ON THE DESIGN OF LARGE FLEXIBLE SPACE STRUCTURES (LFSS)

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On the Design of Large Flexible Space Structures (LFSS)

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Abstract: For a general finite-element model of an LFSS, a strictly passive compensator results in an exponentially stable feedback system, when actuators and sensors are colocated. In the general case (no colocation) we state necessary and sufficient conditions on the parameter Q for stabilizing a certain number of modes. We give conditions for robust stability and show that feedback does not destabilize the unmodeled modes under certain conditions.

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I. INTRODUCTION

We study a general finite-element model for a large flexible space structure (LFSS). When sensors (with suitable gains) and actuators are colocated, strictly passive compensators result in an exponentially stable feedback system. For the general case (no colocation) we use Q-parametrization theory to state necessary and sufficient conditions on Q for stabilizing a certain number of modes which approximate the plant for design purposes. We state a necessary and sufficient condition for stability under additive perturbation (by an unmodeled mode) and, finally, we show that, under certain conditions, the compensator can be chosen so that it does not destabilize the unmodeled modes.

II. THE LFSS MODEL

Following standard practice, we consider a general finite-element (lumped) model for the LFSS (see, e.g., [Wes. 1]). We assume small deformations, linear-elastic materials and neglect gyroscopic coupling and damping. The equation of motion of the LFSS is then:

$$M\ddot{q} + Kq = \tilde{B}u \tag{1}$$

where $M = M^T > 0$ is an "inertia" matrix, $K = K^T \geq 0$ is a "stiffness" matrix, $K,M \in \mathbb{R}^{n\times n}$; $q \in \mathbb{R}^n$ is a vector of generalized coordinates (position and angle); u is a vector of control inputs (forces and torques) and $\tilde{B}$ is determined by the type and location of control actuators.

The <u>modal vectors</u>, η_k, are defined as solutions to the (generalized) eigenvalue problem $\omega_k^2 M\eta_k = K\eta_k$, $k = 1,\dots,n$, with the normalizations

$\eta_k^T K \eta_i = \omega_k^2 \delta_{ik}$ and $\eta_k^T M \eta_i = \delta_{ik}$ where ω_k^2, $k = 1,\ldots,n$ are the eigenvalues. Define the <u>modal matrix</u>, T_0, as the matrix with $\eta_1,\ldots,\eta_n$ as columns [Cou. 1, p. 282 ff.], [Gol. 1]. Then, with $q =: T_0\zeta$, (1) becomes

$$\ddot{\zeta} + \Omega^2\zeta = \hat{B}u \qquad (2)$$

where $\hat{B} := T_0^T\tilde{B}$, $\Omega := \mathrm{diag}(\omega_1,\ldots,\omega_n)$ with $\omega_i \geq 0$, $\forall i$

Let the modal velocities, $\dot{\zeta}_k$, be the measured variables and let the state be $x := [\zeta^T \vdots \dot{\zeta}^T]^T$; then the LFSS is described by

$$A = \left[\begin{array}{c|c} 0 & I \\ \hline -\Omega^2 & 0 \end{array}\right] ; \quad B = \left[\begin{array}{c} 0 \\ \hline \hat{B} \end{array}\right] ; \quad C = [\, 0 \vdots \hat{C} \,] \qquad (3)$$

and from (3) the plant transfer function, P(s), is:

$$P(s) = \hat{C}(sI_{2n}-A)^{-1}\hat{B} = \hat{C} \cdot s(s^2I_n+\Omega^2)^{-1}\ \hat{B} = \sum_{k=1}^{n} \frac{s}{s^2+\omega_k^2}\, \hat{c}_k\hat{b}_k^T \qquad (4)$$

where $\hat{c}_k$ (resp. $\hat{b}_k^T$) are the column (resp. row) vectors of $\hat{C}$ (resp. $\hat{B}$).

III. COLOCATION OF SENSORS AND ACTUATORS

Colocation (i.e., sensors and actuators located at the same place), together with suitable gains in each sensor, implies that $\hat{C} = \hat{B}^T$. Consequently, from (4), $P_c(s)$ (the plant P(s) with <u>colocated</u> sensors) is given by:

$$P_c(s) = \sum_{k=1}^{n} \frac{s}{s^2+\omega_k^2}\ \hat{b}_k\hat{b}_k^T \qquad (5)$$

Since $\hat{b}_k\hat{b}_k^T \geq 0$, $P_c(s)$ has only <u>simple $j\omega$-axis poles</u> with <u>real symmetric positive semi-definite residues</u> of rank 1 and $P_c(s)$ is <u>passive</u> and strictly proper.

In view of standard results on passivity [Des. 1], [Zam. 1] we state the following well-known result:

Theorem 1: For all strictly passive $C(s)$, ${}^1S(P_c,C)$ (Fig. 1), with $P_c(s)$ as in (5), is exponentially stable.

Remark 1: For example, a controller of the form $C(s) = \frac{K_i}{s} + K_p + \sum_\alpha \frac{K_\alpha}{s+\beta_\alpha}$ is strictly passive if K_i, K_p and $\forall\alpha$, K_α are positive semi-definite and K_p and/or at least one K_α is positive definite, and $\beta_\alpha > 0$, $\forall\, \alpha$.

Remark 2: For all strictly passive $C(s)$, all unmodeled dynamics will be stabilized in ${}^1S(P_c,C)$.

Remark 3: Since $C(s)$ is strictly passive, $C^{-1}(s)$ is strictly positive real [New. 1, pp. 117,126], and thus we can justify Theorem 1 as follows. Let s_k be a closed-loop eigenvalue of ${}^1S(P_c,C)$. Then, since $\det(I+P_cC) = \det(I+CP_c)$,

$$\exists\, \gamma \neq \theta_n \quad \text{s.t.} \quad [I + C(s_k)\, P_c(s_k)]\gamma = \theta_n \tag{*}$$

To get a contradiction, assume $Re(s_k) \geq 0$. Multiplying (*) by $[C(s_k)]^{-1}$ gives

$$[C^{-1}(s_k) + P_c(s_k)]\gamma = \theta_n$$

which is the required contradiction with $\gamma \neq \theta_n$.

Remark 4: Since we use velocity feedback we may have non-zero steady state position error but, using the results of [Mor. 1], [Des. 2], we may get around this by introducing an "integrator" block, $I + \frac{K}{s}$, prior to the compensator $C(s)$ (Fig. 2), and for K small it will not affect the exponential stability of the system. More precisely, let

$H_{y_2u_1}(0) = P_cC(I+P_cC)^{-1}(0) \in \mathbb{R}^{n\times n}$ be nonsingular; call UH its polar decomposition, then U is orthogonal and H is real positive definite; hence if we choose $K = \varepsilon U^*$, for ε small and positive, the system of Fig. 2 is exponentially stable.

IV. GENERAL CASE (NO COLOCATION)

We assume that the design is done for resonant frequencies upto ω_m, i.e., for design purposes, the plant P is approximated by

$$P_d(s) := \sum_{k=1}^{m} \frac{s}{s^2+\omega_k^2} \hat{c}_k \hat{b}_k^T \tag{5}$$

Let $C := Q(I-P_dQ)^{-1}$, where Q is the well-known Q-parameter [Zam. 1], [Des. 3]. Then, defining a transfer function to be <u>$\mathcal{U}$-stable</u> iff it has no poles in a symmetric subset $\mathcal{U}$ ($\supset \mathbb{C}_+$) of $\mathbb{C}$, following [Des. 4], we state:

<u>Theorem 2</u>: For the given rational, strictly proper $P_d(s)$, the system ${}^1S(P_d,C)$ ($C := Q(I-P_dQ)^{-1}$) is $\mathcal{U}$-stable if and only if

i) Q is $\mathcal{U}$-stable,

<u>and</u> ii) $\forall k = 1,\ldots,m \begin{cases} Q(j\omega_k)\hat{c}_k = \theta_{n_o} ; \; \hat{b}_k^T Q(j\omega_k) = \theta_{n_o}^T \\ \underline{\text{and}} \;\; \hat{b}_k^T Q'(j\omega_k)\hat{c}_k = 2 \end{cases}$

<u>Remark 4</u>: It can be shown (when $\mathcal{U} = \mathbb{C}_+$ and $\mathcal{U}$-stability $\Leftrightarrow$ exponential stability) that <u>$\exists Q \in H_\infty^{n\times n}$ satisfying i) and ii)</u> by a straightforward generalization of an elementary Lagrange interpolation argument for the s.i.s.o. case (where q must belong to H_∞ and q and q' must have prescribed values at $j\omega_k$, $k=1,2,\ldots,m$).

V. UNMODELED DYNAMICS

Let us consider any mode with resonant frequency ω_i, $\underline{i > m}$ as an additive perturbation (Fig. 3), ΔP. From (5),

$$\Delta P_i = \frac{\frac{1}{2}\hat{c}_i\hat{b}_i^T}{s-j\omega_i} \tag{6}$$

Redrawing ${}^1S(P_d+\Delta P_i,C)$ (i.e., Fig. 3) as in Fig. 4 (i.e., from the "point of view" of the perturbation ΔP_i) and using the Q-parametrization theorem (since Q in Fig. 4 is exponentially stable) we state [Bha. 1]

Theorem 3: ${}^1S(P_d+\Delta P_i,C)$ is exponentially stable iff $\Delta P_i(I+Q\cdot\Delta P_i)^{-1}$ is exponentially stable.

Remark 5: Note that the results of [Doy. 1], [Chen. 1] cannot be used since ΔP_i has a pole at ω_i on the $j\omega$-axis.

Since the residue of ΔP_i at $j\omega_i$ is a rank one matrix (see (6)), Fig. 3 is essentially an s.i.s.o. system. Good design practice requires that Q be small out of band [Zam. 2], [Des. 3]. Assume that $\hat{b}_i^T Q(j\omega_i)c_i$ is therefore small and let $j\omega_i + h$ denote the new location (under feedback) of the open-loop mode at $j\omega_i$. Then, within the first order, we have

$$h = -\frac{1}{2}\hat{b}_i^T Q(j\omega_i)\hat{c}_i \tag{7}$$

Remark 6: Equ. (7) shows that in the case of colocation with suitable sensor gains such that $\hat{c}_i = \hat{b}_i$, if Q is positive definite at the unmodeled resonant frequency ω_i, then, within the first order, the pole at this frequency moves away from the $j\omega$-axis into the open left half plane.

VI. CONCLUSIONS: THE DESIGN PHILOSOPHY

The analysis above suggests the following philosophy for optimization-based CAD: i) choose a truncated plant model P_d that contains all the modes for the required control-bandwidth, ii) select Q to bring the $j\omega$-axis modes of P_d into a suitable region in the open left half-plane and to achieve a suitable I/O transfer function, iii) for the next few unmodeled modes use (7) to ensure that h is negative (say by imposing inequality constraints[†]), iv) for the remaining unmodeled modes, we know that they are more heavily damped [Asw. 1]; the Green's function approach shows that the $\hat{b}_k$'s and $\hat{c}_k$'s decrease rapidly as k increases and, for Q small, the resulting h (see (7)) will be small enough to ensure that irrespective of its sign, the higher order modes will not be made unstable.

Thus, we can achieve, in principle, suitable control over any prescribed bandwidth. The only fundamental constraint on achieving this goal is plant uncertainty [Zam. 2], [Doy. 1], [Chen 2]: from our work on simple examples the uncertainty on the exact resonant frequencies may turn out to be an important problem.

[†]For an example of optimization-based CAD using Q-parametrization see [Gus. 1].

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Footnote

†For an example of optimization-based CAD using Q-parametrization see [Gus. 1].

Figure Captions

Fig. 1. ${}^{1}S(P_c,C)$ -- the feedback system. P_c is the plant transfer functions when actuators and sensors are colocated.

Fig. 2. ${}^{1}S(P_c,C(I+\frac{K}{s}))$ -- the system ${}^{1}S(P_c,C)$ with an "integrator" block $(I+\frac{K}{s})$ preceding compensator C, to achieve zero position error.

Fig. 3. ${}^{1}S(P_d+\Delta P_i,C)$ -- the perturbed system. P_d is the approximate plant model chosen for design (colocation is not assumed) and the ith mode (which is unmodeled) is considered as an additive perturbation ΔP_i.

Fig. 4. ${}^{1}S(Q,\Delta P_i)$ -- this figure is obtained from Fig. 3. The "gain seen by ΔP," going from point a to point b through ${}^{1}S(P,C)$, is equal to $-Q$.

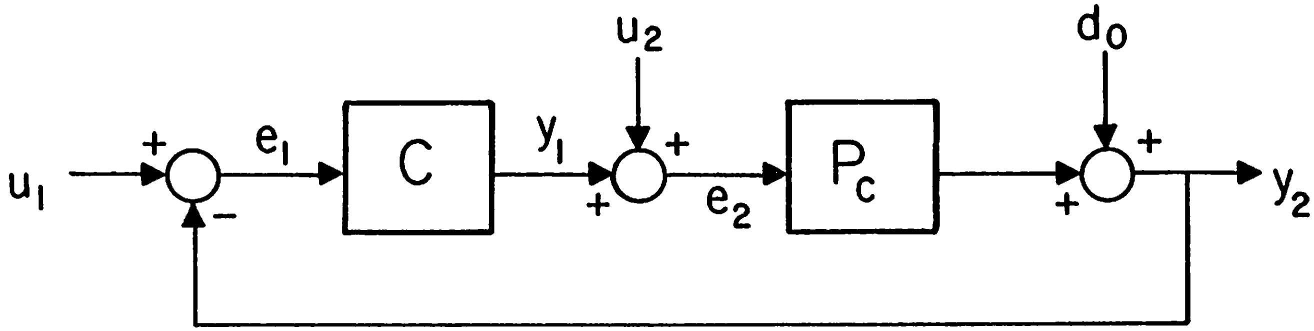

Fig. 1

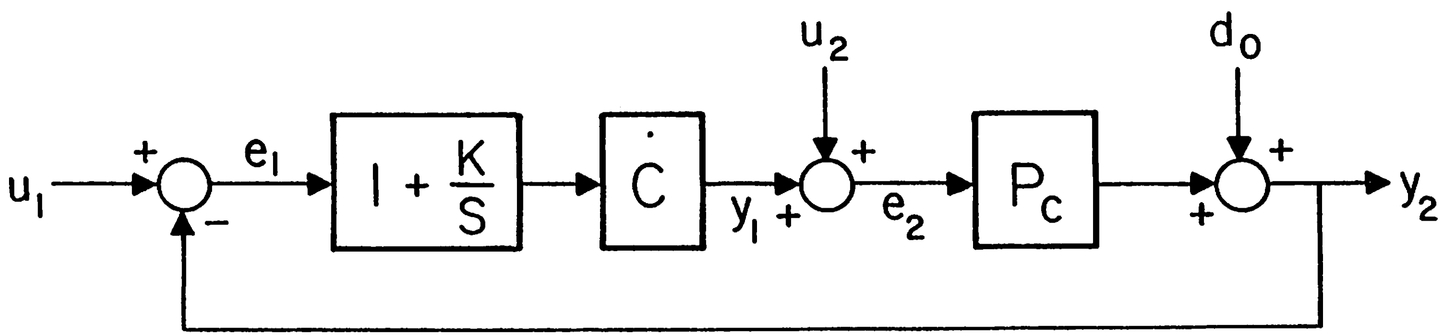

Fig. 2

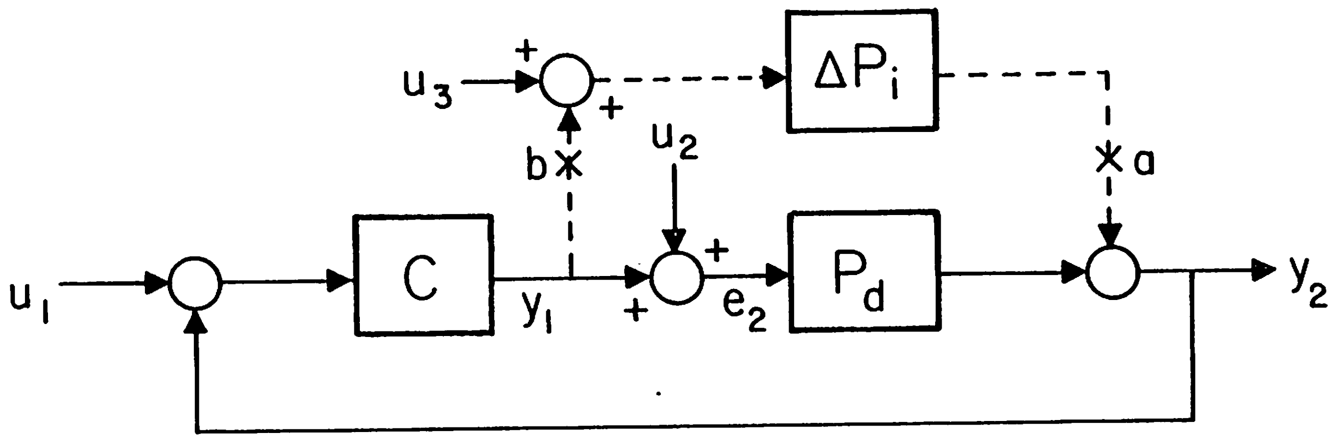

Fig. 3

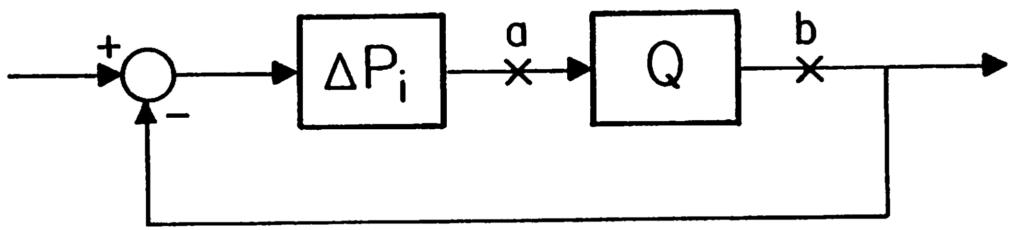

Fig. 4