

Copyright © 1983, by the author(s).  
All rights reserved.

Permission to make digital or hard copies of all or part of this work for personal or classroom use is granted without fee provided that copies are not made or distributed for profit or commercial advantage and that copies bear this notice and the full citation on the first page. To copy otherwise, to republish, to post on servers or to redistribute to lists, requires prior specific permission.

LOW SAMPLE-RATE TIMING RECOVERY  
USING SWITCHED-CAPACITOR TECHNIQUES

by

N-S. Lin

Memorandum No. UCB/ERL M83/72

19 December 1983

COVER PAGE

LOW SAMPLE-RATE TIMING RECOVERY  
USING SWITCHED-CAPACITOR TECHNIQUES

by  
Nan-Sheng Lin

Memorandum No. UCB/ERL M83/72

19 December 1983

ELECTRONICS RESEARCH LABORATORY  
College of Engineering  
University of California, Berkeley  
94720

Nan-Sheng Lin

Author

Low Sample-Rate Timing Recovery

Using Switched-Capacitor Techniques

Title

RESEARCH PROJECT

Submitted to the Department of Electrical Engineering and  
Computer Sciences, University of California, Berkeley,  
in partial satisfaction of the requirements for the degree  
of Master of Science, Plan II.

Approval for the Report and Comprehensive Examination:

Committee:

David G. Hodges

, Research Advisor

December 9, 1983

Date

David M. Messerschmitt

12/13/83

Date

## Table of Contents

<b>Chapter 0:</b>	Introduction .....	2
<b>Chapter 1:</b>	General Description.....	3
1.1	Wave Difference Method.....	3
1.2	The Construction of the System.....	4
1.3	Some Basic Switched Capacitor Filters.....	5
<b>Chapter 2:</b>	System and Circuit Analysis.....	11
2.1	Analysis of the Circuit .....	11
2.2	Some Other View Point.....	13
2.3	Analysis of the Stability of the System.....	15
2.4	Analysis of the Transient Response of the System.....	18
<b>Chapter 3:</b>	The effects of Some OP Amp Non-idealities.....	23
3.1	Effect of Finite Gain of OP Amp .....	23
3.2	Effect of the Offset Voltage of OP Amp .....	24
3.3	Effect of the Finite Bandwidth of OP Amp .....	25
3.4	Conclusion .....	32
<b>References</b>	.....	34
<b>Appendix</b>	.....	35

## INTRODUCTION

Nowadays a lot of efforts have been devoted to the integration of the receiving part of digital subscriber loop into a single chip. Clock or timing recovery circuits are an essential part of such a system.

It has been discussed in [1] that the widely used zero-crossing phase detector has intrinsic pattern jitter due to the multiple zero-crossing points caused by various bipolar patterns. The wave difference method (WDM) phase detector, however, can effectively reduce pattern jitter. This report is devoted entirely to a circuit realization of the combination of phase detector using WDM and loop filter inside the phase locked loop (PLL). This very simple circuit, employing switched capacitor technique, can give a satisfactory performance as was proved by computer simulation using realistic line impulse response. Some nonidealities have also been considered and the performance of the circuit including those nonidealities is discussed and simulated.

# CHAPTER I

## GENERAL DESCRIPTION

### SECTION 1.1 : WAVE DIFFERENCE METHOD

As shown in Fig. 1.1 , we take two samples which are  $T/2$  apart. The difference of these two samples are sent to an integrator which in turn is connected to the VCO. By changing the sampling instants according to the output of VCO, a feedback loop is formed. Now if we assume the input waveform is symmetrical, we should expect the sampling instants converge to two points, one of which is at  $(t-T/4)$  and the other at  $(t+T/4)$  . This will make the difference of the two samples equal zero, and the output of integrator and the input of the VCO will be constant.

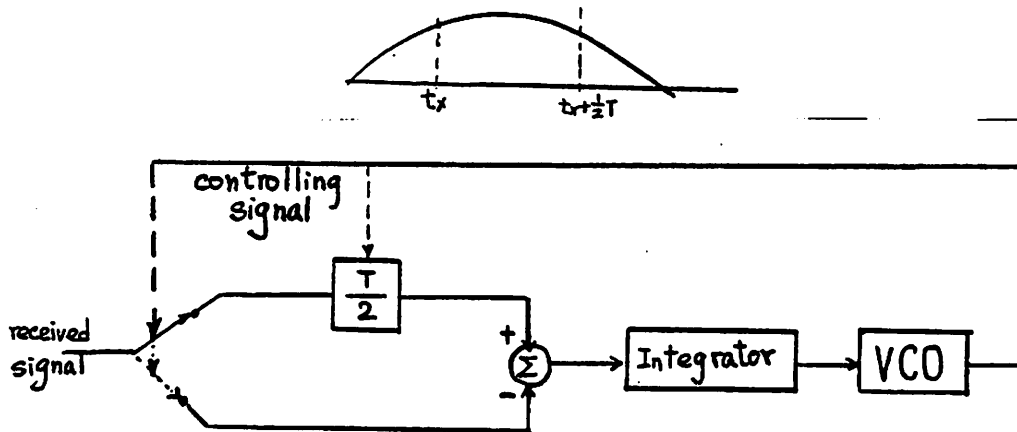


Fig. 1.1

From the discussion above, we can at least feel the advantage of realization the algorithm by using a switched capacitor integrator because of its intrinsic

sampling and integrating characteristics. This is the motivation of using a switched-capacitor circuit realization of WDM phase detector.

### SECTION 1.2 THE CONSTRUCTION OF THE SYSTEM

The switched capacitor (SC) technique which has been widely discussed these days enables the design of filters that can be realized in monolithic integrated circuit form using MOS technology. In the timing recovery application, instead of constant sampling rate, a SC circuit with varying sampling frequency which is determined by the phase error signal is used. With this modification, the phase detector and loop filter part of the PLL can be easily combined and realized by a simple circuit.

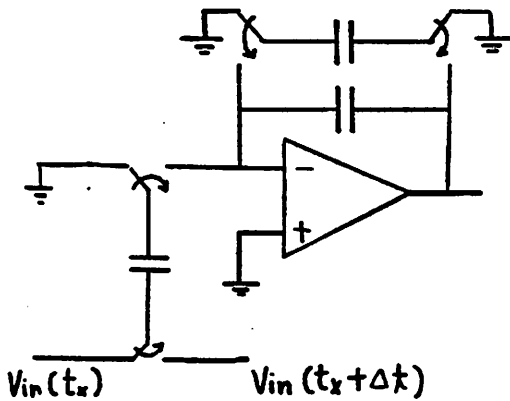


Fig 1.2(a)

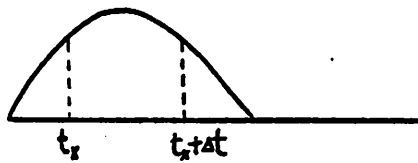


Fig 1.2(b)

As shown in Fig. 1.2, the error signal  $V_{in}' = V_{in}(t_x + \Delta t) - V_{in}(t_x)$  is fed into the integrator ( or low pass filter). Due to stability considerations, the pole of the integrator has to be moved to the left half plane from the origin. In SC



circuit, a termination capacitor is added to achieve the relocation of the pole.

Before exploring the circuit with varying sampling frequency, the constant sampling rate switched-capacitor integrator is described.

**SECTION 1.3 : SOME BASIC SWITCHED CAPACITOR FILTERS**

For the circuit shown in Fig. 1.3 (a), by conservation of charge, we have :

$$C_u V_{in}(n) + C_1 V_o(n) = C_1 V_o(n + \frac{1}{2})$$

$$= C_1 V_o(n+1)$$

$$\rightarrow \frac{V_o(z)}{V_{in}(z)} = \frac{C_u z^{-1}}{1 - z^{-1}}$$

The block diagram of the circuit is shown in Fig. 1.3(b)

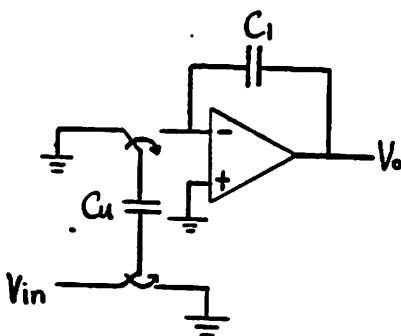


Fig 1.3(a)

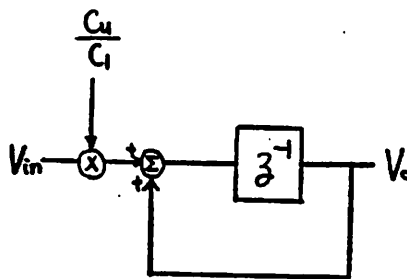


Fig 1.3(b)

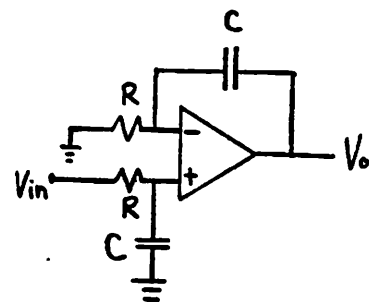


Fig 1.3(c)

This switched capacitor circuit is equivalent to an analog integrator with some phase and magnitude error. The analog counterpart of this circuit is shown in Fig. 1.3(c), that has the transfer function of

$$H(s) = \frac{V_o(s)}{V_{in}(s)} = \left[ \frac{\frac{1}{sC}}{R + \frac{1}{sC}} \right] \left[ \frac{R + \frac{1}{sC}}{R} \right] = \frac{1}{sRC}$$

If we apply lossless digital integrator (LDI) transformation to the above equation, i.e.

$$\frac{1}{s} \rightarrow T_c \frac{z^{-1/2}}{1-z^{-1}}$$

then

$$H(z) = \frac{1}{Rs} T_c \frac{z^{-1/2}}{1-z^{-1}}$$

Compare these two equations, we can get the relationship between the digital and the analog circuits.

$$\frac{1}{RC} \rightarrow \frac{C_u}{C_1} \frac{1}{T_c}$$

By using the same approach, we can get the transfer function of the circuit shown in Fig. 1.4(a):

$$H(z) = -\frac{C_u}{C_1} \frac{1}{1-z^{-1}}$$

The block diagram and the analog circuit which has similar transfer function are shown in Fig. 1.4(b), 1.4(c) respectively.

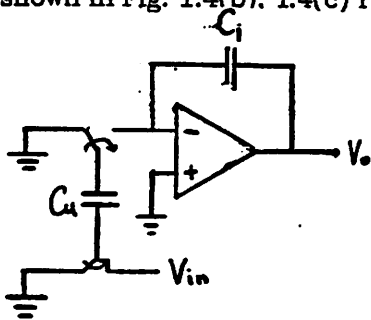


Fig 1.4(a)

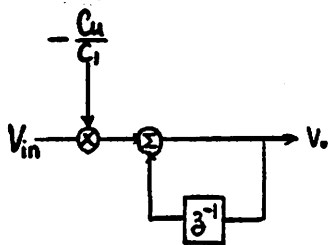


Fig 1.4(b)

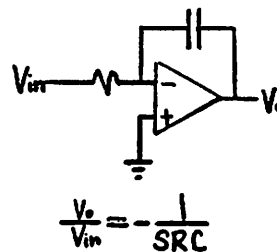


Fig 1.4(c)

$$\frac{V_o}{V_{in}} = -\frac{1}{sRC}$$

As shown in Fig. 1.5, simply by adding a resistor in parallel with the capacitor in the feedback loop of the continuous-time integrator, one can push the pole of the integrator from the origin to the left half plane.

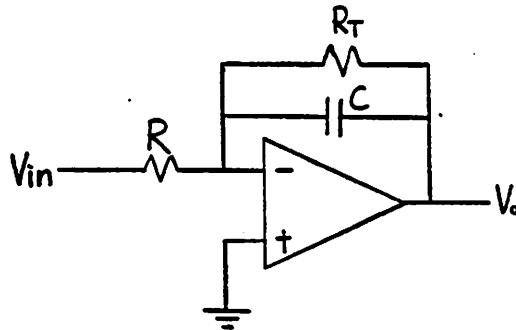


Fig 1.5

The transfer function of the circuit is

$$H(s) = -\frac{\frac{1}{RC}}{s + \frac{1}{R_T C}}$$

which has a pole at  $-\frac{1}{R_T C}$ .

Replace s by :

$$\left(\frac{1}{T}\right) \left(\frac{1-z^{-1/2}}{z^{-1/2}}\right)$$

Then we get:

$$H(z) = -\frac{\frac{1}{RC}}{\frac{1}{T_c} \frac{1-z^{-1}}{z^{-1/2}} + \frac{1}{R_T C}} = -\frac{\frac{T_c}{RC} z^{-1/2}}{1-z^{-1} + \frac{T_c}{R_T C} z^{-1/2}} \quad (*)$$

We should expect to get a SC circuit which can give us a similar transfer function. There are two simple kinds of SC circuits which have similar transfer

function:

(1) SC filter with one-delay termination capacitor:

Again by charge conservation:

$$C_u V_{in}(n - \frac{1}{2}) + C_1 V_o(n - \frac{1}{2}) = C_1 V_o(n)$$

$$(C_1 + C_T) V_o(n - \frac{1}{2}) = C_1 V_o(n-1)$$

$$\rightarrow C_u V_{in}(n - \frac{1}{2}) + \frac{C_1^2}{(C_1 + C_T)} V_o(n-1) = C_1 V_o(n)$$

$$\frac{V_o}{V_{in}} = \frac{\frac{C_u}{C_1} z^{-1/2}}{1 - \frac{C_1}{C_1 + C_T} z^{-1}} = \frac{\frac{C_u}{C_1} z^{-1/2}}{1 - z^{-1} + \frac{C_T}{C_1 + C_T} z^{-1}}$$

Compare this equation with (\*), we can see the relation

$$\frac{1}{T_c} \frac{C_u}{C_1} \rightarrow \frac{1}{RC}$$

$$\frac{1}{T_c} \frac{C_T}{C_1 + C_T} \rightarrow \frac{1}{R_T C}$$

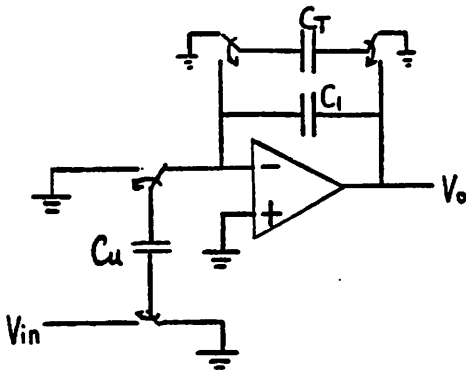


Fig 1.6 (a)

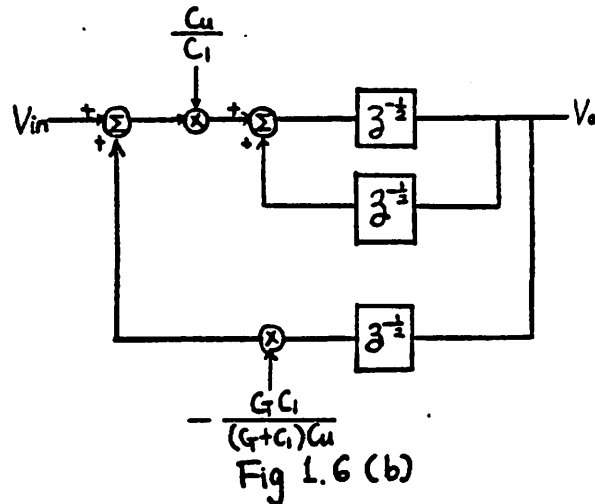


Fig 1.6 (b)

(2) SC filter with zero-delay termination capacitor:

$$C_u V_{in}(n - \frac{1}{2}) + C_1 V_o(n - \frac{1}{2}) = (C_1 + C_T) V_o(n)$$

$$= (C_1 + C_T) V_o(n + \frac{1}{2})$$

$$\rightarrow \frac{V_o}{V_{in}} = \frac{C_u z^{-1/2}}{C_1 + C_T - C_1 z^{-1}}$$

$$= \frac{\frac{C_u}{C_1} z^{-1/2}}{1 - z^{-1} + \frac{C_T}{C_1}}$$

Again

$$\frac{1}{T_c} \frac{C_u}{C_1} \rightarrow \frac{1}{RC}$$

$$\frac{1}{T_c} \frac{C_T}{C_1} \rightarrow \frac{1}{R_T C}$$

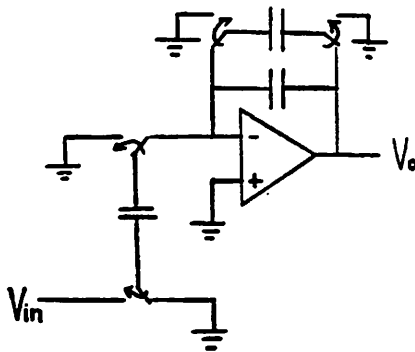


Fig 1.7 (a)

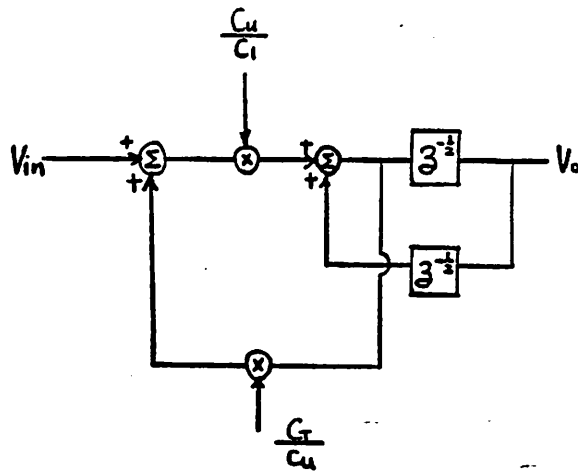


Fig 1.7 (b)

These two types of filter are quite the same except for the phase difference introduced by the last term of the denominator.

As we will see in the next Chapter, instead of using constant sampling rate, we can actually combine the phase detector ( adapting the WDM algorithm ) with loop filter by using a varying- sampling-rate switched capacitor filter.

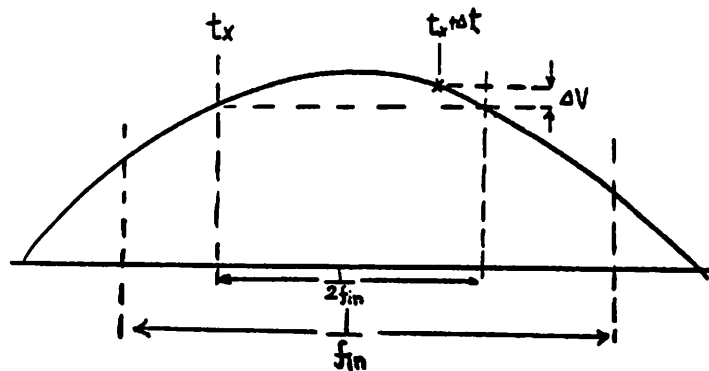
---

**CHAPTER 2**  
**SYSTEM AND CIRCUIT ANALYSIS**

In this Chapter, we are going to analyze the circuit with varying sampling rate. In section 2.3 and 2.4, the stability and transient behavior of the system will be discussed.

*SECTION 2.1 : ANALYSIS OF THE CIRCUIT*

By making the free running frequency of VCO be approximately twice of the input frequency and taking output frequency of VCO as sampling frequency, the WDM algorithm can easily be realized.



As shown in the figure, if we assume the waveform is symmetric, and if  $\Delta t$  is not equal to  $1/(2f_{in})$ , an error voltage  $V = (V(t_x + \Delta t) - V(t_x))$  will be fed to the

integrator. By taking the output of the integrator as input to the VCO, the output frequency of VCO can be adjusted according to the error voltage. Due to the feedback of the circuit,  $\Delta t$  will end up to be equal to  $1/(2f_{in})$  with  $t_x + \frac{1}{2}\Delta t$  at the peak of the wave.

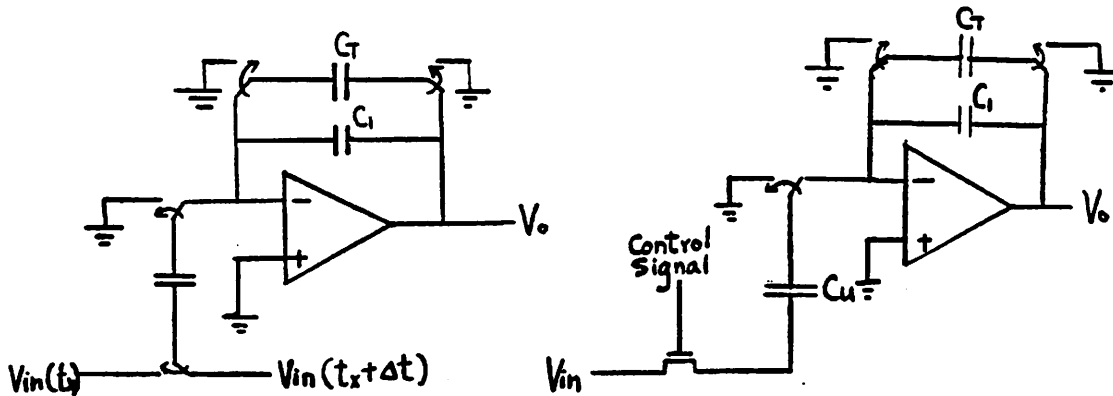
Now let's derive the equation for the varying-sample-rate circuit.

(1) One-delay termination capacitor:

$$\begin{aligned} (C_T + C_1) V_o(t_x + \Delta t + \Delta t') &= (C_T + C_1) V_o(t_x + \Delta t) \\ &= C_1 V_o(t_x) + [-C_u V_{in}(t_x + \Delta t) + V_{in}(t_x)] \end{aligned}$$

or

$$\begin{aligned} (C_T + C_1) V_o(t_x + \Delta t + \Delta t') &= (C_T + C_1) V_o(t_x + \Delta t) \\ &= C_1 V_{in}(t_x) - C_u (V_{in}(t_x + \Delta t) - V_{in}(t_x)) \end{aligned}$$



(2) Zero-delay termination capacitor:

$$\begin{aligned} C_1 V_o(t_x + \Delta t) &= C_1 V_{in}(t_x) - C_u (V_{in}(t_x + \Delta t) + V_{in}(t_x)) \\ (C_T + C_1) V_{in}(t_x) &= V_o(t_x - \Delta t) \end{aligned}$$

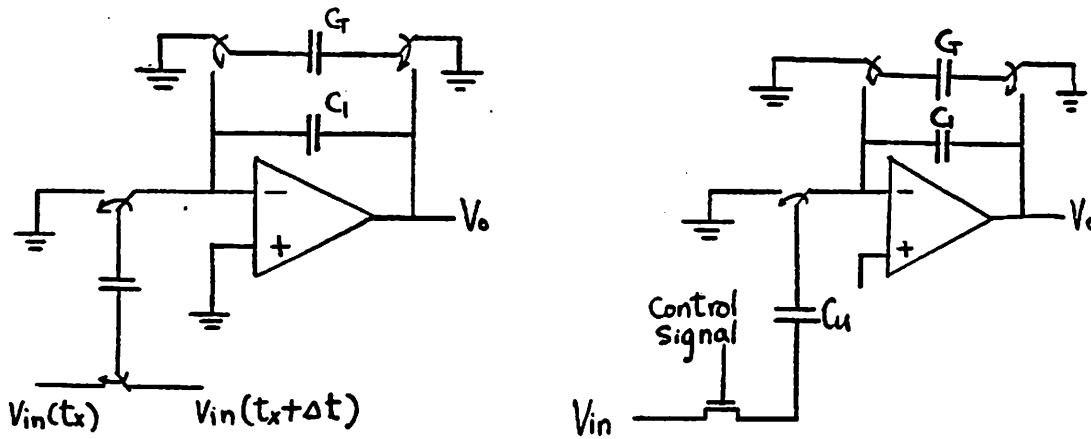


$$\rightarrow V_{in}(t_x) = \frac{C_1}{C_T + C_1} V_o(t_x - \Delta t)$$

or

$$C_1 V_o(t_x + \Delta t) = (C_1 + C_T) V_o(t_x + \Delta t + \Delta t') = V_o(t_x')$$

where  $t_x'$  is the new time axis for the next time period.



By using the equations derived above, several simulations have been carried out and the results are gathered in the appendix.

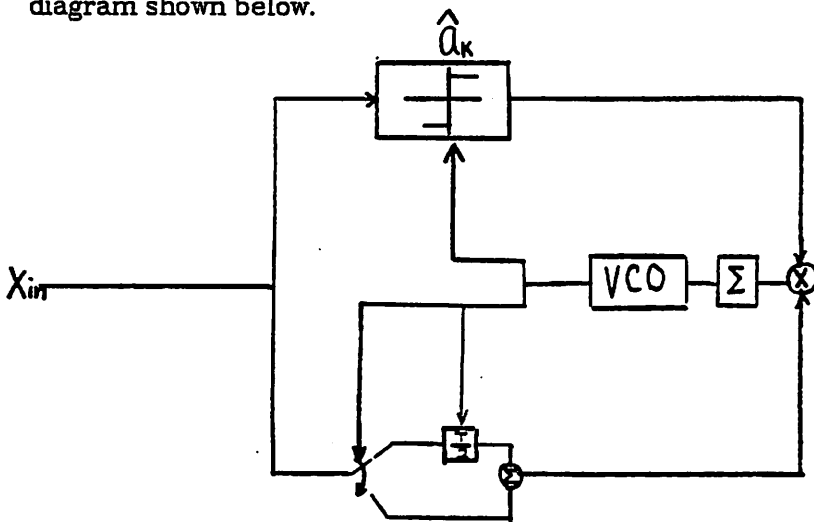
### SECTION 2.2 : SOME OTHER POINT OF VIEW

There are two more things which have to be taken care of before the circuit can really work. As one can see, if we close the feedback loop even when the input signal is zero, the output voltage of the integrator will be reduced because of the flipping of the termination capacitor. One way to avoid that is to use the estimated bit as the controlling bit so that the switches will not close when the

estimated bit is zero.

The other thing is, in order to get the right polarity of the input to the integrator no matter what polarity the input signal is, we have to use either an absolute-value generator or add some extra controlling circuitry.

One totally different point of view is to think of the above idea as the block diagram shown below.



This is one other kind of formal description of the WDM. In order to recover the timing signal, we have to be able to extract the desired frequency from the spectrum of the transmitted signal. One way to do that is to introduce some nonlinearity so that the spectrum of the transmitted signal will have a spectral line ( or at least a large amplitude ) at the desired frequency. The rectifier in the above figure will provide the nonlinearity needed. If we check the signal at the output of the multiplier, we can see that it is simply the absolute value of the input signal which is just what we stated above.

The system is shown in Fig. 2.3 where we connect the output of the filter to an VCO whose output is used as the control signal of switch.

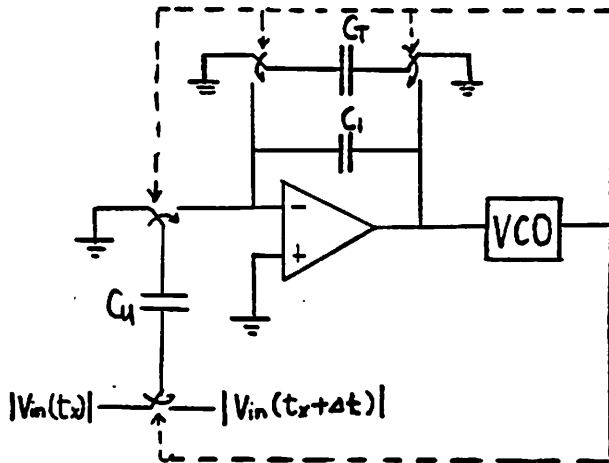


Fig 2.3

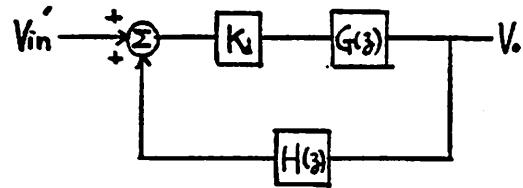


Fig 2.4

One other thing worth of mentioning is that the filter used is in the form of  $\frac{b}{s+a}$  ( or  $\frac{cz^{-1/2}}{1-z^{-1}+dz^{-1/2}}$  ). It is shown in [2] that the active filter with transfer function  $(\alpha + \frac{1}{bs})$ , which is also called proportional plus integral (PPI) filter, will provide better tracking performance. Although it is true, it seems that the sensitivity of ppi filter to the non-idealities of the circuit elements is larger than what we have used here. Also high gain OP Amp is needed for PPI filter. The sensitivity of the filter to the non-idealities of the circuit elements will be analyzed in the next Chapter.

### SECTION 2.3 : ANALYSIS OF THE STABILITY OF THE SYSTEM

The system shown in Fig 2.3 can be approximated by the block diagram shown in Fig 2.4,

where

$$G(z) = \frac{-\frac{C_u}{C_1} z^{-1}}{1 - z^{-1} + \frac{C_T}{C_1}}$$

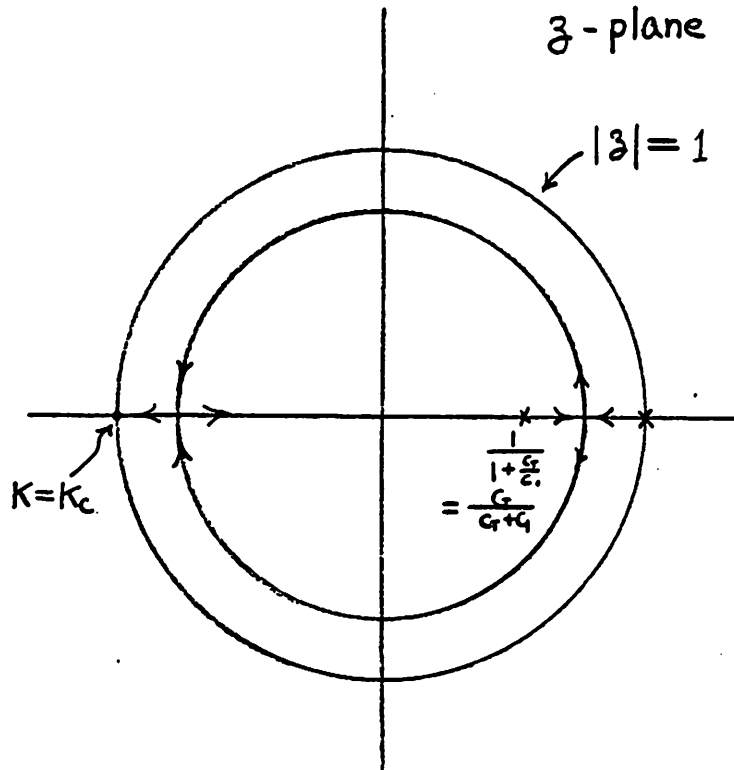
$$H(z) = -\frac{K_0}{f_s} \frac{1}{1 - z^{-1}}$$

Here we consider the SC filter with varying sampling rate as a combination of S.C. filter with constant sampling frequency and a phase detector with gain constant of  $K_d$ . It is quite reasonable since the change of sampling instant is just the change of phase difference. The only thing that should be discussed is the gain constant  $K_d$  which will be considered later.

$$\begin{aligned} \rightarrow K_d G(z)H(z) &= \frac{K_d \frac{C_u}{C_1} z^{-1}}{1 - z^{-1} + \frac{C_T}{C_1}} \frac{K_0}{f_s} \frac{1}{1 - z^{-1}} \\ &= \frac{C_u K_0 K_d}{f_s C_1 (1 + \frac{C_T}{C_1})} \frac{z}{\left[ z - \frac{1}{1 + \frac{C_T}{C_1}} \right] (z - 1)} \\ &= K \frac{z}{\left[ z - \frac{1}{1 + \frac{C_T}{C_1}} \right] (z - 1)} \\ K &= \frac{C_u K_0 K_d}{f_s C_1 (1 + \frac{C_T}{C_1})} \end{aligned}$$

The system is stable for  $K < K_c$  where

$$\begin{aligned}
 K_c &= \left| \frac{\frac{1}{z}}{\frac{(z - \frac{1}{C_T})(z-1)}{1 + \frac{C_T}{C_1}}} \right|_{z=-1} \\
 &= \left| \frac{\frac{1}{-1}}{\frac{(1 + \frac{1}{C_T}) \times 2}{1 + \frac{C_T}{C_1}}} \right| \\
 &= \frac{2(2C_1 + C_T)}{(C_1 + C_T)}
 \end{aligned}$$



Root locus of the (discrete) system

Fig 2.5

SECTION 2.4 : TRANSIENT ANALYSIS OF THE SYSTEM

Again, we use the block diagram to represent the system.

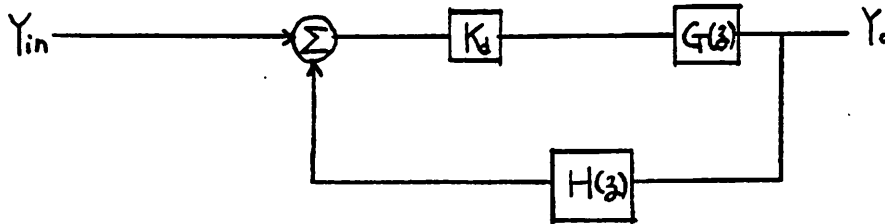


Fig 2.6

$$G(z) = \frac{-\frac{C_u}{C_1} z^{-1}}{1 - z^{-1} + \frac{C_T}{C_1}}$$

$$H(z) = -\frac{K_o}{1 - z^{-1}} \frac{1}{f_s}$$

$$\frac{Y_o}{Y_{in}} = \frac{K_d G(z) H(z)}{1 + K_d G(z) H(z)}$$

$$= \frac{K_d \frac{-\frac{C_u}{C_1} z^{-1}}{1 - z^{-1} + \frac{C_T}{C_1}} \frac{K_o}{(1 - z^{-1}) f_s}}{1 + K_d \frac{\frac{C_u}{C_1} z^{-1}}{1 - z^{-1} + \frac{C_T}{C_1}} \frac{K_o}{(1 - z^{-1}) f_s}}$$

$$= \frac{K_d \frac{K_o}{f_s} \frac{C_u}{C_1} z^{-1}}{(1 - z^{-1} + \frac{C_T}{C_1})(1 - z^{-1}) + K_d \frac{K_o}{f_s} \frac{C_u}{C_1} z^{-1}}$$

$$= \frac{K_d \frac{K_o}{f_s} \frac{C_u}{C_1} z}{\left(1 + \frac{C_T}{C_1}\right) z^2 + \left(-2 + \frac{C_T}{C_1} + K_d \frac{K_o}{f_s} \frac{C_u}{C_1}\right) z + 1}$$

$$= \frac{K_d \frac{K_o}{f_s} \frac{C_u}{C_1 + C_T} z}{z^2 - \left(2 - \frac{C_T}{C_1} - K_d K_o \frac{C_u}{C_1}\right) \frac{C_1}{C_1 + C_T} z + \frac{C_1}{C_1 + C_T}}$$

We can write the denominator of the above equation in the form :

$$z^2 - 2|R| \cos \theta z + |R|^2$$

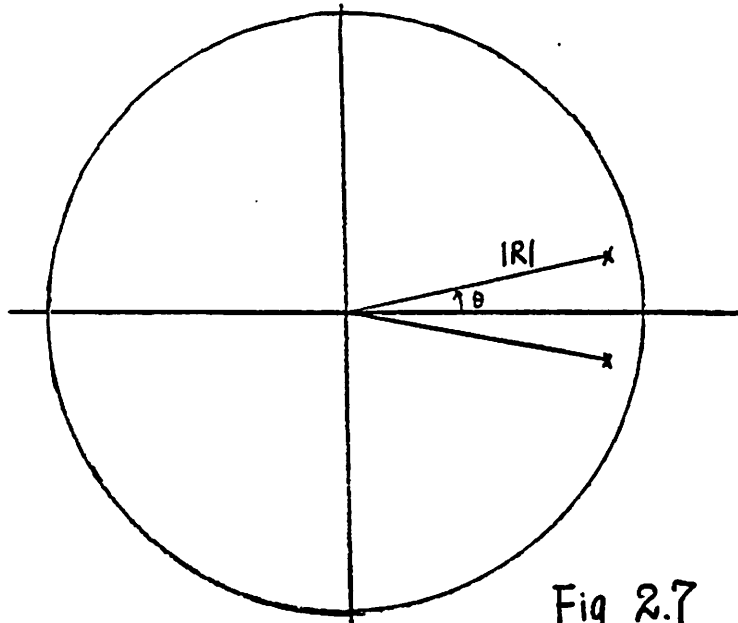


Fig 2.7

where

$$|R|^2 = \frac{C_1}{C_1 + C_T} \approx \left(1 - \frac{C_T}{C_1}\right)$$

$$\rightarrow |R| \approx 1 - \frac{1}{2} \frac{C_T}{C_1}$$

$$\rightarrow 1 - |R| = \frac{1}{2} \frac{C_T}{C_1}$$

$$\rightarrow \frac{\Delta w}{f_s} = \frac{C_T}{C_1}$$

$$2|R| \cos \vartheta = \left( 2 - \frac{C_T}{C_1} - K_d \frac{K_o}{f_s} \frac{C_u}{C_1} \right) \approx 2 \cos \vartheta$$

if  $\vartheta$  is very small, i.e.  $w_o \ll f_s$  . (which is just our case )

$$\rightarrow 2 - 2 \frac{\vartheta^2}{2} = 2 - K_d \frac{K_o}{f_s} \frac{C_u}{C_1}$$

$$\rightarrow \vartheta = \left( K_d \frac{K_o}{f_s} \frac{C_u}{C_1} \right)^{\frac{1}{2}} = \frac{w_o}{f_s}$$

Recall that the Laplace transform of a canonical second order differential equation is in the form of:

$$S^2 + \frac{w_o}{Q} S + w_o^2 = S^2 + \Delta w S + w_o^2$$

If we know  $w_o$  and  $\Delta w$  , we can explore the transient behavior of the system by the analogy between the discrete and analog system in terms of damping ratio  $\xi$

$$\xi = \frac{\frac{1}{2} \Delta w}{w_o} = \frac{1}{2} \frac{C_T}{C_1} \frac{1}{\left( K_d \frac{K_o}{f_s} \frac{C_u}{C_1} \right)^{\frac{1}{2}}}$$

For a step input, which is just our case (  $f_{in} = \text{constant}$  ), the response is shown in Fig. 2.8 where

$$\begin{aligned} T_d &= \frac{2\pi}{w_o \left[ 1 - \xi^2 \right]^{\frac{1}{2}}} \\ &= \frac{2\pi}{w_o \left[ 1 - \frac{1}{4} \left( \frac{C_T}{C_1} \right)^2 \frac{f_s C_1}{K_o K_d C_u} \right]^{\frac{1}{2}}} \end{aligned}$$



$$= \frac{2\pi}{f_s \left[ K_d \frac{K_o}{f_s} \frac{C_u}{C_1} \right]^{\frac{1}{2}} \left[ 1 - \frac{1}{4} \left( \frac{C_T}{C_1} \right)^2 \frac{f_s C_1}{K_d K_o C_u} \right]^{\frac{1}{2}}}$$

$$t_p = \frac{T_d}{2}$$

and

$$f(t)_{\max} = f_o \left[ 1 + \exp \left( - \frac{\xi \pi}{(1-\xi^2)^{\frac{1}{2}}} \right) \right]$$

where  $f_o$  is the steady state value.

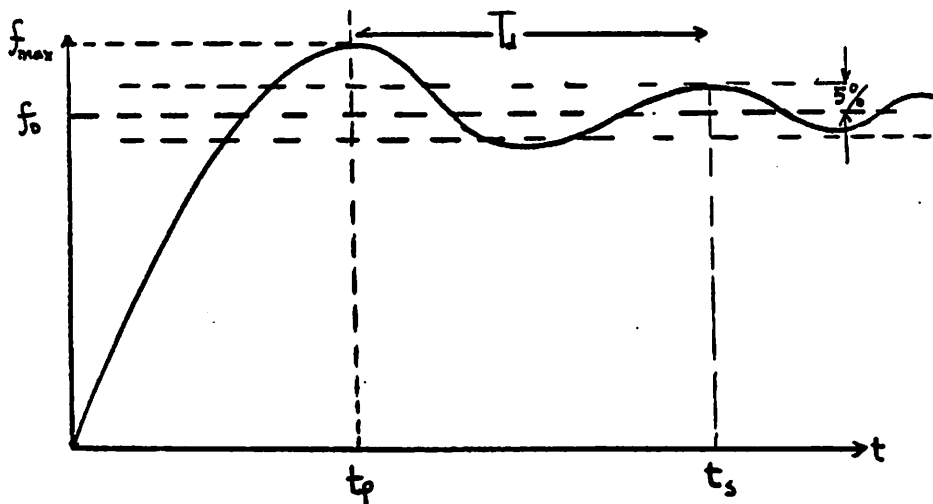
If we define the settling time  $t_s = n t_p$  to be the time when

$$\exp \left( - \frac{\xi n \pi}{(1-\xi^2)^{\frac{1}{2}}} \right) < 0.05$$

then

$$\xi \omega_o n t_p > 3$$

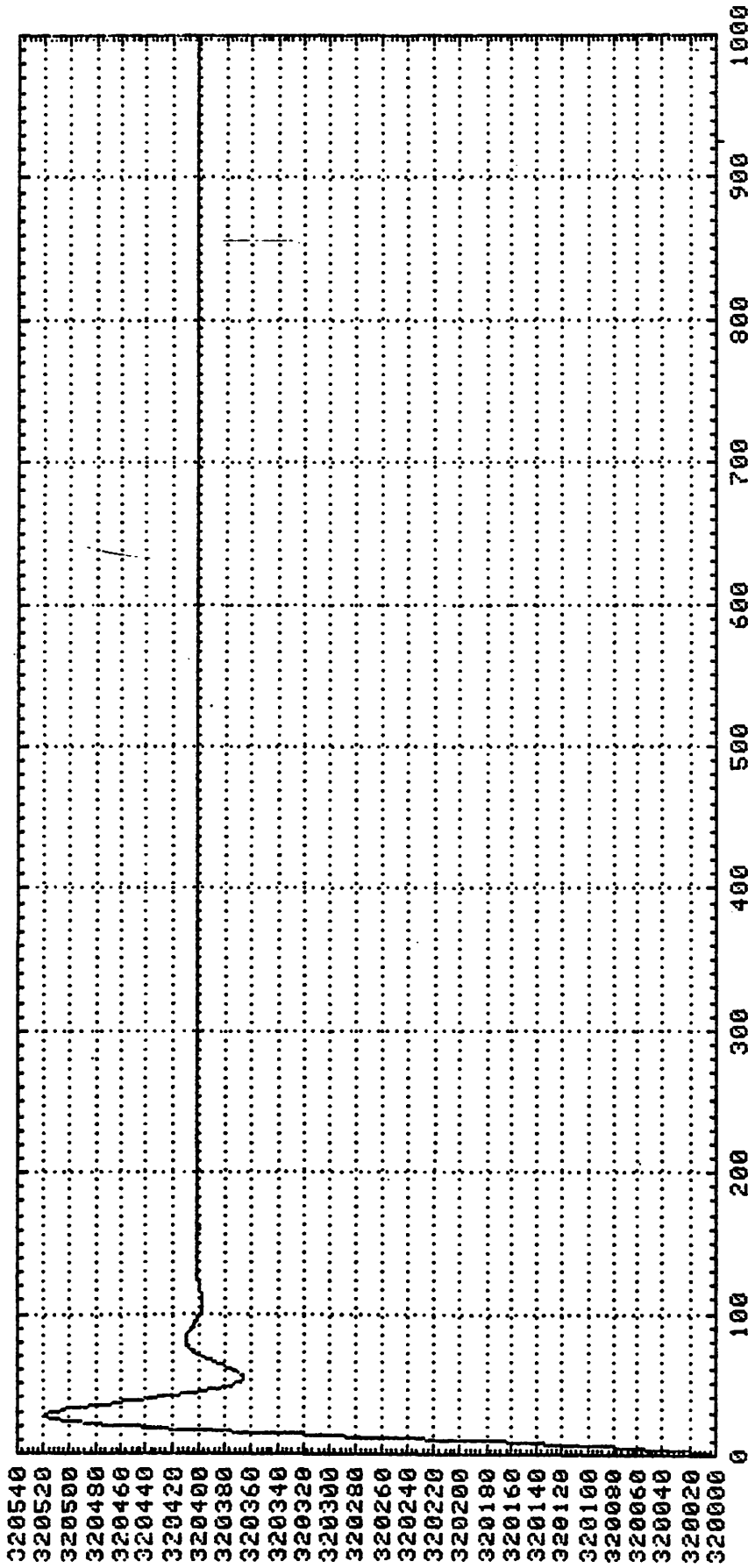
$$\rightarrow t_s = n t_p > \frac{3}{\xi \omega_o}$$



System Response for Step Input

Fig 2.8

The results of computer simulations match the analysis almost exactly. As shown in Fig. 2.9a,b, and c ( where ideal sinusoidal waveform is used as input signal),  $T_d f_s = 54, 17.3$  and  $11$  respectively. The analysis gives  $53.9992, 17.3493$  and  $10.9105$ . Based on the result obtained above, we can conclude that the model used for the system and the analysis which has been developed based on that model are correct.

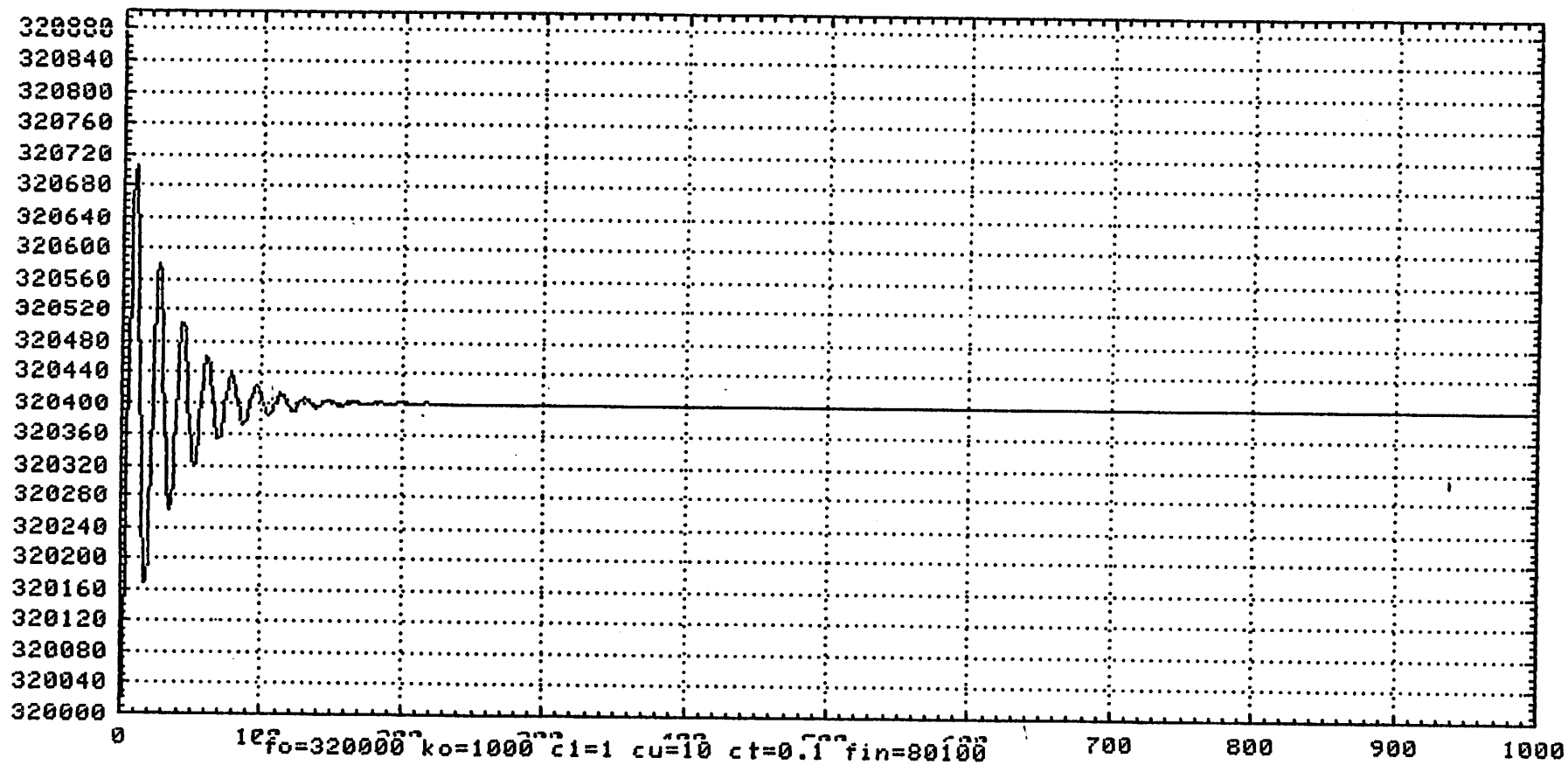


fo=320000 ko=120 c1=1 cu=10 ct=0.1 fin=80100 zero-delay

$T_d = 54$

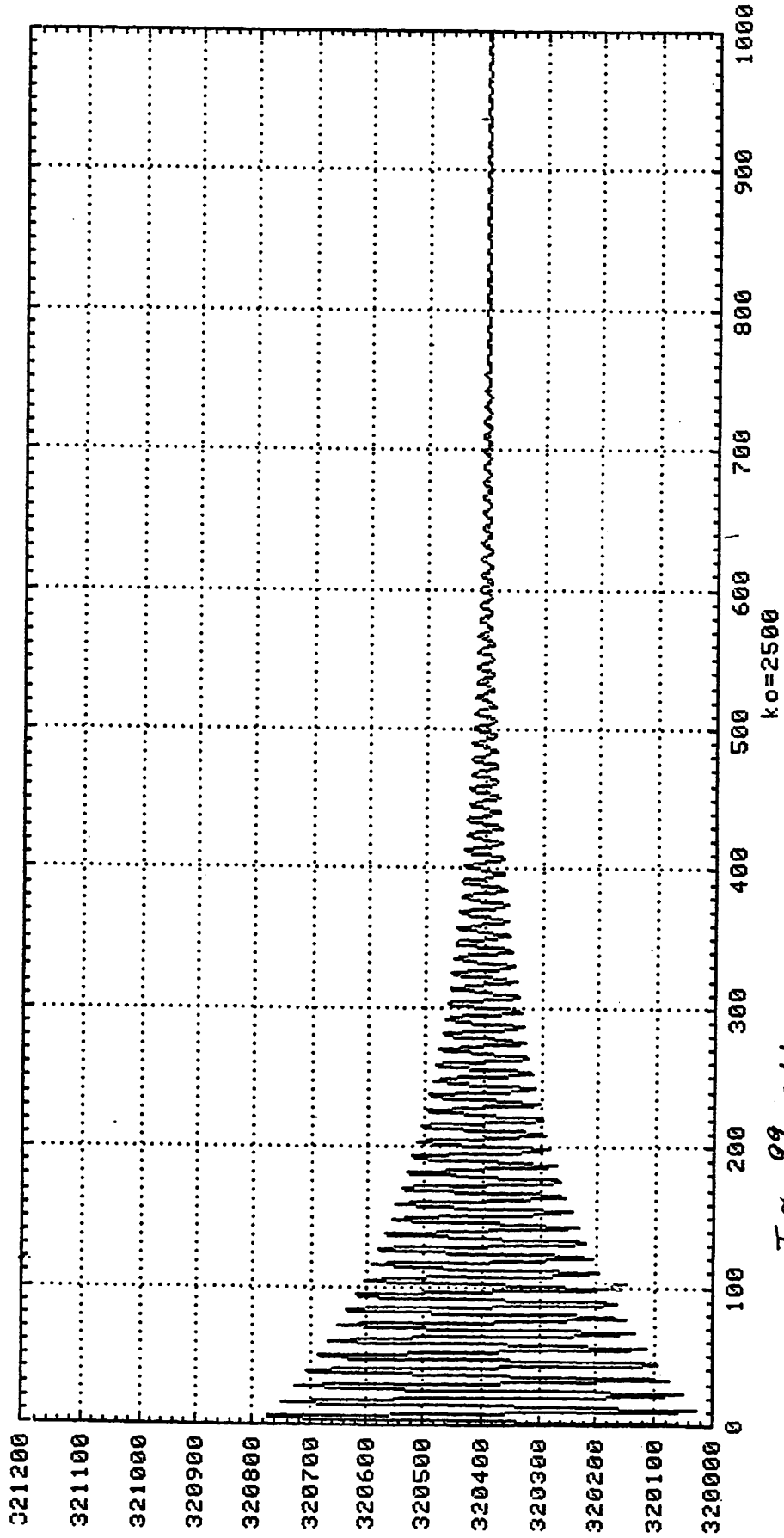
$K_d = 1.0706$

Fig. 2.9a



$$T_d \approx \frac{104}{6} = 17.3$$

Fig 2.9b



$$T_1 \approx \frac{99}{9} = 11$$

$k_0 = 2500$

Fig 2.9c

CHAPTER 3

THE EFFECTS OF SOME OP AMP NON-IDEALITIES

The results and equations derived so far are based on the assumption that the operational amplifier (OP Amp) used is an ideal OP Amp. This chapter, we are going to discuss the effect of the real OP Amp. i.e. OP Amp with finite gain, finite bandwidth, and non-zero offset voltage.

SECTION 3.1 : EFFECT OF FINITE GAIN OF OP AMP

Now we are going to discuss the effect of finite gain of OP Amp.

For ideal OP Amp:

$$\begin{aligned} (C_T + C_1)V_o(t_x + \Delta t + \Delta t') &= (C_T + C_1)V_o(t_x + \Delta t) \\ &= C_1V_o(t_x) - C_u(V_{in}(t_x + \Delta t) - V_{in}(t_x)) \\ &= C_1V_o(t_x) - C_u V_{in}'(t_x) \\ ((C_T + C_1) - C_1z^{-1})V_o &= -C_u V_{in}' \end{aligned}$$

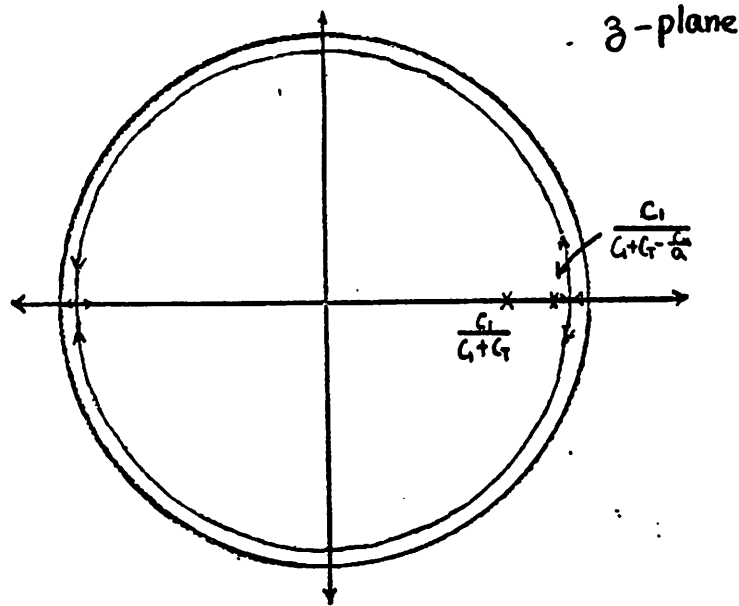
$$\begin{aligned} \rightarrow G(z) = \frac{V_o}{V_{in}'} &= \frac{-\frac{C_u}{C_1}}{1 - z^{-1} + \frac{C_T}{C_1}} \\ &= \frac{-\frac{C_u}{C_1}}{\left(\frac{C_1 + C_T}{C_1}\right) - z^{-1}} \end{aligned}$$

OP Amp with finite gain A:

$$\begin{aligned}
 (C_T + C_1)V_o(t_z + \Delta t + \Delta t') &= (C_T + C_1)V_o(t_z + \Delta t) \\
 &= C_1V_o(t_z) - C_u[V_{in}(t_z + \Delta t) - V_{in}(t_z) - \frac{1}{A}V_o(t_z + \Delta t)] \\
 \rightarrow [(C_T + C_1) - \frac{C_u}{A}]V_o(t_z + \Delta t) &= C_1V_o(t_z) - C_u[V_{in}(t_z + \Delta t) - V_{in}(t_z)] \\
 &= C_1V_o(t_z) - V_{in}'(t_z)
 \end{aligned}$$

$$\rightarrow H_a(z) = \frac{V_o}{V_{in}'} = \frac{-\frac{C_u}{C_1}}{\frac{1}{C_1}(C_1 + C_T - \frac{C_u}{A})z^{-1}}$$

Apparently, the effect of the finite gain of OP Amp is to push the pole toward the unit circle, or from  $\frac{C_1}{C_1 + C_T}$  to  $\frac{C_1}{C_1 + C_T - \frac{C_u}{A}}$ .

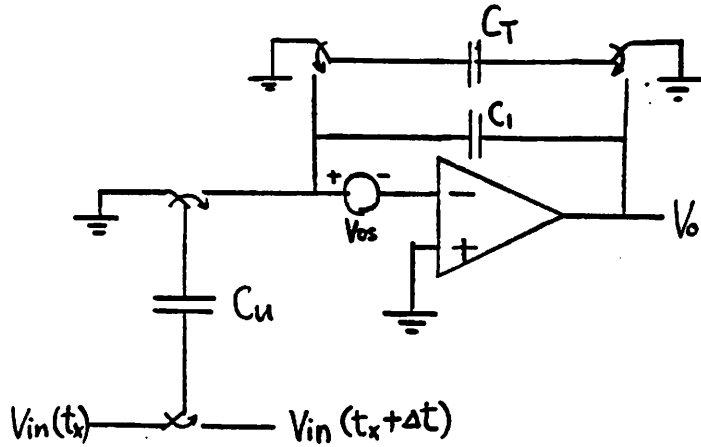


**SECTION 3.2: EFFECT OF THE OFFSET VOLTAGE OF OP AMP**

As shown in the figure

$$(C_T + C_1)V_o(t_z + \Delta t + \Delta t') = (C_T + C_1)V_o(t_z + \Delta t)$$

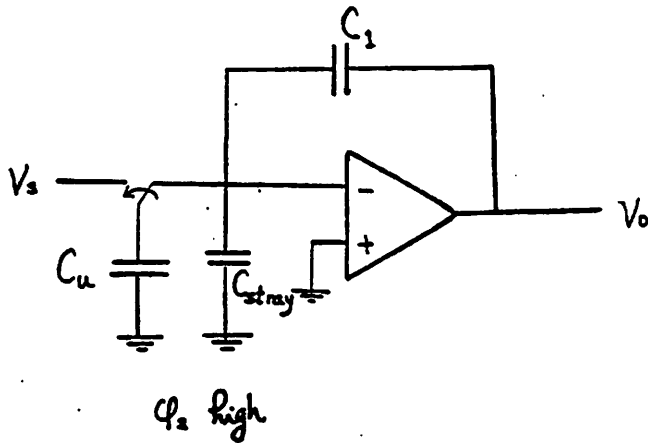
$$= C_1 V_o(t_x) - C_u (V_{in}(t_x + \Delta t) - V_{in}(t_x) - V_{os})$$



Some simulations based on the equation derived above are done. Two of them (with  $V_{os} = 1\text{mV}$  and  $10\text{mV}$  respectively) are shown in the appendix. The results show that the effect of offset voltage of OP Amp is quite negligible.

### SECTION 3.3 : EFFECT OF THE FINITE BANDWIDTH OF OP AMP

Before analyzing the circuit, we first look at one simpler case.



For the circuit shown,

$$V_o(n+1) = V_c(n+1) + V_o(n+1)$$



$$V_c(n+1) = V_c(n) - \frac{C_u}{C_1} [V_s(n + \frac{1}{2}) - V_o(n+1)] + \frac{C_{stray}}{C_1} [V_o(n+1) - V_o(n)]$$

where (n+1) is the very end of the charge integration period.

During  $\phi_2$  high

$$V_c(t) = V_c(n) - \frac{C_u}{C_1} [V_s(n + \frac{1}{2}) - V_o(t)] + \frac{C_{stray}}{C_1} [V_o(t) - V_o(n - \frac{1}{2})]$$

$$V_o(t) = V_c(n) + V_o(t) = V_c(n) - \frac{C_u}{C_1} [V_s(n + \frac{1}{2}) - V_o(t)] + V_o(t)$$

Differentiate the above equation :

$$\rightarrow \frac{dV_o(t)}{dt} = \frac{C_{stray} + C_u + C_1}{C_1} \frac{dV_o(t)}{dt} \quad (a)$$

Let the unity gain frequency of the OP amp be  $w_t$ , then

$$\frac{dV_o(t)}{dt} = -w_t V_o(t) \quad (b)$$

or

$$\frac{V_o(w)}{V_s(w)} = -\frac{w_t}{jw}$$

Compare (a), (b)

$$\rightarrow \frac{dV_o(t)}{dt} = -w_t \frac{C_1}{C_u + C_1 + C_{stray}} V_o(t) = -w_u V_o(t)$$

or

$$V_o(t) = V_o(n + \frac{1}{2})^+ \exp(-w_u t)$$

and

$$V_o(n + \frac{1}{2})^+ = \frac{C_u}{C_u + C_{stray} + C_1} V_s(n + \frac{1}{2})^+ + V_o(n + \frac{1}{2})^-$$

where  $w_u$  is the closed loop bandwidth when  $\phi_2$  is high and

$$w_u = \frac{C_1}{C_u + C_{stray} + C_1} w_t$$

During  $\varphi_2$  low

$$V_o(t)C_1 = (C_{stray} + C_1)[V_s(t) - V_s(n - \frac{1}{2} + mT)]$$

$$\rightarrow \frac{dV_o(t)}{dt} = \frac{C_{stray} + C_1}{C_1} \frac{dV_s(t)}{dt} = -\omega_t V_o(t)$$

$$\rightarrow \frac{dV_o(t)}{dt} = -\omega_{u_o} V_o(t)$$

where  $\omega_{u_o}$  is the open loop bandwidth and

$$\omega_{u_o} = \frac{C_1}{C_{stray} + C_1} \omega_t$$

$$\rightarrow V_o(t) = V_s(n - \frac{1}{2} + mT) \exp(-\omega_{u_o} t)$$

$$\rightarrow V_o(n + \frac{1}{2})^- = V_s(n) \exp[-\omega_{u_o} (1-m)T]$$

$$\rightarrow V_o(n+1) = V_s(n + \frac{1}{2})^+ \exp(-\omega_{u_o} mT)$$

$$= \frac{C_u}{C_u + C_{stray} + C_1} V_s(n + \frac{1}{2}) \exp(-\omega_{u_o} mT)$$

$$+ V_s(n) \exp[-\omega_{u_o} (1-m)T] \exp(-\omega_{u_o} mT)$$

So we can get

$$V_o(n+1) = V_o(n+1) + V_o(n+1)$$

$$= V_o(n) - \frac{C}{C} [V_s(n + \frac{1}{2}) - V_s(n+1)] + \frac{C}{C} (V_s(n + \frac{1}{2}) - V_s(n)) + V_o(n+1)$$

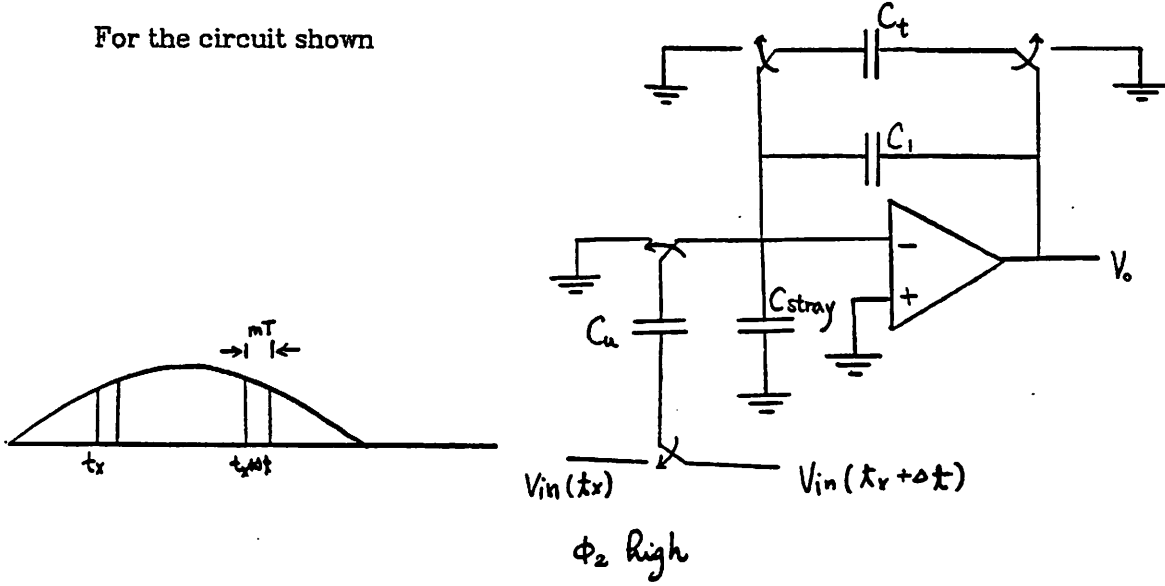
$$= V_o(n) - \frac{C_u}{C_1} V_s(n + \frac{1}{2}) + \frac{C_u + C_{stray} + C_1}{C_1} V_o(n+1) - \frac{C_{stray} + C_1}{C_1} V_o(n)$$

$$= V_o(n) - \frac{C_u}{C_1} V_s(n + \frac{1}{2}) + \frac{C_u}{C_1} V_s(n + \frac{1}{2}) \exp[-\omega_{u_o} mT]$$

$$+ \frac{C_u + C_{stray} + C_1}{C_1} V_o(n) \exp[-\omega_{u_o} (1-m)T] \exp[-\omega_{u_o} mT]$$

$$- \frac{C_{stray} + C_1}{C_1} V_o(n)$$

For the circuit shown



$$V_o(t_x + \Delta t) = V_c(t_x + \Delta t) + V_o(t_x + \Delta t) \quad (a)$$

$$(C_f + C_1)V_c(t_x + \Delta t + t) = C_1 V_c(t_x) - C_u [V_{in}(t_x + \Delta t) - V_{in}(t_x) - V_o(t_x + \Delta t + t)] + C_{stray} [V_o(t_x + \Delta t + t) - V_o(t_x + \Delta t)] \quad (0 \leq t \leq T) \quad (b)$$

$$V_o(t_x + \Delta t + t) = \frac{C_u}{C_u + C_f + C_1 + C_{stray}} [V_{in}(t_x + \Delta t) - V_{in}(t_x)] \exp(-w_u t) + V_o(t_x - \Delta t + mT) \exp(-w_u t) \exp[-(1-m)T w_u] \quad (c)$$

where

$$w_u = \frac{C_1 + C_f}{C_u + C_{stray} + C_1 + C_f} w_t$$

is the closed loop bandwidth when  $\phi_2$  is high.

The expression

$$w_{u_0} = \frac{C_1}{C_{stray} + C_1} w_t$$

is the closed loop bandwidth when  $\varphi_1$  is low and  $w_t$  is the unit gain frequency ( in radius ) of OP Amp.

Now we separate the effect of varying sampling rate ( which can be considered as the function of phase detector) and concentrate ourselves on the filter, then we can neglect  $C_{stray}$  and rewrite equation (a), (b) and (c) as :

$$(a) \rightarrow V_o(n+1) = V_c(n+1) + V_e(n+1) \quad (a1)$$

$$(b) \rightarrow V_c(n+1) = \frac{C_1 V_c(n) - C_u [V_{in}'(n + \frac{1}{2}) - V_e(n+1)]}{C_T + C_1} \quad (b1)$$

$$(c) \rightarrow V_e(t_x + \Delta t + t) = \frac{C_u}{C_u + C_T + C_1} \exp(-w_u t) V_{in}'(t_x + \Delta t) \\ + V_e(t_x - \Delta t + mT) \exp(-w_u t) \exp[-(1-m)T w_{u_0}]$$

$$\rightarrow V_e(n+1) = \frac{C_u}{C_u + C_T + C_1} \exp(-w_u t) V_{in}'(n + \frac{1}{2}) \\ + V_e(n) \exp(-w_u t) \exp[-(1-m)T w_{u_0}] \\ = \frac{C_u}{C_u + C_T + C_1} \exp[-mT w_u] \times \\ \sum_{p=0}^{p=\infty} V_{in}'(n-p + \frac{1}{2}) \exp[-pmT(w_u + (1-m)w_{u_0})] \quad (c1)$$

Take the Z transform

$$(a1) \rightarrow V_o(z) = V_c(z) + V_e(z)$$

$$(b1) \rightarrow V_c(z) = \frac{-\frac{C_u}{C_T + C_1} z^{-1/2}}{1 - \frac{C_1}{C_T + C_1} z^{-1}} V_{in}(z) + \frac{\frac{C_u}{C_T + C_1}}{1 - \frac{C_1}{C_T + C_1} z^{-1}} V_e(z)$$

$$(c1) \rightarrow V_o(z) = \frac{C_u}{C_u + C_T + C_1} \exp(-mT\omega_u) \times \frac{z^{-1/2}}{1 - z^{-1} \exp(-T[m\omega_u + (1-m)\omega_{u_0}])} V_{in}(z)$$

$$V_o(z) = V_c(z) + V_g(z)$$

$$= \frac{-\frac{C_u}{C_T + C_1} z^{-1/2}}{1 - \frac{C_1}{C_T + C_1} z^{-1}} V_{in}(z)$$

$$+ \left[ \frac{\frac{C_u}{C_T + C_1}}{1 - \frac{C_1}{C_T + C_1} z^{-1}} + 1 \right] \frac{C_u}{C_u + C_T + C_1} \exp(-mT\omega_u) \times$$

$$\frac{z^{-1/2}}{1 - z^{-1} \exp[-mT(\omega_u + (1-m)\omega_{u_0})]} V_{in}(z)$$

$$\rightarrow \frac{V_o(z)}{V_{in}(z)} = \frac{-\frac{C_u}{C_T + C_1} z^{-1/2}}{1 - \frac{C_1}{C_T + C_1} z^{-1}} \times$$

$$\left[ \frac{\frac{1}{C_u} (C_T + C_1 - C_1 z^{-1} + C_u) \frac{C_u}{C_u + C_T + C_1} \exp(-mT\omega_u)}{1 - \frac{1}{1 - z^{-1} \exp(-T[m\omega_u + (1-m)\omega_{u_0}])}} \right]$$

We see that the transfer function is modified by the expression inside the braces. If we define it to be  $M(z)$ , then

$$M(z) = 1 - \frac{\frac{1}{C_u} (C_T + C_1 - C_1 z^{-1} + C_u) \frac{C_u}{C_u + C_T + C_1} \exp(-mT\omega_u)}{1 - z^{-1} \exp(-T[m\omega_u + (1-m)\omega_{u_0}])}$$

Now, we want to see the frequency response of  $M$ , i.e. plug

$$z = \exp(j\omega T) \approx 1 + j\omega T \quad (\omega \ll \frac{1}{T})$$

into the above equation, and let

$$\exp(-mT\omega_u) = x$$

$$\exp(-T(m\omega_u + (1-m)\omega_o)) = y$$

then

$$\begin{aligned} M(j\omega) &= 1 - \frac{\frac{1}{C_u}(C_T + C_1 + C_u - C_u(1+j\omega T)) \frac{C_u}{C_u + C_1 + C_T} x}{1 - (1+j\omega T)y} \\ &= 1 - \frac{\left[ \left( 1 - \frac{C_u}{C_u + C_1 + C_T} \right) - j\omega T \frac{C_u}{C_u + C_1 + C_T} \right] x}{1 - y - j\omega T y} \\ &= \frac{1 - y - j\omega T y - x + \frac{C_u}{C_u + C_1 + C_T} x + j\omega T \frac{C_u}{C_u + C_1 + C_T} x}{(1 - y) - j\omega T y} \\ &= \frac{(1 - y - x + \frac{C_u}{C_u + C_1 + C_T}) + j(-\omega T y + \omega T \frac{C_u}{C_u + C_1 + C_T} x)}{(1 - y) - j\omega T y} \\ &= A \exp(j\varphi) \end{aligned}$$

Note that

$$1 \gg x \gg y$$

$$\begin{aligned} \rightarrow A &\approx \frac{(1 - x + \frac{C_u}{C_u + C_1 + C_T}) + \frac{1}{2} \omega^2 T^2 \frac{C_u}{C_u + C_1 + C_T} x^2}{1 - y + \frac{1}{2} \omega^2 T^2 y^2} \\ &\approx \left( 1 - x + \frac{C_u}{C_u + C_1 + C_T} x + \frac{1}{2} \omega^2 T^2 \frac{C_u}{C_u + C_1 + C_T} x^2 \right) \left( 1 + y - \frac{1}{2} \omega^2 T^2 y^2 \right) \\ &\approx 1 - \frac{C_u + C_T}{C_u + C_1 + C_T} x + \frac{1}{2} \omega^2 T^2 \frac{C_u}{C_u + C_1 + C_T} x^2 \end{aligned}$$

$$= 1 - \frac{C_u + C_T}{C_u + C_1 + C_T} \exp(-mT\omega_u)$$


---


$$+ \frac{1}{2} \omega^2 T^2 \frac{C_u}{C_u + C_1 + C_T} \exp(-2mT\omega_u)$$

In order to find  $\varphi$ , we make another approximation:

$$M(j\omega T) \approx \frac{1 - j\omega T y + j\omega T \frac{C_u}{C_u + C_1 + C_T} x}{1 - j\omega T y}$$

$$\approx \frac{1 + \omega^2 T^2 y^2 + j\omega T \frac{C_u}{C_u + C_1 + C_T} x - \omega^2 T^2 \frac{C_u}{C_u + C_1 + C_T} x^2 y}{1 + \omega^2 T^2 y^2}$$

so, the phase error  $\varphi$  :

$$\varphi \approx \omega T \frac{C_u}{C_u + C_1 + C_T} x$$

$$= \omega T \frac{C_u}{C_u + C_1 + C_T} \exp(-mT\omega_u)$$

As we can see from the equations derived above, the magnitude error and phase error are quite small in our case even for small  $m$  and moderate  $f_t$ . So the specification required for OP Amp can be relaxed. Some computer simulations have been done and the results are quite satisfactory. One of them ( with  $m=0.05$  and  $f_t=100K$  ) is shown in the appendix. Also shown in Fig. A7.1 is the comparison of the system with ideal OP Amp and the system with OP Amp of finite bandwidth. From the figure, we can see that the discrepancy is very insignificant and one can hardly distinguish between these two curves. This is just what the analysis expects.

*SECTION 3.4 CONCLUSION*

From the previous sections we know that the circuit is quite insensitive to the typical OP Amp non-idealities. This means that we don't have to use a high performance OP Amp in order to get a satisfactory result. One can draw the conclusion that the timing recovery subsystem which is one of the critical parts of the digital subscriber loop is quite realizable by using a simple switched capacitor circuit adopting WDM without severe demands for precise circuit elements.



REFERENCES

- [1] T. Suzuki, H. Takatori, M. Ogawa, and K. Tomooka, " Line Equalizer for a Digital Subscriber Loop Employing Switched Capacitor Technology," *IEEE Trans. Commun.*, vol. COM-30, pp. 2074-2082, Nov.1982.
- [2] F. M. Gardner, *Phaselock Technique*, John Wiley & Sons (1979).
- [3] R. T. Kaneshiro, *Circuit and Technology Considerations for High Frequency Switched-Capacitor Filters*, Memorandum No. UCB/ERL M83/42, July 1983.
- [4] K. Martin, and A. S. Sedra, " Effects of the Op Amp Finite Gain and Bandwidth on the Performance of Switched-Capcaitor Filters," *IEEE Trans. Circuits and Systems*, vol. CAS-28, pp.822-829, Aug. 1981.

APPENDIX

A. Programs

- (i) Main Program ( Ideal Op Amp )
- (ii) Program simulating the effect of Finite Gain of Op Amp
- (iii) Program simulating the effect of Offset Voltage of Op Amp
- (iv) Program simulating the effect of Finite BW of Op Amp

B. Figures

- A1:  $\delta f >$  lock-in range (with pure sinusoidal input)
- A2: Effect of different gains and pole locations to the rate of convergence (with pure sinusoidal input) using zero-delay termination capacitor filter
- A3: Effect of different gains and pole locations to the rate of convergence (with pure sinusoidal input) using one-delay termination capacitor filter
- A4: System response using simulated channel output signal as input and comparison of the rate of convergence with different gains and pole locations
- A5: Effect of Finite Gain of Op Amp
- A6: Effect of Offset Voltage of Op Amp
- A7: Effect of Finite BW of Op Amp
- A7.1: Comparison of Ideal OP Amp and OP Amp of Finite Bandwidth

## APPENDIX

### I. Main Program -

```
program IWDM (input, output);
{Implementation of WDM using switched capacitor filter with }
{zero-delay termination capacitor }
{with lineout as input signal}
{use b[4] as decision bit}
{assume OP Amp is ideal}
type random = record
    time : real;
    echo : real;
    value : real
end; {end of record 'random'}
storage =array [1..513] of random;
filter =array [1..22] of integer;
var v, tx, f: real;
    fo, ko, c1, cu, ct: real;
    v1, v2, fn, t, ttest: real;
    y1, y2: real;
    ab, fn, bb:integer;
    i, j, k, l, m, n: integer;
    data : storage;
    a, b:filter;
{v:voltage input to vco, tx:time axis, fvco output freq
fo:vco free running freq, kd:gain of phase detector or
cu/ci of switched cap filter, ko:gain of vco, fn:input
freq, t:delta t or 1/2Tin.}
function vin (x, fre : real): real;
var tprime, delta, d0, d1,d2, d3: real;
    index: integer;
{x:time, fre:input fre, table:output pulse of line}
begin {function begin}
    tprime:=0.072/fre;
    delta:=x-trunc(x/tprime)*tprime;
    delta:=delta/tprime;
    index:=trunc(x/tprime)+1;
    d0:=data[index-1].value;
    d1:=data[index].value;
    d2:=data[index+1].value;
    d3:=data[index+2].value;
    vin:=d1+delta*(d2-d1)+delta*(delta-1)*(d3-d2-d1+d0)/4
+(delta-0.5)*delta*(delta-1)*(d3-3*d2+3*d1-d0)/6;
    {vin:=abs(vtemp)}
end;{function}
begin {main}
readln (fo);
readln (ko);
readln (c1);
readln (cu);
readln (ct);
readln (fn);
```

```
for j:=1 to 513 do
  begin {for}
    read (data[j].time);
    read (data[j].echo);
    readln (data[j].value);
  end; {for}
  writeln ('fo=',fo);
  writeln ('c1=',c1);
  writeln ('cu=',cu);
  writeln ('ct=',ct);
  writeln ('ko=',ko);
  writeln ('fn=',fn);
  {initialization}
  v:=0;
  {tx:=0.5/fo;}
  tx:=0.0000072373;
  t:=1/fo;
  ttest:=1/fn;
  ab:=0;
  for k:=1 to 22 do
    begin {initialization of matrix}
      a[k]:=0;
      b[k]:=0;
    end; {matrix}
  {end of initialization}
  for i:=1 to 800 do
    begin
      {generate a new bit if tx > ttest and run the scrambler}
      if tx>=ttest then
        begin {change cycle and run the scrambler}
          tx:=tx-ttest;
          for l:=1 to 19 do
            begin
              a[21-l]:=a[20-l];
            end;
            a[1]:=ab;
            fn:=1+a[3]+a[20];
            ab:=fn-(fn div 2)*2;
          {end of scrambler}
          {bipolar coder}
          bb:=a[1]-a[2];
          {transversal filter}
          for m:=1 to 19 do
            begin
              b[21-m]:=b[20-m];
            end;
            b[1]:=bb;
          end; {if}
        {convolution}
        y1:=0;
        y2:=0;
        for n:=1 to 22 do
          begin
            y1:=y1+b[n]*vin(tx+t+n*ttest-ttest,fn);
```

```
    y2:=y2+b[n]*vin(tx+n*ttest-ttest,fin);
  end; {begin}
  {end of convolution}
  v1:=abs(y1)/0.05545;
  v2:=abs(y2)/0.05545;
  if b[4] <> 0 then
    v:=(c1*v-cu*(v1-v2))/(ct+c1);
    f:=fo+ko*v;
    tx:=tx+t; {shift time axis}
    t:=1/f; {change delta t}
    tx:=tx+t;
    {if i>9000 then}
    writeln (i,' ',f);
  end; {for}
end. {main program}
```

## II. OP Amp With Finite Gain

```
program IWDM (input, output);
{a switched cap WDM with termination advance cap}
{Implementation of WDM using switched capacitor filter with }
{zero-delay termination capacitor }
{with lineout as input signal}
{use b[4] as decision bit}
{finite OP Amp gain}
type random = record
    time : real;
    echo : real;
    value : real
end; {end of record 'random'}
storage =array [1..513] of random;
filter =array [1..22] of integer;
var v, tx, f, gain: real;
    fo, ko, c1, cu, ct: real;
    v1, v2, fn, t, ttest: real;
    y1, y2: real;
    ab, fn, bb:integer;
    i, j, k, l, m, n: integer;
    data : storage;
    a, b:filter;
{v:voltage input to vco, tx:time axis, f:vco output freq
fo:vco free running freq, kd:gain of phase detector or
cu/ci of switched cap filter, ko:gain of vco, fn:input
freq, t:delta t or 1/2Tin.}
function vin (x, fre : real): real;
var tprime, delta, d0, d1,d2, d3: real;
    index: integer;
{x:time, fre:input fre, table:output pulse of line}
begin {function begin}
    tprime:=0.072/fre;
    delta:=x-trunc(x/tprime)*tprime;
    delta:=delta/tprime;
    index:=trunc(x/tprime)+1;
    d0:=data[index-1].value;
    d1:=data[index].value;
    d2:=data[index+1].value;
    d3:=data[index+2].value;
    vin:=d1+delta*(d2-d1)+delta*(delta-1)*(d3-d2-d1+d0)/4
+(delta-0.5)*delta*(delta-1)*(d3-3*d2+3*d1-d0)/6;
    {vin:=abs(vtemp)}
end;{function}
begin {main}
readln (fo);
readln (ko);
readln (c1);
readln (cu);
readln (ct);
readln (fn);
readln (gain); {OP Amp gain}
for j :=1 to 513 do
```

```
begin {for}
  read (data[j].time);
  read (data[j].echo);
  readln (data[j].value);
end; {for}
writeln ('fo=',fo);
writeln ('c1=',c1);
writeln ('cu=',cu);
writeln ('ct=',ct);
writeln ('ko=',ko);
writeln ('fn=',fn);
{initialization}
v:=0;
{tx:=0.5/fo;}
tx:=0.0000072373;
t:=1/fo;
ttest:=1/fn;
ab:=0;
- for k:=1 to 22 do
  begin {initialization of matrix}
    a[k]:=0;
    b[k]:=0;
  end; {matrix}
{end of initialization}
for i:=1 to 800 do
  begin
    {generate a new bit if tx > ttest and run the scrambler}
    if tx>=ttest then
      begin {change cycle and run the scrambler}
        tx:=tx-ttest;
        for l:=1 to 19 do
          begin
            a[21-l]:=a[20-l];
          end;
          a[1]:=ab;
          fn:=1+a[3]+a[20];
          ab:=fn-(fn div 2)*2;
          {end of scrambler}
          {bipolar coder}
          bb:=a[1]-a[2];
          {transversal filter}
          for m:=1 to 19 do
            begin
              b[21-m]:=b[20-m];
            end;
            b[1]:=bb;
          end; {if}
        {convolution}
        y1:=0;
        y2:=0;
        for n:=1 to 22 do
          begin
            y1:=y1+b[n]*vin(tx+t+n*ttest-ttest,fn);
            y2:=y2+b[n]*vin(tx+n*ttest-ttest,fn);
```

```
end; {begin}
{end of convolution}
v1:=abs(y1)/0.05545;
v2:=abs(y2)/0.05545;
if b[4] <> 0 then
{ effect of finite gain of the OP Amp }
v:=(c1*v-cu*(v1-v2))/(ct+c1-cu/gain);
f:=fo+ko*v;
tx:=tx+t; {shift time axis}
t:=1/f; {change delta t}
tx:=tx+t;
{if i>9000 then}
writeln (i,' ',f);
end; {for}
end. {main program}
```



### III. OP Amp With Offset Voltage

```

program IWDM (input, output);
{Implementation of WDM using switched capacitor filter with}
{zero-delay termination capacitor}
{with lineout as input signal}
{use b[4] as decision bit}
{take into consideration OP Amp offset voltage}
type random = record
    time : real;
    echo : real;
    value : real
end; {end of record 'random'}
storage =array [1..513] of random;
filter =array [1..22] of integer;
var v, tx, f, vos: real;
    fo, ko, c1, cu, ct: real;
    v1, v2, fin, t, ttest: real;
    y1, y2: real;
    ab, fn, bb:integer;
    i, j, k, l, m, n: integer;
    data : storage;
    a, b:filter;
{v:voltage input to vco, tx:time axis, f:vco output freq
fo:vco free running freq, kd:gain of phase detector or
cu/ci of switched cap filter, ko:gain of vco, fin:input
freq, t:delta t or 1/2Tin.}
function vin (x, fre : real): real;
var tprime, delta, d0, d1,d2, d3: real;
    index: integer;
{x:time, fre:input fre, table:output pulse of line}
begin {function begin}
    tprime:=0.072/fre;
    delta:=x-trunc(x/tprime)*tprime;
    delta:=delta/tprime;
    index:=trunc(x/tprime)+1;
    d0:=data[index-1].value;
    d1:=data[index].value;
    d2:=data[index+1].value;
    d3:=data[index+2].value;
    vin:=d1+delta*(d2-d1)+delta*(delta-1)*(d3-d2-d1+d0)/4
+(delta-0.5)*delta*(delta-1)*(d3-3*d2+3*d1-d0)/6;
    {vin:=abs(vtemp)}
end;{function}
begin {main}
readln (fo);
readln (ko);
readln (c1);
readln (cu);
readln (ct);
readln (fin);
{readln (gain); OP Amp gain}
readln (vos); {OP Amp gain}
for j :=1 to 513 do

```

```
begin {for}
  read (data[j].time);
  read (data[j].echo);
  readln (data[j].value);
end; {for}
writeln ('fo=',fo);
writeln ('c1=',c1);
writeln ('cu=',cu);
writeln ('ct=',ct);
writeln ('ko=',ko);
writeln ('fn=',fn);
{initialization}
v:=0;
{tx:=0.5/fo;}
tx:=0.0000072373;
t:=1/fo;
ttest:=1/fn;
ab:=0;
for k:=1 to 22 do
  begin {initialization of matrix}
    a[k]:=0;
    b[k]:=0;
  end; {matrix}
{end of initialization}
for i:=1 to 800 do
  begin
    {generate a new bit if tx > ttest and run the scrambler}
    if tx>=ttest then
      begin {change cycle and run the scrambler}
        tx:=tx-ttest;
        for l:=1 to 19 do
          begin
            a[21-l]:=a[20-l];
          end;
        a[1]:=ab;
        fn:=1+a[3]+a[20];
        ab:=fn-(fn div 2)*2;
        {end of scrambler}
        {bipolar coder}
        bb:=a[1]-a[2];
        {transversal filter}
        for m:=1 to 19 do
          begin
            b[21-m]:=b[20-m];
          end;
        b[1]:=bb;
      end; {if}
    {convolution}
    y1:=0;
    y2:=0;
    for n:=1 to 22 do
      begin
        y1:=y1+b[n]*vin(tx+t+n*ttest-ttest,fn);
        y2:=y2+b[n]*vin(tx+n*ttest-ttest,fn);
```

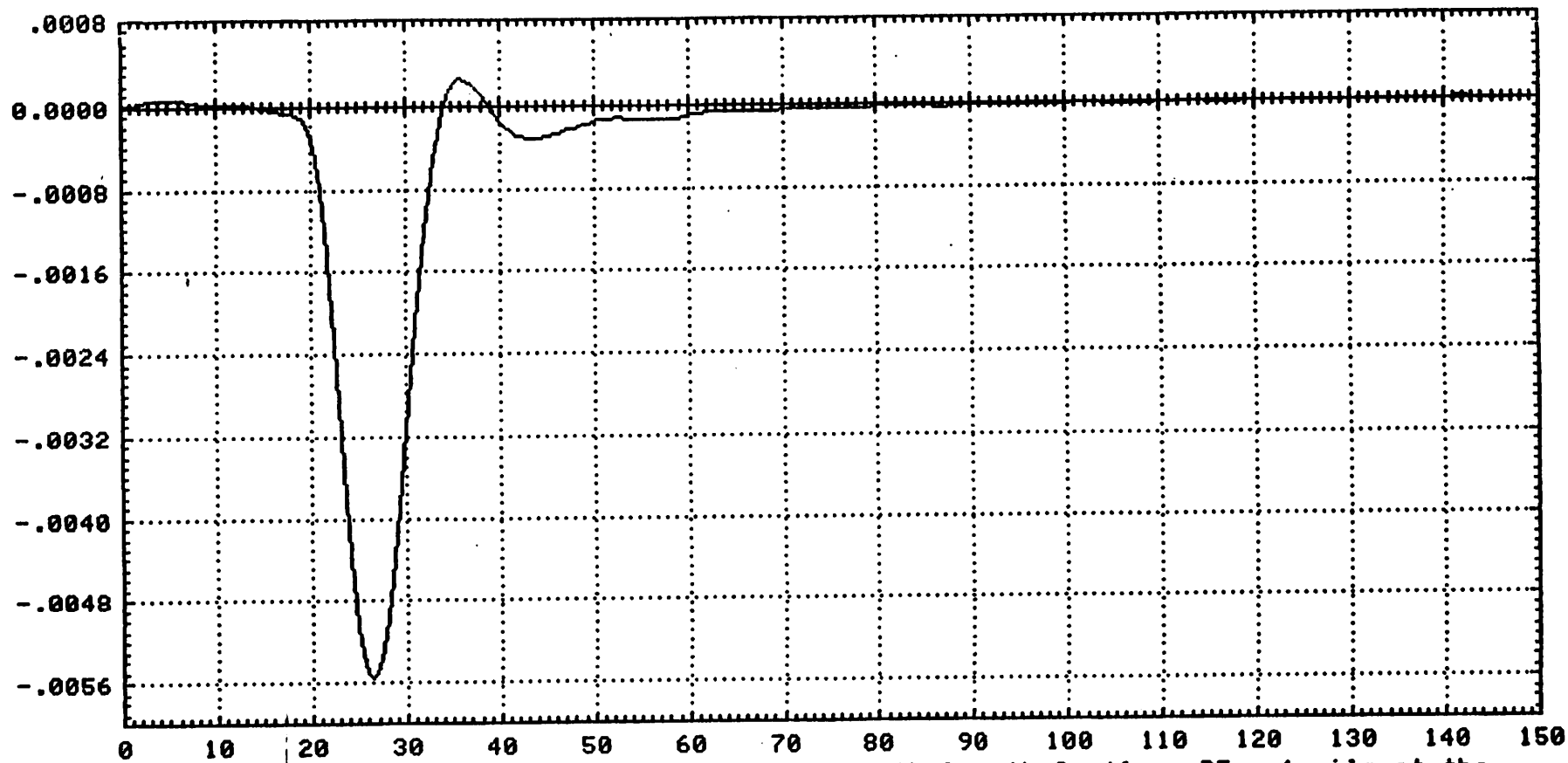
```
end; {begin}
{end of convolution}
v1:=abs(y1)/0.05545;
v2:=abs(y2)/0.05545;
if b[4] <> 0 then
{v:=(c1*v-cu*(v1-v2))/(ct+c1-cu/gain);}
{ effect of OP Amp offset voltage }
v:=(c1*v-cu*(v1-v2))/(ct+c1)+(cu/(ct+c1))*vos;
f:=fo+ko*v;
tx:=tx+t; {shift time axis}
t:=1/f; {change delta t}
tx:=tx+t;
{if i>9000 then}
writeln (i,' ',f);
end; {for}
end. {main program}
```

#### IV. OP Amp With Finite Bandwidth

```
program IWDM (input, output);
{Implementation of WDM using switched capacitor filter with }
{zero-delay termination capacitor }
{with lineout as input signal}
{use b[4] as decision bit}
{with finite OP Amp bandwidth}
type random = record
    time : real;
    echo : real;
    value : real
end; {end of record 'random'}
storage =array [1..513] of random;
filter =array [1..22] of integer;
var v, tx, f, ve, cstray, vc, x, wu, wuo, ft: real;
    fo, ko, c1, cu, ct: real;
    v1, v2, fin, t, ttest: real;
    y1, y2: real;
    ab, fn, bb:integer;
    i, j, k, l, m, n: integer;
    data : storage;
    a, b:filter;
{v:voltage input to vco, tx:time axis, f:vco output freq
fo:vco free running freq, kd:gain of phase detector or
cu/ci of switched cap filter, ko:gain of vco, fin:input
freq, t:delta t or 1/2Tin.}
{ve:error voltage at input, cstray:input cap of OP Amp,}
{vc: voltage across cap, x:duty cycle of phase 2,}
{wu:close loop BW when phi 2 on, wuo:phi 2 off}
{ft: unity gain freq of OP Amp}
function vin (x, fre : real): real;
var tprime, delta, d0, d1,d2, d3: real;
    index: integer;
{x:time, fre:input fre, table:output pulse of line}
begin {function begin}
    tprime:=0.072/fre;
    delta:=x-trunc(x/tprime)*tprime;
    delta:=delta/tprime;
    index:=trunc(x/tprime)+1;
    d0:=data[index-1].value;
    d1:=data[index].value;
    d2:=data[index+1].value;
    d3:=data[index+2].value;
    vin:=d1+delta*(d2-d1)+delta*(delta-1)*(d3-d2-d1+d0)/4
+(delta-0.5)*delta*(delta-1)*(d3-3*d2+3*d1-d0)/6;
    {vin:=abs(vtemp)}
end;{function}
begin {main}
readln (fo);
readln (ko);
readln (c1);
readln (cu);
readln (ct);
```

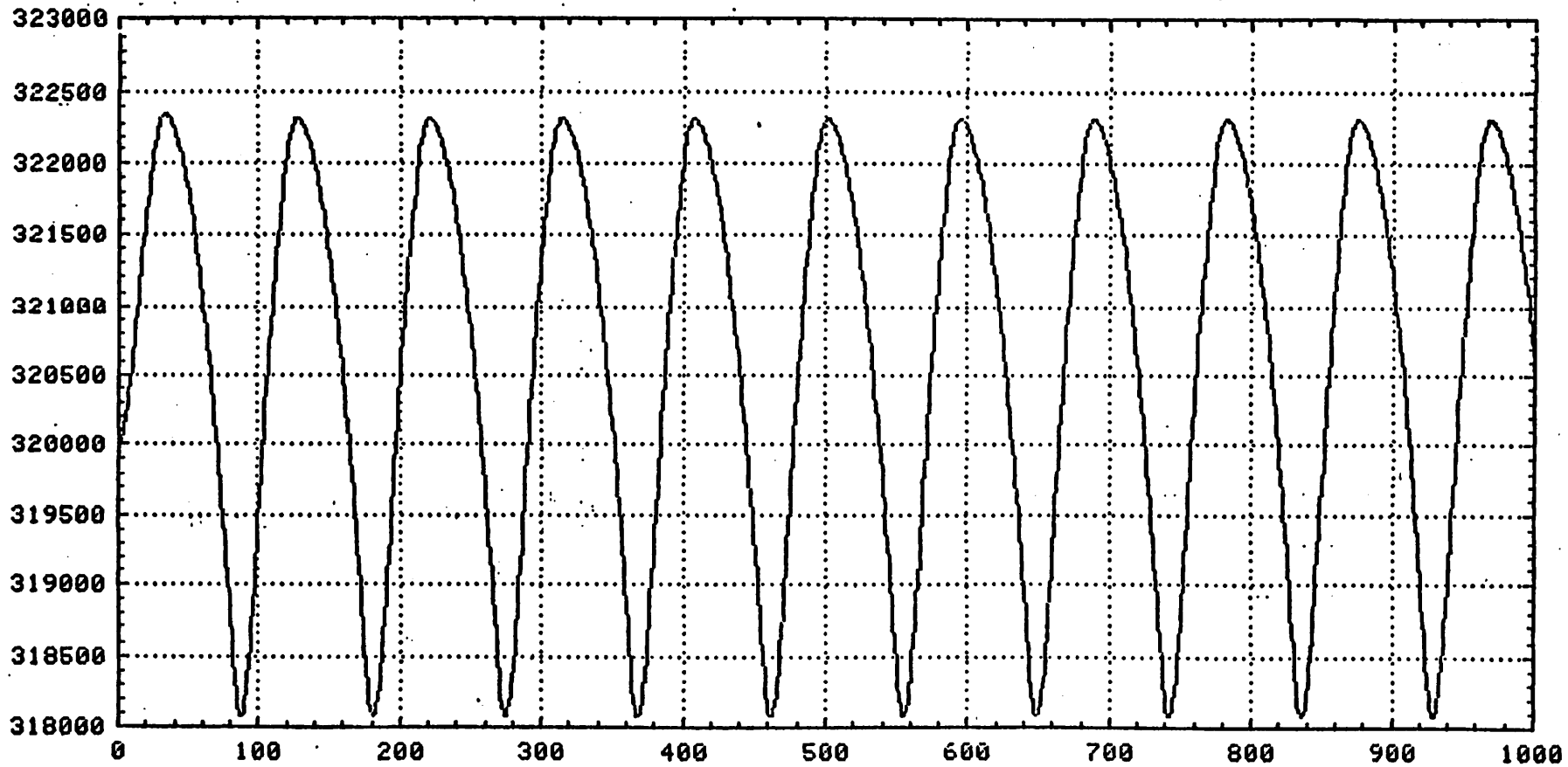
```
readln (fn);
readln (ft);
for j :=1 to 513 do
  begin {for}
    read (data[j].time);
    read (data[j].echo);
    readln (data[j].value);
  end; {for}
writeln ('fo=',fo);
writeln ('c1=',c1);
writeln ('cu=',cu);
writeln ('ct=',ct);
writeln ('ko=',ko);
writeln ('fn=',fn);
writeln ('ft=',ft);
{initialization}
v:=0;
ve:=0;
vc:=0;
cstray:=0;
x:=0.05;
wu:=((c1+ct)/(cu+cstray+c1+ct))*6.283*ft;
wuo:=(c1/(cstray+c1))*6.283*ft;
{tx:=0.5/fo;}
tx:=0.0000072373;
t:=1/fo;
ttest:=1/fn;
ab:=0;
for k:=1 to 22 do
  begin {initialization of matrix}
    a[k]:=0;
    b[k]:=0;
  end; {matrix}
{end of initialization}
for i:=1 to 800 do
  begin
    {generate a new bit if tx > ttest and run the scrambler}
    if tx>=ttest then
      begin {change cycle and run the scrambler}
        tx:=tx-ttest;
        for l:=1 to 19 do
          begin
            a[21-l]:=a[20-l];
          end;
        a[1]:=ab;
        fn:=1+a[3]+a[20];
        ab:=fn-(fn div 2)*2;
        {end of scrambler}
        {bipolar coder}
        bb:=a[1]-a[2];
        {transversal filter}
        for m:=1 to 19 do
          begin
            b[21-m]:=b[20-m];
```

```
end;
b[1]:=bb;
end; {if}
{convolution}
y1:=0;
y2:=0;
for n:=1 to 22 do
begin
y1:=y1+b[n]*vin(tx+t+n*ttest-ttest,fn);
y2:=y2+b[n]*vin(tx+n*ttest-ttest,fn);
end; {begin}
{end of convolution}
v1:=abs(y1)/0.05545;
v2:=abs(y2)/0.05545;
if b[4] <> 0 then
begin
{ effect of finite bandwidth of OP Amp }
ve:=(cu/(cu+ct+c1+cstray))*(v1-v2)*exp(-x*ttest*wu)
+ve*exp(-x*ttest*wu)*exp(-(1-x)*ttest*wuo);
vc:=(c1*vc-cu*(v1-v2))/(ct+c1);
v:=vc+ve;
end
else
{reexame the equation below}
ve:=ve*exp(-ttest*wuo);
{v:=(c1*v-cu*(v1-v2))/(ct+c1);}
f:=fo+ko*v;
tx:=tx+t; {shift time axis}
t:=1/f; {change delta t}
tx:=tx+t;
{if i>9000 then}
writeln (i, ',f);
end; {for}
end. {main program}
```



Impulse response of the line with length 2 miles, BT: .1 mile at the center of the line. ( used as the input data )

Fig. A0

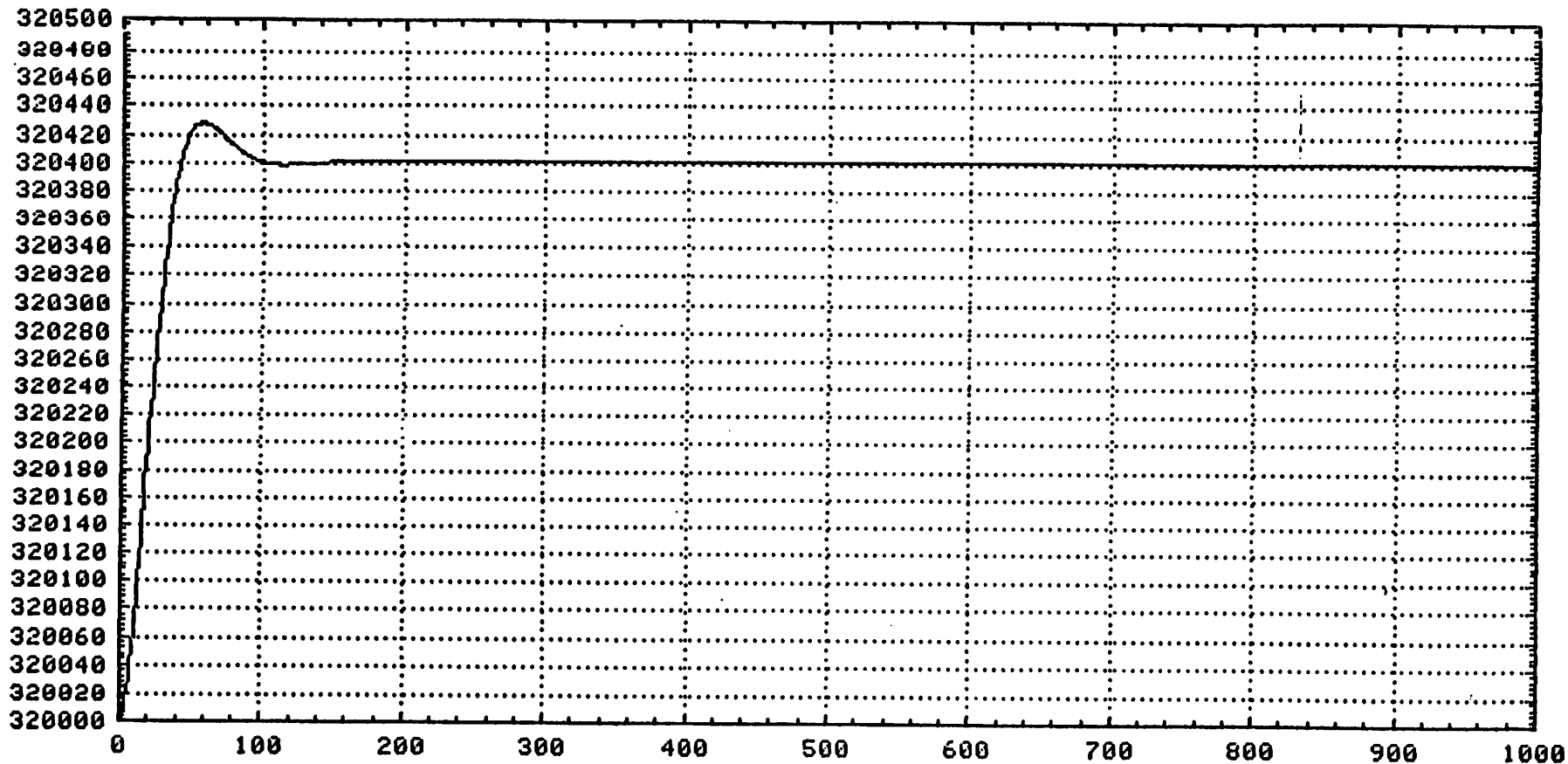


$\Delta f > \text{lock-in range. } f_0=320000, k_0=40, c_1=1, c_u=10, c_t=0.15, f_{in}=810000$

(with pure sinusoidal input)

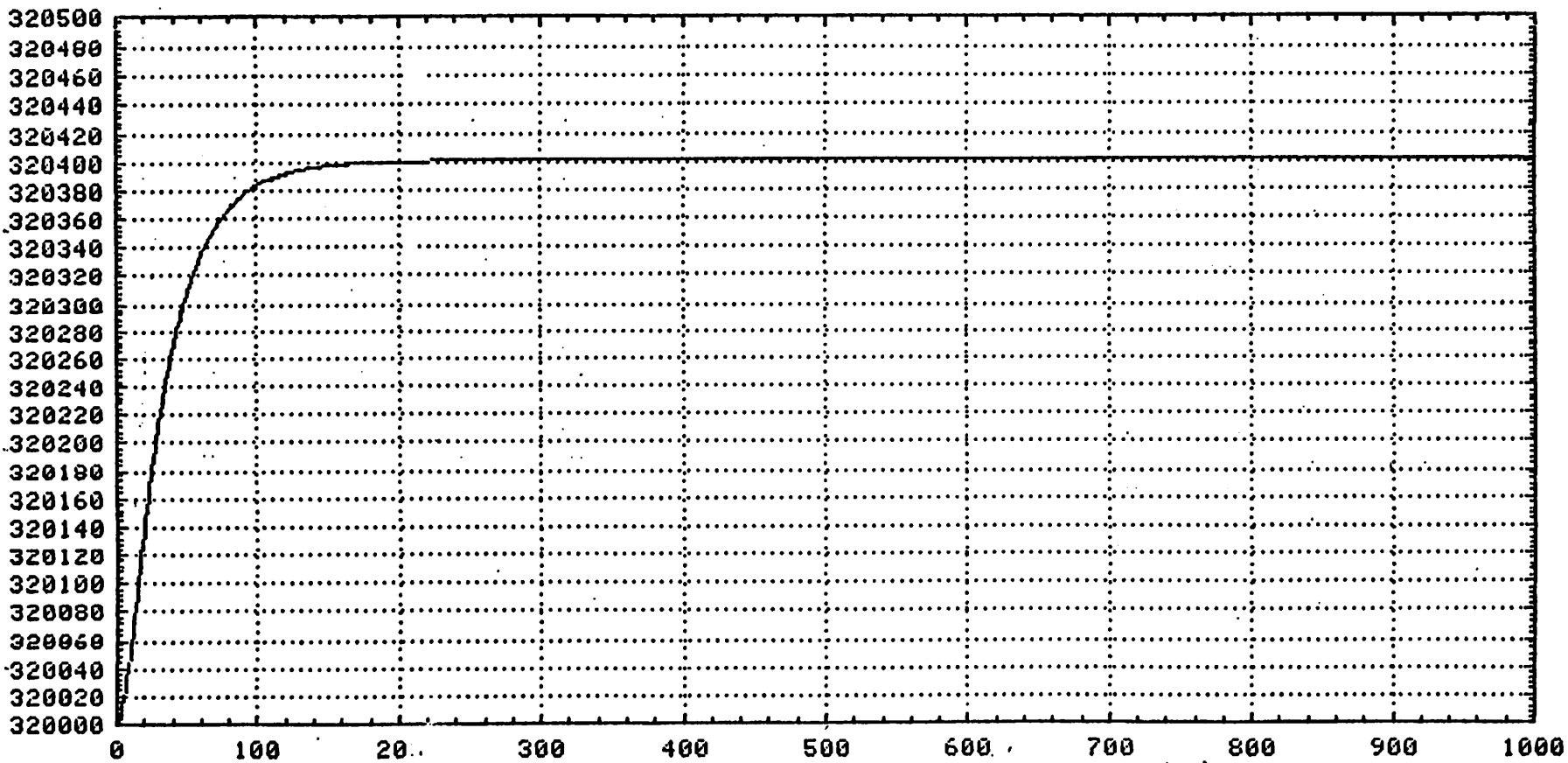
Fig. A 1





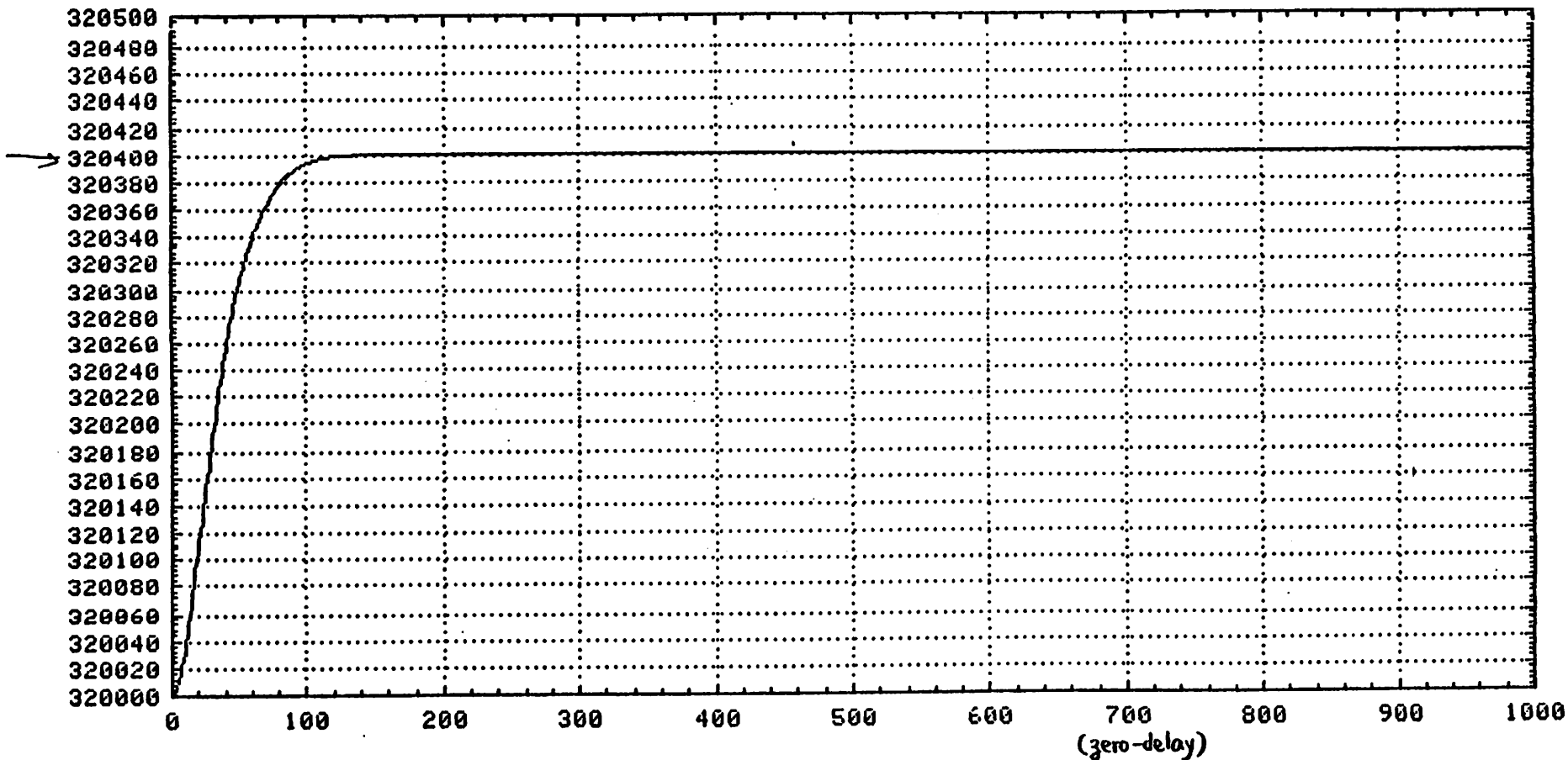
fo=320000, ko=40, cl=1, cu=10, ct=0.1, fin=80100, SC w/ lead termination cap  
 (zero-delay)  
 (with pure sinusoidal input)

Fig A2.1



(zero-delay)  
fo=320000, ko=40, c1=1, cu=10, ct=0.2, fin=80100, SC w/ lead termination cap  
(with pure sinusoidal input)

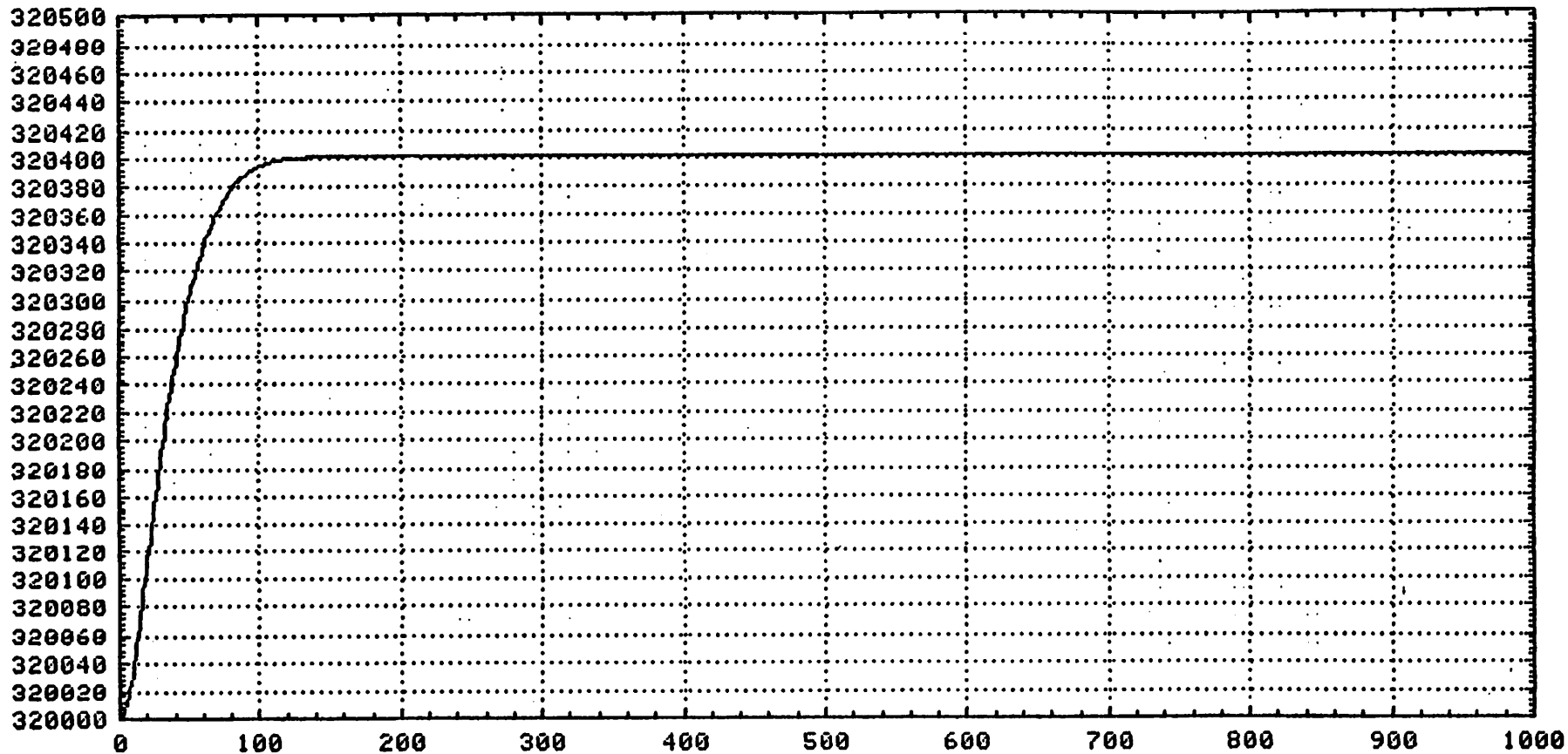
Fig A 2.2.



fo=320000, ko=20, c1=1, cu=10, ct=0.1, fin=80100, SC w/ lead termination cap  
 (with pure sinusoidal input)

2

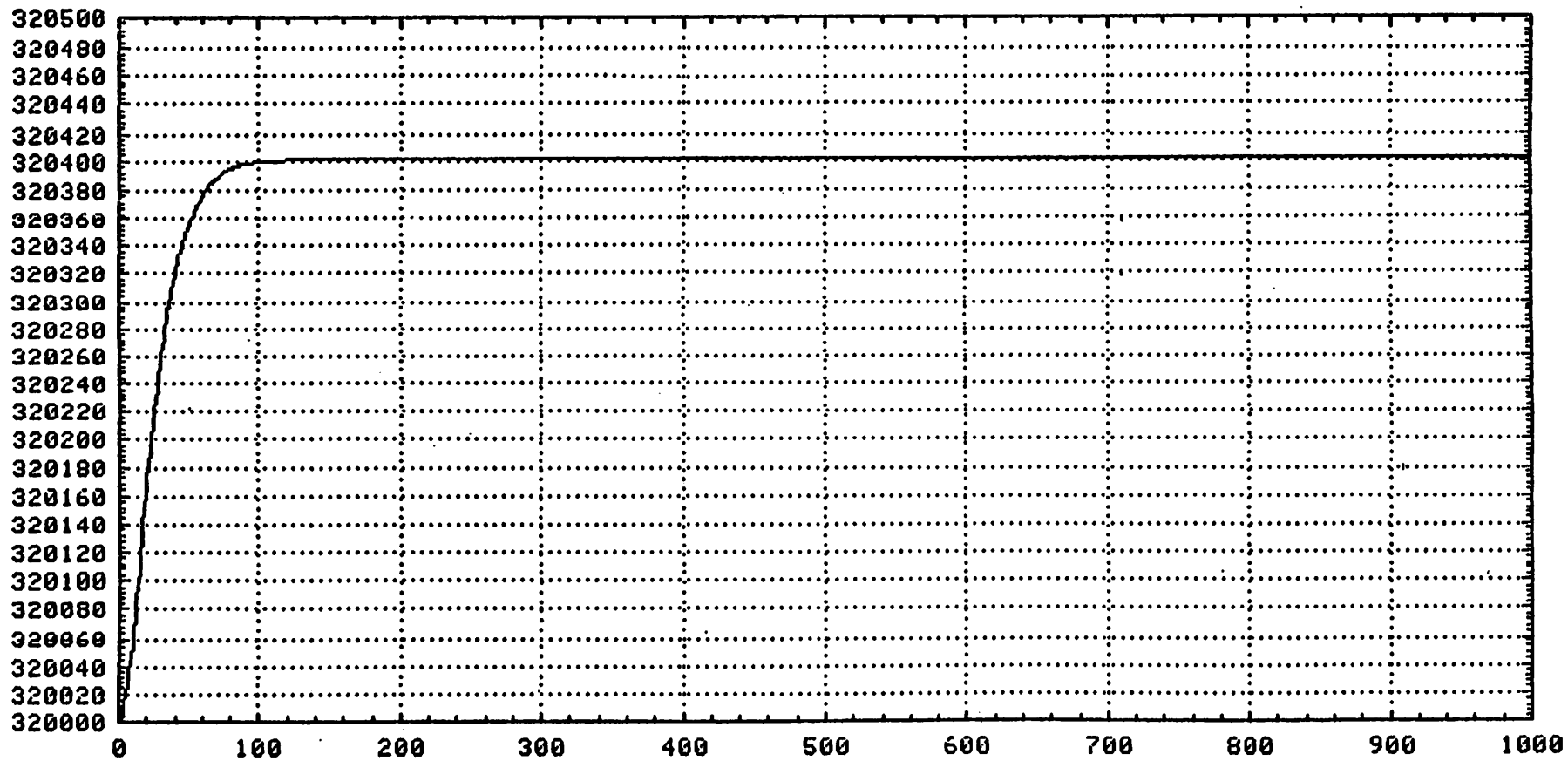
Fig. A2.3



(one-delay)  
fo=320000, ko=20, cl=1, cu=10, ct=0.1, fin=80100, SC w/ lag termination cap

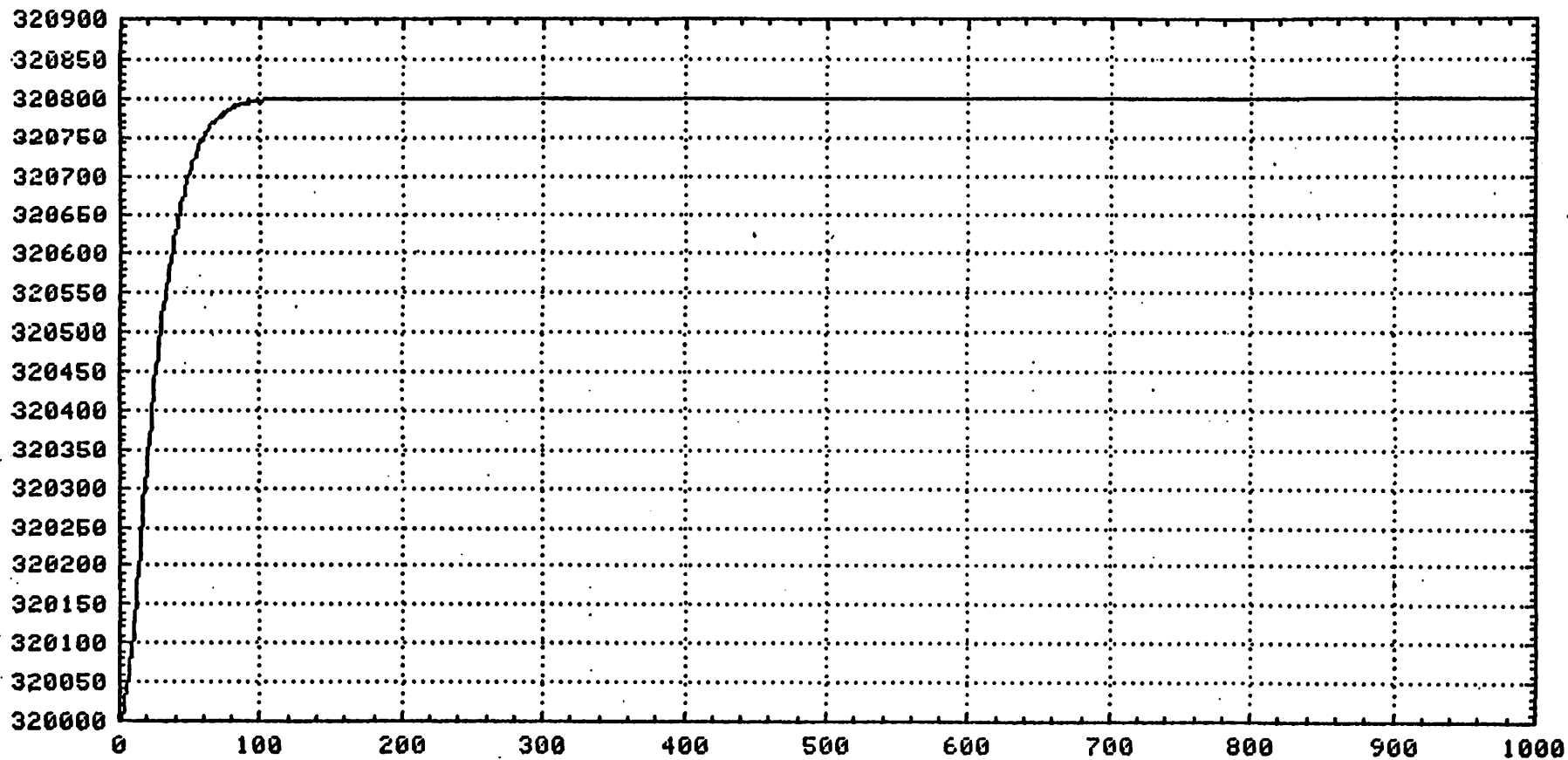
(with pure sinusoidal input)

Fig A 3.1



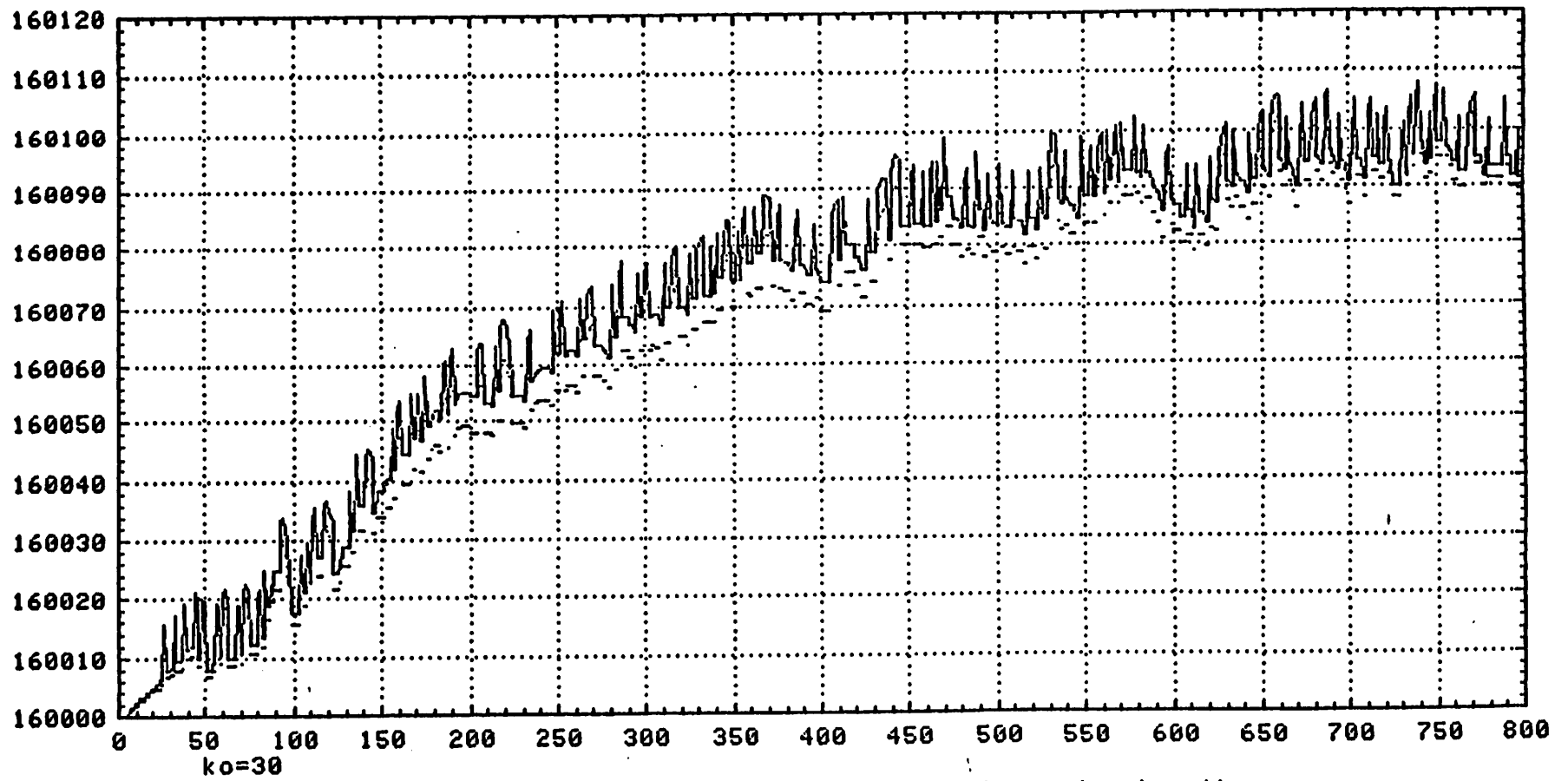
$f_0=320000$ ,  $k_0=40$ ,  $c_1=1$ ,  $c_u=10$ ,  $c_t=0.15$ ,  $f_{in}=80100$ , SC w/ lag termination cap  
 (one delay)  
 (with pure sinusoidal input)

Fig. A 3.2



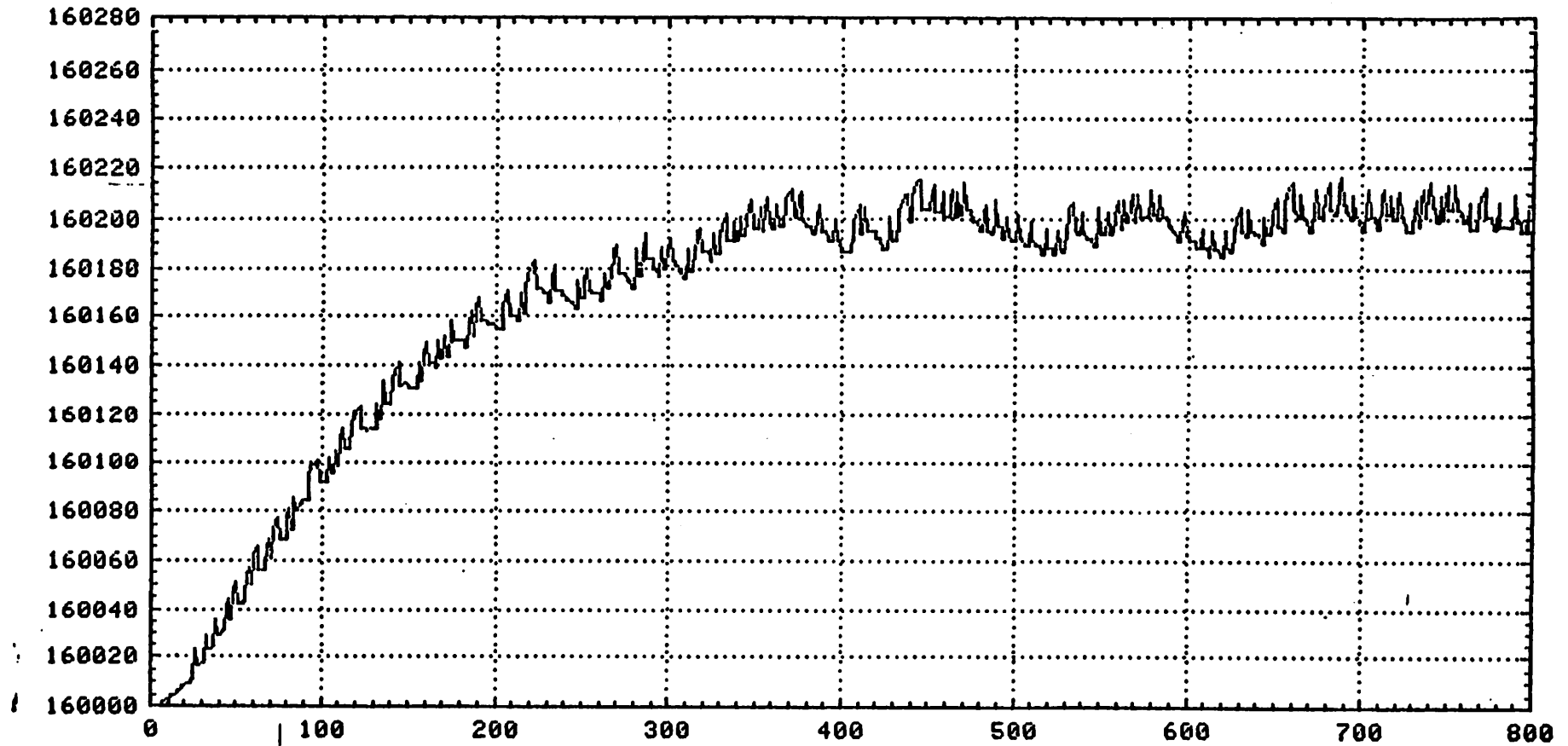
$f_0=320000$ ,  $k_0=40$ ,  $c_1=1$ ,  $c_u=10$ ,  $c_t=0.15$ ,  $f_{in}=80200$ , SC w/ lag termination cap  
(one-delay)  
(with pure sinusoidal input)

Fig A3.3



fo=160000, ko=35, cl=1, cu=10, ct=0.15, fin=80050.(With psudo random input)

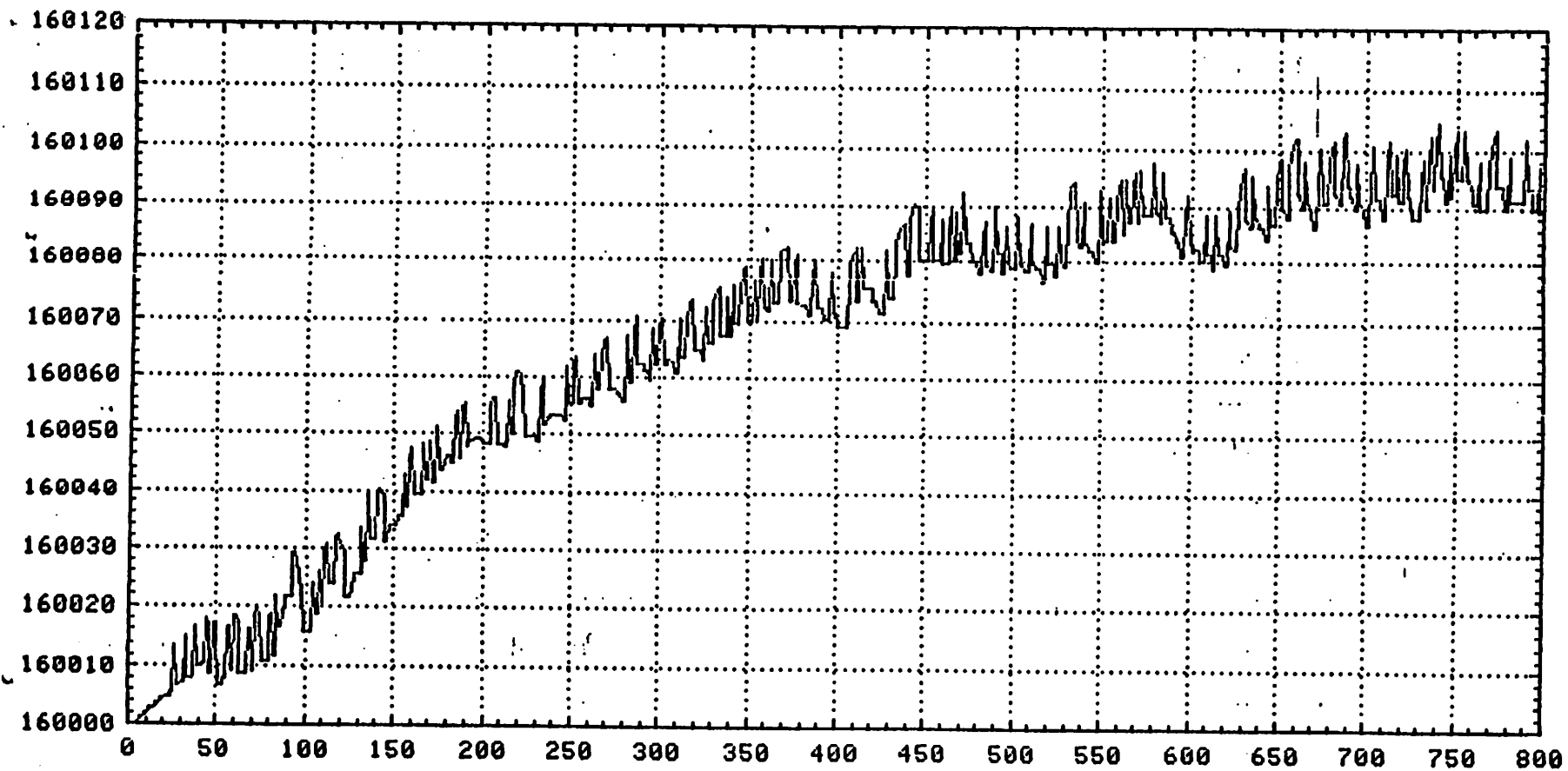
Fig A.4.1



fo=160000, ko=35, cl=1, cu=10, ct=0.08, fin=80100. (With psudo random input)

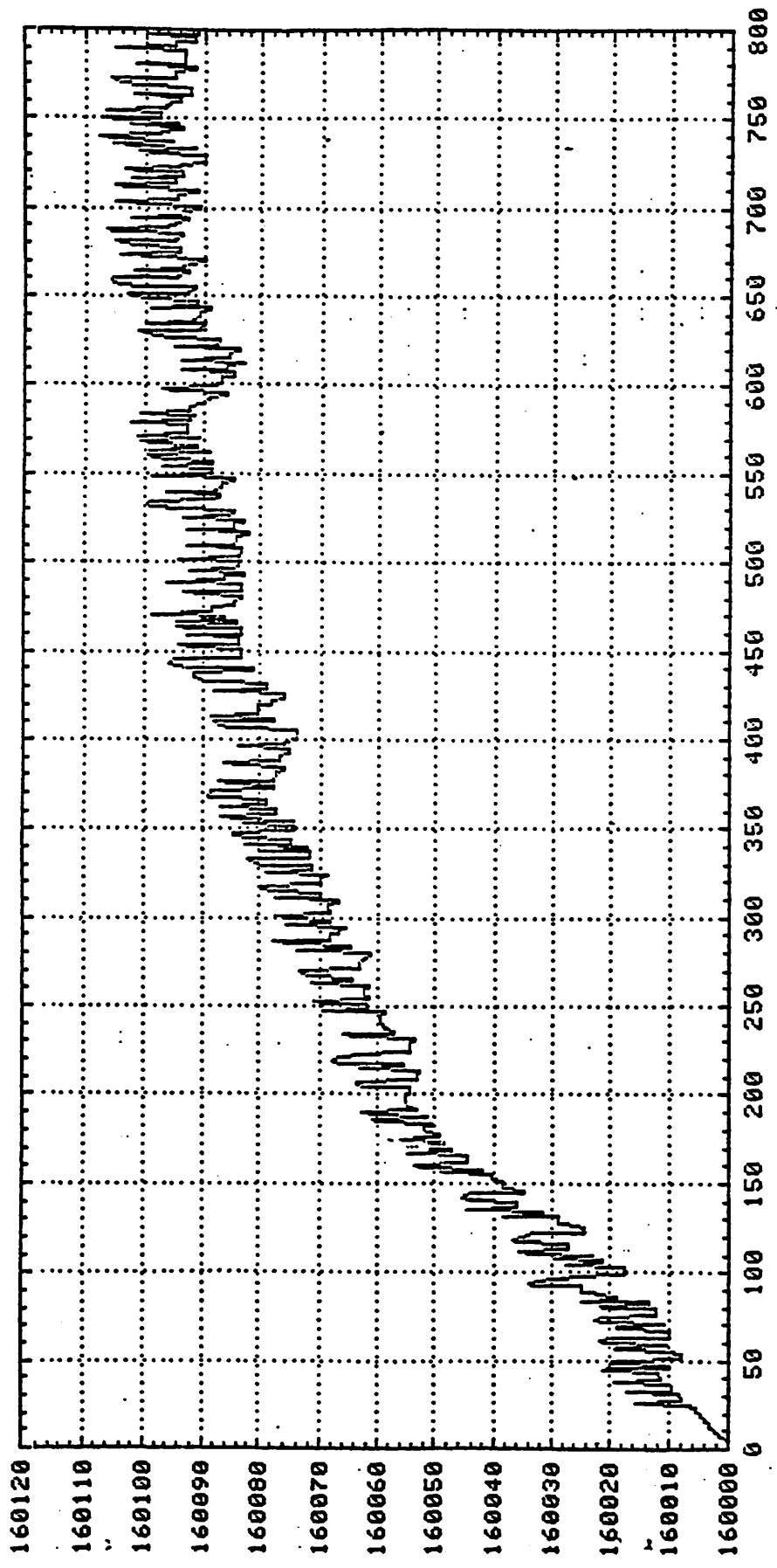
Fig A4.2





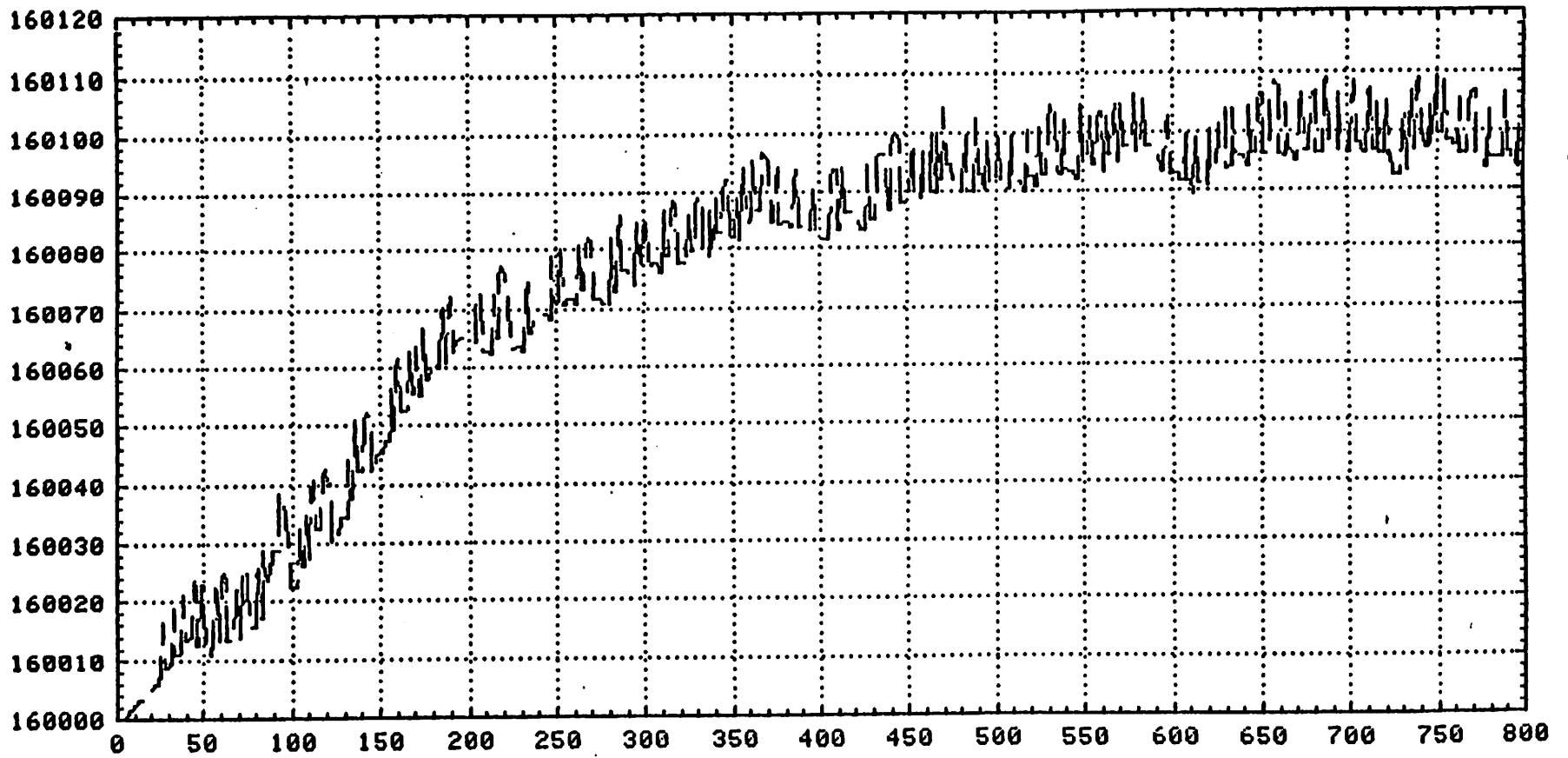
fo=160000, ko=30, c1=1, cu=10, ct=0.15, fin=80050. (With pseudo random input)

Fig A 4.3



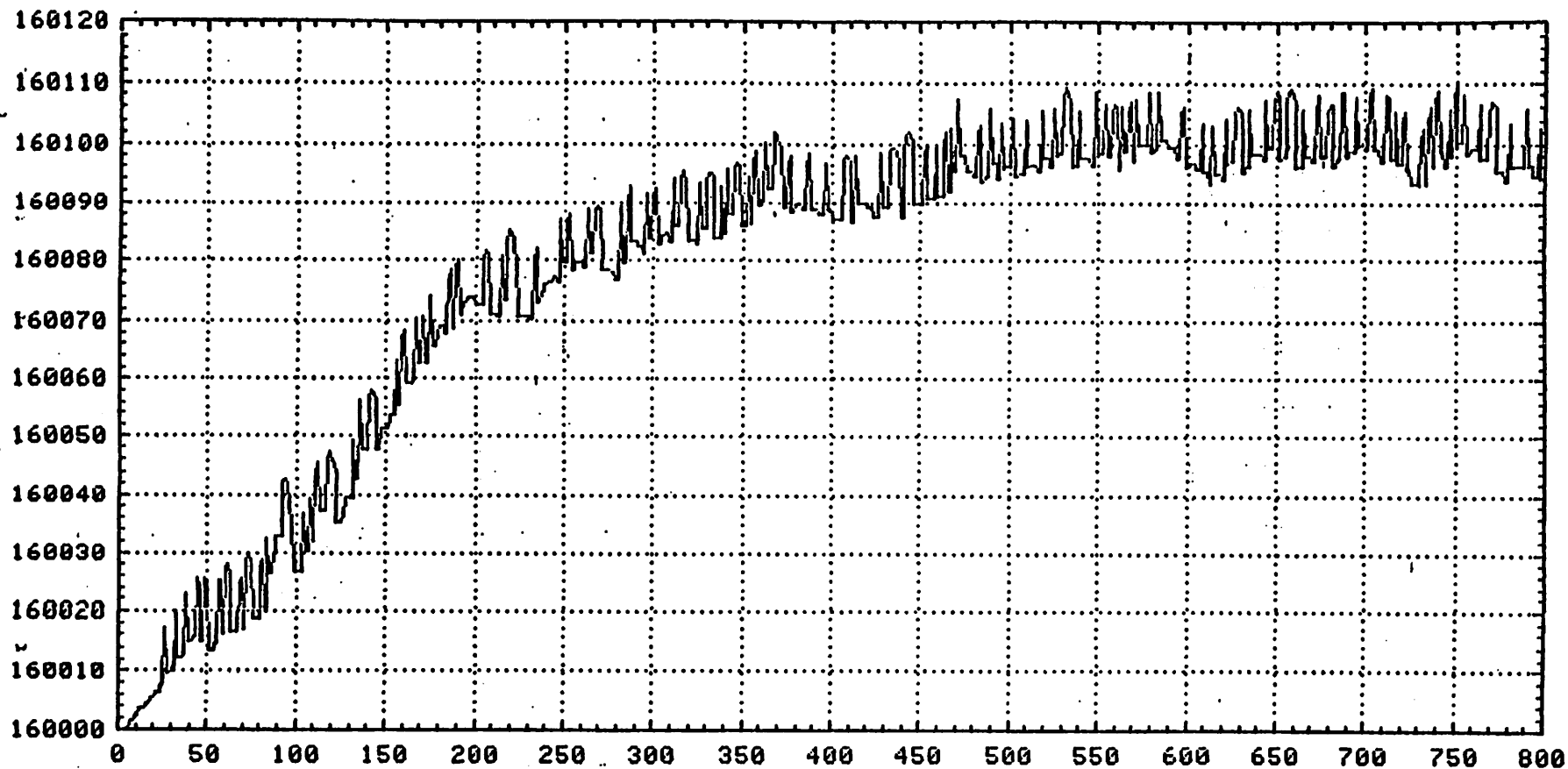
fo=160000, ko=35, c1=1, cu=10, ct=0.15, fin=80050.(with psudo random input)

Fig A 44



fo=160000, ko=35, cl=1, cu=10, ct=0.12 fin=80050.(With psudo random input)

Fig A4.5



fo=160000, ko=35, cl=1, cu=10, ct=0.1, fin=80050.(With psudo random input)

Fig A 4.6

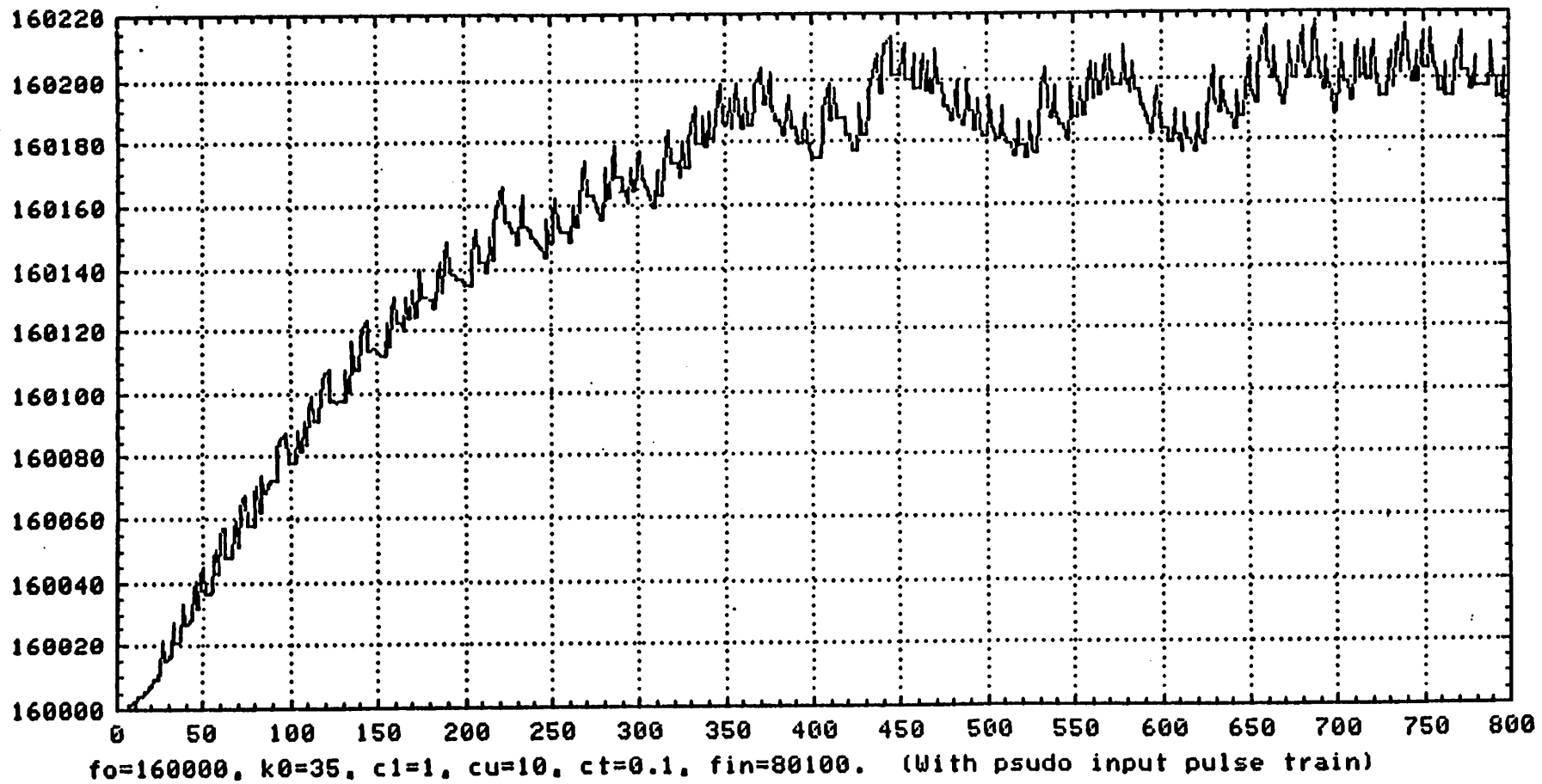
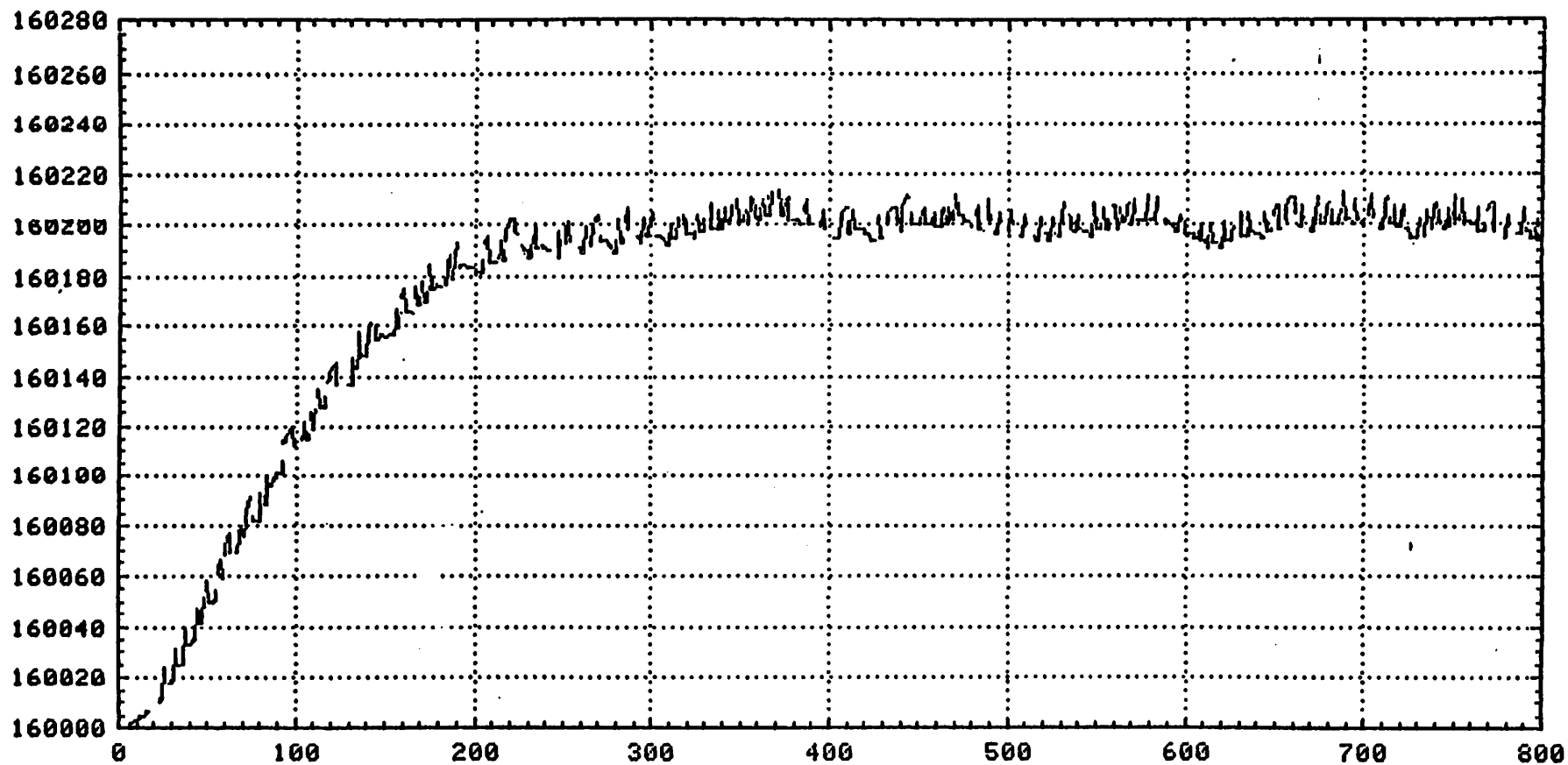
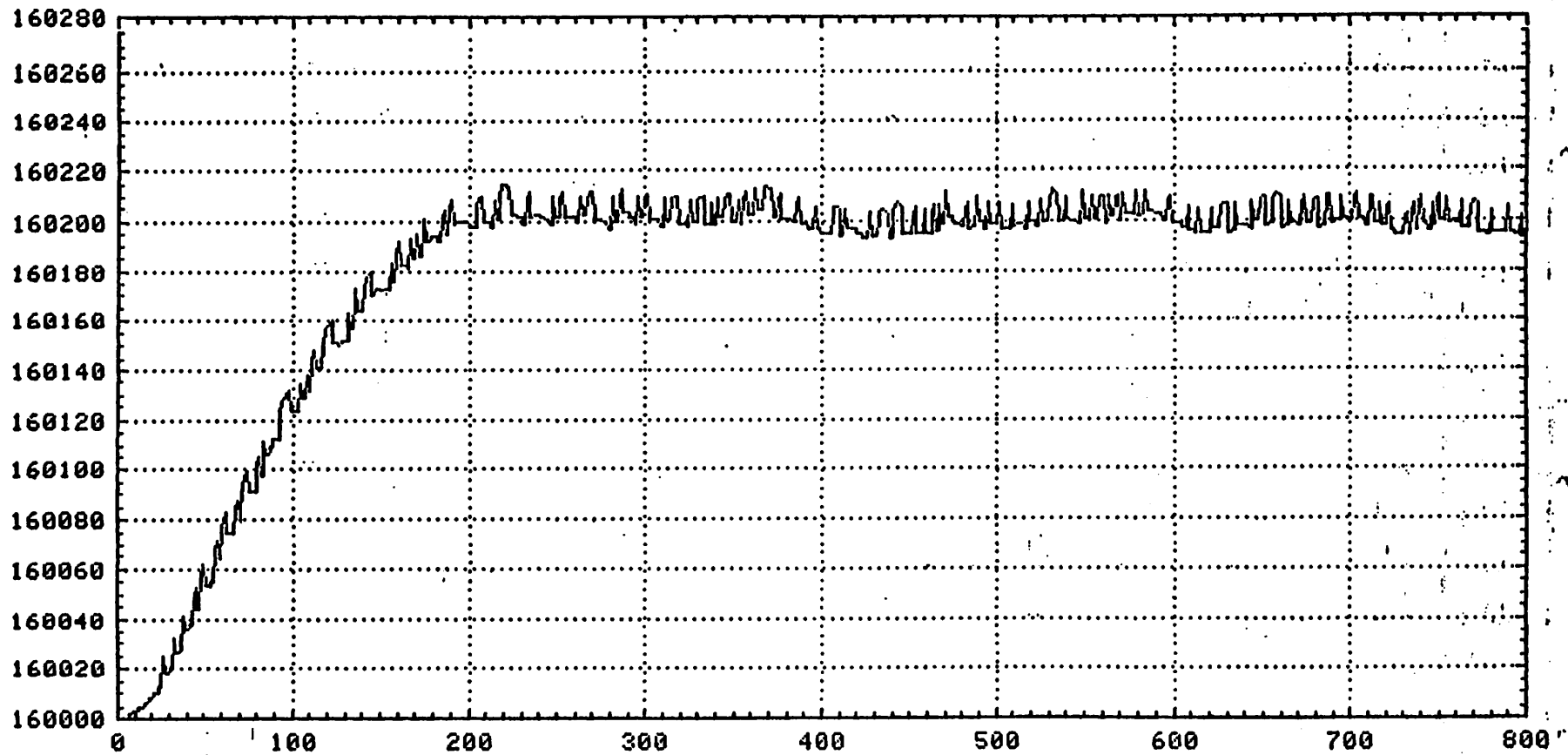


Fig A4.7.



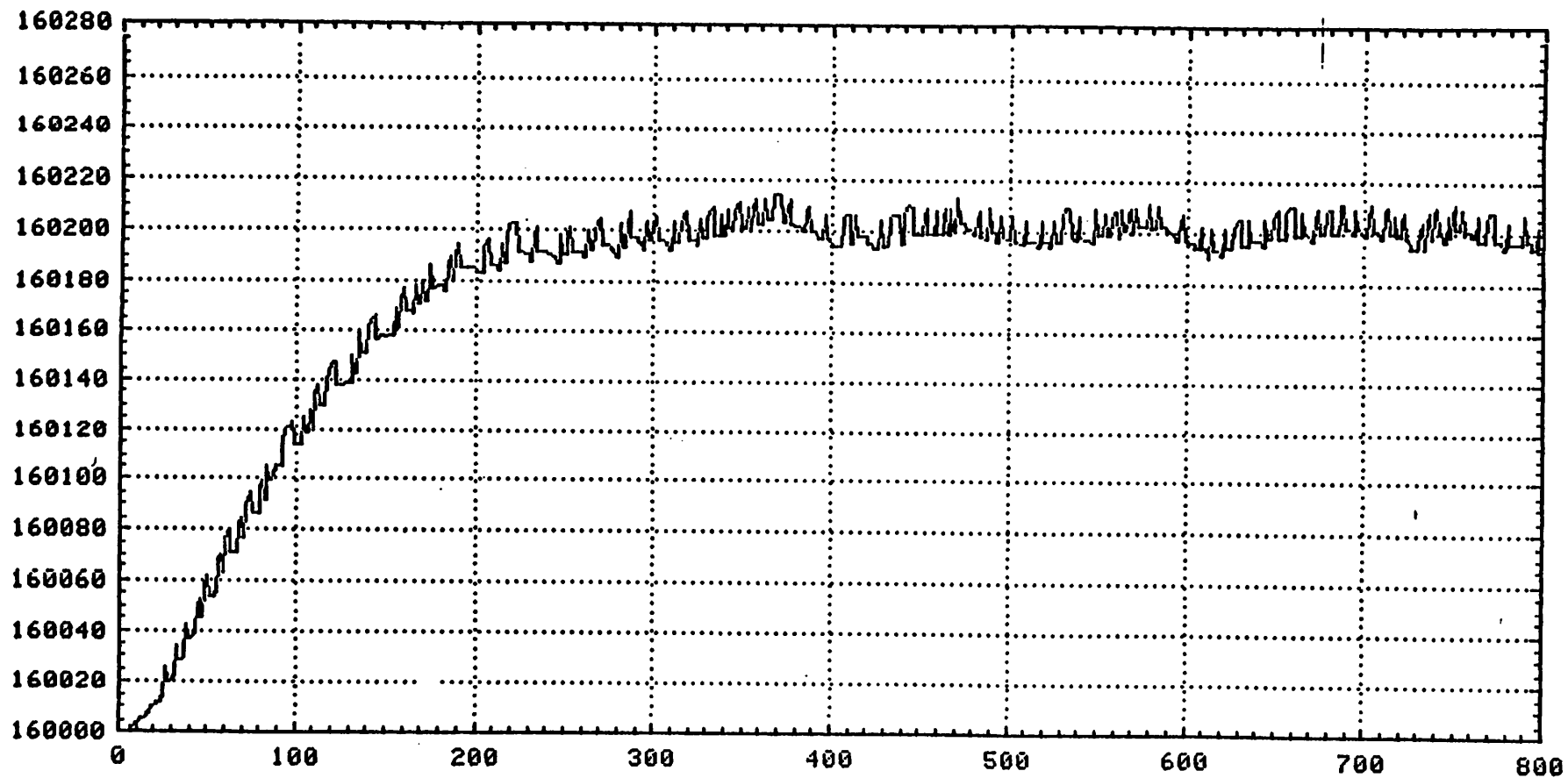
fo=160000, ko=35, c1=1, cu=10, ct=0.06, fin=80100. (with pseudo random input)

Fig A4.9



fo=160000, ko=35, c1=1, cu=10, ct=0.06, fin=80100, OP Amp gain=1000. (With psudo random input)

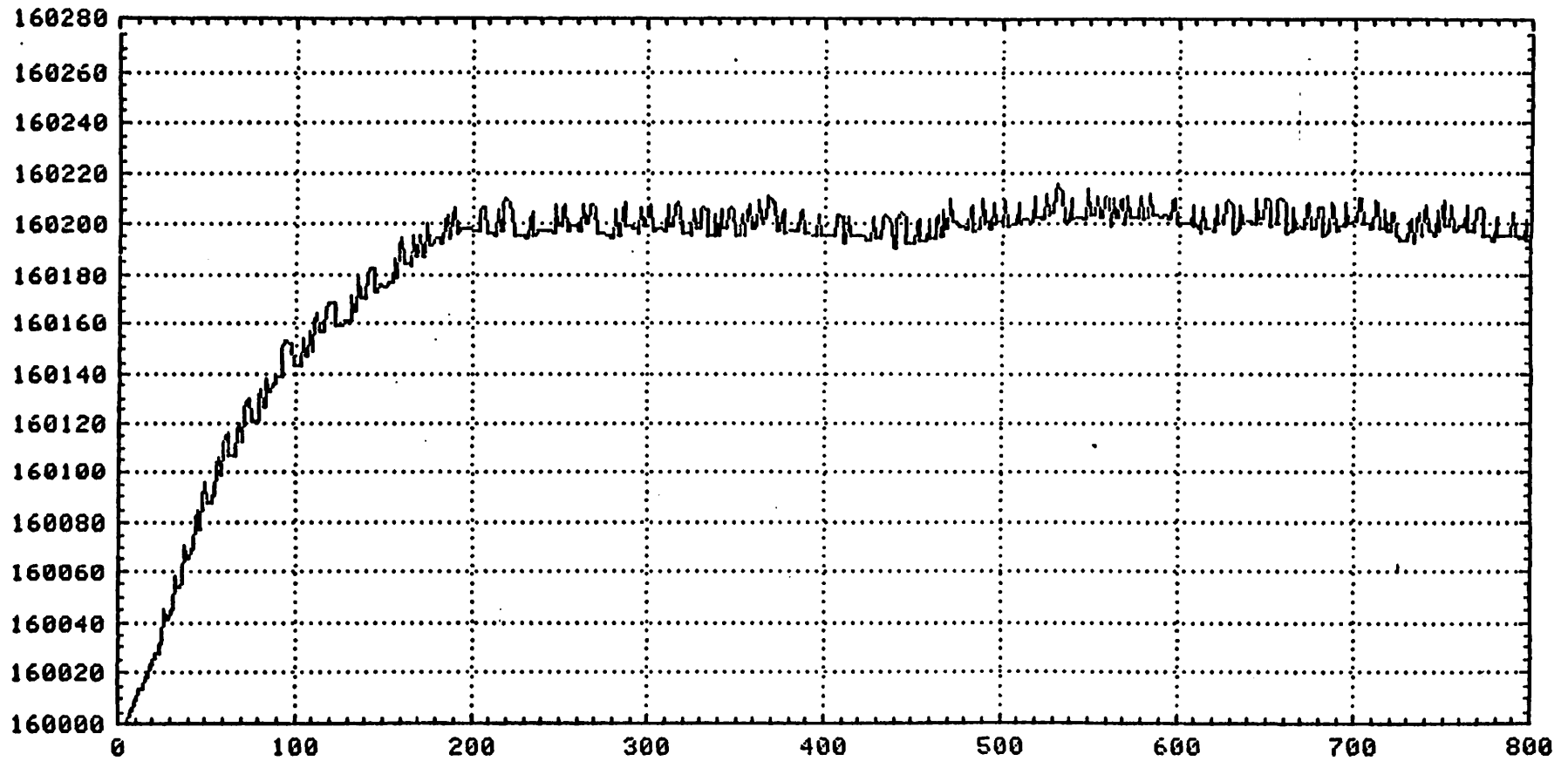
Fig A5



f0=160000, ko=35, ci=1, cu=10, ct=0.06, fin=80100, vos=1mv.(With pseudo random input)

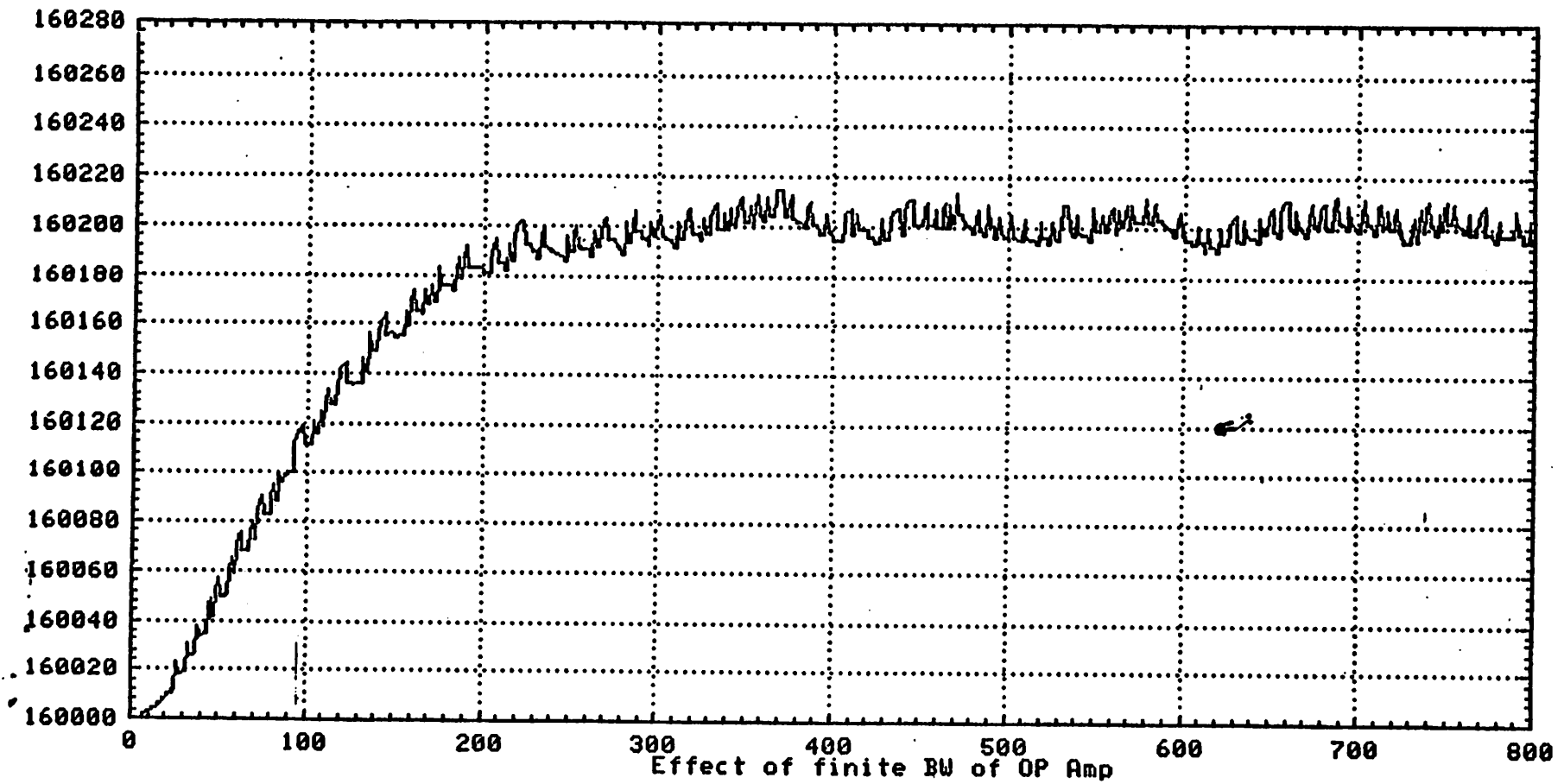
Fig AG.1





fo=160000, cl=1, cu=10, ct=0.06, fin=80100, vos=10mv.(With psudo random input)

Fig A 6.2



$f_0=160000$ ,  $k_0=35$ ,  $c_1=1$ ,  $c_u=10$ ,  $c_t=0.06$ ,  $f_{in}=80100$ ,  $f_t=100k$ .

Fig A7

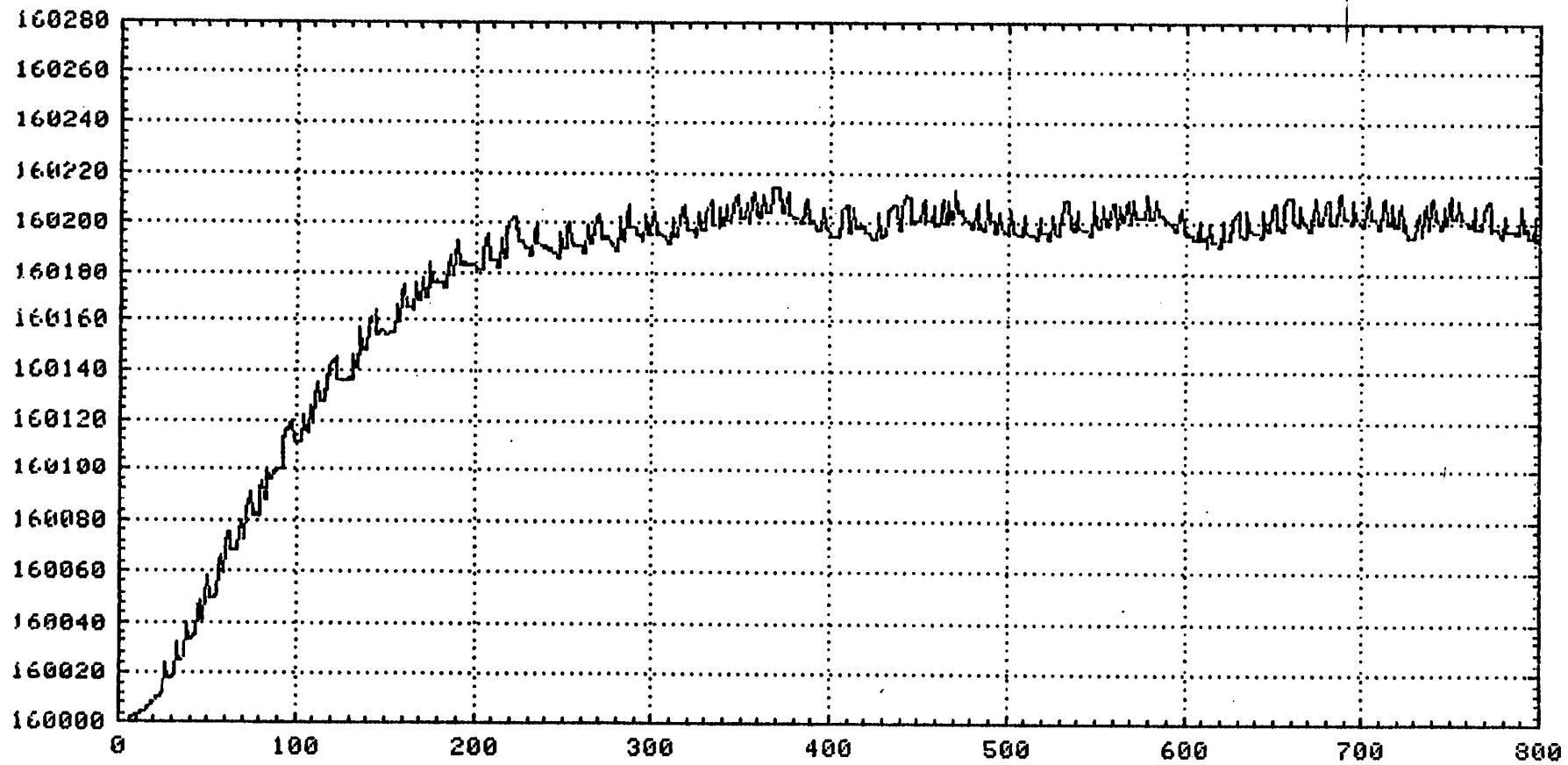


Fig. A7.1.1  $ct = 0.06$

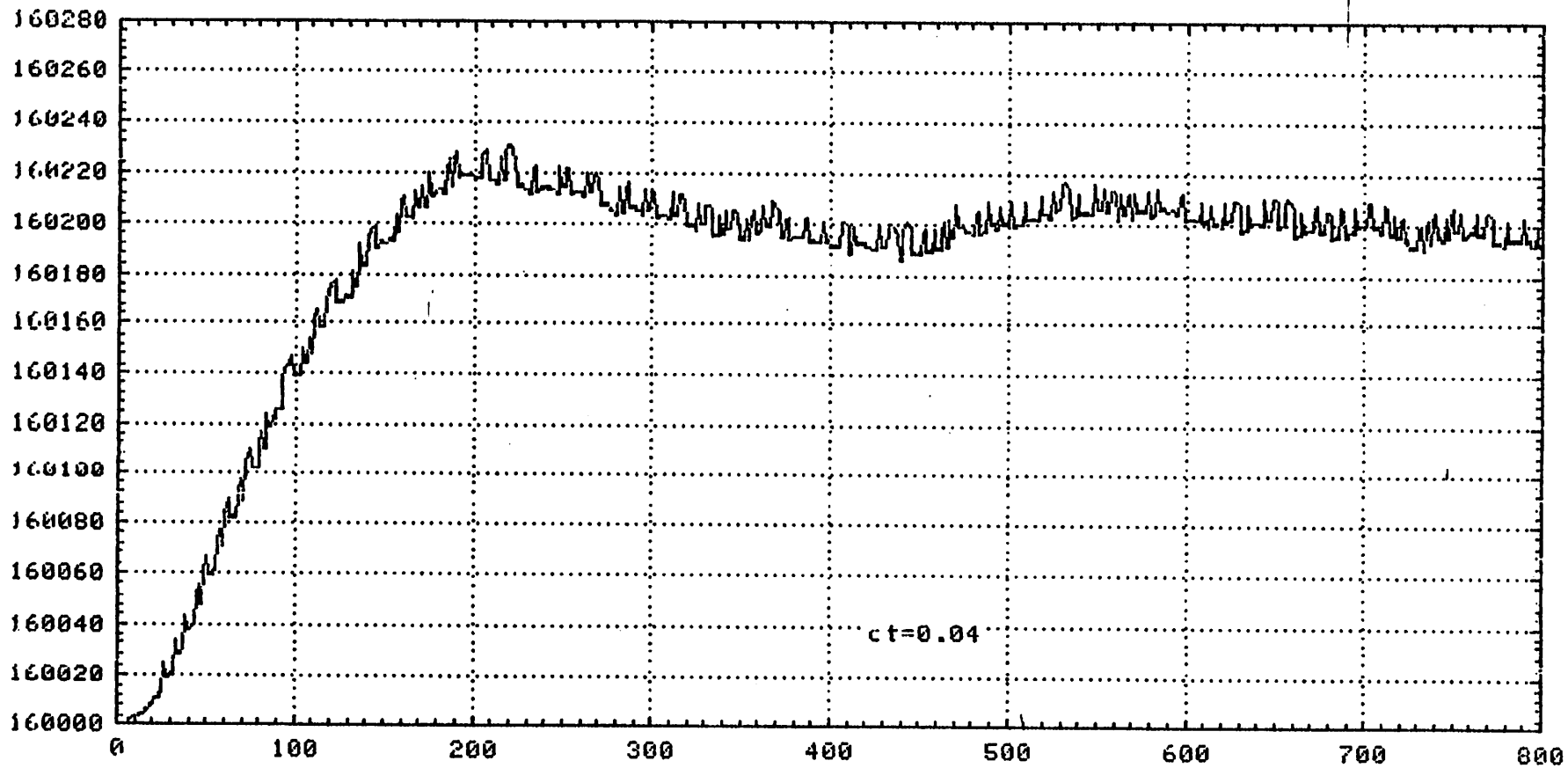


Fig. A7.1.2