STABILITY STUDIES OF MULTIMACHINE POWER SYSTEMS WITH THE EFFECTS OF AUTOMATIC VOLTAGE REGULATORS

by

H. Miyagi and A. R. Bergen

Memorandum No. UCB/ERL M84/39

14 March 1984

ELECTRONICS RESEARCH LABORATORY

College of Engineering
University of California, Berkeley
94720

Stability Studies of Multimachine Power Systems with the Effects of Automatic Voltage Regulators

H. Miyagi* and A. R. Bergen

Department of Electrical Engineering and Computer Sciences
and the Electronics Research Laboratory
University of California, Berkeley, CA 94720

Abstract

The stability of power systems including the effects of automatic voltage regulators is studied via Lyapunov's direct method. The multivariable Popov criterion developed by Moore and Anderson is employed in constructing a Luré type function for a power system consisting of synchronous machines interconnected by a lossless transmission system. The Lyapunov function constructed for the system with only flux decay is regarded as a special case of the result obtained in this paper. The effect of an automatic voltage regulator on power system stability is illustrated by numerical examples, showing some shifts of equipotential curves which are due to changes of voltage regulator gain.

*On leave from the Department of Electronic Engineering and Computer Sciences, Ryukyu University, Nishihara, Okinawa, Japan.

I. Introduction

In power system stability analysis, the direct method of Lyapunov has been utilized by many authors over the last two decades. In their work, several researchers [1]-[8] have discussed power systems, taking into account the effects of control apparatus such as velocity governors (GOV) or automatic voltage regulators (AVR). In spite of its important role in stability, however, there have been comparatively few attempts to consider the effects of AVRs while much of the literature has dealt with GOV. Pai and Rai [6] have obtained a Lyapunov function for a power system considering simple voltage regulator action. For the system including an AVR in a feedback loop, another result has been reported in [8]. However these papers have dealt with single-machine systems. Multimachine systems considering the effects of AVR dynamics have not been discussed.

More recently, Kakimoto et al. [9] have modified the Moore-Anderson theorem [10] according to Desoer-Wu's condition [11] for the stability criteria. The result was applied to a multimachine power system with field flux decays, and a Luré type Lyapunov function was constructed with the method established by Willems [12] and other researchers [13] [14]. In their method, however, one has to choose the function V_1, which is an integration of the nonlinear part of the system. The choice of this function is somewhat intuitive and is not readily extended to systems with AVR dynamics or to other more complex systems.

In this paper, the function V_1 is uniquely obtained by determining an arbitrary matrix $\underline{Q}$ first, namely, all elements of the matrix $\underline{Q}$ are determined so that the curl equations may hold for the nonlinear components of the system given [7]. Once $\underline{Q}$ is determined, V_1 will be given by a line integral of the nonlinear components and is independent of the path of integration. Thus a Luré type Lyapunov function is constructed for the multimachine power system with AVR. The linear part of the Luré type function is determined by the procedure developed by Willems [12].

We note the voltage regulator modifies the original behavior of the system, hence, when we discuss the generator with AVR, we should pay attention to the effect of installation of this control system on the stability of the original system. A poor design will sometimes violate the stability of the original system. The Moore-Anderson theorem yields extra conditions for the stability of the system, in connection with the installation of AVR.

The organization of the paper is as follows. In Section II, we present the dynamic equations of the system, taking into account flux decays and AVR. The AVR dynamics with feedback loop are approximated by first-order responses. In Section III, the stability criteria are given according to the Moore-Anderson theorem, including stability criteria which are due to installations of AVR. The nonlinear part of the Luré type Lyapunov function, i.e., $V_1(\underline{\sigma})$, is also determined in this section, being concerned with Desoer-Wu's condition. The linear part of the Lyapunov function is derived in Section IV by solving matrix equations. The procedure established by Willems is used for determining the unknown matrices. In Section V, we discuss critical values of the Lyapunov function. A discussion related to the choice of some coefficients appearing in the Lyapunov function is also given. Section VI gives examples to show the effect of an AVR on power system stability. The final Section VII summarizes the main results of the paper and briefly outlines several extensions to be developed in future work.

II. Dynamic Model of Power System

In this section, we consider a power system consisting of n synchronous machines including the effects of field flux decays and AVR action. The synchronous machines are modeled by internal voltages which are connected to the machine terminals via transient reactances. A simplifying assumption is that the quadrature reactances are equal to the transient reactances, in which case, neglecting resistances, a dynamic model of the ith machine is given by [16]

$$M_i \frac{d^2\delta_i}{dt^2} + D_i \frac{d\delta_i}{dt} = P_{mi} - \sum_{j=1}^{n} B_{ij} E_i E_j \sin \delta_{ij}$$

$$T'_{doi} \frac{dE_i}{dt} = E_{exi} - E_i + (x_{di} - x'_{di}) I_{di} \tag{1}$$

the AVR is modeled by a linear 1st-order response as follows:

$$T_{vi} \frac{dE_{exi}}{dt} = (E^o_{exi} - E_{exi}) - \mu_i (V_{ti} - V^o_{ti}) \tag{2}$$

In (1) and (2)

δ_i: machine internal voltage phase

δ_{ij}: $\delta_i - \delta_j$

E_i: machine internal voltage magnitude

E_{exi}: excitation voltage magnitude

E^o_{exi}: E_{exi} at equilibrium

V_{ti}: terminal voltage magnitude

V^o_{ti}: V_{ti} at equilibrium

T'_{doi}: open circuit time constant of machine

T_{vi}: time constant of voltage regulating system

μ_i: feedback gain of voltage regulating system

I_{di}: d-axis current

B_{ij}: susceptance parameters of reduced system

In this system, some of the machines may act as motors while the rest act as generators. Despite this fact, we simplify by supposing each machine has an AVR system. In addition, as the terminal voltage has a root square form of direct and quadrature components, a feature which frustrates the proposed analytical method of constructing Lyapunov functions, we assume simply $V_{ti} \simeq k_i V_{tqi}$, where k_i is a constant [8]. This assumption is reasonable for a lightly loaded system.

Under these assumptions and taking into account the condition that $d^2\delta_i/dt^2$, $d\delta_i/dt$, dE_i/dt and dE_{exi}/dt are equal to zero at equilibrium, the system dynamics (1) and (2) may be rewritten as

$$
\begin{aligned}
& M_i \frac{d^2\delta_i}{dt^2} + D_i \frac{d\delta_i}{dt} = \sum_{j=1}^{n} B_{ij}\{E_i^o E_j^o \sin \delta_{ij}^o - E_i E_j \sin \delta_{ij}\} \\
& \gamma_i \frac{dE_i}{dt} = - \alpha_i (E_i - E_i^o) + \beta_i (E_{exi} - E_{exi}^o) \\
& \qquad - \sum_{\substack{j=1 \\ j\neq i}}^{n} B_{ij}\{E_j^o \cos \delta_{ij}^o - E_j \cos \delta_{ij}\} \qquad\qquad (3) \\
& \xi_i \frac{dE_{exi}}{dt} = - \phi_i (E_{exi} - E_{exi}^o) - \eta_i (E_i - E_i^o) \\
& \qquad + \sum_{\substack{j=1 \\ j\neq i}}^{n} B_{ij}\{E_j^o \cos \delta_{ij}^o - E_j \cos \delta_{ij}\}
\end{aligned}
$$

where the superscript "o" denotes the value at the equilibrium state and

$$\alpha_i = \frac{1}{x_{di}-x'_{di}} - B_{ii}\ , \qquad \beta_i = \frac{1}{x_{di}-x'_{di}}$$

$$\gamma_i = \frac{T'_{doi}}{x_{di}-x'_{di}}\ , \qquad \eta_i = \frac{1}{x'_{di}} + Y_{ii} \tag{4}$$

$$\phi_i = \frac{1}{\mu_i k_i x'_{di}}\ , \qquad \xi_i = \frac{T_{vi}}{\mu_i k_i x'_{di}}$$

We define $\Delta\delta_i \triangleq \delta_i-\delta_i^o$, $\omega_i \triangleq \Delta\dot{\delta}_i$, $\Delta E_i \triangleq E_i-E_i^o$ and $\Delta E_{exi} \triangleq E_{exi}-E_{exi}^o$ for $i = 1,2,\cdots,n$, and $\Delta\theta_i \triangleq \Delta\delta_i-\Delta\delta_n$ for $i = 1,2,\cdots,n-1$, choosing the nth machine as reference. Furthermore, we define a (4n-1) vector $\underline{x}$, (m+n) vector $\underline{\sigma}$ and (m+n) vector $\underline{F}(\underline{\sigma})$ as

$$\underline{x} \triangleq \begin{bmatrix} \Delta\underline{\theta} \\ \underline{\omega} \\ \Delta\underline{E} \\ \Delta\underline{E}_{ex} \end{bmatrix}\ , \qquad \underline{\sigma} \triangleq \begin{bmatrix} \underline{\sigma}_1 \\ \underline{\sigma}_2 \end{bmatrix} \tag{5}$$

$$\underline{F}(\underline{\sigma}) \triangleq \begin{bmatrix} \underline{f}_1(\underline{\sigma}) \\ \underline{f}_2(\underline{\sigma}) \end{bmatrix}$$

where $\underline{\sigma}_1$ and $\underline{f}_1(\underline{\sigma})$ are m (=n(n-1)/2) vectors, $\underline{\sigma}_2$ and $\underline{f}_2(\underline{\sigma})$ are n vectors and

$$\sigma_{1k} = \Delta\delta_{ij}$$

$$\sigma_{2k} = \Delta E_i$$

$$f_{1k} = B_{ij}\{(E_i^0+\Delta E_i)(E_j^0+\Delta E_j)\sin(\delta_{ij}^0+\Delta\delta_{ij}) - E_i^0E_j^0 \sin \delta_{ij}^0\}$$

$$f_{2k} = \sum_{\substack{j=1 \\ j\neq i}}^{n} B_{ij}\{E_j^0 \cos \delta_{ij}^0 - (E_j^0+\Delta E_j)\cos(\delta_{ij}^0+\Delta\delta_{ij})\} \tag{6}$$

Then the state model is given by

$$\begin{aligned} \dot{\underline{x}} &= \underline{A}\,\underline{x} - \underline{B}\,\underline{F}(\underline{\sigma}) \\ \underline{\sigma} &= \underline{C}^T\underline{x} \end{aligned} \tag{7}$$

where

$$\underline{A} = \begin{bmatrix} \underline{0} & \underline{K}^T & \underline{0} & \underline{0} \\ \underline{0} & -\underline{M}^{-1}\underline{D} & \underline{0} & \underline{0} \\ \underline{0} & \underline{0} & -\underline{\gamma}^{-1}\underline{\alpha} & \underline{\gamma}^{-1}\underline{\beta} \\ \underline{0} & \underline{0} & -\underline{\xi}^{-1}\underline{\eta} & -\underline{\xi}^{-1}\underline{\phi} \end{bmatrix}, \quad \underline{B} = \begin{bmatrix} \underline{0} & \underline{0} \\ \underline{M}^{-1}\underline{T} & \underline{0} \\ \underline{0} & \underline{\gamma}^{-1} \\ \underline{0} & -\underline{\xi}^{-1} \end{bmatrix}$$

$$\underline{C}^T = \begin{bmatrix} \underline{G}^T & \underline{0} & \underline{0} & \underline{0} \\ \underline{0} & \underline{0} & \underline{I} & \underline{0} \end{bmatrix}, \qquad \underline{K} = \begin{bmatrix} \underline{I} \\ -\underline{e}^T \end{bmatrix} \tag{8}$$

$$\underline{T} = \begin{bmatrix} \underline{e}^T & \underline{0} \\ -\underline{I} & \underline{T}' \end{bmatrix}, \qquad \underline{G} = \begin{bmatrix} \underline{e}^T & \underline{I} & \underline{0} \\ -\underline{I} & \underline{0} & \underline{G}' \end{bmatrix}$$

in which $\underline{A}$ is a (4n-1)x(4n-1) matrix, $\underline{B}$ is a (4n-1)x(n+m) matrix, $\underline{C}$ is a (4n-1)x(n+m) matrix, $\underline{K}$ is a nx(n-1) matrix, $\underline{T}$ is a nxm matrix, $\underline{G}$ is a (n-1)xm matrix, $\underline{T}'$ is a (n-1)x(m-n+1) matrix, $\underline{G}'$ is a (n-2)x(m-n+1) matrix, $\underline{e}$ is a (n-1) vector with unity entries and $\underline{M}$, $\underline{D}$, $\underline{\alpha}$, $\underline{\beta}$, $\underline{\gamma}$, $\underline{\eta}$, $\underline{\phi}$ and $\underline{\xi}$ are nxn diagonal matrices. We notice a relationship $\underline{T} = \underline{K}\,\underline{G}$.

III. Stability Criteria of the System

The transfer function of the linear part of the system (7) relating

$\underline{\sigma}$ to $-\underline{F}(\underline{\sigma})$ is given by

$$\underline{W}(s) = \underline{C}^T(s\underline{I}-\underline{A})^{-1}\underline{B}$$

$$= \begin{bmatrix} \frac{1}{s}\,\underline{T}^T(s\;\underline{I}+\underline{M}^{-1}\underline{D})^{-1}\underline{M}^{-1}\underline{T} & \underline{0} \\ \underline{0} & \underline{U}_1\underline{\gamma}^{-1} - \underline{U}_2\underline{\xi}^{-1} \end{bmatrix} \tag{9}$$

where

$$\underline{U}_1 = \text{diag}\left[\frac{\frac{\xi_i}{\eta_i}\,(s+\frac{\phi_i}{\xi_i})}{(s+\frac{\alpha_i}{\gamma_i})(s+\frac{\phi_i}{\xi_i})\;\frac{\xi_i}{\eta_i}+\frac{\beta_i}{\gamma_i}}\right]$$

$$\underline{U}_2 = \text{diag}\left[\frac{\frac{\xi_i\beta_i}{\eta_i\gamma_i}}{(s+\frac{\alpha_i}{\gamma_i})(s+\frac{\phi_i}{\xi_i})\;\frac{\xi_i}{\eta_i}+\frac{\beta_i}{\gamma_i}}\right] \tag{10}$$

The nonlinearity $\underline{F}(\underline{\sigma})$ is assumed to satisfy the following conditions [11]:

(i) $\underline{F}(\underline{\sigma})$ is continuous, and maps R^{m+n} into R^{m+n}

(ii) For some constant real symmetric matrix $\underline{N}$,

$$\underline{F}^T(\underline{\sigma})\;\underline{N}\;\underline{\sigma} \geq 0 \quad \text{for all } \underline{\sigma} \in R^{m+n}$$

and

$$\underline{F}(\underline{\sigma}) = \underline{0} \qquad \text{if } \underline{\sigma} = \underline{0}$$

(iii) There is a scalar function $V_1(\underline{\sigma})$ such that $V_1(\underline{\sigma}) \geq 0$ for all $\underline{\sigma} \in R^{m+n}$, $V_1(\underline{\sigma}) = 0$ for $\underline{\sigma} = \underline{0}$ and for some constant real matrix $\underline{Q}$

$$\nabla V_1(\underline{\sigma}) = \underline{Q}^T\;\underline{F}(\underline{\sigma}) \quad \text{for all } \underline{\sigma} \in R^{m+n}$$

Using the above conditions in [11], we obtain a version of the Moore-Anderson theorem as follows:

Theorem

If there exist real matrices $\underline{N}$ and $\underline{Q}$ such that

$$\underline{Z}(s) = (\underline{N}+\underline{Q}s)\underline{W}(s) \tag{11}$$

is positive real, then, assuming there are no pole-zero cancellations, (i) there exist real matrices $\underline{P}$, $\underline{L}$, $\underline{W}_0$ with $\underline{P}$ positive definite and symmetric, satisfying

$$\underline{P}\,\underline{A} + \underline{A}^T\underline{P} = -\underline{L}\,\underline{L}^T \tag{12}$$

$$\underline{P}\,\underline{B} = \underline{C}\,\underline{N}^T + \underline{A}^T\underline{C}\,\underline{Q}^T - \underline{L}\,\underline{W}_0 \tag{13}$$

$$\underline{W}_0^T\underline{W}_0 = \underline{Q}\,\underline{C}^T\underline{B} + \underline{B}^T\underline{C}\,\underline{Q}^T \tag{14}$$

and

(II) these matrix relations along with (i), (ii) and (iii) may be used to establish that the null solution of (7) is asymptotically stable in the large. □

To establish this result we use the Lyapunov function

$$V(\underline{x}) = \frac{1}{2}\,\underline{x}^T\underline{P}\,\underline{x} + V_1(\underline{\sigma}) \tag{15}$$

with

$$\dot{V}(\underline{x}) = -\frac{1}{2}(\underline{x}^T\underline{L}-\underline{F}^T\underline{W}_0^T)(\underline{L}^T\underline{x}-\underline{W}_0\underline{F}) - \underline{x}^T\underline{C}\,\underline{N}^T\underline{F} \tag{16}$$

Note that under the conditions stated we obtain a global stability result. For our application, however, (ii) and (iii) are not satisfied globally and we will use $V(\underline{x})$ to estimate a region of asymptotic stability.

In this paper, $\underline{Q}$, which is required to satisfy Desoer-Wu's condition (iii), is specified so that $\nabla V_1(\underline{\sigma}) = \underline{Q}^T\underline{F}(\underline{\sigma})$ satisfies the curl equations

$$\frac{\partial[\underline{Q}^T\underline{F}(\underline{\sigma})]_i}{\partial\sigma_j} = \frac{\partial[\underline{Q}^T\underline{F}(\underline{\sigma})]_j}{\partial\sigma_i} \tag{17}$$

In our case (17) implies the result

$$\underline{Q} = q\,\underline{I} \tag{18}$$

where q is an arbitrary constant. Thus we can obtain $V_1(\underline{\sigma})$ by a line integral which is independent of the path as

$$\begin{aligned} V_1(\underline{\sigma}) &= \int_{\underline{0}}^{\underline{\sigma}} [\underline{Q}^T\,\underline{F}(\underline{\sigma})]^T d\underline{\sigma} \\ &= q\int_{\underline{0}}^{\underline{\sigma}_1} \underline{f}_1^T\, d\underline{\sigma}_1 + q\int_{\underline{0}}^{\underline{\sigma}_2} \underline{f}_2^T\, d\underline{\sigma}_2 \\ &= q\sum_{i=1}^{n-1}\sum_{j=i+1}^{n} B_{ij}[(E_i^0+\Delta E_i)(E_j^0+\Delta E_j)\{\cos\delta_{ij}^0 - \cos(\delta_{ij}^0+\Delta\delta_{ij})\} \\ &\quad - \Delta E_i\Delta E_j\cos\delta_{ij}^0 - \Delta\delta_{ij}E_i^0E_j^0\sin\delta_{ij}^0] \end{aligned} \tag{19}$$

Now let us define $\underline{N}$ with a scalar constant n as

$$\underline{N} \doteq \begin{bmatrix} n\underline{I} & \underline{0} \\ \underline{0} & \underline{0} \end{bmatrix} \tag{20}$$

Desoer-Wu's condition (ii) with this $\underline{N}$ implies $n\,\underline{\sigma}_1^T\,\underline{f}_1(\underline{\sigma}) \geq 0$. In this particular problem, however, the term $\underline{\sigma}_1^T\,\underline{f}_1(\underline{\sigma})$ can have negative values around the origin as pointed out by Pai and Rai [6]. Hence it is desirable to remove this condition, letting n=0. On the other hand, however, this selection of n may lead to a pole-zero cancellation between $(\underline{N}+\underline{Q}s)$ and $\underline{W}(s)$. In order to avoid the pole-zero cancellation, we give n a nonzero value in constructing the Lyapunov function, and after the function is obtained we let n=0, according to the procedure given in [9]. Substituting (18) and (20) into (11) gives

$$\underline{Z}(s) = \begin{bmatrix} \underline{Z}_1(s) & \underline{0} \\ \underline{0} & \underline{Z}_2(s) \end{bmatrix}$$

$$= \begin{bmatrix} (n+qs)\,\frac{1}{s}\,\underline{T}^T(s\underline{I}+\underline{M}^{-1}\underline{D})^{-1}\underline{M}^{-1}\underline{T} & \underline{0} \\ \underline{0} & qs(\underline{U}_1\underline{\gamma}^{-1}-\underline{U}_2\underline{\xi}^{-1}) \end{bmatrix} \tag{21}$$

The conditions for $\underline{Z}(s)$ to be positive real are:

(1) $\underline{Z}(s)$ has elements which are analytic for $Re(s) > 0$

(2) $\underline{Z}^*(s) = \underline{Z}(s^*)$ for $Re(s) > 0$

(3) $\underline{Z}^T(s^*) + \underline{Z}(s)$ is positive semi-definite for $Re(s) > 0$

The first two conditions clearly hold. In this case condition (3) will be satisfied if both $\underline{Z}_1^T(s^*) + \underline{Z}_1(s)$ and $\underline{Z}_2^T(s^*) + \underline{Z}_2(s)$ are positive semi-definite for $Re(s) > 0$.

We have

$$\begin{aligned} \underline{Z}_1^T(s^*) + \underline{Z}_1(s) &= 2\underline{T}^T\left[\mathrm{diag}\left\{\frac{qD_i-nM_i}{M_i^2\omega^2+D_i^2}\right\}\right]\underline{T} \\ \underline{Z}_2^T(s^*) + \underline{Z}_2(s) &= 2q\ \mathrm{diag}\left[\frac{\omega^2\{\psi_1\omega^2+\psi_2\psi_3-\psi_1\psi_4\}}{(\psi_4-\omega^2)^2+\psi_3^2\omega^2}\right] \end{aligned} \tag{22}$$

where

$$\psi_1 = \frac{1}{\gamma_i}\ , \quad \psi_2 = \frac{(\phi_i-\beta_i)}{\gamma_i\xi_i} \tag{23}$$

$$\psi_3 = \frac{\alpha_i}{\gamma_i}+\frac{\phi_i}{\xi_i}\ , \quad \psi_4 = \frac{\alpha_i\phi_i+\eta_i\beta_i}{\gamma_i\xi_i}$$

Hence if there exist constants

$$q > n\,\frac{M_i}{D_i} \qquad \text{for all } i \tag{24}$$

and

$$\gamma_i > 0$$

$$\phi_i - \beta_i > \frac{\beta_i \xi_i}{\gamma_i \phi_i}(\alpha_i + \eta_i) \quad \text{for all } i \tag{25}$$

$\underline{Z}^T(s^*) + \underline{Z}(s)$ is positive semi-definite, which implies $\underline{Z}(s)$ is positive real. This means the system is asymptotically stable under the conditions (24) and (25). The extra condition (25) is due to the installation of the AVR.

IV. Lyapunov Function

In our case, it is not enough to simply find conditions guaranteeing the existence of a Lyapunov function. We need to find the function to estimate the region of asymptotic stability. For this purpose we use the Willems technique to find the matrix $\underline{P}$ which satisfies (12), (13) and (14).

Let $\underline{P}$, $\underline{L}$ and $\underline{W}_0$ be partitioned as

$$\underline{P} = \begin{bmatrix} \underline{P}_{11} & \underline{P}_{12} & \underline{P}_{13} & \underline{P}_{14} \\ \underline{P}_{21} & \underline{P}_{22} & \underline{P}_{23} & \underline{P}_{24} \\ \underline{P}_{31} & \underline{P}_{32} & \underline{P}_{33} & \underline{P}_{34} \\ \underline{P}_{41} & \underline{P}_{42} & \underline{P}_{43} & \underline{P}_{44} \end{bmatrix}, \quad \underline{L} = \begin{bmatrix} \underline{L}_{11} & \underline{L}_{12} \\ \underline{L}_{21} & \underline{L}_{22} \\ \underline{L}_{31} & \underline{L}_{32} \\ \underline{L}_{41} & \underline{L}_{42} \end{bmatrix} \tag{26}$$

$$\underline{W}_0 = \begin{bmatrix} \underline{W}_{11} & \underline{W}_{12} \\ \underline{W}_{21} & \underline{W}_{22} \end{bmatrix}$$

Then as a consequence (14) leads to

$$\underline{W}_{11} = \underline{0} \quad , \underline{W}_{21} = \underline{0} \tag{27-1}$$

$$\underline{W}_{12}^T\underline{W}_{12} + \underline{W}_{22}^T\underline{W}_{22} = 2q\underline{\gamma}^{-1} \tag{27-2}$$

From (13), we have

$$\underline{P}_{32} = \underline{P}_{23}^T = \underline{0}, \quad \underline{P}_{42} = \underline{P}_{24}^T = \underline{0} \tag{28-1}$$

$$\underline{P}_{13}\underline{\gamma}^{-1} - \underline{P}_{14}\underline{\xi}^{-1} = -(\underline{L}_{11}\underline{W}_{12}+\underline{L}_{12}\underline{W}_{22}) \tag{28-2}$$

$$\underline{L}_{21}\underline{W}_{12} + \underline{L}_{22}\underline{W}_{22} = \underline{0} \tag{28-3}$$

$$\underline{P}_{12}\underline{M}^{-1}\underline{T} = n\underline{G} \tag{28-4}$$

$$\underline{P}_{22}\underline{M}^{-1}\underline{T} = q\underline{K}\ \underline{G} \tag{28-5}$$

$$\underline{P}_{33}\underline{\gamma}^{-1} - \underline{P}_{34}\underline{\xi}^{-1} = -q(\underline{\gamma}^{-1}\underline{\alpha})^T - (\underline{L}_{31}\underline{W}_{12}+\underline{L}_{32}\underline{W}_{22}) \tag{28-6}$$

$$\underline{P}_{43}\underline{\gamma}^{-1} - \underline{P}_{44}\underline{\xi}^{-1} = q(\underline{\gamma}^{-1}\underline{\beta})^T - (\underline{L}_{41}\underline{W}_{12}+\underline{L}_{42}\underline{W}_{22}) \tag{28-7}$$

Furthermore we obtain the following relations from

$$\underline{L}_{11} = \underline{0}, \ \underline{L}_{12} = \underline{0}, \ \underline{P}_{13} = \underline{P}_{31}^T = \underline{0}, \ \underline{P}_{14} = \underline{P}_{41}^T = \underline{0} \tag{29-1}$$

$$\underline{P}_{11}\underline{K}^T - \underline{P}_{12}\underline{M}^{-1}\underline{D} = \underline{0} \tag{29-2}$$

$$\underline{K}\ \underline{P}_{12} - (\underline{M}^{-1}\underline{D})^T\underline{P}_{22} + \underline{P}_{21}\underline{K}^T - \underline{P}_{22}\underline{M}^{-1}\underline{D} = -(\underline{L}_{21}\underline{L}_{21}^T+\underline{L}_{22}\underline{L}_{22}^T) \tag{29-3}$$

$$-\underline{P}_{33}\underline{\gamma}^{-1}\underline{\alpha} - \underline{P}_{34}\underline{\xi}^{-1}\underline{\eta} - (\underline{\gamma}^{-1}\underline{\alpha})^T\underline{P}_{33} - (\underline{\xi}^{-1}\underline{\eta})^T\underline{P}_{43} = -(\underline{L}_{31}\underline{L}_{31}^T+\underline{L}_{32}\underline{L}_{32}^T) \tag{29-4}$$

$$-\underline{P}_{43}\gamma^{-1}\underline{\alpha} - \underline{P}_{44}\underline{\xi}^{-1}\underline{\eta} + (\underline{\gamma}^{-1}\underline{\beta})^T\underline{P}_{33} - (\underline{\xi}^{-1}\underline{\phi})^T\underline{P}_{43} = -(\underline{L}_{41}\underline{L}_{31}^T+\underline{L}_{42}\underline{L}_{32}^T) \tag{29-5}$$

$$\underline{P}_{43}\underline{\gamma}^{-1}\underline{\beta} - \underline{P}_{44}\underline{\xi}^{-1}\underline{\phi} + (\underline{\gamma}^{-1}\underline{\beta})^T\underline{P}_{34} - (\underline{\xi}^{-1}\underline{\phi})^T\underline{P}_{44} = -(\underline{L}_{41}\underline{L}_{41}^T+\underline{L}_{42}\underline{L}_{42}^T) \tag{29-6}$$

We also have

$$\underline{L}_{31}\underline{L}_{21}^{T} + \underline{L}_{32}\underline{L}_{22}^{T} = \underline{0}$$

$$\underline{L}_{41}\underline{L}_{21}^{T} + \underline{L}_{42}\underline{L}_{22}^{T} = \underline{0} \tag{29-7}$$

$$\underline{L}_{21}\underline{L}_{31}^{T} + \underline{L}_{22}\underline{L}_{32}^{T} = \underline{0}$$

$$\underline{L}_{21}\underline{L}_{41}^{T} + \underline{L}_{22}\underline{L}_{42}^{T} = \underline{0}$$

which may be satisfied with

$$\underline{L}_{31} = \underline{0}, \quad \underline{L}_{41} = \underline{0}, \quad \underline{L}_{22} = \underline{0} \tag{30}$$

Thus, the solution of (12), (13) and (14) reduces to the solution of (27), (28), and (29).

Now, substituting (30) into (28-3) gives $\underline{L}_{21}\underline{W}_{12} = \underline{0}$ which leads to $\underline{W}_{12} = \underline{0}$.

Solving (28-4), (28-5), and (29-2), we obtain well-known results [14] [15] [16] .

$$\underline{P}_{11} = \rho \underline{D}_{n-1} \underline{1} \underline{D}_{n-1} + n \underline{D}_{n-1}$$

$$\underline{P}_{12} = \underline{P}_{11}\underline{K}^{T}\underline{D}^{-1}\underline{M} \tag{31}$$

$$\underline{P}_{22} = q\underline{M} + u\underline{M}\,\underline{1}\,\underline{M}$$

where $\underline{D}_{n-1}$ is the matrix $\underline{D}$ with nth row and nth column deleted, ρ and μ are scalar constants and $\underline{1}$ is a matrix with all elements equal to 1. Substituting (31) into (29-3), we have the matrix inequality

$$2(-\underline{D} + \frac{n}{q}\underline{M}) - u'(\underline{M}\,\underline{1}\,\underline{D} + \underline{D}\,\underline{1}\,\underline{M}) \leq 0 \tag{32}$$

where $u' = (u-\rho)/q$

The determinant of the left side of (32) is a quadratic in u'. Hence (32) will be satisfied if u' lies between the two roots of this quadratic equation given by [14]

$$(u')^2 \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \frac{(D_i M_j - D_j M_i)^2}{4(D_i - \frac{n}{q} M_i)(D_j - \frac{n}{q} M_j)}$$

$$- u' \sum_{i=1}^{n} \frac{D_i M_i}{D_i - \frac{n}{q} M_i} - 1 = 0 \tag{33}$$

Next we determine $\underline{P}_{33}$, $\underline{P}_{34}$, and $\underline{P}_{44}$, by solving (27-2), (28-6), (28-7), (29-4)-(29-6).

Substituting (28-6) and (28-7) into (29-4) and (29-6), respectively, we obtain the following equations:

$$(\underline{L}_{32}+\underline{\alpha}^T\underline{W}_{22}^T)(\underline{L}_{32}^T+\underline{W}_{22}\underline{\alpha}) - (\underline{P}_{34}\underline{\xi}^{-1})(\underline{\alpha}+\underline{\eta}) - (\underline{\alpha}+\underline{\eta})^T(\underline{P}_{34}\underline{\xi}^{-1})^T = \underline{0} \tag{34}$$

and

$$(\underline{L}_{42}-\underline{\beta}^T\underline{W}_{22}^T)(\underline{L}_{42}^T-\underline{W}_{22}\underline{\beta}) - (\underline{P}_{44}\underline{\xi}^{-1})(\underline{\phi}-\underline{\beta}) - (\underline{\phi}-\underline{\beta})^T(\underline{P}_{44}\underline{\xi}^{-1})^T = \underline{0} \tag{35}$$

In addition, substituting (28-6) and (28-7) into (29-5) gives

$$(\underline{L}_{42}-\underline{\beta}^T\underline{W}_{22}^T)(\underline{L}_{32}^T+\underline{W}_{22}\underline{\alpha}) - (\underline{P}_{44}\underline{\xi}^{-1})(\underline{\alpha}+\underline{\eta}) - (\underline{\alpha}-\underline{\beta})^T(\underline{P}_{34}\underline{\xi}^{-1})^T = \underline{0} \tag{36}$$

In these derivations, one should remember that $\underline{P}$ is a symmetric matrix and $2q\underline{\gamma}^{-1} = \underline{W}_{22}^T\underline{W}_{22}$ which is the solution given by (27-2)

We define

$$\begin{aligned} \underline{H}_1 &\triangleq \underline{L}_{32} + \underline{\alpha}^T\underline{W}_{22}^T \\ \underline{H}_2 &\triangleq \underline{L}_{42} - \underline{\beta}^T\underline{W}_{22}^T \\ \underline{\Phi}_1 &\triangleq \underline{P}_{34}\underline{\xi}^{-1} \\ \underline{\Phi}_2 &\triangleq \underline{P}_{44}\underline{\xi}^{-1} \end{aligned} \tag{37}$$

Now it is convenient to assume $\underline{\Phi}_1(\underline{\alpha}+\underline{\eta})$, $\underline{\Phi}_2(\underline{\phi}-\underline{\beta})$ and $\underline{H}_2\underline{H}_1^T(\underline{\alpha}+\underline{\eta})^{-1}(\underline{\phi}-\underline{\beta})$ are symmetric matrices. Then (34) and (35) reduce to

$$\begin{aligned}\underline{\Phi}_1 &= \frac{1}{2}\,\underline{H}_1\underline{H}_1^T(\underline{\alpha}+\underline{\eta})^{-1}\\ \underline{\Phi}_2 &= \frac{1}{2}\,\underline{H}_2\underline{H}_2^T(\underline{\alpha}-\underline{\beta})^{-1}\end{aligned} \tag{38}$$

Substituting (38) into (36) gives

$$\frac{1}{2}\,[\underline{H}_2 - \{(\underline{\alpha}+\underline{\eta})^{-1}(\underline{\phi}-\beta)\}^T\underline{H}_1][\underline{H}_2 - \{(\underline{\alpha}+\underline{\eta})^{-1}(\underline{\phi}-\underline{\beta})\}^T\underline{H}_1]^T = \underline{0} \tag{39}$$

which leads to

$$\underline{H}_2 = \{(\underline{\alpha}+\underline{\eta})^{-1}(\underline{\phi}-\underline{\beta})\}^T\underline{H}_1 \tag{40}$$

The above equation gives the relationship between $\underline{L}_{32}$ and $\underline{L}_{42}$.

Next we derive the relationship between $\underline{P}_{44}$ and $\underline{P}_{34}$, by substituting (34) and (40) into (35). Thus we obtain

$$\underline{\Phi}_2 = \{(\underline{\alpha}+\underline{\eta})^{-1}(\underline{\phi}-\underline{\beta})\}^T\underline{\Phi}_1 \tag{41}$$

Substituting (34), (40), and (41) into (28-7), we can obtain the following equation with respect to $\underline{H}_1$:

$$\begin{aligned}(\underline{\phi}-\underline{\beta})^T[\underline{H}_1^T(\underline{\alpha}+\underline{\eta})^{-1}]^T[\underline{H}_1^T(\underline{\alpha}+\underline{\eta})^{-1}] - \underline{\xi}^T[\underline{H}_1^T(\underline{\alpha}+\underline{\eta})^{-1}]^T[\underline{H}_1^T(\underline{\alpha}+\underline{\eta})^{-1}](\underline{\alpha}+\underline{\eta})\underline{\gamma}^{-1}\\ - 2(\underline{\phi}-\underline{\beta})^T[\underline{H}_1^T(\underline{\alpha}+\underline{\eta})^{-1}]^T\underline{W}_{22} - \underline{\beta}^T\underline{W}_{22}^T\underline{W}_{22} = \underline{0}\end{aligned} \tag{42}$$

In order to solve (42) easily, we will assume $\underline{L}_{32}$ and $\underline{L}_{42}$ are diagonal matrices which implies $\underline{H}_1$ and $\underline{H}_2$ are also diagonal matrices. Thus we have

$$\begin{aligned}\underline{L}_{32} &= \mathrm{diag}\{\ell_{1k}\}\\ \underline{L}_{42} &= \mathrm{diag}\{\ell_{2k}\}\\ \underline{H}_1 &= \mathrm{diag}\{h_{1k}\}\\ \underline{H}_2 &= \mathrm{diag}\{h_{2k}\}\end{aligned} \tag{43}$$

Solving (42) with respect to h_{1k}, we can obtain

$$h_{1k} = \frac{w_k[(\phi_k-\beta_k) \pm \sqrt{\phi_k\{(\phi_k-\beta_k) - \frac{\beta_k\xi_k}{\phi_k\gamma_k}(\alpha_k+\eta_k)\}}]}{\frac{\phi_k-\beta_k}{\alpha_k+\eta_k} - \frac{\xi_k}{\gamma_k}} \tag{44}$$

for $k = 1,2,\cdots,n$, where $w_k = \sqrt{2q/\gamma_k}$.

In order for h_{1k} to be a real number, it is required that $\phi_k-\beta_k > \beta_k\xi_k(\alpha_k+\eta_k)/(\phi_k\gamma_k)$. This requirement is identical to the condition that the positive real matrix $\underline{Z}(s)$ exists for the system, as shown in inequality (25). Although we have two solutions h^+_{1k} and h^-_{1k} in (44), $h^-_{1k} = \min|h_{1k}|$ is chosen. The reason for this choice will be given later.

From (34), we obtain

$$\underline{P}_{34} = \frac{1}{2}\,\mathrm{diag}\left[\frac{\xi_k}{\alpha_k+\eta_k}\,h^2_{1k}\right] \tag{45}$$

Using (37), (41), and (45), $\underline{P}_{44}$ is given as

$$\underline{P}_{44} = \frac{1}{2}\,\mathrm{diag}\left[\frac{\xi_k(\phi_k-\beta_k)}{(\alpha_k+\eta_k)^2}\,h^2_{1k}\right] \tag{46}$$

Finally using (28-6), (37), (42) and (45), we can derive $\underline{P}_{33}$ as

$$\underline{P}_{33} = \frac{1}{2}\,\mathrm{diag}\left[\frac{\xi_k}{\phi_k-\beta_k}\,h^2_{1k} + \frac{2q}{(\phi_k-\beta_k)}\,(\eta_k\beta_k+\phi_k\alpha_k)\right] \tag{47}$$

On the other hand, $\underline{H}_2$ which is related to $\underline{L}_{42}$, is determined by substituting $\underline{H}_1$ into (40) as

$$\underline{H}_2 = \mathrm{diag}\left[\frac{\phi_k-\beta_k}{\alpha_k+\eta_k}\,h_{1k}\right] \tag{48}$$

Thus, $\underline{P}$, $\underline{L}$ and $\underline{W}_0$ are all determined.

Now the Lyapunov function given in (21) with $n = 0$ reduces to

$$V(\underline{x}) = \frac{1}{2}\left[\Delta\underline{\theta}^T, \underline{\omega}^T\right]\begin{bmatrix} \underline{P}_{11} & \underline{P}_{12} \\ \underline{P}_{21} & \underline{P}_{22} \end{bmatrix}\begin{bmatrix} \Delta\underline{\theta} \\ \underline{\omega} \end{bmatrix}$$

$$+ \frac{q}{2}\sum_{i=1}^{n}\left\{\frac{\eta_i\beta_i+\phi_i\alpha_i}{\phi_i-\beta_i}\,\Delta E_i^2\right\}$$

$$+ \sum_{i=1}^{n}\left(\frac{h_{1i}}{2}\right)^2\left[\sqrt{\frac{\xi_i}{\phi_i-\beta_i}}\,\Delta E_i + \frac{\sqrt{\xi_i}(\phi_i-\beta_i)}{(\alpha_i+\eta_i)}\,\Delta E_{exi}\right]^2$$

$$+ q\sum_{i=1}^{n-1}\sum_{j=i+1}^{n} B_{ij}\Big[(E_i^0+\Delta E_i)(E_j^0+\Delta E_j)\{\cos\delta_{ij}^0-\cos(\delta_{ij}^0+\Delta\delta_{ij})\}$$

$$- \Delta E_i\Delta E_j\cos\delta_{ij}^0 - \Delta\delta_{ij}\cdot E_i^0E_j^0\sin\delta_{ij}^0\Big] \qquad (49)$$

with

$$\dot{V}(\underline{x}) = -\frac{1}{2}\underline{\omega}^T\underline{L}_{21}\underline{L}_{21}^T\underline{\omega}$$

$$-\frac{1}{2}\sum_{i=1}^{n}\left[\left(\Delta E_i + \frac{\phi_i-\beta_i}{\alpha_i+\eta_i}\,\Delta E_{exi}\right)h_{1i} + \sqrt{2q\gamma_i}\,\Delta\dot{E}_i\right]^2 \qquad (50)$$

The extra terms, which are due to the AVR, are obvious in (49) and (50).

If one let β_i be zero, i.e., $h_{1i} = 0$, in (49) the Lyapunov function which includes only the effects of flux decays is obtained.

V. Critical Value

The region of asymptotic stability R is defined in terms of the critical value V_c as follows:

$$R = \{\underline{x} | V(\underline{x}) < V_c\} \tag{51}$$

The technique of determining V_c by evaluating V at the "closest" unstable equilibrium point may be employed.

The unstable equilibrium points of the Lyapunov function may be determined by solving

$$\frac{\partial V}{\partial \underline{x}} = \underline{0}$$

subject to the constraint $V_1(\underline{\sigma}) \geq 0$.

In case of $\rho = 0$, the critical value V_c is given by

$$V_c = \frac{q}{2} \sum_{i=1}^{n} \left\{ \frac{\eta_i \beta_i + \phi_i \alpha_i}{\phi_i - \beta_i} (\Delta E_i^u)^2 \right\}$$

$$+ q \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} B_{ij} [(E_i^0 + \Delta E_i^u)(E_j^0 + \Delta E_j^u)\{\cos \delta_{ij}^0 - \cos(\delta_{ij}^0 + \Delta\delta_{ij}^u)\}$$

$$- \Delta E_i^u \Delta E_j^u \cos \delta_{ij}^0 - \Delta\delta_{ij}^u \cdot E_i^0 E_j^0 \sin \delta_{ij}^0] \tag{53}$$

where "u" denotes the closest equilibrium point to the origin. As found in (53), V_c does not depends on h_{1i} $(i = 1,2,\cdots,n)$, while V in (49) depends on them. Hence, if one choose h_{1i} so that its effect on V is small, a wider stability region will be guaranteed. This is the reason why we have chosen the minimum value of $|h_{1i}|$ in (44).

VI. Example

Let us consider a three-machine system as shown in Fig. 1. In this system M_1 and M_2 act as generators while M_3 acts as a load. We investigate the effect of AVR action on power system stability from the standpoint of modification of the equipotential curves with the gain parameter of the AVR system. In order to clarify the effect, we impose the flux decay and AVR action on only the No. 1 machine. In this connection, the system constants for the No. 1 machine including the AVR system are listed in Table 1.

Figure 2 shows the equipotential curves for the system without the AVR i.e., with only flux decay, under the constraints $\Delta\delta_{23} = 0$ and $\Delta\dot{E}_1 = 0$. Equipotential curves of the system including the AVR with $\mu = 1.1$ and $\mu = 2.2$ are given in Fig. 3 and Fig. 4, respectively, under the constraints $\Delta\delta_{23} = 0$, $\Delta\dot{E}_1 = 0$ and $\Delta\dot{E}_{exi} = 0$. While flux decay is seen to make the stability region narrower, voltage regulator action corrects this situation. The degree of correction depends on μ, the magnitude of the feedback gain of the voltage regulating system. The comparison between Fig. 3 and Fig. 4 shows how the AVR gradually changes the figures of potential energy level of the system. For reference, the equipotential curves of the system neglecting both flux decay and AVR action is also given in Fig. 5. It is found that with the increase in magnitude of μ the curves approach the situation shown in Fig. 5.

VII. Conclusions

This paper has given a Lyapunov function for a multimachine power system taking automatic voltage regulator action into consideration. The nonlinear function $V_1(\underline{\sigma})$ has been obtained uniquely by determining the matrix $\underline{Q}$ first. All the elements of the matrix were determined so that the curl equations might hold for the nonlinear components of the system equation. The Moore-Anderson theorem yielded extra conditions on the power system, in connection with the installation of the AVR which may have some implications on system design. The effect of AVR action on power system stability was investigated from the standpoint of changes in the equipotential curves.

An inclusion of load characteristics, which also plays an important role in power system stability, is currently investigated and will be discussed on another occasion.

Acknowledgement

The research was supported in part by Department of Energy Contract DE-AC01-82CE76221.

References

[1] N. Dharma Rao, "Routh-Hurwitz conditions and Lyapunov method for the transient-stability problem," Proc. IEE, vol. 116, no. 4, 1969, pp. 539-547.

[2] J. L. Willems and J. C. Willems, "The application of Lyapunov methods to the computation of transient stability regions for multimachine power systems," IEEE Trans. Power Appar. & Syst., vol. PAS-89, 1970. pp. 795-801.

[3] M. A. Pai, M. Ananda Mohan and J. Gopala Rao, "Power system transient stability regions using Popov's method," IEEE Trans. Appar. & Syst., vol. PAS-89, 1970, pp. 788-794.

[4] T. K. Mukherjee, B. Bhattacharyya, and A. K. Choudhury, "Transient stability of a power system by the second method of Lyapunov," Int. J. Syst. Sci., vol. 3, 1972, pp. 439-448.

[5] F. S. Prabhakara, A. H. El-Abiad, and A. J. Koivo, "Application of generalized Zubov's method to power system stability," Int. J. Contr., vol. 20, no. 2, 1974, pp. 203-212.

[6] M. A. Pai and Vishwanatta Rai, "Lyapunov-Popov stability analysis of synchronous machine with flux decay and voltage regulator," Int. J. Contr., vol. 19, no. 4, 1974, pp. 817-829.

[7] H. Miyagi and T. Taniguchi, "Construction of Lyapunov function for power systems," Proc. IEE, vol. 124, no. 12, 1977.

[8] T. Taniguchi and H. Miyagi, "A method of constructing Lyapunov functions for power systems," Trans. IEE of Japan, vol. 97, no. 5, 1977, pp. 271-278.

[9] N. Kakimoto, Y. Ohsawa, and M. Hayashi, "Transient stability analysis of multimachine power systems with field flux decays via Lyapunov's direct method," IEEE Trans. Power Appar. & Syst.,

vol. PAS-99, no. 5, 1980, pp. 1819-1827.

[10] J. B. Moore and B. D. O. Anderson, "A generalization of the Popov criterion," J. Franklin Inst., vol. 285, no. 6, 1968, pp. 488-492.

[11] C. A. Desoer and M. Y. Wu, "Stability of a nonlinear time-invariant feedback system under almost constant inputs," Automatica, vol. 5, 1969, pp. 231-233.

[12] J. L. Willems, "Optimal Lyapunov functions and stability regions for multimachine power systems," Proc. IEE, vol. 117, no. 3, 1970, pp. 573-577.

[13] M. A. Pai and P. G. Murthy, "New Lyapunov functions for power systems based on minimal realization," Int. J. Control, vol. 19, no. 2, 1974, pp. 401-415.

[14] V. E. Henner, "A multimachine power system Lyapunov function using the generalized Popov criterion," Int. J. Control, vol. 19, no. 5, 1974, pp. 969-976.

[15] D. J. Hill and A. R. Bergen, "Stability analysis of multimachine power networks with linear frequency dependent loads," IEEE Trans. Circuits and Systems, vol. CAS-29, no. 12, 1982, pp. 840-848.

[16] M. A. Pai, Power System Stability (Analysis by the Direct Method of Lyapunov), North-Holland Publishing Co., 1981.

Table 1. System constants (p.u.)

$x_{d1} = 1.15$,	$x'_{d1} = 0.3$	
$T'_{d01} = 6.6$ [s],	$T_{v1} = 0.1$ [s],	$k_1 = 1.021$

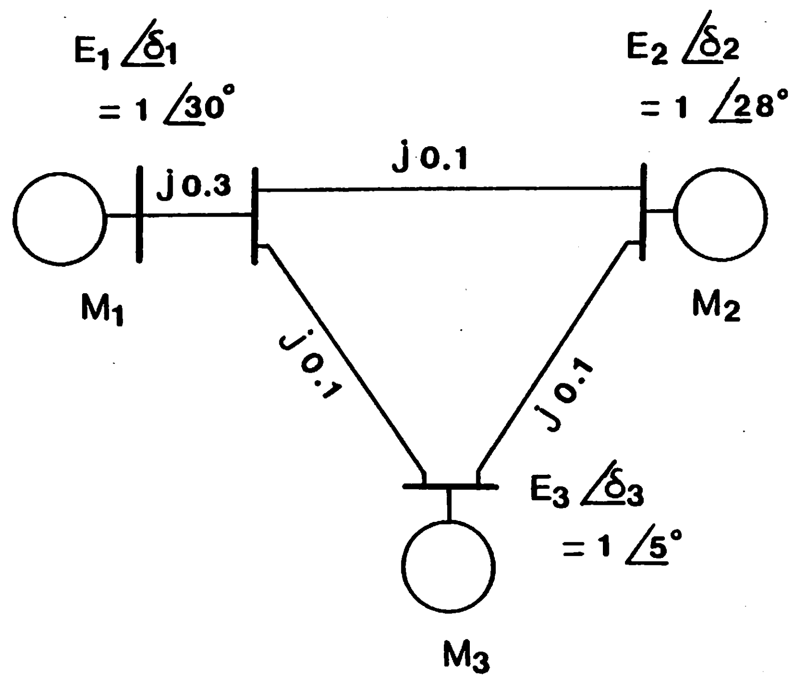

Fig. 1 Sample system

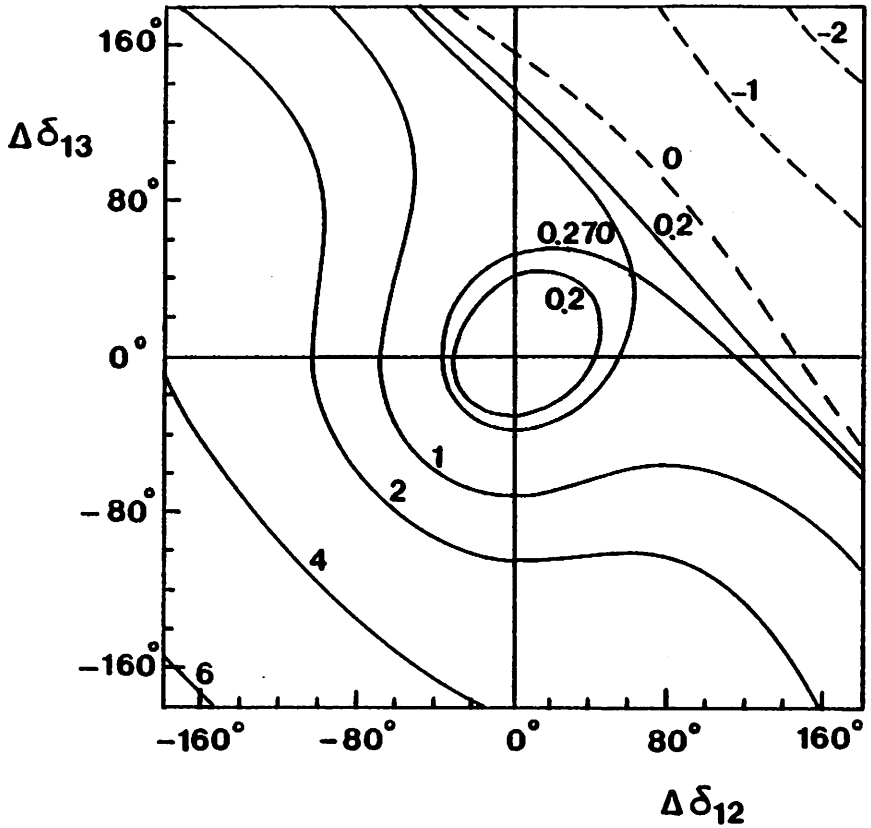

Fig.2 Equipotential curves without AVR

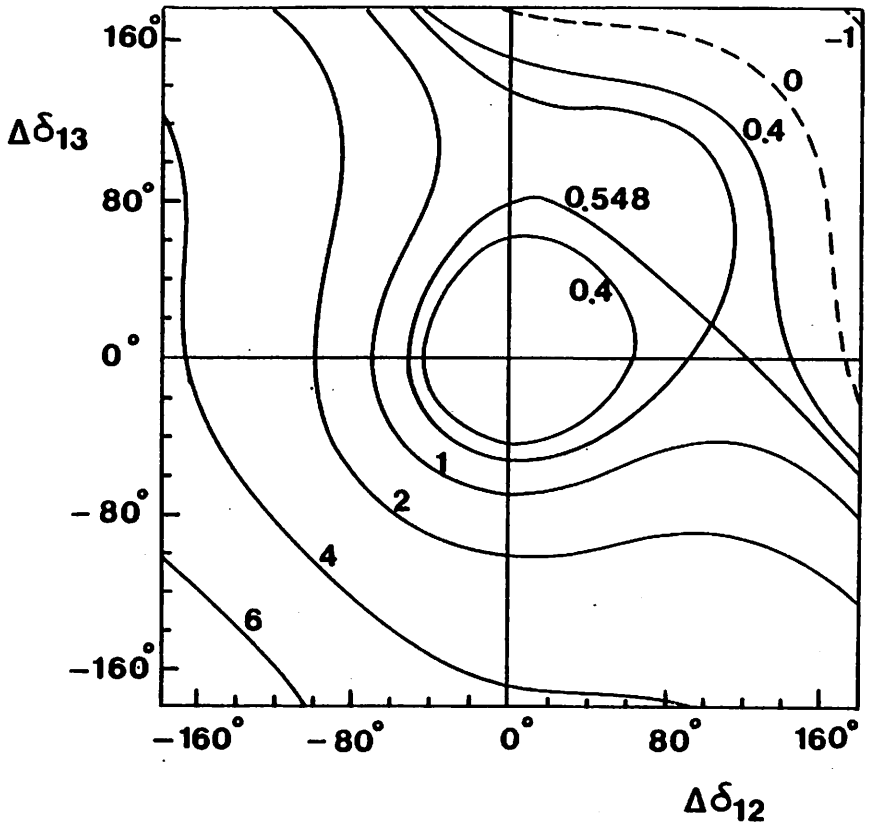

Fig. 3 Equipotential curves with AVR ($\mu = 1.1$)

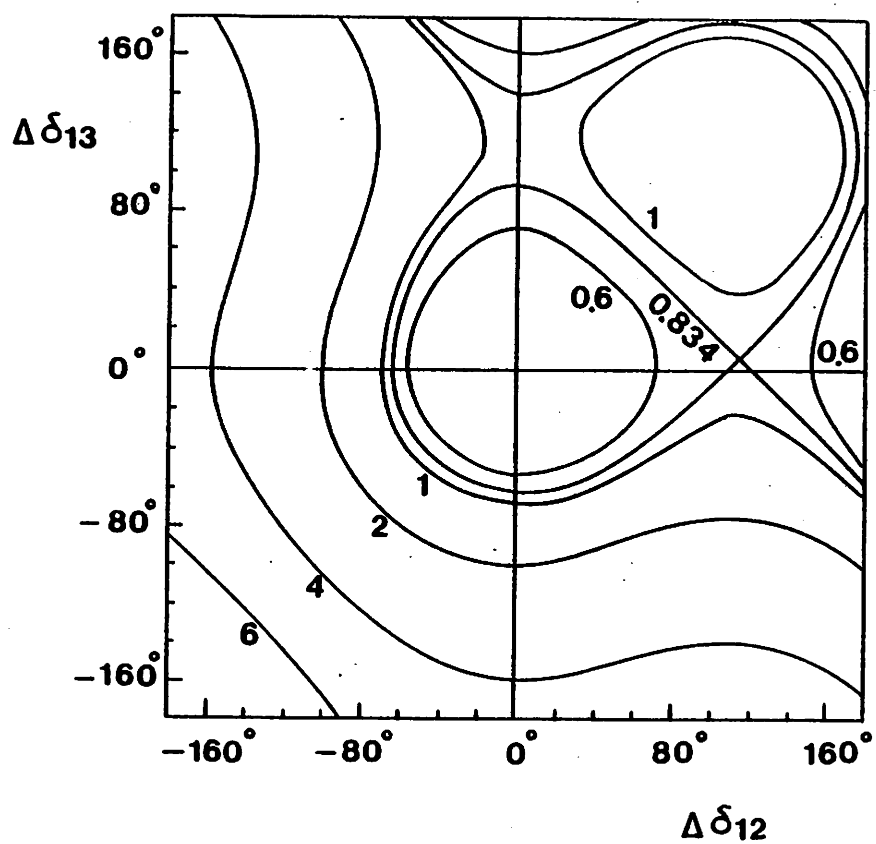

Fig. 4 Equipotential curves with AVR (μ =2.2)

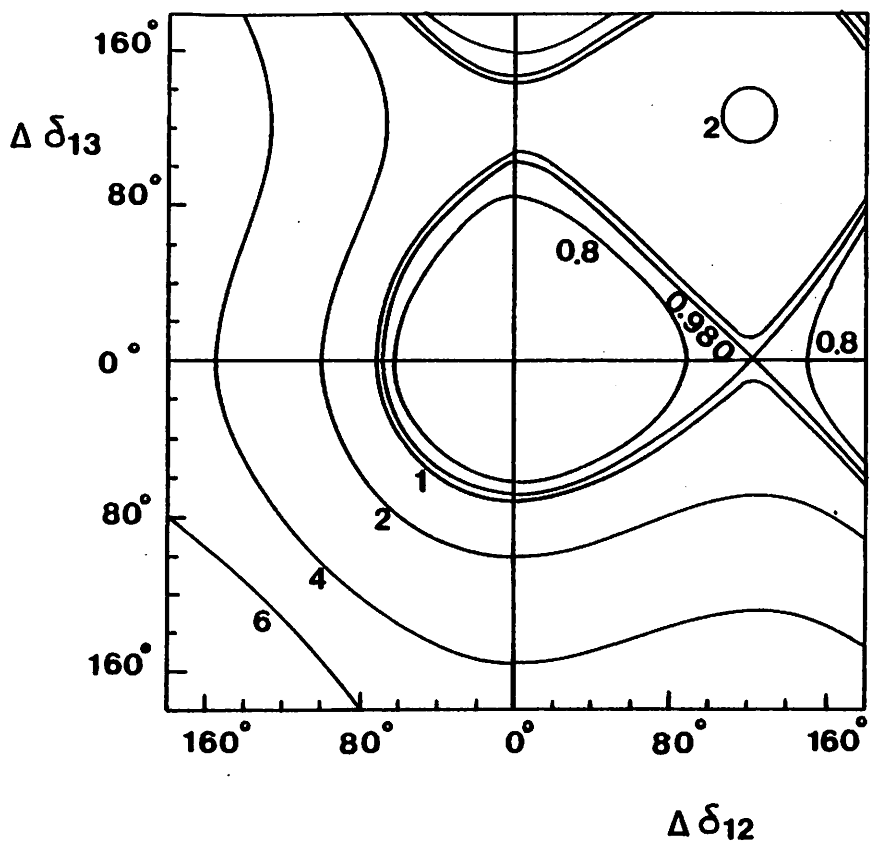

Fig. 5 Equipotential curves without flux decay and AVR