to occur for realistic parameter values, and the stability of the oscillations is studied both analytically and numerically.

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1. Introduction

An electric power system normally functions at a stable operating point. However, the operating point can lose its stability due, say, to a disturbance and the subsequent change in system parameters. When this happens, it is not uncommon for the power system to exhibit oscillatory behavior. We use the Hopf bifurcation theorem to study this phenomenon, paying particular attention to the stability of the ensuing oscillations. We show that the classical swing equation for a power generator undergoes a Hopf bifurcation to periodic solutions if it is augmented to include any of a number of usually unmodeled effects.

Consider a system modeled by a vector differential equation $\dot{x} = f(x,p)$ where p is a real parameter. Let $x_0(p)$ be an equilibrium. Suppose that for a critical parameter value $p = p_c$ the Jacobian $D_x f(x_0(p_c),p_c)$ has a pair of purely imaginary eigenvalues. Then the linear approximation suggests the presence of small-amplitude oscillations for p near p_c. To determine whether oscillations do in fact occur in the original nonlinear model is the object of the Hopf bifurcation theorem [1,2,3]. To apply the theorem requires some knowledge of the behavior of the Jacobian $D_x f(x_0(p),p)$ in a neighborhood of p_c. The precise statements are given in the next section.

In Section 3 these results are applied to four models of a synchronous generator. Each model extends the classical swing equation to account for one of the following effects: variable damping, frequency dependence of the electrical torque, lossy transmission line, or excitation control. In each case the results of Section 2 indicate the presence of oscillations and enable us to study the stability of these oscillations.

2. Hopf Bifurcation

2.1 The Hopf Bifurcation Theorem

The Hopf bifurcation theorem describes the emergence of a peridic solution from an equilibrium point of a vector differential equation

$$\dot{x} = f(x,p) \tag{2.1}$$

where $x \in \mathbb{R}^n (n \geq 2)$ and $p \in \mathbb{R}$. Existence of a family of bifurcating periodic solutions is assured (under further technical assumptions) if the linearization of (2.1) possesses a pair of complex conjugate eigenvalues which cross the imaginary axis as the controlling parameter p is varied through a critical value $p = p_c$. The theorem also gives information on the range of parameter values for which the periodic solutions arise, as well as their amplitude, frequency and stability. Hypotheses (H1)-(H4) are in force in this section.

(H1) System (2.1) has an isolated equilibrium $x_0 = x_0(p)$.

(H2) $f(x,p)$ is $C^r (r \geq 4)$ in a neighborhood of $(x_0(p_c),p_c)$.

(H3) The Jacobian $D_x f(x_0(p),p)$ possesses a pair of complex conjugate, simple eigenvalues $\lambda(p) =: \alpha(p) + i\omega(p)$, $\overline{\lambda(p)}$, $\alpha'(p_c) > 0$ and $\omega_c := \omega(p_c) > 0$.

(H4) Besides $\pm i\omega_c$, eigenvalues of the critical Jacobian $D_x f(x_0(p_c),p_c)$ lie in the open left half complex plane.

For definiteness, we have required $\alpha'(p_c) > 0$ although clearly $\alpha'(p_c) \neq 0$ is sufficient. This so-called Hopf condition means that the eigenvalues $\lambda(p)$, $\overline{\lambda(p)}$ cross the imaginary axis transversally at $p = p_c$.

Linearized analysis now suggests the presence of small-amplitude oscillations for p near p_c. Theorem 2.1 below (adapted from Theorem II of [2]) asserts that this is indeed the case. For definitons of the terms in the theorem statement relating to the stability of the periodic solutions, see Hale [4].

Theorem 2.1 (The Hopf Bifurcation Theorem)

(a) (Existence) There is a $\nu_0 > 0$ and a C^{r-1} function $p(\nu) = p_c + p_2\nu^2 + O(\nu^3)$ such that for each $\nu \in (0,\nu_0]$ there is a nonconstant periodic solution $\gamma_\nu(t)$ of (2.1) near $x_0(p)$ for $p = p(\nu)$. The period of γ_ν is a C^{r-1} function $T(\nu) = 2\pi\omega_c^{-1}[1 + T_2\nu^2] + O(\nu^3)$, and its amplitude grows as $O(\nu)$.

(b) (Uniqueness) If $p_2 \neq 0$ then there is a $\nu_1 \in (0,\nu_0]$ such that for each $\nu \in (0,\nu_1]$ the periodic orbit γ_ν is the only periodic solution of (2.1) for $p = p(\nu)$ lying in a neighborhood of $x_0(p(\nu))$.

(c) (Stability) Exactly one of the characteristic exponenets of $\gamma_\nu(t)$ approaches 0 as $\nu \downarrow 0$, and it is given by a real C^{r-1} function $\beta(\nu) = \beta_2\nu^2 + O(\nu^3)$. The relationship

$$\beta_2 = -2\alpha'(p_c)p_2 \tag{2.2}$$

holds. Moreover, the periodic solution $\gamma_\nu(t)$ is orbitally asymptotically stable with asymptotic phase if $\beta(\nu) < 0$ but is unstable if $\beta(\nu) > 0$. $\square$

For a linear system of ordinary differential equations satisfying the hypotheses of the Hopf bifurcation theorem, a little reflection shows that the predicted family of periodic solutions occurs only for $p = p_c$, so that $p(\nu) \equiv p_c$. In fact, each of p_2, T_2 and β_2 must vanish in the linear case. If $p_2 \neq 0$, then the periodic solutions $\gamma_\nu(t)$, $0 < \nu \leq \nu_1$, occur either for $p > p_c$ or for $p < p_c$. The bifurcation is said to be supercritical in the former case and subcritical in the latter. If p_2, β_2 are both nonzero, then the direction of bifurcation, i.e., $p > p_c$ or $p < p_c$, and the stability of the oscillations are determined, for small ν, by the coefficients p_2 and β_2, respectively. In light of (2.2), therefore, the oscillations are stable, resp. unstable, if they are supercritical, resp. subcritical. For this reason, (2.2) is referred to as Hopf's exchange of stability formula. Figure 1 depicts the two possibilities for a two-dimensional system with an equilibrium at the origin ($x_0(p) \equiv 0$). Note that the amplitude of the oscillations grows as $|p-p_c|^{1/2}$, by Theorem 2.1(a).

Ascertaining that a given system undergoes a Hopf bifurcation to periodic solutions is a straightforward application of Theorem 2.1. Indeed, such a bifurcation is certain if the easily checked conditions (H1)-(H4) are valid.

Determining the stability of the bifurcated periodic solutions is akin to evaluating β_2 and requires more effort. In Hassard, Kazarinoff and Wan [2] explicit formulae (called bifurcation formulae) are obtained for the coefficients p_2, T_2 and β_2 by

(i) finding the formulae for two-dimensional systems in the so-called Poincare normal form

$$\begin{aligned}\dot{Y} &= A(p)Y + \sum_{j=1}^{s} B_j(p)Y|Y|^{2j} + O(|Y||(Y,p-p_c)|^{r-1}) \qquad (2.3)\\ &= F(Y,p),\end{aligned}$$

where

$$A(p) = \begin{pmatrix} \alpha(p) & -\omega(p) \\ \omega(p) & \alpha(p) \end{pmatrix}, \quad B_j(p) = \begin{pmatrix} \mathrm{Re}\ c_j(p) & -\mathrm{Im}\ c_j(p) \\ \mathrm{Im}\ c_j(p) & \mathrm{Re}\ c_j(p) \end{pmatrix},$$

(ii) determining a transformation reducing a general two-dimensional system satisfying (H1)-(H3) to Poincare normal form, and

(iii) using the center manifold theorem to reduce a general n-dimensional system satisfying (H1)-(H4) to a two-dimensional "essential model" for the study of the bifurcated periodic solutions.

For easy reference, we give the formula for β_2 for two-dimensional systems here. Suppose $n = 2$ and (2.1) has been put in real canonical form. That is, $f(0,p_c) = 0$ and the critical Jacobian is

$$D_x f(0,p_c) = \begin{pmatrix} 0 & -\omega_c \\ \omega_c & 0 \end{pmatrix}$$

Let $f = (f^1, f^2)$, $f^i_{pq} := \frac{\partial^2 f^i}{\partial x_p \partial x_q}(0,p_c)$ and $f^i_{pqr} := \frac{\partial^3 f^i}{\partial x_p \partial x_q \partial x_r}(0,p_c)$. Then β_2 is given by the formula

$$8\beta_2 = \frac{1}{\omega_c}\{f^1_{11}(f^1_{12}-f^2_{11}) + f^2_{22}(f^1_{22}-f^2_{12}) + f^2_{12}f^1_{22} - f^2_{11}f^2_{12}\} + (f^1_{111}+f^1_{122}+f^2_{112}+f^2_{222}). \tag{2.4}$$

From this formula, β_2 is seen to be twice the "curvature coefficient" σ_0 of Marsden and McCracken [3].

The algorithm obtained in [2] for computing p_2, T_2 and β_2 for general n-dimensional system has been programmed by B. Hassard (SUNY at Buffalo), and we make use of his program BIFOR2 in our study of oscillations in power systems. We now go on to briefly discuss BIFOR2. A more detailed discussion along with a listing of the program can be found in Hassard, Kazaninoff and Wan [2].

2.2 The Program BIFOR2

The user of BIFOR2 must supply a calling program which identifies the system $\dot{x} = f(x,p)$ of interest (and hence also the bifurcation parameter p) and gives analytical expressions for the elements of the

Jacobian matrix $D_x f(x,p)$. The user also supplies estimates for the critical parameter value p_c and the equilibrium $x_0(p_c)$, as well as the values of certain machine-dependent and logical parameters.

The first step taken by the program is to find the critical value p_c. This is done by solving the equation $\alpha(p) = 0$ by means of the secant method. Here $\alpha(p) = \mathrm{Re}\,\lambda_1(p)$ where $\lambda_1(p)$ denotes the eigenvalue of the Jacobian matrix $A(p) = D_x f(x_0(p),p)$ given by

$$\mathrm{Re}\,\lambda_1 := \max\{\mathrm{Re}\,\lambda \,|\, \lambda \in \sigma(A)\}$$
$$\mathrm{Im}\,\lambda_1 := \max\{\mathrm{Im}\,\lambda \,|\, \lambda \in \sigma(A),\ \mathrm{Re}\,\lambda = \mathrm{Re}\,\lambda_1\}.$$

Here $\sigma(A)$ is the set of eigenvalues of A.

The secant iteration is given by

$$p_{k+1} = p_k - \gamma_k(p_k - p_{k-1}), \quad k = 1,2,111,$$

where

$$\gamma_k = \alpha(p_k)/(\alpha(p_k) - \alpha(p_{k-1})) \ .$$

At each iterate p_k in the location of p_c by the secant method, Newton's method is used to compute $x_0(p_k)$ and the eigenvalues of $A(p_k)$ are computed by the double-step QR algorithm (cf. [2]).

Once p_c, $x_0(p_c)$ and $\omega_c = \omega(p_c)$ have been determined, BIFOR2 evaluates the coefficent $c_1(0)$ of the Poincare normal form (2.3). Numerical differencing is used to evaluate the second and third partial derivatives needed in this computation (cf. the algorithm in [2, Ch. 2]. To evaluate the coefficients μ_2, T_2, β_2, one needs $\alpha'(p_c)$ and $\omega'(p_c)$, in addition to $c_1(0)$. The derivative

$$\lambda_1'(p_c) = \alpha'(p_c) + i\omega'(p_c)$$

is approximated by the symmetric difference quotient

$$\lambda_1'(p_c) = (\lambda_1(p_c+\Delta p) - \lambda_1(p_c-\Delta p))/2\Delta p,$$

where the choice of Δp depends on the machine precision and the initial guess for p_c.

3. Application to Power Systems

3.1 Oscillations Due to Nonlinear Damping

The classical swing equation model for a synchronous machine connected to an infinite bus by a lossy transmission line is

$$M\ddot{\delta} + D\dot{\delta} = P - B\sin\delta + G\cos\delta, \tag{3.1}$$

where $M, D > 0$ are the machine's moment of inertia and damping coefficient, respectively, $G + iB$ is the transmission line admittance, $P > 0$ is the (adjusted) mechanical power intput to the machine, and δ is the rotor angle measured with respect to a synchronously rotating reference. An infinite bus is defined as a source of constant frequency and voltage. The system is depicted in Figure 2. For a derivation and detailed discussion of (3.1), see Anderson and Fouad [14].

Equation (3.1) admits no periodic solutions, as can be checked by multiplying through by $\dot{\delta}$ and integrating over the period of a hypothesized oscillation. To explain the occurrence of oscillations one must resort to more complex models. Our first modification of (3.1) stems from the observation that the damping D is in reality a function of the state of the system, and may take on both positive and negative values during normal operation. It is easy to check that (3.1) admits no periodic solutions if the damping does not change sign. Concordia and Carter [8] and Liwschitz [7] have proposed models to take into account this variable nature of damping in real machines. We use the model

$$D(\delta) = [a \sin^2\delta + b \cos^2\delta] - r[c(1-\tan^2\delta) + d(1-\sin 2\delta)] \tag{3.2}$$

derived by Liwschitz since it captures the essential nonlinearities present and is tractable. In this equation, a, b, c and d are positive constants and $r \geq 0$ is the armature resistance of the machine. Setting $a = b$ and $r = 0$ gives a constant positive damping as in (3.1). For appreciable values of the armature resistance, however, (3.2) can give rise to a negative overall damping. This qualitative dependence of damping on armature resistance was also noted by Kimbark [10]. Liwschitz refers to the first (bracketed) component of $D(\delta)$ in (3.2) as the rotor damping and the remaining term as the stator damping.

We now proceed to show that the model (3.2) can give rise to oscillations in (3.1) using the Hopf bifurcation theorem. Equation (3.1) with the modified damping (3.2) can be expressed in the form (2.1) by defining $\omega := \dot{\delta}$ and writing

$$\begin{aligned} \dot{\delta} &= \omega =: f^1(\delta,\omega,p) \\ \dot{\omega} &= P - D^p(\delta)\omega - B \sin\delta + G\cos\delta =: f^2(\delta,\omega,p) \end{aligned} \tag{3.3}$$

where $p := -r$ is the bifurcation parameter, M has been set to 1 and $D^p(\delta) = [a \sin^2\delta + b \cos^2\delta] + p[c(1-\tan^2\delta) + d(1-\sin 2\delta)]$. Let $x_0 := (\delta=\delta_0, \omega=0)$ be an equilibrium of (3.3). Note that hypothesis (H1) of the Hopf bifurcation theorem is satisfied since $x_0(p) \equiv x_0$. The Jacobian of (3.3) at x_0 is

$$Df(x_0,p) = \begin{pmatrix} 0 & 1 \\ -B\cos\delta_0 - G\sin\delta_0 & -D^p(\delta_0) \end{pmatrix} \tag{3.4}$$

and its eigenvalues are

$$\lambda_{\pm}(p) = \frac{1}{2}\left(-D^p(\delta_0) \pm \sqrt{(D^p(\delta_0))^2 - e(B\cos\delta_0 + G\sin\delta_0)}\,\right). \qquad (3.5)$$

Now suppose there is a p_c such that $D^{p_c}(\delta_0) = 0$ and assume that $B\cos\delta_0 + G\sin\delta_0 =: \omega_c^2$ is positive. Then hypotheses (H1)-(H4) of Theorem 2.1 are satisfied at x_0 for the critical value $p = p_c$, and we are assured of the existence of oscillations. To investigate their stability, we bring (3.3) to real canonical form by letting $y_1 := \delta - \delta_0$, $y_2 := -\omega/\omega_c$. This gives

$$\begin{aligned} \dot{y}_1 &= -\omega_c y_2 =: F^1(y,p) \\ \dot{y}_2 &= -D^p(y_1+\delta_0)y_2 - \omega_c^{-1}[P - B\sin(y_1+\delta_0) + G\cos(y_1+\delta_0)] \\ &=: F^2(y,p) \end{aligned} \qquad (3.6)$$

Equation (2.4) now gives

$$8\beta_2 = -\omega_c^{-1}F^2_{11}F^2_{12} + F^2_{112} \qquad (3.7)$$

where all partial derivatives are evaluated at $y = 0$, $p = p_c$. This gives

$$8\beta_2 = -\omega_c^{-2}(B\sin\delta_0 - G\cos\delta_0)\,\frac{d}{d\delta}D^{p_c}(\delta)\Big|_{\delta_0} - \frac{d^2}{d\delta^2}D^{p_c}(\delta)\Big|_{\delta_0}. \qquad (3.8)$$

Now (taking $r_c := -p_c$)

$$\frac{d}{d\delta}D^{p_c}(\delta)\Big|_{\delta_0} = (a-b)\sin 2\delta_0 + 2r_c(c\tan\delta_0\sec^2\delta_0 + d\cos 2\delta_0) \qquad (3.9a)$$

and

$$\frac{d^2}{d\delta^2}D^{p_c}(\delta)\Big|_{\delta_0} = 2(a-b)\cos 2\delta_0 + 2r_c[c\sec^4\delta_0(1+2\sin^2\delta_0) - 2d\sin 2\delta_0]. \qquad (3.9b)$$

In a specific example, the stability of the bifurcated oscillations is determined by the sign of β_2 (if $\beta_2 \neq 0$). However, we have the following general result.

Theorem 3.1. Suppose $a = b$, $2d < c$ and $\tan^{-1}(\frac{G}{B}) < \delta_0 < \frac{\pi}{4}$. Then (3.3) has a stable periodic solution for each $r < r_c$ sufficiently close to r_c. The period is near $2\pi\omega_c^{-1} = 2\pi(B\cos\delta_0 + G\sin\delta_0)^{-1/2}$, and each such solution is unique in a neighborhood of $(\delta_0, 0)$.

Proof. The assumed conditions imply $B\sin\delta_0 - G\cos\delta_0 > 0$, $\frac{d}{d\delta} D^{p_c}(\delta)|_{\delta_0} > 0$ and $\frac{d^2}{d\delta^2} D^{p_c}(\delta)|_{\delta_0} > 0$. Hence $\delta_2 < 0$, by (3.8). Theorem 2.1 now applies, guaranteeing that the oscillations are stable, locally unique at $(\delta_0, 0)$ and occur for $p > p_c$, i.e., $r < r_c$. $\square$

The oscillations we have studied in this section have been observed in practice and are an example of the so-called "hunting" oscillations. See Wagner [9] for an early study of the appearance of hunting oscillations in synchronous machines having significant armature resistance.

3.2 Frequency Dependence of the Electrical Torque

Equation (3.1) is derived from the torque balance equation

$$M\ddot{\delta} + D\dot{\delta} = T_m - T_e \tag{3.10}$$

where $T_m(T_e)$ is the mechanical driving (electrical retarding) torque on the generator shaft. The delivered electrical power $P_e = B\sin\delta + G(1-\cos\delta)$ is related to the electrical torque by $P_e = (\omega_0 + \dot{\delta})T_e$ where ω_0 is the synchronous frequency and $\dot{\delta}$ is the instantanteous deviation from ω_0. Take $p = -D$ as the bifurcation parameter and rewrite (3.10) as (M has been normalized to 1):

$$\begin{aligned} \dot{\delta} &= \omega \\ \dot{\omega} &= p\omega + T_m - \frac{B\sin\delta + G(1-\cos\delta)}{\omega + \omega_0} \end{aligned} \tag{3.11}$$

At the equilibrium $(\delta_0,\omega=0)$ we find that hypotheses (H1)-(H4) are satisfied for $p_c = -T_m/\omega_0$ and $\omega_c^2 = (B \cos \delta_0 + G \sin \delta_0)/\omega_0$, which we assume to be positive. The system is transformed into real canonical form by again defining $y_1 = \delta-\delta_0$, $y_2 = -\omega/\omega_c$. Equation (2.4) of Theorem 2.2 will then yield

$$\beta_2 = \omega_c^2 T_m/(2\omega_0^3) \tag{3.12}$$

giving

<u>Theorem 3.2</u>. Suppose $B \cos \delta_0 + G \sin \delta_0 > 0$ and $T_m > 0$. Then (3.11) has an unstable periodic solution for each $D > T_m/\omega_0$ sufficiently near T_m/ω_0. The oscillations are unique in a neighborhood of $(\delta_0,0)$, and their period is near $2\pi\sqrt{\omega_0/(B \cos \delta_0 + G \sin \delta_0)}$.

Note that bifurcation to stable orbits would occur in the set up above if $T_m < 0$ (so the machine is a motor), but the critical damping would be negative.

3.3 <u>Lossy Transmission Line</u>

Next we consider the case of two generators connected by a lossy transmission line. If the generators each have constant damping, the swing equations governing their dynamics take the form

$$\begin{aligned}\ddot{\delta}_1 + D_1\dot{\delta}_1 &= \omega_0(P_1 - B \sin(\delta_1-\delta_2) + G \cos(\delta_1-\delta_2)) \\ \ddot{\delta}_2 + D_2\dot{\delta}_2 &= \omega_0(P_2 + B \sin(\delta_1-\delta_2) + G \cos(\delta_1-\delta_2))\end{aligned} \tag{3.13}$$

upon dividing each equation by the corresponding machine's inertia (ω_0 is a scaling factor of order M_i^{-1}, $i = 1,2,$; $\omega_0 \equiv 2\pi f_0$). It has been shown in Arapostathis, Sastry, and Varaiya [11] that if the line is lossless, i.e., $G = 0$, then (3.13) has no periodic solutions. The direct state-space formulation of (3.13) as a set of four first-order differential equations has a zero eigenvalue in its spectrum, precluding use of the Hopf bifurcation theorem as stated. Instead, let $x_1 := \delta_1-\delta_2$, $x_2 := \dot{x}_1$ and $x_3 := \dot{\delta}_2$, yielding the equivalent three-dimensional system

$$\begin{aligned}\dot{x}_1 &= x_2\\ \dot{x}_2 &= \omega_0(P_1-P_2) - D_1x_2 + (D_2 - D_1)x_3 - 2B\omega_0 \sin x_1\\ \dot{x}_3 &= \omega_0P_2 - D_2x_3 + \omega_0(B \sin x_1 + G \cos x_1)\ .\end{aligned} \tag{3.14}$$

The Jacobian of (3.14) at the equilibrium (x_1^0,x_2^0,x_3^0) is

$$J = \begin{pmatrix} 0 & 1 & 0\\ -2B\omega_0\cos x_1^0 & -D_1 & D_2-D_1\\ \omega_0(B \cos x_1^0-G \sin x_1^0 & 0 & -D_2\end{pmatrix}$$

and its characteristic equation is

$$\lambda^3 + (D_1+D_2)\lambda^2 + (D_1D_2+2B\omega_0\cos x_1^0)\lambda + B\omega_0(D_1+D_2)\cos x_1^0 + G\omega_0(D_2-D_1)\sin x_1^0 = 0.$$

Hence J will have eigenvalues $\pm i\omega_c$ if and only if

$$\begin{aligned}\omega_c^2 &= D_1D_2 + 2B\omega_0\cos x_1^0\\ &= [B\omega_0(D_1+D_2)\cos x_1^0 + G\omega_0(D_2-D_1)\sin x_1^0]/(D_1+D_2)\end{aligned} \tag{3.15}$$

The third eigenvalue of J would then be $\lambda_3 = -(D_1+D_2)$. Clearly, for (3.15) to hold with $G = 0$ it is necessary that $\cos x_1^0 < 0$. In normal operation, however, we should have $|\delta_1^0-\delta_2^0| < \frac{\pi}{2}$ so that $\cos x_1^0 > 0$. Therefore, require $G > 0$, i.e., assume the transmission line is lossy. Now, letting $\theta := \tan^{-1} (\frac{G}{B})$, (3.15) implies

$$\frac{D_1D_2(D_1+D_2)}{G\omega_0\sqrt{1+(B/G)^2}} = -D_2 \cos(x_1^0+\theta) - D_1 \cos(x_1^0 - \theta). \tag{3.16}$$

For (3.16) to hold with $x_1^0 < \frac{\pi}{2}$, it is necessary that $D_2 > D_1$ and

$x_1^0 + \theta > \frac{\pi}{2}$. For the realistic choice $B = 10G$, we must require $x_1^0 > 1.47$ radians (84.3°).

The calculations involved in determining the stability of the bifurcated oscillations of (3.14) are prohibitively tedious to carry out by hand. Instead, we analyze specific cases numerically using the computer program BIFOR2. For the realistic values $D_1 = 0.01$, $D_2 = 0.25$, $P_1 = 0.6416$, $P_2 = -0.653$, $B = 0.65$, $\omega_0 = 2\pi f_0 = 2\pi(60) \cong 377.0$, and letting $p := \sqrt{G}$ be the bifurcation parameter, BIFOR2 gives

$$\omega_c = 6.638,\ p_2 = -0.638,\ T_2 = 3.850,\ \beta_2 = 0.987,$$

$$p_c = 0.252 \text{ (so } G_c = 0.064),\ \alpha'(p_c) = 0.774,$$

(using the notation of Theorem 2.1) at the equilibrium ($x_1^0 = 1.481$, $x_2^0 = 0$, $x_3^0 = 0.142$). Recall that p_2 is the first coefficient in the expansion of $p(\nu)$, and is not to be confused with the mechanical power P_2. Since $\beta_2 > 0$ and $\alpha'(p_c) > 0$, the oscillations are in this case unstable and occur for $p < p_c$, i.e., for $G < 0.064$. In fact, we have been unable to find an instance in which $\beta_2 < 0$ except in the unrealistic situation $G_c > B$. We therefore conjecture that any Hopf bifurcation for the system (3.14) with $|x_1^0| < \frac{\pi}{2}$, $D_1, D_2 < 1$ and $G < B$ is a bifurcation to unstable periodic orbits. Similar results have been obtained in the numerical study of a network of three generators connected by lossy transmission lines.

3.4 The Effect of Excitation Control

Van Ness, et al. [13] and many others (cf. [12,14,15]) have considered the effect of excitation system parameters on power system stability using a combination of simulation and linear analysis. In [13] it was observed that the stability of a power system was sensitive to changes in the amplifier gain K_A of the exciter. Oscillations were also observed to take place, but no rigorous analysis was given. We now proceed to study this situation in the framework of Hopf bifurcation theory.

Given a synchronous machine connected to an infinite bus of fixed voltage $E<0$, if the transmission line has impedance $R_\ell + jX_\ell$ and we neglect amortisseur effects, armature resistance, armature $\dot{\psi}$ terms and saturation, we have (see de Mello and Concordia [12])

$$
\begin{aligned}
V_t^2 &= v_d^2 + v_q^2 \\
-v_d = \psi_q &= -X_q i_q \\
v_2 = \psi_d &= E_q' - X_d' i_d \\
E_q &= E_q' + (X_q - X_d') i_d \\
P_e &= E_q i_q \\
i_d &= x(E_q - E\cos\delta) - r\,E\sin\delta \\
i_q &= r(E_q - E\cos\delta) + x\,E\sin\delta \\
\tau_{d0}' \dot{E}_q' &= E_{FD} - E_q' - (X_d - X_d') i_d \\
2H\ddot{\delta} + D\dot{\delta} &= \omega_0 (P_m - P_e),
\end{aligned}
\tag{3.17a}
$$

where

$$
\begin{aligned}
x &:= (X_\ell + X_q)/[R_\ell^2 + (X_\ell + X_q)^2], \text{ and} \\
r &:= R_\ell/[R^2 + (X_\ell + X_q)^2] .
\end{aligned}
$$

The notation used here is standard [14] and so it will not be elaborated; d and q refer to the direct and quadrature axes. The equations above describe the behavior of the synchronous machine if it is not equipped with an exciter, i.e., if the field voltage E_{FD} is fixed. Suppose that a continuously acting exciter is present, however. Then E_{FD} will no longer be constant, but will be governed by the dynamics of the exciter. Following Van Ness, et al. [13], suppose the exciter has the block diagram representation in Figure 3 (the IEEE Type 1 excitation system). The input signal to the exciter is then the machine terminal voltage V_t and its output is E_{FD}. Neglecting the limiter, the block diagram translates into the system

$$\begin{aligned}
\tau_E \dot{E}_{FD} &= -K_E E_{FD} + V_R - E_{FD} S_E(E_{FD}) \\
\tau_F \dot{V}_3 &= -V_3 + \frac{K_F}{\tau_E}\left(-K_E E_{FD} + V_R - E_{FD} S_E(E_{FD})\right) \\
\tau_A \dot{V}_R &= -V_R + K_A(V_{REF} - V_t - V_3).
\end{aligned} \tag{3.17b}$$

The saturation function $S_E(E_{FD})$ is usually approximated as $S_E(E_{FD}) = A_{EX}\exp(B_{EX}E_{FD})$ where the coefficients A_{EX}, B_{EX} are computed from saturation data.

To study oscillations in the system (3.17a-b) we assign the values quoted in Table 1 to the corresponding parameters in the model, and let $p := \sqrt{K_A}$ be the bifurcation parameter (this ensures that the critical K_A will be positive). The program BIFOR2 then searches for a critical value p_c and an equilibrium point of (3.17a-b) for which the hypotheses (H1-H4) of Theorem 2.1 are satisfied. (Of course, (3.17a-b) must first be expressed explicitly in the form (2.1), but this is an elementary exercise in algebra.) Since the stability of the bifurcating oscillations is of crucial importance, we study numerically a large number of cases enabling us to draw some general conclusions on this issue. Each case is determined by specifying an initial guess for an equilibrium point of (3.17a-b) and a parameter λ which has the effect of changing the length of the transmission line: $R_\ell + jX_\ell = \lambda(R_\ell^0 + jX_\ell^0)$ where R_ℓ^0, X_ℓ^0 are the nominal values given in Table 1. The mechanical power P_m and the terminal reference voltage V_{REF} are adjusted to ensure that the estimate for an equilibirum is accurate.

We will now give four sets of results which show the qualitatively distinct possibilities that may arise. The first two cases correspond to generator action ($P_m > 0$) and the last two to motor action ($P_m < 0$). We give an example with $\beta_2 > 0$ and one with $\beta_2 < 0$ for each case. Denote by $x^0 := (\delta^0, \omega^0, E_q'^0, E_{FD}^0, V_3^0, V_R^0)$ an equilibrium point of (3.17a-b).

Example 1. ($P_m > 0$, $\beta_2 > 0$) Set $P_m = 0.937$, $V_{REF} = 1.130$, $\lambda = 2$ and consider the equilibrium $x^0 = (1.351, 0, 1.105, 2.316, 0, 0.548)$ of (3.17a-b). BIFOR2 computes the values

$$\omega_c = 7.569,\ p_2 = -4.710,\ T_2 = 0.164,\ \beta_2 = 0.359,$$

$$p_c = 13.919 \text{ (so } K_{A_c} = 193.74),\ \alpha'(p_c) = 0.038,$$

using the notation of Theorem 2.1. Hence a bifurcation to periodic orbits occurs at the critical value $K_{A_c} = 193.74$, and (3.17a-b) has a locally unique unstable periodic solution in the vicinity of x^0 for each $K_A < 193.74$ with $(193.74 - K_A)$ sufficiently small, by Theorem 2.1.

Example 2. ($P_m > 0$, $\beta_2 < 0$) Set $P_m = 2.272$, $V_{REF} = 1.244$, $\lambda = 0.8$ and consider the equilibrium $x^0 = (1.363, 0, 1.420, 4.525, 0, 3.686)$. We have

$$\omega_c = 11.474,\ p_2 = 2.612,\ T_2 = 0.129,\ \beta_2 = -0.206,$$

$$p_c = 18.975 \text{ (so } K_{A_c} = 360.05),\ \alpha'(p_c) = 0.039.$$

Hence (3.17a-b) has a stable periodic solution, locally unique at x^0, for each $K_A > 360.05$ with $(K_A - 360.05)$ sufficiently small.

Example 3. ($P_m < 0$, $\beta_2 > 0$) Set $P_m = -0.891$, $V_{REF} = 1.098$, $\lambda = 2$. At the equilibrium $x^0 = (-1.353, 0, 1.103, 2.359, 0, 0.573)$, BIFOR2 yields

$$\omega_c = 6.895,\ p_2 = -3.120,\ T_2 = 0.211,\ \beta_2 = 0.601,$$

$$p_c = 11.319 \text{ (so } K_{A_c} = 128.12),\ \alpha'(p_c) = 0.096.$$

Hence (3.17a-b) has an unstable periodic solution, locally unique at x^0, for each $K_A < 128.12$ with $(128.12 - K_A)$ sufficiently small.

Example 4. ($P_m < 0$, $\beta_2 < 0$) Set $P_m = -1.939$, $V_{REF} = 1.230$, $\lambda = 0.8$. At the equilibrium $x^0 = (-1.229, 0, 1.436, 4.292, 0, 3.088)$, BIFOR2 gives

$$\omega_c = 11.457,\ p_2 = 1.765,\ T_2 = 0.121,\ \beta_2 = -0.103,$$

$$p_c = 19.561 \text{ (so } K_{A_c} = 382.63),\ \alpha'(p_c) = 0.0292.$$

Hence (3.17a-b) has a stable periodic solution, locally unique at x^0, for each $K_A > 382.63$ with $(K_A - 382.63)$ sufficiently small.

The examples above show that bifurcation to either stable or unstable oscillations can occur in each of the cases $P_m > 0$ and $P_m < 0$. However, a very large number of numerical tests has indicated the following general tendency: if the system (3.17a-b) undergoes a Hopf bifurcation as K_A is varied, then $\beta_2 > 0$. That is, Examples 1 and 3 above are indicative of the vast majority of numerical results we have obtained for this system using BIFOR2. Bifurcation to stable oscillations ($\beta_2 < 0$) was also observed frequently in the motor case ($P_m < 0$), but the values of V_R^0 were unacceptably large; see Examples 2 and 4. It should also be noted that imaginary axis eigenvalue crossings were obtained only for large values of the equilibrium angle magnitude $|\delta_0|$. This agrees with the results of Van Ness, et al. [13] and is indicated in Examples 1-4.

4. Conclusions

The prediction of nonlinear oscillations in power systems has invariably been based on linearized models with subsequent validation by simulation of the nonlinear model. The Hopf bifurcation theorem confirms that such a prediction will usually be true. However, to get a more detailed account of the amplitude and stability of the oscillations, linearized analysis is insufficient. Such an account is available using more recent elaborations of the Hopf Theorem. Most important for engineering is the development of computer packages that claculate the various coefficients which determine the properties of the oscillations. In this paper we have demonstrated the usefulness of the theory and the BIFOR2 program through a study of oscillations in models that are more detailed than the classical swing equation. We have confirmed the existence of oscillations that have been observed by others and determined their hitherto unknown stability properties. We hope that the paper will stimulate others to use the Hopf theory and associated computer packages.

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Figure Captions

1. Illustration of Theorem 2.1.
2. Machine connected to infinite bus.
3. IEEE Type 1 excitation system.

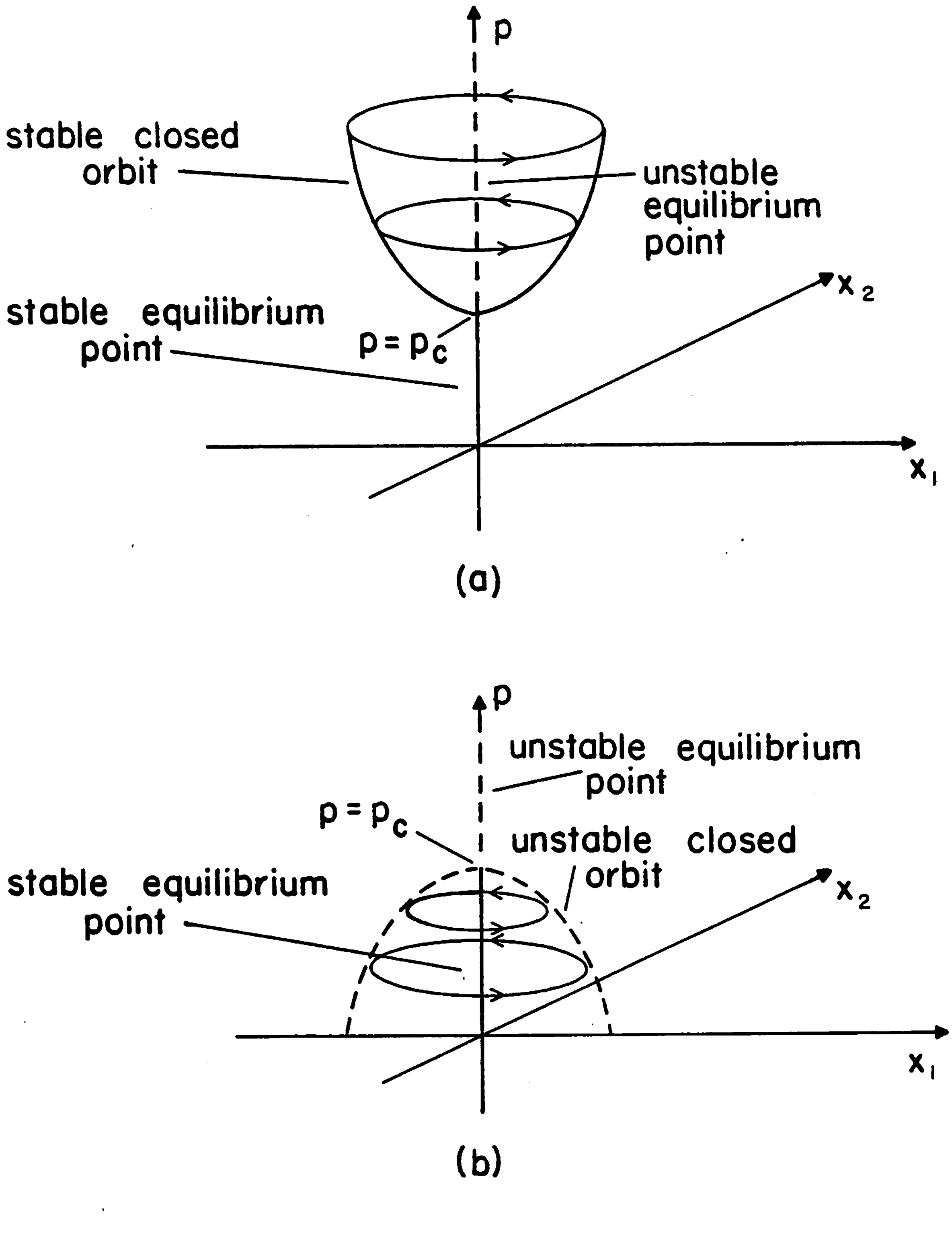
p
stable closed orbit
unstable equilibrium point
stable equilibrium point
p = p_c
x_2
x_1
(a)
p
unstable equilibrium point
p = p_c
unstable closed orbit
stable equilibrium point
x_2
x_1
(b)

Figure 1

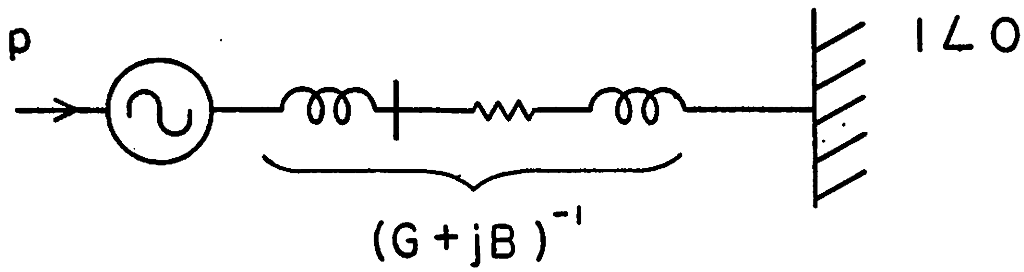

Figure 2

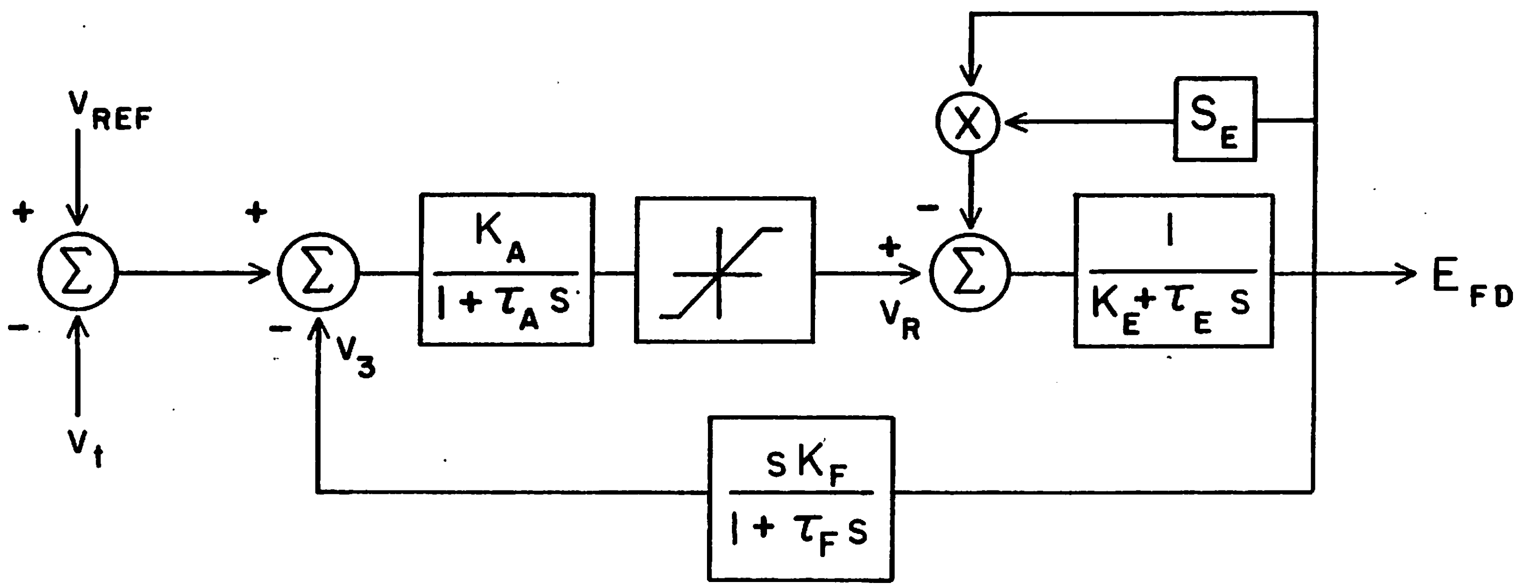

Figure 3

Synchronous Machine	Exciter	Transmission Line
$H = 2.37$ sec	$K_E = -0.05$	$R_\ell^o = 0.02$
$D = 1$ p.u.	$K_F = 0.02$	$X_\ell^o = 0.40$
$X_d = 1.7$	$\tau_E = 0.50$ sec	$R_\ell = \lambda R_\ell^o$
$X_d' = 0.245$	$\tau_F = 0.60$ sec	$X_\ell = \lambda X_\ell^o$
$X_q = 1.64$	$\tau_A = 0.10$ sec	
$\omega_o = 377.0$ rad/sec	$A_{EX} = 0.09$	
$\tau_{do}' = 5.9$ sec	$B_{EX} = 0.50$	

Table 1