

Copyright © 1983, by the author(s).
All rights reserved.

Permission to make digital or hard copies of all or part of this work for personal or classroom use is granted without fee provided that copies are not made or distributed for profit or commercial advantage and that copies bear this notice and the full citation on the first page. To copy otherwise, to republish, to post on servers or to redistribute to lists, requires prior specific permission.

A CRITICAL REVIEW ON EXTERNAL NETWORK
MODELING FOR ON-LINE SECURITY ANALYSIS

by

F. F. Wu and A. Monticelli

Memorandum No. UCB/ERL M83/46

8 June 1983

ELECTRONICS RESEARCH LABORATORY

College of Engineering
University of California, Berkeley
94720

A CRITICAL REVIEW ON EXTERNAL NETWORK MODELING
FOR ON-LINE SECURITY ANALYSIS

Felix F. Wu and A. Monticelli*

Department of Electrical Engineering and Computer Sciences
and the Electronics Research Laboratory
University of California, Berkeley, California, 94720

ABSTRACT

A critical review on various external network modeling methods for on-line security analysis is presented. Motivations and derivations of the methods are presented. Assessment is made on each method from the accuracy, computational, and load-flow compatibility considerations. A unified approach to external network modeling which encompasses the desired features of different methods is proposed.

Keywords: electric power systems, load flow, network equivalents,
security assessment

* On leave from Departamento de Engenharia Elétrica, UNICAMP, Campinas, S.P. Brazil

I. EXTERNAL NETWORK MODELING

I.1. What Is It and Where Is It Used

Power systems are interconnected. An energy control center for a member system of the interconnection is responsible for the control of a part of the interconnected system. The control center receives telemetered data of real-time measurements. The monitored part of the power system that these measurements cover normally consists of one's own system; we call it the internal system. The system is connected to neighboring systems; we call that the external system. The effect of external system on the internal system in steady-state is reflected by the real and reactive power flows from the external system into the boundary buses -- these flows usually are not part of the measurements. The set of real-time measurements is processed in the control center through the state estimator to obtain the best estimate of the present state of the internal system. Once the state is obtained other quantities of interest -- for example, real and reactive power generations, line flows, bus voltages -- can be calculated and their values can be checked against the specified limits. In particular, the state estimator can give the values of the power flows from the external system into the boundary buses at the present time. With a state estimator, it is not necessary to know more about the external system for the purpose of determining the present situation of the internal system.

Security of a power system is defined as the ability of a system to withstand imminent disturbances (possible next contingencies), such as line outages or generator outages [1]. In other words, a power system is said to be secure if no contingency in the next-contingency list will cause line overload, abnormal voltage, etc. To evaluate the consequence of each contingency, which is a postulated

future case, state estimation can no longer be used because there are no measurements available. Therefore a load flow will have to be simulated for contingency evaluation. A load flow model of the internal system for the contingent case is available from the state estimator; thus it would be possible to run a load flow just for the internal system if the flows from the external system to the boundary buses are known. However, the response of the external system to the contingency makes the flows from the external system to the boundary buses different from their present values. Therefore we have to add to the internal system a model that accounts for the response of the external system called the external network model, for on-line contingency evaluation. Depending on the system, sometimes it is possible (from the storage and computational considerations) to use the unreduced load flow (ULF) model of the external system. More often is the case where a reduced model has to be used. A reduced model of the external system that approximately represents the steady-state effect of the external system on the internal system for contingency evaluation is commonly called an external network equivalent or simply external equivalent for short.

I.2. How Is It Used

An external model may be constructed either on-line or off-line, or a combination of both. Once it is constructed it is attached to the load flow model of the internal system for contingency evaluation. The external model should be a good representation of the effect of the external system on the internal system. Since from the state estimation, we have the load flow solution of the internal system at the present time (we shall refer to this case without contingency as the base case), we require that the solution of the load flow model consisting of the internal system plus the external model for the base case be the same as the one obtained from the state estimation. This is accomplished by

the so-called boundary matching as described below:

- (i) From the state estimation, calculate the flows from the boundary buses into the internal system
- (ii) Use the external system model together with the complex voltages of the boundary buses from the state estimation to compute the flows from the external network into the boundary buses.
- (iii) For each boundary bus, add power injections so that the flows into and out of that bus are balanced.

After boundary matching, the adjusted external model (the original external model plus the boundary injections) is attached to the internal system as the load flow model for evaluating internal system response to contingencies.

It is sometimes preferable to extend the detail load flow model beyond the internal system. In such a case, the external system is divided into two parts, one with detail load flow model and one without. We shall refer to the part of the external system for which the detail load flow model is retained inner external system and the remaining part for which a reduced model is used outer external system (Fig. 1). The inner external system is called buffer zone by some authors. We use the terms inner and outer external systems to facilitate the presentation of a unified approach to external equivalent proposed in this paper.

I.3. How Is It Judged

An external model is judged by how well does it serve its intended purpose, namely, as a model representing the effect of the external system on the internal system for contingency evaluation. Thus external models are compared by their accuracy for contingency evaluation, and the computational efficiency. Since load flow model is used for

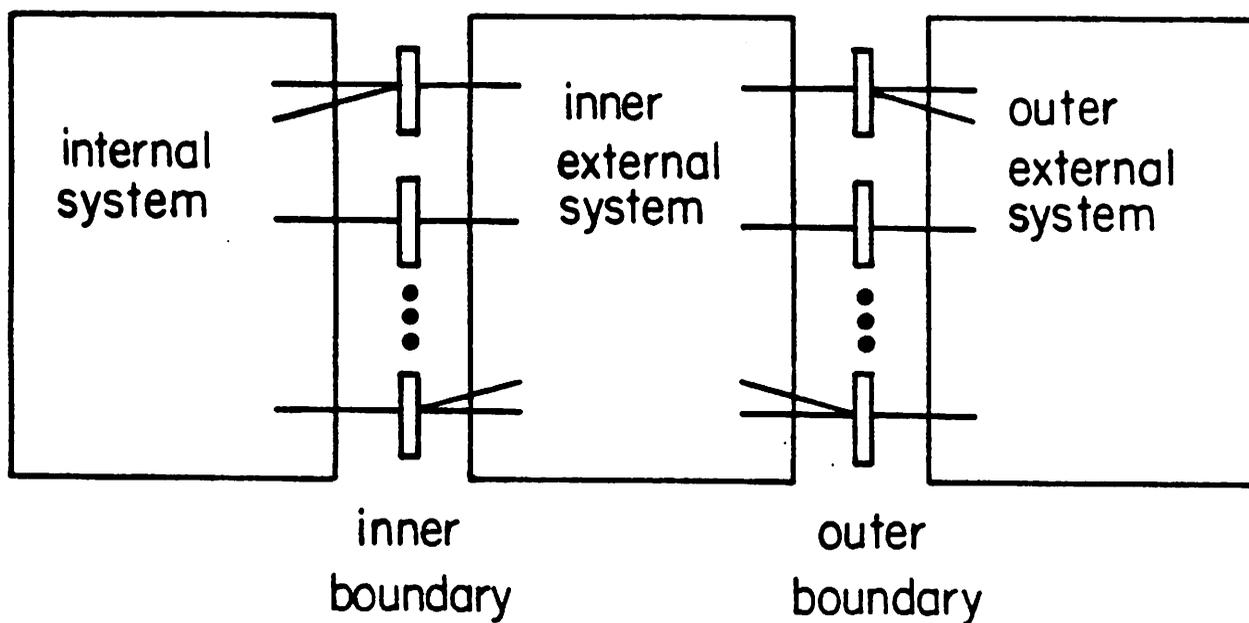


Fig. 1. The external system is divided into inner external and outer external systems.

contingency evaluation, it is desirable that the external model be compatible to a standard load flow. These points are further elaborated below.

(i) The accuracy should be judged by the difference in real and reactive power flows, bus voltages, etc. of the results obtained by using the external equivalent and by using the simulated complete load flow model of the external system for the contingency cases. If the state estimation of the contingent case is actually available, which is rarely the case, it should be used for comparison.

(ii) The computational considerations involve the sparsity of the resulting equivalents, programming simplicity, ease of updating, data requirement, etc.

(iii) Since the external model is to be incorporated as part of the load flow for contingency evaluation, it is desired to have the usual properties found in a standard load flow model, for example, large X/R ratio, bus voltages close to one per unit, etc.

I.4. About This Paper

This paper presents a survey of methods for external network modeling. A rather comprehensive review of external equivalents with numerical testing was presented in 1980 by Deckman, Pizzolante, Monticelli, Stott and Alsac [2,3]. This paper will concentrate on methods that have new developments since then. The emphasis is placed on methods for external modeling that can be directly incorporated into a standard load flow program. The reason is that only those methods are likely to be generally adopted. As a result, the linearization method proposed by Alvarado and Elkonyaly [4,5], though accurate, is not discussed here. The external modeling methods discussed in this paper are divided into three categories, namely, the unreduced load flow model (Sec. 11),

Ward-type equivalents (Sec. III), and REI-type equivalents (Sec. IV). The unreduced load flow model may be viewed in terms of Fig. 1 as stretching out the inner external system and cutting off the outer external system. The original Ward and REI equivalents may be viewed as squeezing out the inner external system, hence there is only the outer external system. For each category, we first present the original method (basic theme) and then various modifications (thematic variations). Discussions (comments) on accuracy, computational, and load flow compatibility issues follow each method. Some observations (remarks) are made throughout the paper. After the survey of the state-of-the-art in external equivalents, we suggest a unified approach to external network modeling in Sec. V.

Only work that are directly mentioned in this paper are referenced. Readers should consult Refs. 2 and 3 for a more complete bibliography.

I.5. Load Flow Equations and Notation

Vectors and matrices are underlined, e.g., \underline{E} , \underline{Y} . E_k denotes the k-th component of the vector \underline{E} , and Y_{kj} denotes the kj-th element of the matrix \underline{Y} . E_k^* is the complex conjugate of E_k . $\left(\frac{\Delta P}{\underline{V}}\right)$ represents a vector whose k-th component is $\frac{\Delta P_k}{V_k}$. $[\underline{E}]$ represents a diagonal matrix whose k-th diagonal is E_k .

Let $E_k = V_k e^{j\theta_k}$ be the complex (phasor) voltage at bus k, I_k and $S_k = P_k + jQ_k$ be, respectively, the complex (phasor) current injection and complex power injection into the network from bus k. We have in matrix notation

$$\underline{S} = [\underline{E}] \underline{I}^* \quad (1)$$

The complex current injection can be expressed using the network bus admittance matrix $\underline{Y} = \underline{G} + j\underline{B}$,

$$\underline{I} = \underline{Y} \underline{E} \quad (2)$$

Substituting (2) into (1) and separating the real and imaginary parts, we obtain the standard load flow equations in polar coordinates [6],

$$P_k = \sum_j V_k V_j \{G_{kj} \cos(\theta_k - \theta_j) + B_{kj} \sin(\theta_k - \theta_j)\} \quad (3)$$

$$Q_k = \sum_j V_k V_j \{G_{kj} \sin(\theta_k - \theta_j) - B_{kj} \cos(\theta_k - \theta_j)\} \quad (4)$$

There are three basic types of buses, namely, (i) swing (slack) bus, for which V and θ are specified, (ii) PQ (load) bus, for which P and Q are specified, and (iii) PV (generator) bus, for which P and V are specified. The basic types of buses may evolve into other types of buses in the load flow model when the modeling of real situation demands. For example a PV bus A is connected to a PQ bus B, and the reactive power injection at bus A is used to control the voltage at bus B, then bus A becomes a type P bus (only P is specified) and bus B becomes a type PQV bus (P , Q , and V are all specified). Similarly, when a transformer tap is used to regulate the voltage at an adjacent load bus, since the tap setting is a variable we can thus specify the voltage at the load bus making it a type PQV bus. Another case is when multi-area load flow studies are conducted, each area can have one or more generation buses whose real power generations become dependent variables to keep the areas net interchange to a specified value.

There are two kinds of data that are required to set up the load flow equations (3)-(4). (i) The data regarding the network configuration and the branch admittances, i.e., to specify the bus admittance matrix Y . We shall call then the network data. The network data is based on the breaker status and line impedances, etc. (ii) The data (e.g., real and reactive power demand at a PQ bus, the real power generation and the voltage magnitude at a PV bus) that is used to determine the operating

point of a given network. We shall call them the operating-point data. The network data changes rather infrequently, whereas the operating-point data may change from minute to minute.

The incremental form of the load flow equations (3)-(4) can be approximated by the following decoupled load flow equations [7],

$$\left(\frac{\Delta P}{V}\right) = \underline{B}' \Delta \theta \quad (5)$$

$$\left(\frac{\Delta Q}{V}\right) = \underline{B}'' \Delta V \quad (6)$$

where, for systems consist of only basic bus types, \underline{B}' has the dimension of total number of buses minus one and is obtained from $-\underline{B}$ (of the bus admittance matrix $\underline{Y} = \underline{G} + j\underline{B}$) by (i) neglecting the shunts, (ii) using the reciprocal of the branch reactance in place of the branch susceptance (iii) setting taps to nominal values; and \underline{B}'' has the dimension of the total number of PQ buses and is obtained from $-\underline{B}$ by (i) deleting rows and columns corresponding to PV buses, (ii) doubling the shunts [2, App. 4, p. 2299]. (iii) ignoring phase shifters. For systems having other types of buses, some modifications may be needed.

II. UNREDUCED LOAD FLOW MODEL

II.1. Basic Theme

The external model in this case is the load flow model of the external system itself. In other words, there is no model reduction of the external system. In terms of the picture of Fig. 1, the "inner external" becomes the whole external system and the "outer external" is simply cut off.

The network data and the operating-point data of the external system are needed for the external load flow model. Recall that in Sec. I.5, we define the network data to be network configuration and branch admittances that specify the bus admittance matrix and we define

the operating-point data to be a set of data which includes P and Q injections at a PQ bus, and P injection and V at a PV bus that determine the operating point. (If there are other types of buses, the specified quantities are part of the operating-point data). Complete data of the external system even for the base case is usually not available in a real-time environment. Therefore the construction of an unreduced load flow model for the external system is not a trivial problem. Direct data telemetering from the external system is very limited, if available at all. Therefore the external network configuration and branch admittances are determined by whatever current information available. Some, if not all, required external system operating-point data to complete the external load flow model may have to be manufactured. This is done by some form of extrapolation from the internal system data. Exactly how the extrapolation is carried out will be elaborated later. To summarize, there are four basic steps in the construction of an unreduced load flow model of the external system:

1. Determine the external system network data
 - by using latest available information
2. Determine the external system operating-point data
 - by extrapolation from the internal system data
3. Solve the load flow for the system consisting of the external system and the boundary buses.
 - by treating all boundary buses as swing buses, whose $V-\theta$ values are specified by the base case internal system state estimator.
4. Boundary matching.

In Step 2, the extrapolation from the internal system data to the external system operating-point data is based on the assumption that

they are congruent. A simple extrapolation can be done based on a typical load flow solution of the entire (internal plus external) system. For example, one calculates from this typical case solution the ratio of the internal and external system total loads and the ratios (distribution factors) of various injections at external buses to the total external load. Assuming these ratios are constant, from the real-time internal system data one may come up with external operating point data. A more sophisticated extrapolation, for example, involve several steps as described below [8].

1) Determine external MW, MVAR bus load

- The ratio of the total external system load and the total internal system load is assumed to be a constant. The total external system load is thus obtained from the total internal system load.
- The MW and MVAR demand at each bus are determined using the load distribution factors from a typical load profile.

2) Determine external MW generations

- With the external system bus loads, the Economic Dispatch is used to determine the MW outputs required at all generator buses in the external system.

3) Determine regulated-bus voltages

- Regulated bus voltage levels are determined from input data curves describing desired voltage as a function of bus loading.

In addition to the above, a lot of other specific available information about the external system that are believed to affect the internal system response, such as key flows, interchange schedules, etc., can be incorporated and updated.

Comments (Accuracy issues)

1. To determine how far to include in the external load flow model, off-line load flow contingency studies of the entire system are used in practice.
2. An advantage of having a unreduced load flow model for the external system is that certain load flow information (e.g., net interchange, key flows) can easily be incorporated.
3. In Ref. 2 simulation experiments are conducted to determine the effect of errors in external operating-point data of an unreduced load flow model on internal system contingency studies when boundary matching is employed. A rather interesting result is reported, namely, that the inaccuracy of external operating-point data does not seem to influence very much the results of contingency studies as long as the boundary matching is done for the base case. For example, an external unreduced load flow model with all injections set to zero and voltages of PV buses set to one (designated ULF-0 in Ref. 2) after boundary matching gives rather low percentage of error for contingent studies [2, p. 2296]. This may be explained by noting that if the changes due to external system response is small so that the incremental form of the load flow equations (5)-(6) provide reasonable approximation for contingent studies, it is seen that the incremental response depends mainly on the network data (B' and B'') rather than the external operating-point data (Δ -change of them are zero in either case, exact or ULF-0).

Comments (Computational issues)

4. Clearly if it is necessary to go rather far into the external system for the unreduced load flow model, the computational burden for contingency studies of always carrying the external system model is considerable.

However, using an unreduced load flow model rather than a reduced equivalent (Secs. III-IV), one saves software and database costs associated with these equivalents.

Comments (Compatibility issues)

5. There is no load flow compatibility problem since a true load flow model is used for the external system.

II.2. Thematic Variation

II.2.1. State estimation based ULF model

The external ULF model constructed by solving an external load flow may result in large boundary mismatches. It is reasonable to try to adjust external operating-point data in order to minimize the boundary mismatches. Since the external operating point is characterized by its state variables, finding the external operating point amounts to finding the external system state. We formulate here the problem of determining the external state that minimizes the boundary mismatches. Consider the enlarged external system \mathcal{E} consisting of the external system and the boundary buses. Let

\underline{x} = state variables (bus voltage magnitudes and phase angles) of the enlarged external system \mathcal{E}

\underline{z}^{SE} = a vector, of power injections from the internal system to the boundary buses, i.e., each component is the sum of line flows from the internal system to a boundary bus, calculated from the internal state estimator

$\underline{f}(\underline{x})$ = the expressions for the sum of flows from the boundary buses to the external system using the load flow model for \mathcal{E}

\underline{y}^{SP} = the specified values (by extrapolation or otherwise) of external operating point data

$\underline{g}(\underline{x})$ = the expressions for the external operating-point data

$\underline{\epsilon}$ = boundary mismatches

\underline{e} = errors

\underline{W} = weighting factors, larger value indicates it is more important to match that line flow

The problem is:

$$\min_{\underline{x}} \sum_k W_k (\epsilon_k)^2 \quad (7)$$

$$\text{s.t. } \underline{z}^{SE} = \underline{f}(\underline{x}) + \underline{\epsilon} \quad (8)$$

$$\underline{y}^{SP} = \underline{g}(\underline{x}) + \underline{e}$$

The problem formulated above has precisely the same form as the state estimation. Therefore a state estimation program can be employed to solve it. It is possible to further generalize the cost function (7) to include a term which reflects the confidence one has on the specified external operating-point data, i.e.,

$$\min_{\underline{x}} \sum_k W_k (\epsilon_k)^2 + \sum_j W_j (e_j)^2 \quad (9)$$

For a piece of operating-point data y_j that one has high confidence a large weighting factor W_j should be assigned. This state estimation based external network modeling is proposed by Geisler and Bose [8].

The set of external system "measurements" for the state estimation program \underline{z}^{SE} and \underline{y}^{SP} may be further enlarged to include other "measurements." Indeed, the following is a list of candidate measurements.

(i) boundary bus voltage magnitudes and phase angles and the boundary bus power injections from the internal system calculated from the internal state estimation

(ii) zero injections

(iii) telemetered data from the external system

(iv) extrapolated external system data from the internal system data

For boundary matching, highest weighting factors W_j should be assigned to measurement set (i). Also high confidence, hence high weighting factors, should be given to (ii). The W_j for the measurement set (iii) should receive medium values. Lower values should be assigned to (iv).

Unlike the usual state estimator, the solution algorithm developed for the external system modeling must have the ability to perform load flow tasks, such as generation MVAR and voltage limit enforcement. It is suggested [8] to accomplish this through the manipulation of the weighting factors, i.e., when the calculated value of a "measurement" is outside of the range, its corresponding weighting factor is increased in order to force it back to the limit.

Comments (Accuracy issues)

1. The state estimation formulation of minimizing boundary mismatches by adjusting external operating-point data can be considered as a refinement on boundary matching. In view of Comment 3 in Sec. II.1, the gain in accuracy for contingency evaluation may not be significant.

2. If there is an error in the external network data, minimizing boundary mismatch by adjusting external operating-point data does not necessarily result in a better load flow model. As a matter of fact, a large mismatch can be used as an indication as to where one should look for more external information to improve the model.

Comments (Computational issues)

3. The adjustment of weighting factors to enforce limits increases the number of external system state estimation iterations. The situation becomes even worse when there are other types of buses in addition to the basic types.

Comments (Compatibility issues)

4. Once the weighting factors are changed (for enforcing limits) in the external system state estimation, the gain matrix will have to be changed even if the flat-voltage approximation is used. A direct use of a fast decoupled state estimation program [9,10] becomes not possible because it uses a constant gain matrix.

II.2.2. Observability based ULF model.

The state estimation solution for external network modeling presented in Sec. II.2.1 results in minimal boundary mismatches. The approach may be modified using the concept of observability in state estimation to have perfect boundary matching. The idea was first pointed out in Ref. 3 [Sec. III. c., p. 2302], and recently used in Ref. 11.

The idea is to form a minimal set of nonredundant "measurement" of the enlarged external system \mathcal{E} for the state estimator. The algorithm suggested by Clements, Krumpholz, and Davies [12] may be utilized for this purpose. For the minimal set of nonredundant measurements, one starts from the boundary bus power injections from the internal system, and then adds to them available or extrapolated external operating-point data as discussed in the previous section. The criteria for selecting data to be included in the set of nonredundant measurements are based on the observability and the confidence level of the data. With a minimal set of nonredundant measurements, a state estimation can be solved. The solution matches the input "measurements," because of nonredundancy [13]. In particular the boundary flows will match perfectly. Of course, a complete load flow model can be derived from the state estimation results.

As a matter of fact there is no need to perform two state estimations, one for the internal system and one for the external system. Once a minimal set of nonredundant measurements for the

external system is identified by the algorithm in Ref. 12, one can run a state estimation for the entire system with internal system measurements and this minimal set of nonredundant "measurements" of the external system. The latter will not affect the result of the internal system state estimation in any way.

A problem, the same one as pointed out in the previous section, may arise, namely, the state estimation solution may result in generation MVAR and voltage outside limits. In this case a set of different nonredundant measurements should be selected. Exactly how to proceed in the selection of measurements warrants further investigation.

III. WARD-TYPE EQUIVALENT

III.1. Basic Theme

The Ward equivalent can be derived using the injection current and voltage relationship (2). In the basic version of Ward equivalent, the whole external system is equivalenced, i.e., in terms of Fig. 1, there is no inner external system (or buffer zone). Hence the set of buses can be partitioned into external (subscript e), boundary (subscript b), and internal (subscript i) buses. By Gaussian elimination to eliminate the variables \underline{E}_e , we obtain

where the off-diagonal elements of \underline{Y}_{bb}^i are the negative of the branch boundary-boundary admittances, the diagonal elements of \underline{Y}_{bb}^i include admittances of branches connecting boundary-boundary buses and boundary-internal buses, \underline{Y}_{bb}^e is a diagonal matrix whose elements are the sum of admittances of branches connecting boundary to external buses, and

$$\underline{Y}_{eq} = \underline{Y}_{bb}^e - \underline{Y}_{be} \underline{Y}_{ee}^{-1} \underline{Y}_{eb} \quad (11)$$

$$\underline{I}_{eq} = \underline{Y}_{be} \underline{Y}_{ee}^{-1} \underline{I}_e \quad (12)$$

By pre-multiplying the reduced equations in (10) (the last two block rows by the complex voltages $[\underline{E}_b]$ and $[\underline{E}_i]$ as in eq. (1), eq. (10) can then be transformed into the load flow model. The resulting boundary injection from the external injection $\underline{S}_e = [\underline{E}_e] \underline{I}_e^*$ can be obtained from (12):

$$\underline{S}_{eq} = [\underline{E}_b] \underline{Y}_{be}^* (\underline{Y}_{bb}^*)^{-1} [\underline{E}_e^*]^{-1} \underline{S}_e^* \quad (13)$$

The corresponding equivalent network of the reduced equations in (10) is illustrated in Fig. 2.

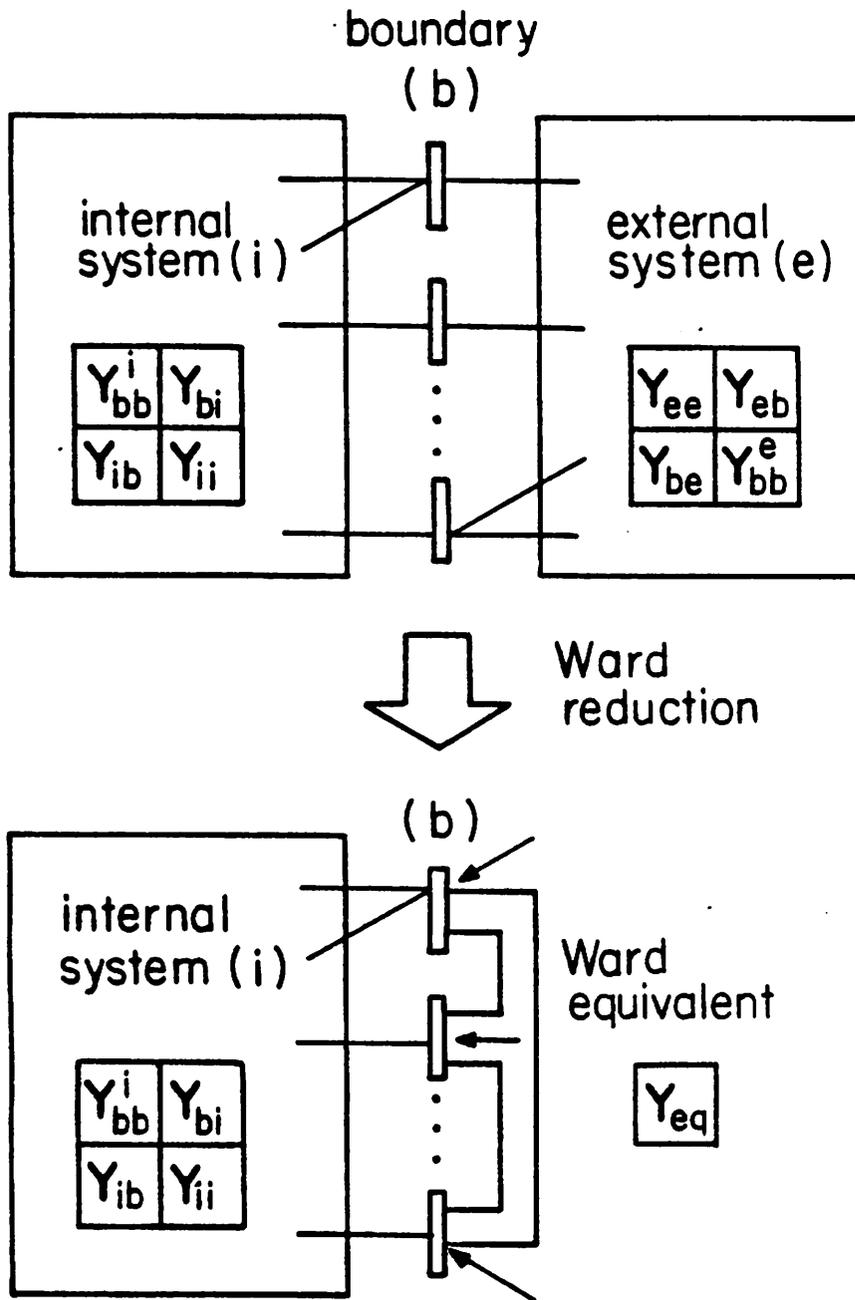


Fig. 2. Ward reduction and the Ward equivalent network.

Note that the part of the bus admittance matrix corresponding to the internal system is not affected by the reduction process. One needs only to perform Gaussian elimination on the part of the bus admittance matrix corresponding to the external system, i.e.,

$$\begin{array}{|c|c|} \hline Y_{ee} & Y_{eb} \\ \hline Y_{be} & Y_{bb}^e \\ \hline \end{array} \xrightarrow{\text{Gaussian elimination}} \begin{array}{|c|c|} \hline \text{shaded triangle} & \\ \hline \text{O} & Y_{eq} \\ \hline \end{array} \quad (14)$$

Further, since boundary matching is to be performed for on-line applications, there is no need actually to compute \underline{S}_{eq} as in eq. (13).

To summarize, the basic steps in the construction of a Ward equivalent consist of the following.

1. Determine the external network data from available information.
2. Obtain Ward equivalent network \underline{Y}_{eq} by Gaussian elimination as in (14).
3. Using the values for the complex voltages at the boundary buses from the internal state estimation to compute the flows in the Ward equivalent branches.
4. Boundary matching.

Comments (Accuracy issues)

1. If the external bus voltages \underline{E}_e are dependent variables, depending on boundary bus voltages \underline{E}_b and external injections \underline{S}_e , then the approximation involved in applying Ward-equivalent lies in the use of \underline{S}_{eq} (which is a function of \underline{E}_b , \underline{E}_e , and \underline{S}_e) calculated from the base case quantities for the contingent cases. When the external buses are PQ buses, \underline{E}_e are indeed dependent variables and \underline{S}_e remain unchanged. In this case it is observed that the Ward equivalent gives acceptable results. However, when the external buses are PV buses, then \underline{E}_e contain independent variables and \underline{S}_e no longer remain constant. In this case it is observed that Ward equivalent gives poor results.

2. For contingency studies, it is generally observed that the Ward-equivalent gives pretty good result for real power flows even if there are external PV buses. The poor results usually occur in the reactive power flows when there are external PV buses. This is due to the fact that the change in reactive power injection to maintain constant voltage at a PV bus is not accounted for in the Ward equivalent.

Comments (Computational issues)

3. Normally the number of boundary buses is much smaller than the number of external buses. The equivalent admittance matrix \underline{Y}_{eq} is rather full. In other words the resulting equivalent network of the external system is almost a complete graph (there is a branch between any pair of boundary buses). The impedance of some equivalent branches may be rather high. A common practice is to discard from the equivalent line list all lines with high impedance (say for example larger than 2-5 p.u.).

Comments (Compatibility issues)

4. In Ref. 2 (p. 2297), it is shown that Ward reduction process tends to amplify the effect of external shunts at the boundary buses. The external shunts may be line charging, reactive compensation, or equivalents from lower voltage networks. At any rate the reduction process produces unrealistic load-flow models for the shunts. The problem comes again from the failure of Ward equivalent to represent PV buses. It is therefore recommended that prior to reduction the external shunts are converted to injections. (Since boundary matching is to be preformed, this is equivalent to simply discard shunts.)

5. It is recommended [3] to represent external bus load demand as power injections rather than impedance to the ground. We have seen the problem with shunts in load flow modeling in Comment 4. Moreover, the elimination of impedance load will result in high resistance in the equivalent lines, thus low X/R ratios. The low X/R ratio may cause serious convergence problem when the fast decoupled load flow is applied [14]. DyLiacco [15] has devised a series compensation scheme to alleviate the problem, a similar parallel compensation scheme is suggested in [2, App. 5, p. 2299].

The Ward equivalent presented in this section is actually the Ward Injection method [2], where the shunts and loads are converted into injections. The classical Ward method, called Ward Admittance method, has the bus injections converted to shunt admittances. In view of Comments 4 and 5 and the test results in [2], it is concluded that the Ward Injection method is superior.

III.2. Thematic Variations

III.2.1. Ward-PV equivalent

The Ward equivalent presented in the last section is unable to represent faithfully the effect of external PV buses. Therefore it is suggested that for more accurate results, the Ward reduction process is applied only to external PQ buses. The resulting external equivalent has a Ward equivalent of PQ buses and the retained external PV buses, it will be referred to as Ward-PV equivalent.

Modification is needed on Step 3 in the basic Ward equivalent process. A load flow solution of the system consisting of the external PV buses, the Ward equivalent, and the boundary buses is called for. In the load flow the boundary buses are treated as swing buses with $V-\theta$ given by the internal state estimation (same as Step 3 in Sec. II.1).

Comments (Accuracy issues)

1. The Ward-PV equivalents give excellent result for contingency evaluation [2].

Comments (Computational issues)

2. If there is a large number of external PV buses, carrying them as part of external network model for contingency evaluation is a heavy computational burden. Housos, Irisarri, Porter, and Sasson [16] have reported that, in order to achieve acceptable results, it is necessary only to retain a small number of generator (PV) buses that are capable of producing large amounts of reactive power whenever needed. This may be suitable for planning studies, but for on-line applications a clear criterion for the selection of PV buses to retain, under different operating conditions, is needed.

III.2.2. Extended Ward

It has been pointed out that:

(i) The Ward equivalent gives pretty accurate results for real power flows, whereas the accuracy for reactive power flows is rather poor.

(ii) The Ward-PV equivalent gives pretty accurate results for both real and reactive power flows.

Monticelli, Deckmann, Garcia, and Stott [17, 2, 3] have derived a Ward-type equivalent, called the extended Ward equivalent, with the intention of having the combination of the simplicity of the Ward equivalent and the response of the Ward-PV equivalent.

The extended Ward equivalent is a Ward equivalent with additional reactive support at the boundary buses such that its reactive response is close to that of the Ward-PV equivalent. The reactive support in the extended Ward is derived so that the incremental response (linearized response from the base case) for the reactive power flows is almost the same as that from a Ward-PV equivalent. In what follows, we shall first analyze the reactive response of the Ward-PV equivalent using incremental version of the decoupled load flow equations. We then compare the reactive response of the Ward-PV equivalent with the Ward-equivalent and find the amount of reactive support that should be added at the boundary buses of the Ward equivalent so that the equivalent will behave like a Ward-PV equivalent.

Recall that the incremental form of the decoupled load flow equations are given by eqs. (5)-(6), where the matrices \underline{B}' and \underline{B}'' are derived from \underline{Y} .

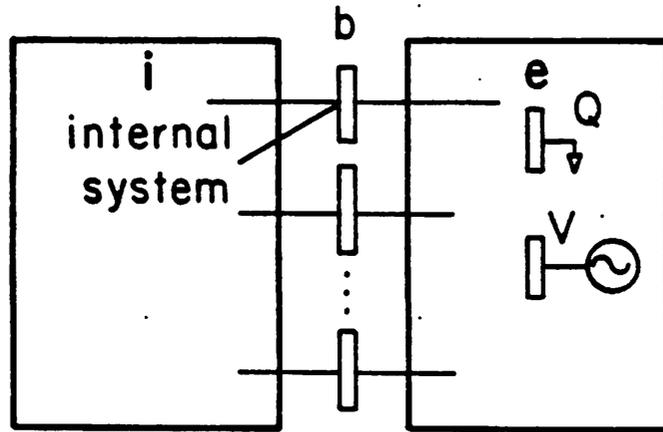
The original unreduced system and its corresponding bus admittance matrix \underline{Y} are shown in Fig. 3(a) and (b), respectively, where the external buses are partitioned into PQ buses (subscript Q) and PV buses (subscript V).

The Ward-PV equivalent shown in Fig. 4(a) is derived from Fig. 3(a) by eliminating all external PQ buses (Q). The corresponding bus admittance matrix of the Ward-PV equivalent shown in Fig. 4(b) is obtained from the matrix in Fig. 3(b) by Gaussian elimination on Q. Recall that the \underline{B}'' matrix in the decoupled reactive power flow equation (6) is derived from $-\underline{B}$ of $\underline{Y} = \underline{G} + j\underline{B}$ by (i) deleting rows and columns corresponding to PV buses, (ii) doubling the shunts, (iii) ignoring phase shifters. Thus after deleting the rows and columns of V, the incremental decoupled reactive power flow equation of the Ward-PV equivalent takes the form

$$\begin{array}{|c|c|} \hline \mathbf{B}_{wv}'' + (\mathbf{B}_{bb}^i)'' & \mathbf{B}_{bi}'' \\ \hline \mathbf{B}_{ib}'' & \mathbf{B}_{ii}'' \\ \hline \end{array}
 \begin{array}{|c|} \hline \Delta \mathbf{V}_b \\ \hline \Delta \mathbf{V}_i \\ \hline \end{array}
 =
 \begin{array}{|c|} \hline \frac{\Delta \mathbf{Q}_b}{\mathbf{V}_b} \\ \hline \frac{\Delta \mathbf{Q}_i}{\mathbf{V}_i} \\ \hline \end{array}
 \quad (15)$$

where the same subscripts and superscripts are used for the corresponding \underline{B}'' and \underline{Y} , and the necessary modifications on the retained PV buses are not performed because they are not of direct interest here. From eq. (15), it is seen that the reactive power response of the Ward-PV equivalent to the changes in boundary bus voltages $\Delta \mathbf{V}_b$ is

$$\Delta \underline{Q}_b^{wv} = [\underline{V}_b] \underline{B}_{wv}'' \Delta \underline{V}_b \quad (16)$$

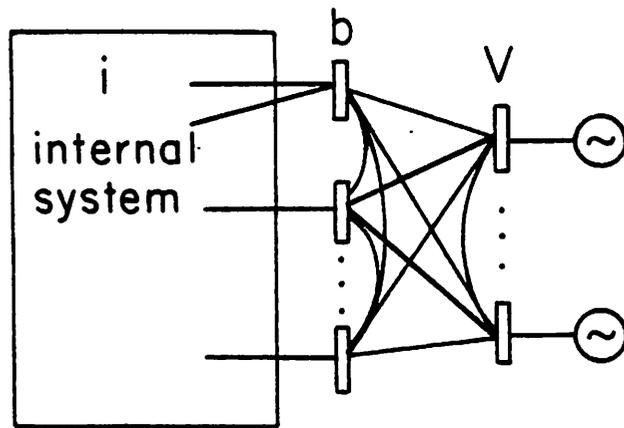


(a)

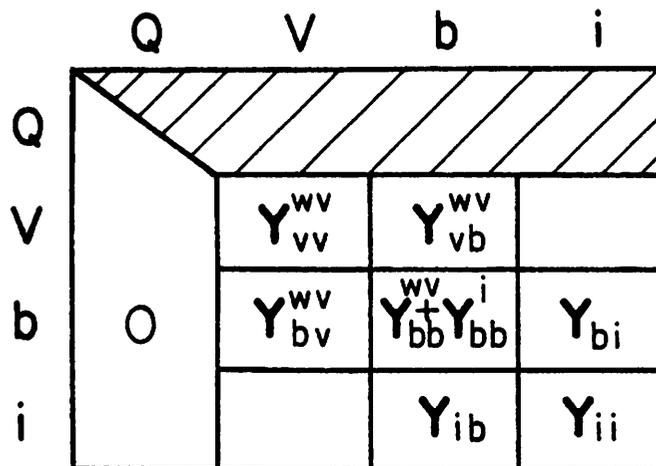
	Q	V	b	i
Q	Y_{QQ}	Y_{QV}	Y_{Qb}	
V	Y_{VQ}	Y_{VV}	Y_{Vb}	
b	Y_{bQ}	Y_{bV}	$Y_{bb}^e + Y_{bb}^i$	Y_{bi}
i			Y_{ib}	Y_{ii}

(b)

Fig. 3 (a) The original system before reduction and (b) the associated bus admittance matrix.



(a)



(b)

Fig. 4. The system after eliminating the external PQ buses (a) the Ward-PV equivalent network and (b) the associated bus admittance matrix.

For the Ward equivalent, we proceed further to eliminate the external PV buses (V). The resulting Ward equivalent is shown in Fig. 5(a) and its corresponding bus admittance matrix in Fig. 5(b) is obtained from the matrix in Fig. 4(b) by Gaussian elimination on V. Since there is no PV buses in Ward-equivalent, the incremental decoupled reactive power flow equation (6) of the Ward equivalent is given by

$$\begin{array}{|c|c|} \hline \mathbf{B}_w'' + (\mathbf{B}_{bb}^i)'' & \mathbf{B}_{bi}'' \\ \hline \mathbf{B}_{ib}'' & \mathbf{B}_{ii}'' \\ \hline \end{array} \begin{array}{|c|} \hline \Delta \mathbf{V}_b \\ \hline \Delta \mathbf{V}_i \\ \hline \end{array} = \begin{array}{|c|} \hline \frac{\Delta \mathbf{Q}_b}{\mathbf{V}_b} \\ \hline \frac{\Delta \mathbf{Q}_i}{\mathbf{V}_i} \\ \hline \end{array} \quad (17)$$

From eq. (17), we find the reactive power response of the Ward equivalent to be

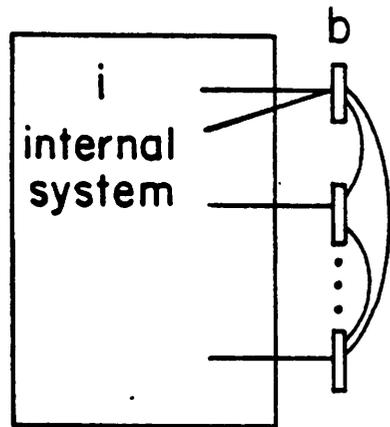
$$\Delta \underline{Q}_b^W = [\underline{V}_b] \underline{B}_{wv}'' \Delta \underline{V}_b \quad (18)$$

By comparing eqs. (16) and (18), clearly if we want the Ward equivalent to have the same reactive response of the Ward-PV equivalent we should add reactive injections at the boundary buses of the amount

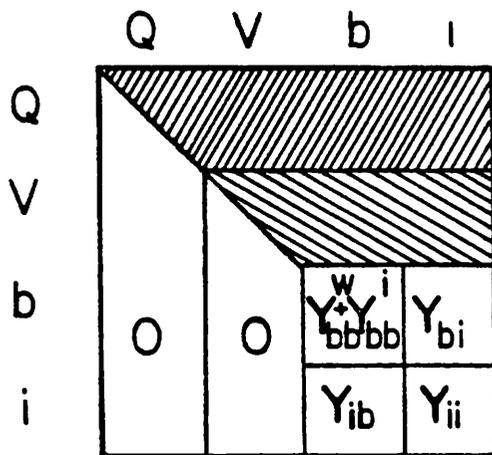
$$\Delta \tilde{\underline{Q}}_b = [\underline{V}_b] (\underline{B}_{wv}'' - \underline{B}_w'') \Delta \underline{V}_b \quad (19)$$

Unfortunately the reactive support at the boundary buses as required by eq. (19) can not be directly incorporated in a load flow program. We now derive an approximation to eq. (19) that can be directly implemented in a load flow program.

Let $\hat{\underline{B}} = (\underline{B}_{wv}'' - \underline{B}_w'')$. We want to construct a network whose corresponding \underline{B}'' matrix is $\hat{\underline{B}}$. The k-th component of eq. (19) is



(a)



(b)

Fig. 5. The system after eliminating all external buses (a) The Ward equivalent network and (b) the associated bus admittance matrix.

$$\begin{aligned}
\Delta \tilde{Q}_k / V_k &= \hat{B}_{kk} \Delta V_k + \sum_{m \neq k} \hat{B}_{km} \Delta V_m \\
&= \hat{B}_k \Delta V_k + \sum_{m \neq k} \hat{B}_{km} (\Delta V_m - \Delta V_k)
\end{aligned} \tag{20}$$

where

$$\hat{B}_k = \hat{B}_{kk} + \sum_{m \neq k} \hat{B}_{km} \tag{21}$$

In other words, the corresponding network has a "shunt" ($-\hat{B}_k$). Note that for incremental reactive power response the PV buses act as if they were grounded ($\Delta V=0$). Therefore this "shunt" can be represented as a susceptance to a PV-bus (Fig. 6(b)).

Let the km-th element of \underline{B}_{wv}'' and \underline{B}_w'' be B_{km}^{wv} and B_{km}^w , respectively. The k-th diagonal element of \underline{B}_{wv}' is

$$- \sum_{\substack{m \neq k \\ m \in \mathcal{B}}} B_{km}^{wv} - \sum_{j \in \mathcal{V}} B_{kj}^{wv} - B_{k,shunt} \tag{22}$$

The k-th diagonal element of \underline{B}_w'' is

$$- \sum_{\substack{m \neq k \\ m \in \mathcal{B}}} B_{km}^w - B_{k,shunt} \tag{23}$$

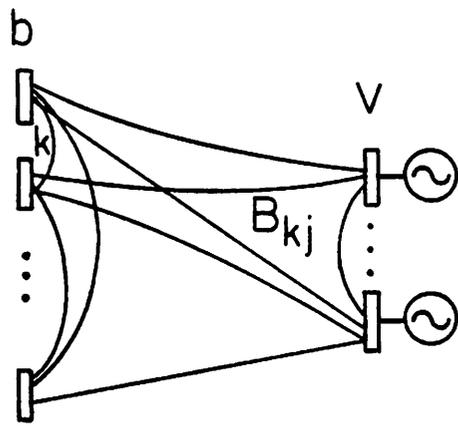
Substituting (22), (23), B_{km}^{wv} , and B_{km}^w into (21), we obtain

$$\hat{B}_k = - \sum_{j \in \mathcal{V}} B_{kj}^{wv} \tag{24}$$

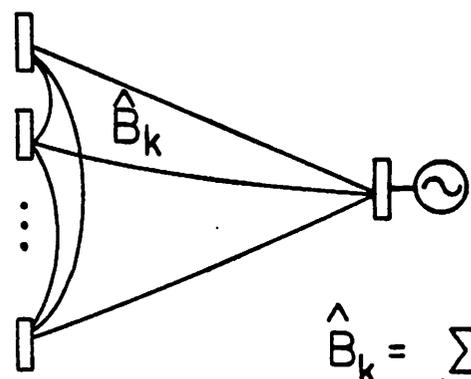
That is, the susceptance ($-\hat{B}_k$) from bus k to the fictitious PV bus is equal to the sum of the susceptances from bus k to the external PV buses in Ward-PV equivalent.

Consider again eq. (20). The following observations are in order.

1) The combined effect of the second term to the internal system is zero, i.e.,

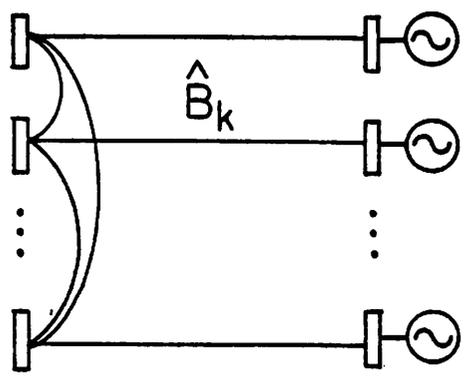


(a)

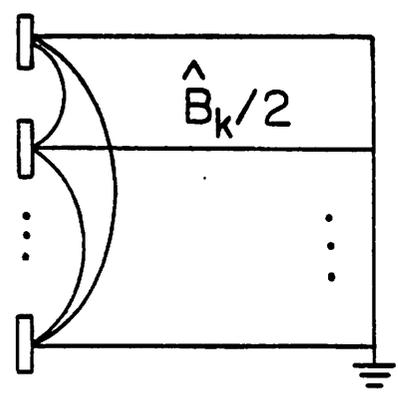


(b)

$$\hat{B}_k = \sum_{j \in V} B_{kj}$$



(c)



(d)

Fig. 6. (a) The Ward-PV equivalent, where B_{kj} is the susceptance between the boundary bus k and an external PV bus j . (b) A network whose incremental reactive response is the same as in a. (c) Extended Ward equivalent with PV buses (d) Extended Ward equivalent with shunts.

$$\sum_k \sum_{m \neq k} \hat{B}_{km} (\Delta V_m - \Delta V_k) = 0 \quad (25)$$

- 2) When $(\Delta V_m - \Delta V_k)$ is small, the second term is small.
- 3) When $(\Delta V_m - \Delta V_k)$ is appreciable, it means ΔV_m and ΔV_k respond very differently. Physically this happens when m and k are not tightly coupled, i.e., the impedance between them is large, or \hat{B}_{km} is small.
- 4) When B_{km}^{WV} and B_{km}^W are equal, the second term becomes zero.

In view of the above observations, we neglect the second term in eq. (20) and approximate the reactive response from the external system of (19) by

$$\Delta \tilde{Q}_k = V_k \hat{B}_k \Delta V_k \quad (26)$$

The derivation of eq. (26) may be seen by way of networks. The Ward-PV equivalent is shown again in Fig. 6(a). Recall that for incremental reactive power response the PV buses can be treated as if they were grounded ($\Delta V=0$). Thus both networks in Fig. 6(b) and (c) are equivalent to the one in Fig. 6(a), as far as the incremental reactive response is concerned. The equation for the incremental reactive response of Fig. 6(c) is eq. (26).

A further simplification of the model in Fig. 6(c) is possible. Note that the networks in Fig. 6(c) and Fig. 6(d) have the same incremental reactive response. (Recall that the shunts are to be doubled in forming \underline{B}''). This is the commonly implemented version of the extended Ward equivalent.

To summarize, the construction of an extended Ward equivalent has the following basic steps:

1. Obtain a Ward equivalent of the external system. (External shunts are converted to injections).
2. Start again from the original system. Ground all external PV buses. Apply Gaussian elimination on the bus admittance matrix \underline{Y}

to eliminate all external buses to obtain the equivalent shunts at the boundary buses, which are the admittances $j\hat{B}_k$.

3. Augment the Ward equivalent by inserting a shunt $j\hat{B}_k/2$ at each boundary bus.

Comments (Accuracy issues)

1. The extended Ward equivalent has been found to give accurate results for contingency evaluation [2].

Comments (Computational issues)

2. For extended Ward equivalent, as well as other Ward-type equivalents, there are two distinct steps involved, namely, (i) the construction of the equivalent network, and (ii) the boundary matching. It should be pointed out that the Ward-type equivalent network is constructed using only the external network data and the external operating-point data affects only the boundary injections. Recall that the network data changes rather infrequently, whereas the operating point data changes from minute to minute. With this separation property of Ward-type equivalents with respect to network and operating-point, it is possible to track in real-time the changes in operating point frequently (by boundary matching).

Comments (Compatibility issues)

3. The extended Ward equivalent with shunt compensation (Fig. 6d) combines simplicity and good VAR response. It also has the advantage that a standard Ward equivalent program can be easily adapted for usage as an extended Ward.

Remarks

Most external network models are derived under the implicit assumption that the changes in external system real power generation

as the result of the internal system contingency are negligible. This is usually true for a line or transformer outage, or a generator outage when the internal system has enough spinning reserve to make up for the generation loss. Aschmoneit and Verstege [18] have extended the Ward-type equivalents to cases where a generator outage in the internal system results in the real power support being supplied to a large extent by the external system.

III.2.3. Generalized Ward

Amerongen and Meeteren [19] have derived a generator bus model for load flow studies based on a linearized synchronous machine model with automatic voltage regulation. The model is a PQ bus connected to a PV bus as shown in Fig. 7, where x_{eq} is derived from the machine parameters, as well as the operating-point data. Using this model for generator bus, proceeding with the elimination process, they have derived an equivalent boundary injection representing the AVR effect of external generators. The equivalent boundary injection has a form similar to eq. (19). Good results are reported on the testing of the equivalent on the 220-bus Dutch system. Load flow program has to be modified to accommodate the additional boundary injections.

IV REI-TYPE EQUIVALENT

IV.1. Basic Theme

The REI (Radial Equivalent Independent) approach to external equivalents was invented by Dima [20]. It was brought to the attention of power engineers largely by Tinney and Powell [21]. The main idea in the REI equivalent is to aggregate the injections of a group of buses into one bus. The aggregated injection is distributed back to these buses through a radial network called REI network. After the aggregation

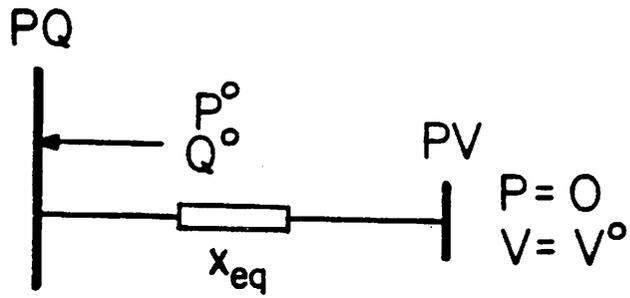


Fig. 7. The generator bus model for the generalized Ward equivalent.

all buses with zero injection are eliminated by Gaussian elimination.

The basic steps in the REI equivalencing process consist of the following:

1. Remove the injections from all buses to be aggregated (Fig. 8a).
2. Create an REI network and attach it to these buses (Fig. 8b).

Aggregate all the injections $S_R = \sum_k S_k$ to the REI node R.

3. Eliminate all buses k and node G by Gaussian elimination (Fig. 8c).

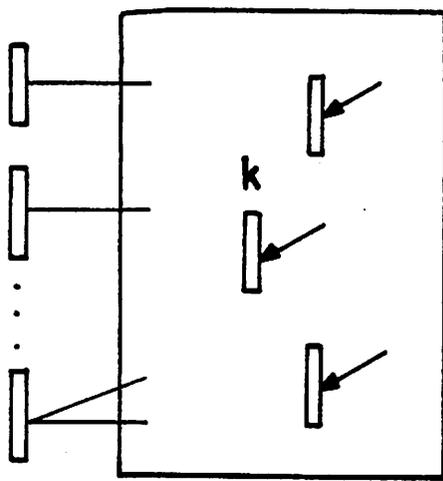
The values of the admittances y_k and y_R in the REI network are selected based on a solved load flow of the external system in such a way that the injections into buses k from the REI network are exactly the same as the original injections in the solved load flow. Conventionally, V_G for the base case is set to be zero. Therefore, the values of these admittances should be

$$y_k = - \frac{S_k^*}{V_k^2} \quad (27)$$

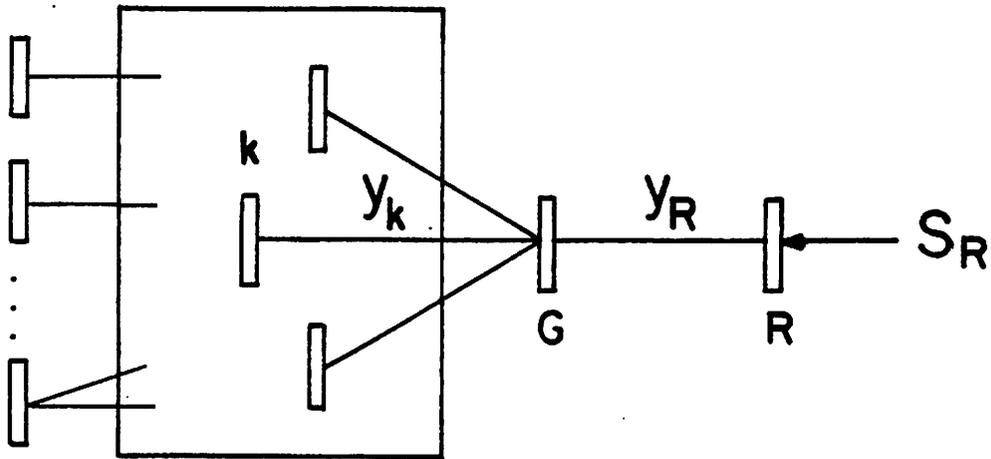
$$y_R = \frac{S_R^*}{V_R^2} \quad (28)$$

It can easily be checked that the total losses in the REI network for the base case is zero, consequently Dimo calls it the zero power balance network. However, the total losses in the REI network for a contingent case may be different from zero.

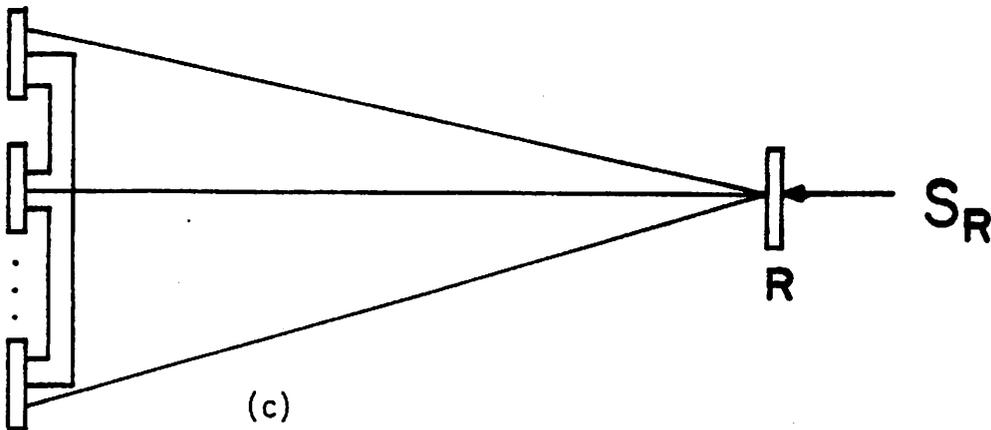
It is often desirable to construct more than one REI network for the external system. The grouping of buses into an REI network may be based on bus type (PQ or PV), geographical proximity, sparsity considerations, or other criteria.



(a)



(b)



(c)

Fig. 8. The REI equivalent. (a) The original system with injections to be aggregated. (b) The attachment of an REI network. (c) The resulting REI equivalent after eliminating zero injection buses.

Comments (Accuracy issues)

1. The REI equivalent is derived as an equivalent that matches the solved load flow. For contingency evaluation, cases other than the base case are of interest. A mathematical analysis has shown [22,23] that very stringent necessary conditions have to be satisfied in order for the REI equivalent to match the incremental behavior of the original unreduced load flow model.
2. The accuracy of the REI equivalent for contingency evaluation in practice depends on how the grouping is done. Various heuristic criteria for grouping have been proposed [16,24].
3. The admittances of the REI network (eqs. 27-28) are functions of the operating condition at which the equivalent is constructed. Let us refer to this operating condition A. Suppose that for on-line application the actual operation condition is B. Then even after boundary matching, the REI equivalent constructed using operating condition A is not the same REI equivalent one would construct using operating condition B. If the operating condition (A) at which the REI equivalent is constructed and the operating condition (B) that is used as the base case for contingency evaluation are far apart, one could expect the accuracy of the equivalent would deteriorate. Therefore the updating of the equivalent is perhaps essential for on-line application of REI equivalents. On the contrary for Ward-type equivalents (except for the Ward Admittance method) there is no difference between the equivalents constructed for different operating conditions after boundary matching.

Comments (Computational issues)

4. The REI network creates extra interconnections, therefore the REI equivalent tends to be more dense. It is suggested to choose more REI

nodes thus separating the equivalent into clusters of high connectivity in order to preserve sparsity [21].

5. We have seen that for Ward-type equivalents the equivalent network is constructed using only the external network data and the external system operating-point data affects only the boundary injections. With this nice separation property, when the current operating condition is different from the condition the equivalent was constructed, one needs merely to change the boundary matching to update the equivalent. This is, however, not the case for REI equivalent. The operating-point data of the external system is built into the branch admittances of the REI network (eqs. 27-28). This makes updating the REI equivalent for changing operating conditions a difficult task.

Comments (Compatibility issues)

6. The admittance of the REI network branches may assume values very much different from that of the usual branch admittances. They may have very unusual X/R ratios, negative resistance, capacitive series branch, etc.

IV.2. Thematic Variations

IV.2.1. X-REI

The X-REI equivalent for on-line application was developed by Dy Liacco, Savulescu, Ramarao [25]. Two new ideas were introduced in the X-REI, (i) the retaining of certain nodes and branches in the external system, called essential nodes and essential branches, (ii) the introduction of a calibrating network and a calibrating node (Fig. 9) whose purpose is for boundary matching. The basic steps in the construction of an X-REI are the following:

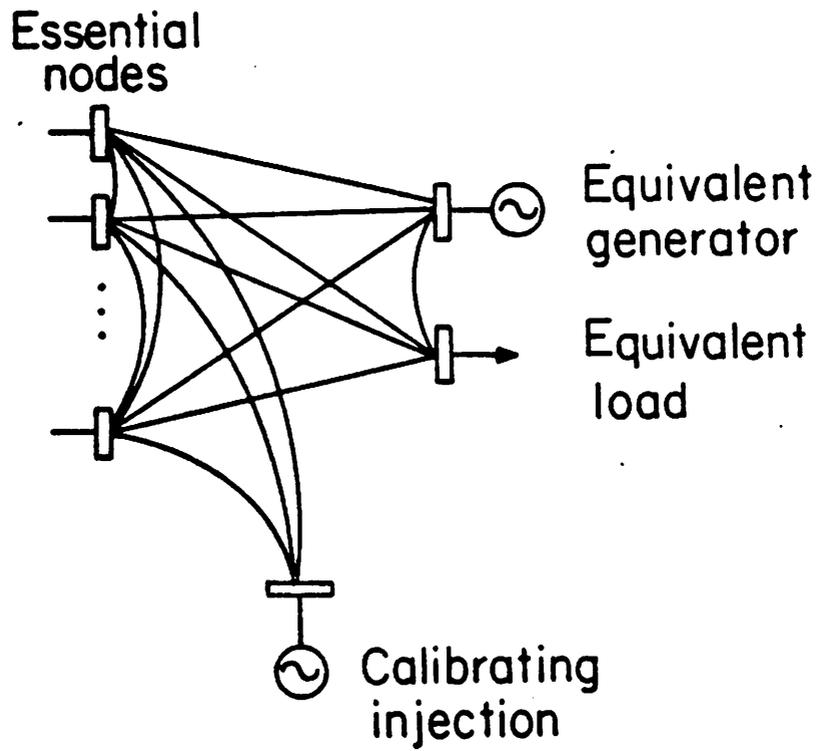


Fig. 9. The X-REI equivalent.

1. Select the essential nodes of the external system, which should include, as a minimum, all the boundary nodes.

2. Aggregate the injections at the non-essential nodes through REI networks, and eliminate all non-essential nodes with zero injection.

3. Calculate required injections for boundary matching.

4. Replace these injections by a calibrating network similar to an REI network.

The function of the calibrating network is precisely boundary matching. The retaining of external essential nodes and essential branches is for the incorporation of vital real-time information. The X-REI equivalent may be adjusted in several ways depending on new operating information available

Comments (Accuracy and computational issues)

1. Savulescu [24] suggested that loads should be aggregated to an REI network and generations should be aggregated to a different REI network for accuracy. Thus, for example, one bus that has both generation and load is connected to two REI networks. If there are many buses having both generation and load, the sparsity of the equivalent suffers.

IV. 2.2. S-REI

The S-REI (S for stochastic) was introduced by Dopazo, Irisarri and Sasson [26]. The novel idea in the S-REI is the scheme for REI-node voltage update and equivalent network update. The basic steps in the construction of S-REI are the following:

1. Construct REI equivalent as in Sec. IV.1.
2. Determine boundary mismatch
3. Adjust the voltages of the REI nodes to minimize the boundary mismatch.

4. Adjust equivalent branch admittances to further minimize the boundary mismatch.

The criterion used in the selection of the REI nodal voltages is to minimize the boundary mismatch. Note that the same problem was formulated in Sec. II.2.1. for the ULF model. For the particular case with REI equivalent, this problem, called redundant load flow by the authors, has a closed form solution [27, p. 4837]. Note that finding a solution (REI nodal voltages) for the redundant load flow problem essentially is equivalent to readjusting the REI injections (external operating-point data) for boundary matching. The network update scheme uses a system identification approach which is a time-sequential method [28,29].

Comments (Accuracy issues)

1. Good performance of the S-REI is reported for a particular outage case of the AEP system [27]. Comprehensive testing with more outage cases is under way to further evaluate the method. It seems that the credit of success is largely due to the voltage update (on-line boundary matching). The voltage updating approach can be applied to Ward-PV and extended Ward equivalents. It should also be pointed out that the network updating method is applicable to all Ward-type equivalents as well.

Comments (Compatibility issues)

2. The REI nodal voltages, obtained from the solution of the redundant load flow, may be rather abnormal (as high as 7 pu was observed) [27]. Various grouping schemes to alleviate this problem have been suggested [16,27], for example, all PV buses with negative net power are assigned to the PQ-REI bus and all PQ buses with positive net power are assigned

to the PV-REI bus. Deterioration in accuracy is also reported as a result of these grouping schemes [27, p. 483].

IV.2.3. GRANEQ

Step 3 (elimination of zero injection nodes) of the REI equivalencing process in Sec. IV.1 usually results in a graph which is almost complete. Hager and Glavitsch [30] have developed a method, called GRANEQ (General RAdial Network Equivalent), to construct instead a radial network that matches the transfer impedances between the boundary-boundary buses and boundary buses-REI node.

V. A UNIFIED APPROACH

The review of the methods for external network modeling in the preceding sections show that different methods have different desired features. In particular, very different modeling philosophies are involved in the ULF and equivalencing (Ward-type or REI-type) approaches. The following is a recapitulation of some important observations made in the preceding sections.

a) Certain external system condition may affect the internal system response in a significant way. Sometimes data that manifest the changes of these conditions, for example, certain external network data (mainly breaker status) or external operating-point data (key flows), are available--albeit intermittently. In such cases the ULF model has the advantage of being able to incorporate these data directly into the external network model.

b) The Ward-type or the REI-type equivalents are approximate models of the external system. The inclusion of an equivalent in the load flow study for contingency evaluation saves considerable computation. The accuracy of the results with the use of an equivalent, e.g., the

extended Ward equivalent, may be reasonably good, especially when the external system does not play the leading role in the internal system response to the contingency.

c) The boundary matching is a very important aspect of external network modeling. Its inclusion is essential. The state estimation formulation of adjusting external operating-point data to minimize boundary mismatch can be considered as a refinement of boundary matching. The REI-nodal voltage update is a special case of this.

We propose in the following a unified approach to external network modeling which combines the desired features of all methods, and encompasses all these methods. The external system is divided into inner and outer external systems (Fig. 1). A complete unreduced load flow model is used for the inner external system. The outer external system is reduced and represented by an equivalent. The major steps of the proposed approach are stated first, followed by a more detailed explanation of each step. To facilitate presentation, we assume, as usually is the case, that the internal system is well-defined (one's own company) and it is observable for state estimation. The application of external network modeling for unobservable islands will be commented on later. The basic steps of the unified approach are the following:

1. Define the inner external system.
2. Construct an equivalent network for the outer external system.

Replace the outer external network by an equivalent network.

3. Update the injections at the outer boundary buses.
4. Perform internal system state estimation.
5. Boundary matching at the inner boundary buses.
6. Update external network model whenever necessary.

Elaboration

1(a) The determination of which part of the external system to be retained in the load flow model (inner external system) may be based on a number of considerations, e.g., the desire to retain explicitly representation of certain elements in the external system (for which telemetered data or other information is available), the knowledge that the response of that part is likely to be significant for contingency evaluation, etc. Off-line load flow contingency studies of the entire system may be used (Comment 1, Sec. II.1).

(b) We maintain that if the outage of a line connecting two inner boundary buses is to be studied, the external buses adjacent to these two buses should not be equivalenced (hence retained as part of the inner external system). This is argued as follows. Suppose the outage line ℓ connects buses A and B. If A and B are boundary buses and the system external to them are equivalenced, then there would be an equivalent line q in the external equivalent network also connecting A and B. The outage of line ℓ would be similar to the outage of line q . Since one would not leave a line (that will become part of q) whose outage is to be studied in the system that is to be equivalenced, the lines incident with buses A and B, and the adjacent external buses should be retained.

2. Either a Ward-type or an REI-type equivalent may be a candidate for the outer external equivalent. Since the voltage update scheme of the top REI-type equivalent, the S-REI, is not applicable here, one may choose to employ the extended Ward equivalent or the Ward-PV equivalent if the selection of PV buses to retain is obvious.

3. Unlike the inner boundary buses for which boundary matching with real-time data is possible, the "boundary matching" for the outer boundary

buses has to take the form with assumed boundary flows, i.e., using extrapolated values similar to the scheme in Sec. II.1.

4. This is a standard procedure.

5. The standard scheme for boundary matching (Sec. I.2) may be used here. Alternatively the state-estimation formulation (Sec. II.2.1) of minimizing boundary mismatches or the observability test based boundary matching (Sec. II.2.2) may be employed.

6. If either the ULF model or the Ward-type equivalents is used, the external equivalent network depends only on the external network data and can be constructed off-line. The external network update consists simply of retrieving the appropriate copy of the external system equivalent network. Of course in any case the sequential scheme for external network parameter estimation [26] may be used.

Remark

In the case where the monitored part of the system is not observable, then the proposed method is modified. An observability test [12] is performed to determine the observable subsystem of the monitored system. The observable subsystem now becomes the internal system. The rest of the monitored system, including all unobservable islands, now becomes part of the inner external system.

VI. CONCLUSION

In this paper a critical survey on the state-of-the-art of external network modeling is presented. The motivation and derivation of the commonly adopted methods are presented. Assessment is made on each method from the accuracy, computational, and load flow compatibility considerations. Conclusions are drawn in the Comments on each method, which are not repeated here. After the review, we propose a unified

approach to external network modeling. The method is flexible and encompasses the desired features of the existing methods.

There are three major aspects of on-line security functions, namely, monitoring, analysis, and control. As the result of the progress made in the last few years, the state estimation for security monitoring is pretty well-developed. Now the major development work is concentrated in the area of security analysis. External network modeling is an important component of security analysis. Current efforts in the development and testing of external network models certainly are contributing to the rapid progress towards a good and reliable external network model. Looking into the future, for security control using an optimal power flow, it is even more crucial to have a good external network model.

The external network modeling methods are reviewed in this paper with on-line security analysis applications in mind, because of the nature of this Special Issue. There is a definite need for external network modeling in the emerging field of Dispatcher Training Simulator [31]. The load flow studies for system planning have been the cradle for the development, and will continue to be an important area of application, of external network equivalents. Most of the methods discussed in this paper, however, with proper modifications, can be utilized for planning applications too.

Tinney has remarked [32, p. 17] that some aspects of power system external network reduction for planning studies have been satisfactorily resolved by the sparsity-oriented reduction and the extended Ward equivalent, but there still are areas that need improvements. We believe that there are even more room for improvements in the external network modeling for on-line applications.

VII. ACKNOWLEDGEMENT

The research is supported by the Department of Energy, Division of Electric Energy Systems, under contracts DE-AC01-79-ET29364 and DE-AC01-82-CE76221. The authors are grateful to A. M. Athay, A. Bose, for providing valuable information, and D. Denzel, M. Gorenberg, G. Irisarri, A. K. Subramanian for sharing their experience. We also thank Chen-Ching Liu for his helpful comments.

VIII. REFERENCES

1. T. E. Dy Liacco, "System security: The computer's role," IEEE Spectrum Vol. 15, pp. 43-50, June 1978.
2. S. Deckmann, A. Pizzolante, A. Monticelli, B. Stott, O. Alsac, "Numerical testing of power system load flow equivalencing," IEEE Trans. Power Apparatus and Systems, Vol. PAS-99, pp. 2292-2300, November 1980.
3. S. Deckmann, A. Pizzolante, A. Monticelli, B. Stott, O. Alsac, "Studies on power system load flow equivalencing," IEEE Trans. Power Apparatus and Systems, Vol. PAS-99, pp. 2301-2310, November 1980.
4. F. L. Alvarado and E. J. Elkonyaly, "Reduction in Power Systems," A77 507-7, IEEE PES Summer Meeting, Mexico City, July 1977.
5. E. H. Elkonyaly and F. L. Alvarado, "External System Static Equivalent for On-Line Implementation," A78 060-6, IEEE PES Winter Meeting, New York, January 1978.
6. B. Stott, "Review on load flow calculation methods," Proceedings of the IEEE, Vol. 62, pp. 916-929, July 1974.
7. B. Stott and O. Alsac, "Fast decoupled load flow," IEEE Trans. Power Apparatus and Systems, Vol. PAS-73, pp. 859-869, May/June 1974.
8. K. I. Geisler and A. Bose, "State estimation based external network solution for on-line security analysis," 83 WM 077-5, IEEE PES Winter Meeting, New York, January 1983.
9. A. Garcia, A. Monticelli and P. Abreu, "Fast decoupled state estimation and bad data processing," IEEE Trans. Power Apparatus and Systems, Vol. PAS-98, pp. 1645-1652, September 1979.

10. J. J. Allemong, L. Radu, and A. M. Sasson, "A fast and reliable state estimation algorithm for AEP's new control center," IEEE Trans. Power Apparatus and Systems, Vol. PAS-101, pp. 933-944, April 1982.
11. N. Inoue, H. Daniels, R. Kilmer, V. Echave, "Implementation, testing and field installation of three state estimators," Proc. 1983 PICA Conf., pp. 11-18, Houston, May 1983.
12. K. A. Clements, G. R. Krumpholz, and P. W. Davis, "Power system state estimation with measurement deficiency: an observability/measurement placement algorithm," 83 WM 058-5, IEEE PES Winter Meeting, New York, January 1983.
13. K. A. Clements, G. R. Krumpholz, and P. W. Davis, "Power system state estimation residual analysis: an algorithm using network topology," IEEE Trans. Power Apparatus and Systems, Vol. PAS-100, pp. 1779-1787, April 1981.
14. F. F. Wu, "Theoretical study of the convergence of the fast decoupled load flow," IEEE Trans. Power Apparatus and Systems, Vol. PAS-96, pp. 268-275, January/February 1977.
15. T. E. Dy Liacco and K. A. Ramarao, Discussion of [14], *ibid*, p. 273.
16. E. C. Housos, G. Irisarri, R. M. Porter and A. M. Sasson, "Steady-state network equivalents for power system planning applications," IEEE Trans. Power Apparatus and Systems, Vol. PAS-99, pp. 2113-2120, November 1980.
17. A. Monticelli, S. Deckmann, A. Garcia and B. Stott, "Real-time external equivalents for static security analysis," IEEE Trans. Power Apparatus and Systems, Vol. PAS-98, pp. 498-508, March/April 1979.

18. F. C. Aschmoneit and J. F. Verstege, "An external system equivalent for on-line steady-state generator outage simulation," IEEE Trans. Power Apparatus and Systems, Vol. PAS-98, pp. 770-779, May 1979.
19. R. A. M. van Amerongen and H. P. van Meeteren, "A generalized Ward equivalent for security analysis," IEEE Trans. Power Apparatus and Systems, Vol. PAS-101, pp. 1519-1526, June 1982.
20. P. Dimo, "Nodal analysis of power systems," Abacus Press, England 1975.
21. W. F. Tinney and W. L. Powell, "The REI approach to power network equivalents," Proc. PICA Conf., pp. 314-320, Toronto, May 1977.
22. F. F. Wu and N. Narasimhamurthi, "Necessary conditions for REI reduction to be exact," A79 065-4, IEEE PES Winter Meeting, New York, January 1979.
23. S. N. Talukdar and F. F. Wu, "Computer-aided dispatch for electric power systems," Proc. of the IEEE, Vol. 69, pp. 1212-1231, October 1981.
24. S. C. Savulescu, "Equivalents for security analysis of power systems," IEEE Trans. Power Apparatus and Systems, Vol. PAS-100, pp. 2672-2682, May, 1981.
25. T. E. Dy Liacco, S. C. Savulescu and K. A. Ramarao, "An on-line topological equivalent of a power system," IEEE Trans. Power Apparatus and Systems, Vol. PAS-97, pp. 1550-1563, September/October 1978.
26. J. F. Dopazo, G. Irisarri and A. M. Sasson, "Real-time external system equivalent for on-line contingency analysis," IEEE Trans. Power Apparatus and Systems, Vol. PAS-98, pp. 2153-2171, November 1979.
27. E. Housos and G. Irisarri, "Real time results with on-line network equivalents for control center applications," IEEE Trans. Power Apparatus and Systems, Vol. PAS-100, pp. 4830-4837, December 1981.

28. T. G. Deville and F. C. Schweppe, "On-line identification of interconnected network equivalents from operating data," C72 464-6, IEEE PES Winter Meeting, San Francisco, July 1972.
29. J. F. Dopazo, M. H. Dwarakanath, J. J. Li and A. M. Sasson, "An external system equivalent model using real-time measurements for system security evaluation," IEEE Trans. Power Apparatus and Systems, Vol. PAS-96, pp. 431-439, March 1977.
30. H. Hager and H. Glacitsch, "GRANEQ - a radial, internal network equivalent for power system security calculations," IEEE Trans. Power Apparatus and Systems, Vol. PAS-102, pp. 452-462, February 1983.
31. R. D. Masiello and D. M. Morrison, "Dispatcher training simulation: present and future," prepared for the 1981 PICA Conference.
32. W. F. Tinney, "Mathematical challenges in power system planning," in Electric Power Problems: The Mathematical Challenge, ed. by A. M. Erisman, K. W. Neves and M. H. Dwarakanath, Philadelphia, SIAM, 1980, pp. 3-20.