EXPERIMENTAL CONFIRMATION OF CHAOS FROM CHUA'S CIRCUIT

by

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Memorandum No. UCB/ERL M84/56

6 July 1984

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The purpose of this paper is to present the first laboratory confirmation of chaos as measured from Chua's circuit. Oscilloscope tracings of the chaotic attractors associated with several piecewise-linear resistor characteristics are consistent with those obtained recently by Matsumoto via computer simulation.

[†]Research sponsored by the Office of Naval Research Contract N00014-76-C-0572 and Semiconductor Research Corporation SRC82-11-008.

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1. Introduction

Recently chaos has been observed by computer simulation of a very simple 3rd order reciprocal autonomous circuit containing only one 2-terminal nonlinear resistor [1]. However, a typical property of chaotic systems is that the solution of the associated differential equations is very sensitive to initial conditions [2]. Hence, due to truncation errors inherent in any computer simulation, this chaotic behavior must be interpreted with great care and caution [3]. In order to confirm that the circuit is truly chaotic (i.e. the chaos is free of numerical artifacts) it is highly desirable to build an accurate laboratory model of this circuit and observe its behavior experimentally.

This letter presents two circuit realizations for demonstrating chaos from Chua's circuit. The first realization uses 2 op amps and 6 resistors to realize a specific v-i characteristic (Fig. 1(b)). At the cost of 2 additional resistors, circuit realization II allows other piecewise-linear characteristics with any prescribed parameters to be synthesized. Using various combinations of circuit parameters several chaotic attractors have been experimentally measured from this circuit.

2. Circuit Realization I

Consider the circuit shown in Fig. 1(a) and the piecewise-linear v-i characteristic shown in Fig. 1(b). The 5.6 Ω resistor is inserted in series with the inductor to sense the current i_L and has negligible effects on the measured waveforms and chaotic attractors. The op amp circuit used to realize this nonlinear resistor and its measured v-i characteristics are shown in Figs. 2(a)-(b). Figures 3(a), (b) and (c) show the measured chaotic attractors in the (i_L, V_{C_1})-plane, (i_L, V_{C2})-plane and $(V_{C1}-V_{C2})$-plane, respectively. The measured signals for V_{C1}, V_{C2} and i_L along with their measured spectra are shown in Figs. 4(a)-(b), (c)-(d) and (e)-(f), respectively. These measurements confirm that the circuit in Fig. 1(a) with the nonlinear resistor realized by Fig. 2(a) is indeed chaotic.

3. A More General Realization

In this section we present the design procedure for synthesizing a 5-segment odd-symmetric v-i characteristic with any prescribed set of parameters m_0, m_1, m_2, E_{B1} and E_{B2} as shown in Fig. 5 using the circuit shown in Fig. 6. This flexibility allows us to observe a large variety of chaotic attractors different from that of Fig. 3.

Design Procedure: Circuit Realization II

Given (m_0, m_1, m_2, E_{B1}, E_{B2}) choose any convenient $m_{11} < 0$ such that the following two inequalities are satisfied

$$\frac{m_0-m_1+m_{11}}{m_0-m_1} > \frac{E_{B1}}{E_s} \tag{1}$$

$$\frac{m_1-m_{11}}{m_1-m_2} > \frac{E_{B2}}{E_s} \tag{2}$$

where E_s denotes the saturation voltage of the op amp. (We assume the positive and negative saturation voltage of the op amp to be equal in magnitude. For details and further generalization see [4].) The resistance values can then be obtained using the following formulas:

Step 1

Choose any convenient $R_2 > 0$ and calculate

$$R_1 = \frac{E_s-E_{B1}}{E_{B1}} R_2$$

$$R_4 = \frac{R_1+R_2}{(m_0-m_1)R_2}$$

$$R_3 = \frac{R_4}{(m_0-m_1+m_{11})R_4-1}$$

Step 2

Choose any convenient $R_6 > 0$ and calculate

$$R_5 = \frac{E_s-E_{B2}}{E_{B2}} R_6$$

$$R_8 = \frac{R_5+R_6}{(m_1-m_2)R_6}$$

$$R_7 = \frac{R_8}{(m_1-m_{11})R_8-1}$$

Remark

Conditions (1) and (2) must be satisfied to insure positive resistance values.

As an example let us realize the v-i characteristic shown in Fig. 1(b) using this procedure. For this characteristic we have $m_0 = 1$, $m_1 = -.5$, $m_2 = -.8$, $E_{B1} = 7.5V$ and $E_{B2} = 2.5V$. Choosing a typical op amp (National/8035 741 CN) with $E_s = 13V$ and $m_{11} = -.6$ we find

$$\frac{1+.5-.6}{1+.5} > \frac{7.5}{13}$$

and

$$\frac{-.5+.6}{-.5+.8} > \frac{2.5}{13}$$

Therefore, conditions (1) and (2) are satisfied. Calculating the resistance values we obtain:

$R_1 = 733\ \Omega$, $R_2 = 1\ K\Omega$, $R_3 = 28.9\ K\Omega$, $R_4 = 1.15\ K\Omega$

$R_5 = 4.20\ K\Omega$, $R_6 = 1\ K\Omega$, $R_7 = 23.6\ K\Omega$, $R_8 = 17.3\ K\Omega$

The measured v-i characteristic shown in Fig. 7 is virtually identical to that shown in Fig. 1(b).

References

[1] T. Matsumoto, "A chaotic attractor from Chua's circuit," Memorandum No. UCB/ERL M84/36, University of California, Bekeley, April 17, 1984.

[2] J. Guckenheimer and P. Holmes, Nonlinear Oscillations, Dynamical Systems, and Bifurcation of Vector Fields, Springer-Verlag, Inc., New York, 1983.

[3] L. O. Chua, M. Hasler, J. Neirynck and P. Verbaugh, "Dynamics of a piecewise-linear resonant circuit," IEEE Trans. on Circuits and Systems, Vol. CAS-29, pp. 535-547, August 1982.

[4] L. O. Chua and F. Ayrom, "Exploiting the op amp nonlinearity in circuit design," Memorandum No. UCB/ERL M84/50, University of California, Berkeley, April 23, 1984.

Figure Captions

Fig. 1. (a) Chua's chaotic circuit. (b) v-i characteristic of the nonlinear resistor in (a).

Fig. 2. (a) Op amp circuit realization I of the v-i characteristic in Fig. 1.
(b) Measured v-i characteristic of (a). Scale i : 2ma/div, v : 4v/div.

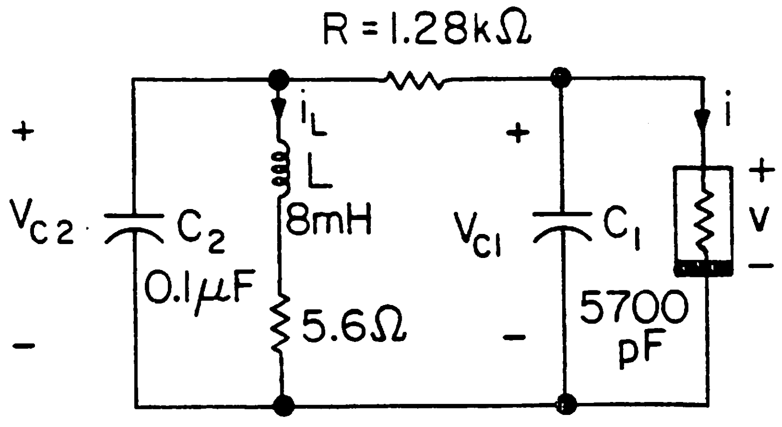
Fig. 3. (a) Chaotic attractor projected onto (i_L, v_{C1})-plane. Scale V_{C1} : 2v/div, i_L : $\frac{.1V}{5.6\Omega}$ /div.
(b) Chaotic attractor projected onto (i_L, V_{C2})-plane. Scale V_{C2} : .5V/div, i_L : $\frac{.1V}{5.6\Omega}$/div.
(c) Chaotic attractor projected onto (V_{C1}, V_{C2})-plane. Scale V_{C1} : 2V/div, V_{C2} : 2V/div.

Fig. 4. (a)-(b) Steady state waveform and spectrum for V_{C1}.
(c)-(d) Steady state waveform and spectrum for V_{C2}.
(e)-(f) Steady state waveform and spectrum for i_L.

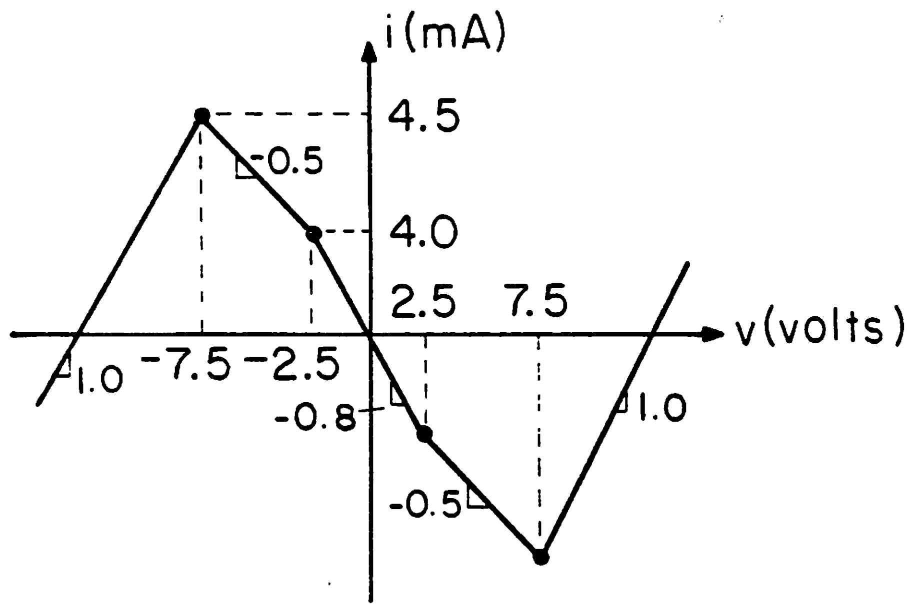
Fig. 5. General form of v-i characteristic to be realized.

Fig. 6. Op amp circuit realization II for the v-i characteristics of the form shown in Fig. 5.

Fig. 7. Measured v-i characteristic using op amp circuit realization II.
Scale i : 2 ma/div, V : 4v/div.

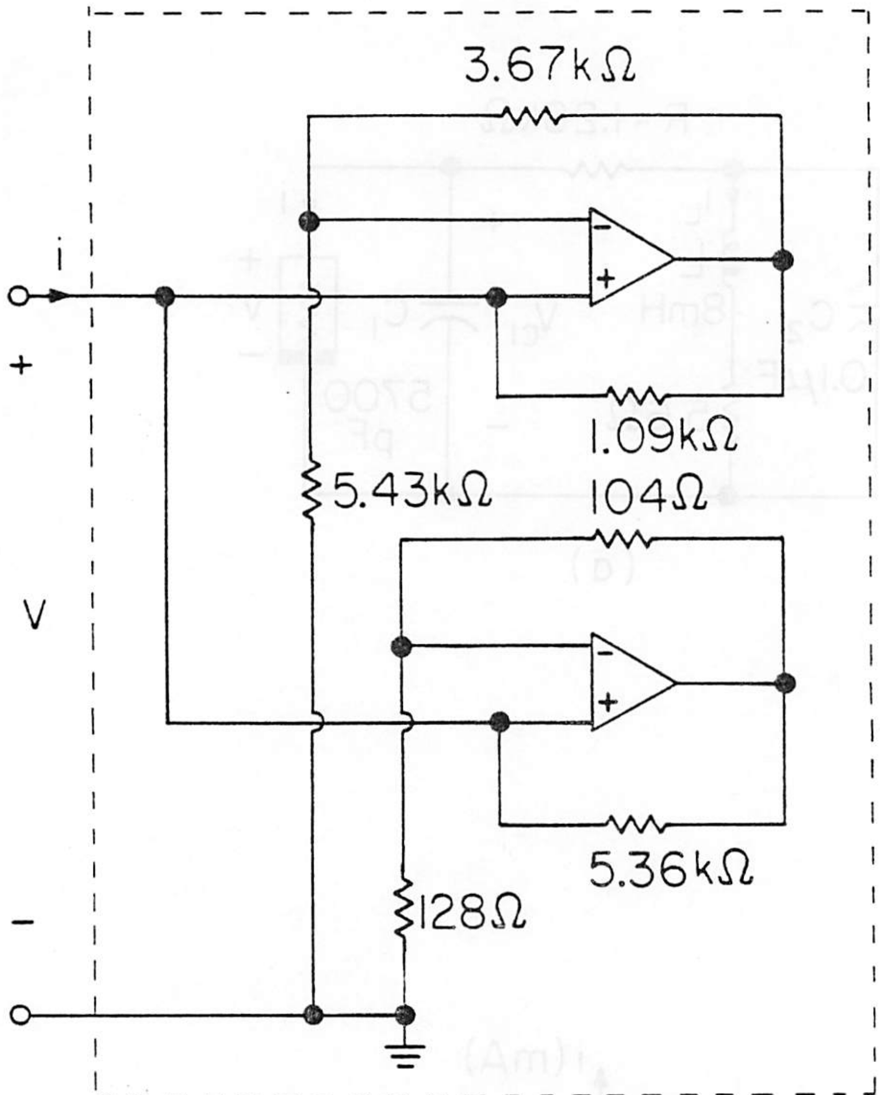

R = 1.28kΩ
i_L
L
8mH
5.6Ω
V_{C2}
C_2
0.1μF
V_{C1}
C_1
5700 pF
i
v
+
−

(a)

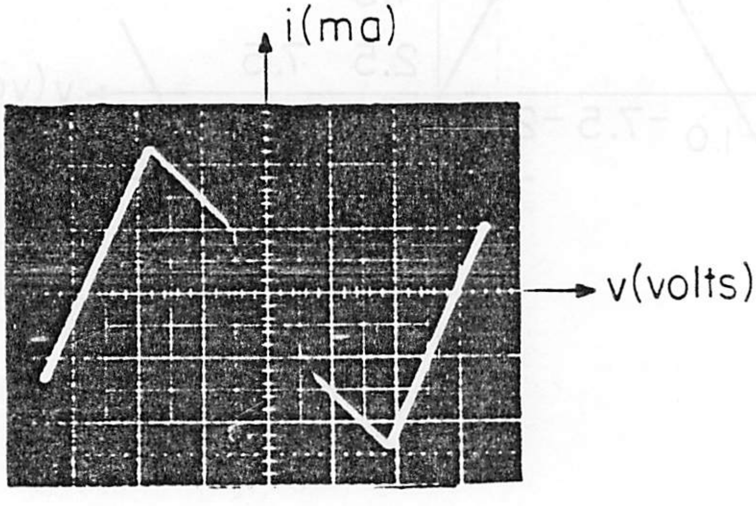

i(mA)
v(volts)
4.5
4.0
2.5
7.5
-7.5
-2.5
-0.8
-0.5
-0.5
1.0
1.0

(b)

Fig. 1

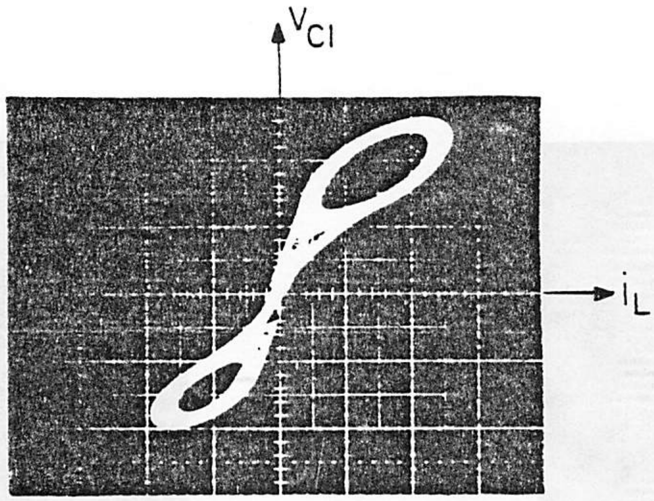


(a)

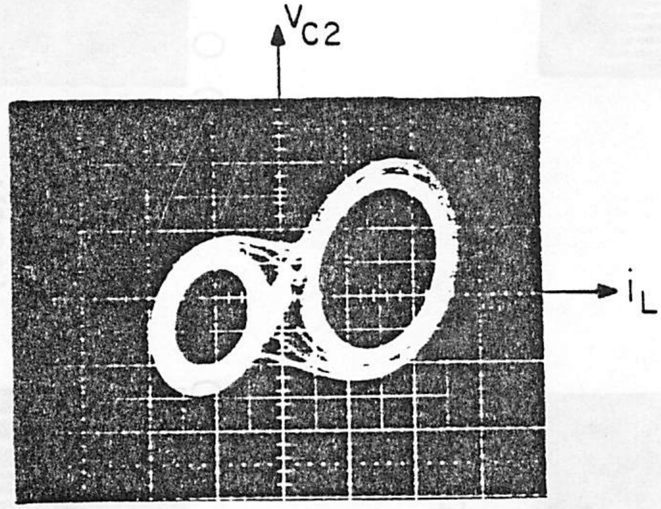


(b)

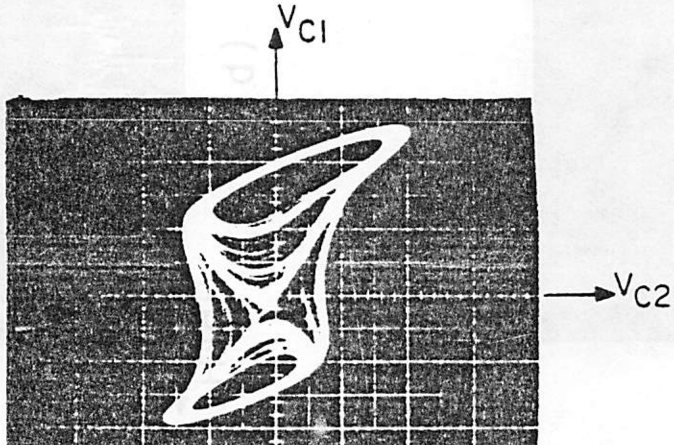
Fig. 2



(a)

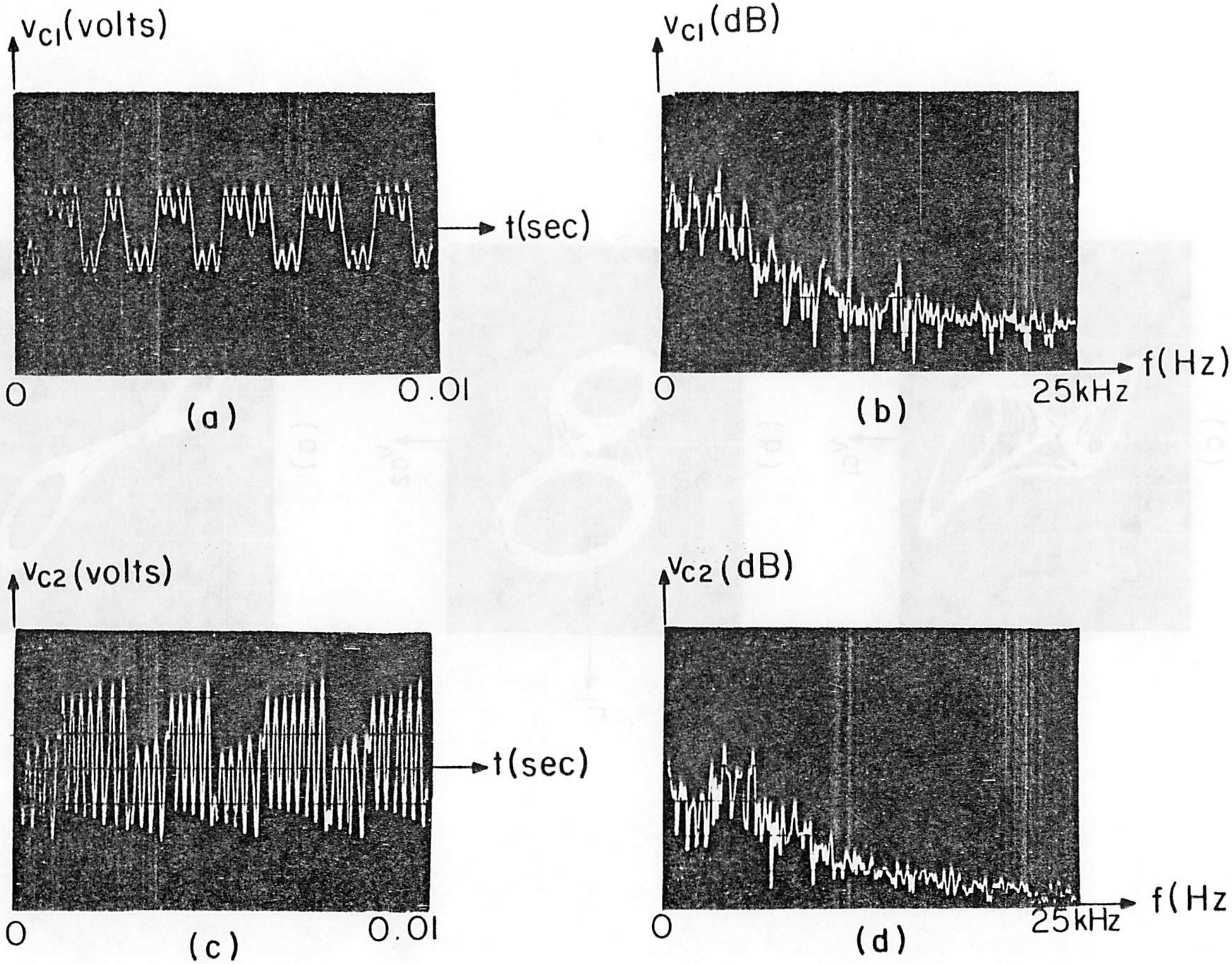


(b)



(c)

Fig. 3



v_{C1}(volts)
t(sec)
0
0.01
(a)
v_{C1}(dB)
f(Hz)
0
25kHz
(b)
v_{C2}(volts)
t(sec)
0
0.01
(c)
v_{C2}(dB)
f(Hz)
0
25kHz
(d)

Fig. 4

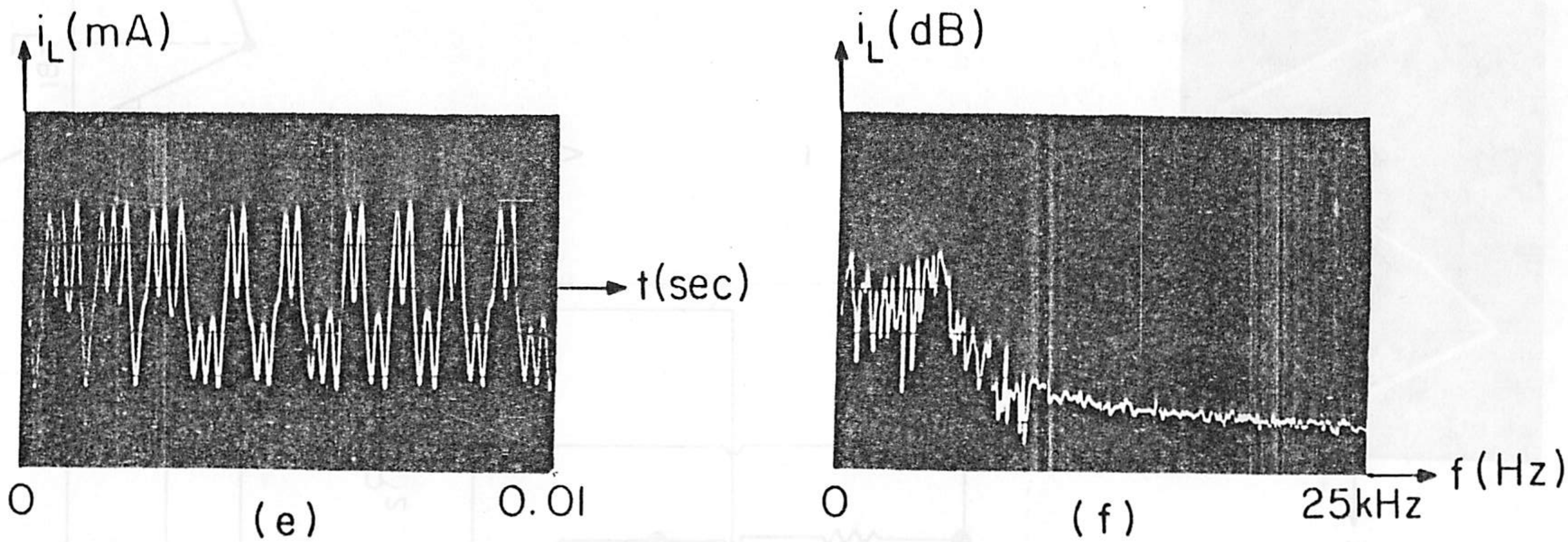


Fig. 4 (Cont'd)

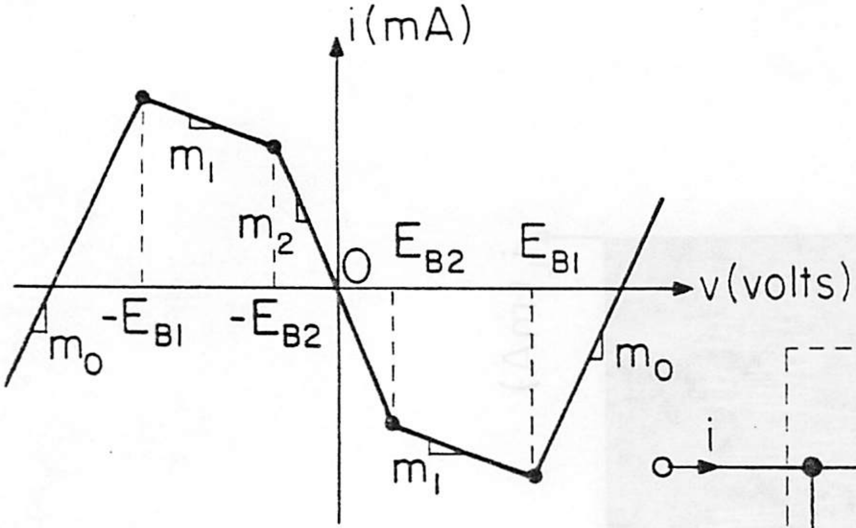


Fig. 5

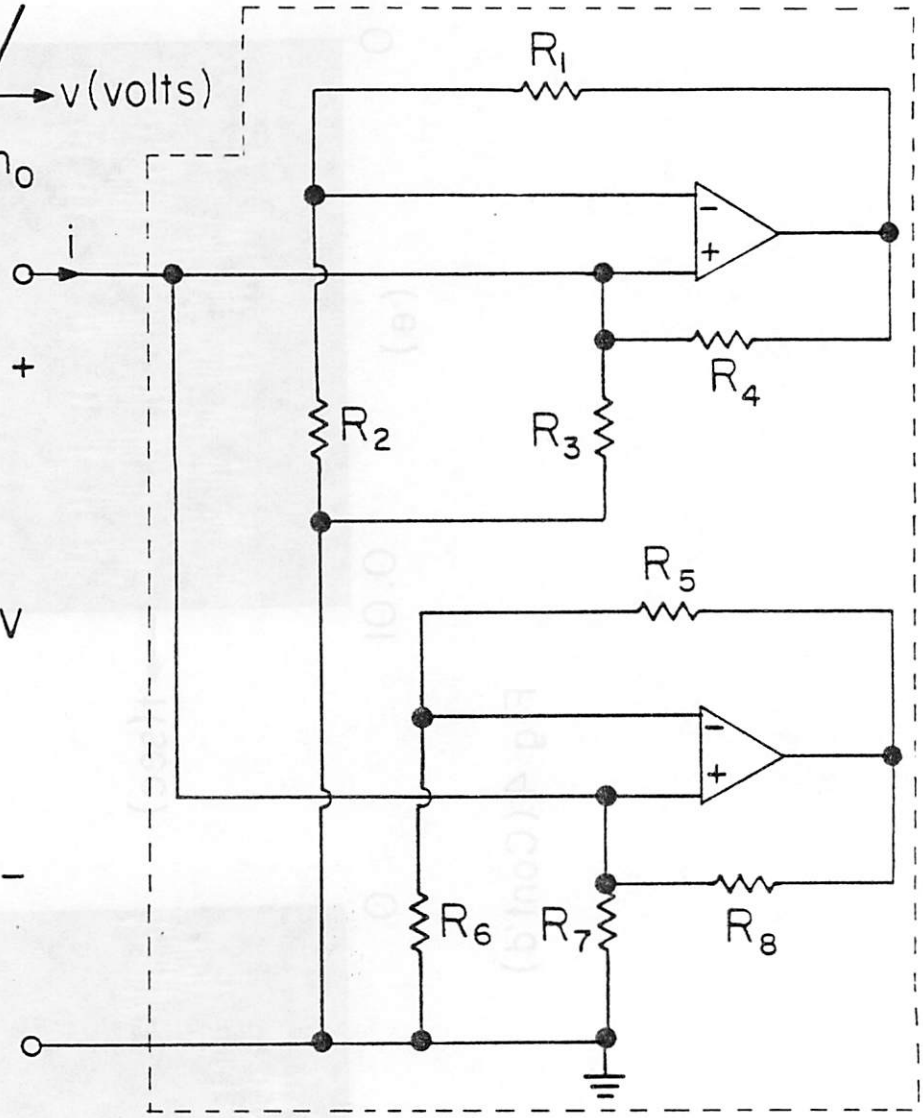


Fig. 6

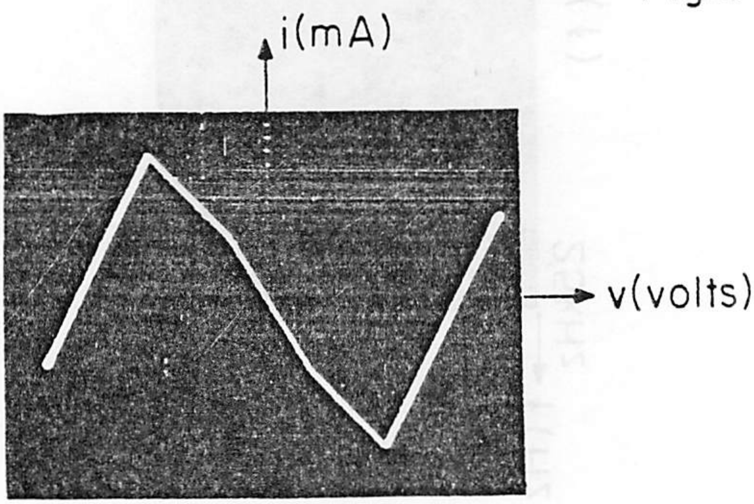


Fig. 7