TEST-SCORE SEMANTICS AS A BASIS FOR A COMPUTATIONAL APPROACH TO THE REPRESENTATION OF MEANING

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Abstract

A basic idea which underlies test-score semantics is that a proposition in a natural language may be interpreted as a system of elastic contraints which is analogous to a nonlinear program. Viewed in this perspective, meaning representation may be regarded as a process which (a) identifies the variables which are constrained, and (b) characterizes the constraints to which they are subjected. In test-score semantics, this is accomplished through the construction of a test procedure which tests, scores and aggregates the elastic constraints which are implicit in the proposition, yielding an overall test score which serves as a measure of compatibility between the proposition, on the one hand, and what is referred to as an explanatory database, on the other.

Test-score semantics provides a framework for the representation of the meaning of dispositions, that is, propositions which are preponderantly, but not necessarily always, true. Another important concept which is a part of test-score semantics is that of a canonical form, which may be viewed as a possibilistic analog of an assignment statement. The concepts of a disposition and canonical form play particularly important roles in the representation of -- and reasoning with -- commonsense knowledge.

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To Professor Abe Mamdani

1. Introduction

During the past decade, the emergence of natural language processing as one of the major areas of research in artificial intelligence has provided a strong stimulus for the development of computationally-oriented theories of meaning, knowledge representation and approximate reasoning.

The traditional approaches to the theory of meaning -- among which the best known are truth-conditional semantics, possible-world semantics, Montague semantics, procedural semantics and, in a broader sense, model-theoretic semantics -- are based on two-valued logic and the concomitant assumption that the extension of a predicate is a crisply defined set in a universe of discourse. It may be argued, however, that, in the case of natural languages, the extension of a predicate is, in general, a fuzzy set in which the transition from membership to nonmembership is gradual rather than abrupt. This is true, for example, of such commonly used predicates as ***tall man***, ***loud music***, ***attractive woman***; quantifiers[1] such as ***most***, ***several***, ***few***; temporal quantifiers such as ***frequently***, ***once in a while***, ***almost always***; and qualifiers such as ***very likely***, ***quite true***, ***almost impossible***, etc. Indeed, it is evident that almost all concepts in or about natural languages are fuzzy in nature. Viewed in this perspective, it is hard to rationalize the almost exclusive use of two-valued logic in the traditional approaches to the semantics of natural languages.

In a departure from this tradition, we have described in a series of papers starting in 1971, an approach to the semantics of natural languages based on the theory of fuzzy sets and, more particularly, on possibility theory. During the past several years, this approach has evolved into a meaning-representation system termed ***test-score semantics*** (Zadeh, 1978b, 1981), which is computational in nature and which is based on the premise that almost everything that relates to natural languages is a matter of degree. Like almost all theories of meaning, test-score semantics is referential in nature in the sense that it deals with the correspondence between expressions in a language and their denotations in a universe, or family of universes, of discourse. However, unlike the traditional approaches to the theory of meaning, test-score semantics does not make use of the machinery of first-order or intensional logic, and is based, instead, on ***fuzzy logic*** -- the logic of approximate or fuzzy reasoning. In what follows, we shall present an informal exposition of some of the basic ideas underlying test-score semantics and illustrate its application to the representation of the meaning of various types of semantic entities, among them propositions, predicates, dispositions and commands.

2. Composition of Meaning

The point of departure in test-score semantics is the assumption that the problem of meaning representation is that of composing the meaning of a given semantic entity, s, from a collection of fuzzy relations -- termed the *composants* of s -- whose meaning is assumed to be known by the addressee of the representation process.

As an illustration, assume that the semantic entity in question is the proposition $p \triangleq$ *Over the past few years Stan made a lot of money.* Furthermore, assume that the relations whose meaning is assumed to be known are:

(a) *INCOME* [*Name*; *Amount*; *Year*] ,

which is a collection of tuples, e.g., (*Ted, 150,000, 1982*), whose first element is the name of an individual; whose second element is the income of that individual; and whose third element is the year in which the income was made;

(b) *FEW* [*Number*; μ] ,

which serves to calibrate the meaning of the fuzzy number *few* by associating with each real value of the variable *Number*, the degree, μ, to which that number fits the intended meaning of *few*; and

(c) *LOT.OF.MONEY* [*Amount*; μ] ,

which calibrates the meaning of *lot of money* by associating with each value of *Amount*, the degree, μ, to which the value of *Amount* fits the intended meaning of *lot of money*.

We shall show at a later point how the meaning of p may be composed from the meaning of its composants. What is important to note at this juncture is that the choice of the relations in question reflects our assumption concerning the knowledge profile of the addressee of the representation process. In this sense, then, the representation of the meaning of p is strongly dependent on the choice of relations from which the meaning of p is composed.

In its strict interpretation -- which is the interpretation underlying Montague semantics -- Frege's principle of compositionality (Hintikka, 1982) (Janssen, 1978) requires that the meaning of a proposition be composed from the meanings of its constituents. In test-score semantics, the composants of s are, in general, implicit rather than explicit in s. Furthermore, their number is, in general, much smaller than the number of constituents in s. As will be seen later, by allowing the meaning of s to be composed from its composants rather than constituents, test-score semantics achieves a higher expressive power than is possible under a strict interpretation of Frege's principle of compositionality (Zadeh, 1983c).

Since the composants of s serve to explain the meaning of s, they constitute, collectively, a collection of fuzzy relations which is referred to as an *explanatory database*, or ED, for short. In general, the procedure which describes the composition of the meaning of s involves only the *frames* of the relations in ED, that is, their names, their attributes and the domains of their attributes. In test-score semantics, the collection of frames in ED is referred to as the *explanatory database frame*, or, more simply, as EDF. In effect, the role played by EDF in test-score semantics is analogous to that of a collection of possible worlds in possible-world, or, more generally, model-theoretic semantics (McCawley, 1981).

3. Testing and Scoring of Elastic Constraints

A basic idea which underlies test-score semantics -- and motivates its name -- is that any semantic entity -- and, especially, a proposition -- may be viewed as a system of implicitly defined elastic or, equivalently, fuzzy constraints whose domain is the collection of fuzzy relations in the explanatory database. Viewed in this perspective, the meaning of a semantic entity, s, may be represented as a procedure which tests, scores and aggregates the elastic constraints which are induced by s. In more concrete terms, assume, for simplicity, that s is a proposition, p. Representation of the meaning of p through the use of test-score semantics involves, in general, the following steps.

1. Selection of an appropriate explanatory database, that is, a set of fuzzy relations which collectively constitute the composants of p.

2. Explicitation of the elastic constraints $C_1, \ldots, C_m$ which are induced by p. For example, in the case of the proposition $p \triangleq$ *Susan is young*, the implicit constrained variable is the age of Susan and the associated elastic constraint is characterized by the fuzzy predicate *young*. Less obviously, in the case of the proposition $p \triangleq$ *Most students are young*, the implicit constrained variable is the proportion of *young students* among *students*, and the associated elastic constraint is characterized by the fuzzy quantifier *most*.

3. Characterization of each constraint C_i, $i = 1, \ldots, m$, by a test which yields the test score, τ_i, representing the degree to which the constraint is satisfied. Usually, the test score is represented as a number in the interval $[0,1]$. More generally, however, a test score may be a fuzzy number or a probability/possibility distribution over the unit interval. The test scores in test-score semantics play a role which is somewhat analogous to that of truth values in truth-conditional semantics.

4. Aggregation of the partial test scores $\tau_1, \ldots, \tau_m$ into a smaller number of test scores $\bar{\tau}_1, \ldots, \bar{\tau}_k$, represented as an *overall vector test score* τ,

$$\tau = (\bar{\tau}_1, \ldots, \bar{\tau}_k) \quad . \tag{3.1}$$

In most cases $k = 1$, so that the overall test score is a scalar. The overall test score serves as a measure of the *compatibility* of p with the explanatory database, ED. Equivalently, τ may be interpreted as the *truth* of p given ED or, equivalently, as the *possibility* of ED given p. What is important to note is that the meaning of p is represented not by the overall test score τ, but by the test procedure which computes τ for any given ED.

As a simple illustration of the concept of a test procedure, consider the proposition $p \triangleq$ *Joan is young and attractive*. The EDF in this case will be assumed to consist of the following relations:

$$EDF \triangleq POPULATION\ [Name\,;\, Age\,;\, \mu Attractive\,] + YOUNG\ [Age\,;\, \mu] \quad , \tag{3.2}$$

in which + should be read as "and."

The relation labeled *POPULATION* consists of a collection of triples whose first element is the name of an individual; whose second element is the age of that individual; and whose third element is the degree to which the individual in question is attractive. The relation *YOUNG* is a collection of pairs whose first element is a value of the variable *Age* and whose second element is the degree to which that value of *Age* satisfies the elastic constraint characterized by the fuzzy predicate *young*. In effect, this relation serves to calibrate the meaning of the fuzzy predicate *young* in a particular context by representing its denotation as a fuzzy subset, *YOUNG*, of the interval [0,100].

With this EDF, the test procedure which computes the overall test score may be described as follows:

1. Determine the age of Joan by reading the value of *Age* in *POPULATION*, with the variable *Name* bound to Joan. In symbols, this may be expressed as

$$Age\ (Joan) = {}_{Age}\ POPULATION\ [Name\ = Joan\,] \quad . \tag{3.3}$$

In this expression, we use the notation ${}_Y R[X = a]$ to signify that X is bound to a in R and the resulting relation is projected on Y, yielding the values of Y in the tuples in which $X = a$ (Zadeh, 1978b).

2. Test the elastic constraint induced by the fuzzy predicate *young*:

$$\tau_1 = {}_{\mu}\ YOUNG[Age\ = Age\,(Joan\,)] \quad . \tag{3.4}$$

3. Determine the degree to which Joan is attractive:

$$\tau_2 = {}_{\mu Attractive}\, POPULATION[Name = Joan] \ . \tag{3.5}$$

4. Compute the overall test score by aggregating the partial test scores τ_1 and τ_2. For this purpose, we shall use the min operator $\wedge$ as the aggregation operator, yielding

$$\tau = \tau_1 \wedge \tau_1 \ , \tag{3.6}$$

which signifies that the overall test score is taken to be the smaller of the operands of $\wedge$. The overall test score, as expressed by (3.6), represents the compatibility of $p \triangleq$ *Joan is young and attractive* with the data resident in the explanatory database.

Remark. In the example under consideration, Joan has two attributes -- *Youth* and *Attractiveness* -- of which *Youth* is measurable in objective terms via *Age* while *Attractiveness* is subjective in nature. For this reason, the degree of attractiveness in the relation *POPULATION* is tabulated directly rather than through a measurable attribute. In so doing, we are implicitly taking advantage of the human ability to assign a (possibly fuzzy) grade of membership in a class without a conscious employment of quantitative criteria to arrive at its value.

As an additional illustration which relates to a semantic entity other than a proposition, assume that s is a second-order fuzzy predicate, namely,

$$s \triangleq many\ large\ balls \ . \tag{3.7}$$

In this case, we shall employ the following EDF:

$$\begin{aligned} EDF \triangleq\ & POPULATION\,[Identifier;\ Size\,] + \\ & LARGE\,[Size;\ \mu] + \\ & MANY[Number;\ \mu] \ . \end{aligned} \tag{3.8}$$

In this EDF, *POPULATION* is a collection of pairs in which the first element is a label which identifies a ball, i.e., an identifier, while the second element is the size of that ball; *LARGE* is a relation which calibrates the meaning of the fuzzy predicate *large*; and *MANY* is a relation which calibrates the fuzzy number *many* by associating with each numerical value of the variable *Number* the degree to which that value fits the intended meaning of *many*.

1. Assume that *POPULATION* consists of a collection of n balls, $b_1, \ldots, b_n$, of various sizes. Determine the size of each ball:

$$Size(b_i) = {}_{Size}\ POPULATION[Identifier = b_i]\ ,\ i = 1, \ldots, n\ . \tag{3.9}$$

2. For each ball, test the constraint induced by *large*:

$$\tau_i = {}_{\mu}LARGE[Size = Size(b_i)]\ ,\ i = 1, \ldots, n\ . \tag{3.10}$$

3. Count the number of large balls by adding the τ_i: [2]

$$\Sigma Count\ (LARGE.BALL) = \Sigma_i \tau_i\ . \tag{3.11}$$

4. Test the constraint induced by *many*:

$$\tau = {}_{\mu}MANY[Number = \Sigma Count\ (LARGE.BALL)]\ , \tag{3.12}$$

which represents the overall test score yielded by the test procedure.

The intent of these simple examples is to provide an idea of how the meaning of a semantic entity, s , may be represented by a test procedure which associates with each instance of an explanatory database, ED, the degree to which that ED is compatible with s . In the examples which we considered, the degree of compatibility is a single number. More generally, however, and particularly in the case of a proposition which is associated with presuppositions, the degree of compatibility may be vector-valued.

4. Aggregation and Composition

In test-score semantics, the manner in which the partial test scores are aggregated is left to the discretion of the constructor of the test procedure. It is helpful, however, to have a collection of standardized rules for dealing with the aggregation and combination of elastic constraints which are associated with conjunction, disjunction, implication, quantification and modification. These standardized rules should be regarded as default rules, that is, rules to be used when the context does not require special-purpose rules which may provide a better fit to the intended mode of aggregation. The basic standardized rules [3] in test-score semantics may be summarized as follows.

Modification rules

If the test score for an elastic constraint C in a specified context is τ, then in the same context:

(*a*) the test score for *not* C is $1 - \tau$ (negation) (4.1)

(*b*) the test score for *very* C is τ^2 (intensification) (4.2)

(*c*) the test score for *more or less* C is $\tau^{1/2}$. (diffusion) (4.3)

Composition rules

If the test scores for elastic constraints C_1 and C_2 in a specified context are τ_1 and τ_2, respectively, then in the same context:

(*a*) the test score for C_1 and C_2 is $\tau_1 \wedge \tau_2$ (conjunction) (4.4)

(*b*) the test score for C_1 *or* C_2 is $\tau_1 \vee \tau_2$ (disjunction) (4.5)

(*c*) the test score for if C_1 *then* C_2 is $1 \wedge (1-\tau_1+\tau_2)$, (implication) (4.6)

where $\wedge \triangleq$ min and $\vee \triangleq$ max.

Quantification rules

These rules apply to semantic entities which contain fuzzy quantifiers, e.g., *several tall men, a few successes, it rained often, many more cats than dogs, most Frenchmen are not very tall,* etc. In such semantic entities, a fuzzy quantifier serves to provide a fuzzy characterization of an absolute or relative count of elements in one or more fuzzy sets.

Since membership in a fuzzy set is a matter of degree, it is not obvious how the count of elements in a fuzzy set may be defined. Among the various ways in which this can be done, the simplest is to employ the concept of the power (DeLuca and Termini, 1970) or, equivalently, the *sigma-count* of a fuzzy set (Zadeh 1975, 1983), which is defined as the arithmetic sum of the grades of membership.[4] More specifically, assume that F is a fuzzy subset of a finite universe of discourse $U = \{u_1, \ldots, u_n\}$ which comprises the objects $u_1, \ldots, u_n$. For convenience, F may be represented symbolically as the linear form

$$F = \mu_1/u_1 + \mu_2/u_2 + \ldots + \mu_n/u_n \quad , \tag{4.7}$$

in which the term μ_i/u_i ,$i = 1, \ldots, n$, signifies that μ_i, $0 \le \mu_i \le 1$, is the grade of membership of u_i in F, and the plus sign should be read as "and."

The *sigma-count* of F, denoted as $\Sigma Count(F)$, is defined as the arithmetic sum

$$\Sigma Count(F) \triangleq \Sigma_i \mu_i \ , \ i = 1, \ldots, n \quad , \tag{4.8}$$

with the understanding that the sum may be rounded if necessary to the nearest integer. Furthermore, one may stipulate that terms with grades of membership below a specified threshold be excluded from the summation in order to keep a large number of terms with low grades of membership from becoming count-equivalent to a small number of terms with high grades of membership.

In the case of a pair of fuzzy sets, (F, G), the ***relative sigma-count***, denoted by $\Sigma Count(F/G)$, may be interpreted as the proportion of elements of F which are in G. More explicitly,

$$\Sigma Count(F/G) = \frac{\Sigma Count(F \cap G)}{\Sigma Count(G)}, \tag{4.9}$$

where $F \cap G$, the intersection of F and G, is defined by

$$\mu_{F \cap G}(u) = \mu_F(u) \wedge \mu_G(u), \quad u \in U . \tag{4.10}$$

Thus, in terms of the membership functions of F and G, the relative sigma-count of F and G may be expressed as

$$\Sigma Count(F/G) = \frac{\Sigma_i \mu_F(u_i) \wedge \mu_G(u_i)}{\Sigma_i \mu_G(u_i)} . \tag{4.11}$$

The concept of a relative sigma-count provides a basis for computing the test score for the elastic constraint induced by the proposition

$$p \triangleq Q\,A\text{'s are } B\text{'s} , \tag{4.12}$$

where A and B are fuzzy sets -- or, equivalently, fuzzy predicates -- and Q is a fuzzy quantifier such as *most*, *many*, *almost all*, *few*, etc. More specifically, if μ_Q is the membership function of Q, then

$$p \to \tau = \mu_Q(\Sigma Count(B/A)) , \tag{4.13}$$

where τ is the compatibility of p with an explanatory database whose constituents are the fuzzy relations A, B and Q, and the arrow $\to$ should be read as "induces" or "translates into."

As an illustration of the rules discussed above, consider the proposition

$$p \triangleq \textit{Most Frenchmen are neither very tall nor very fat.} \tag{4.14}$$

The EDF in this case will be assumed to have the following relations as constituents:

$EDF \triangleq POPULATION.FRENCHMEN\,[Name\,;\ Weight\,;\ Height\,] +$

$FAT\,[Weight;\ Height;\ \mu]$ +

$TALL\,[Height;\ \mu]$ +

$MOST\,[Proportion;\ \mu]$.

In this EDF, the first relation is a listing of a representative group of Frenchmen, with *Weight* and *Height* being the attributes of *Name*; the second relation serves to calibrate the fuzzy predicate ***fat*** as a function of *Weight* and *Height*; the third relation calibrates the fuzzy predicate ***tall***; and the last relation calibrates the fuzzy quantifier ***most*** as a function of *Proportion*. Correspondingly, the test procedure with computes the overall test score and thus represents the meaning of p assumes the following form:

1. Let $Name_i$ be the name of ith Frenchman in POPULATION. For each $Name_i$, $i = 1, \ldots, m$, find the weight and height of $Name_i$:

$$Weight\ (Name_i) = {}_{Weight}\ POPULATION[Name = Name_i]\ . \tag{4.15}$$

$$Height\ (Name_i) = {}_{Height}\ POPULATION[Name = Name_i]\ . \tag{4.16}$$

2. For each $Name_i$, compute the test scores for the constraints induced by ***fat*** and ***tall***:

$$\alpha_i = {}_{\mu}FAT[Weight = Weight(Name_i)\ ; \quad Height = Height(Name_i)] \tag{4.17}$$

$$\beta_i = {}_{\mu}TALL[Height = Height(Name_i)]\ . \tag{4.18}$$

3. Intensify the constraints induced by ***fat*** and ***tall*** to account for the modifier ***very***. Using (4.2), the corresponding test scores may be expressed as:

$$\alpha_i^{*} = \alpha_i^2 \ , \quad i = 1, \ldots, m \tag{4.19}$$

$$\beta_i^{*} = \beta_i^2 \ , \tag{4.20}$$

4. Modify the constraints induced by ***very fat*** and ***very tall*** to account for the negation ***not***:

$$\alpha_i' = 1 - \alpha_i^{*} = 1 - \alpha_i^2 \tag{4.21}$$

$$\beta_i' = 1 - \beta_i^{*} = 1 - \beta_i^2\ . \tag{4.22}$$

5. For each $Name_i$, aggregate conjunctively the test scores for the constraints induced by ***not very fat*** and ***not very tall***:

$$\tau_i = (1-\alpha_i^2) \wedge (1-\beta_i^2) \quad , \quad i = 1, \ldots, m \ . \tag{4.23}$$

The resulting test score represents the degree to which $Name_i$ is ***not very fat and not very tall.***

6. By using the relative sigma-count, compute the proportion of Frenchmen who are ***not very fat and not very tall:***

$$\rho = \frac{1}{m}\Sigma_i \tau_i \tag{4.24}$$
$$= \frac{1}{m}\Sigma_i (1-\alpha_i^2) \wedge (1-\beta_i^2) \ .$$

7. Compute the test score for the constraint induced by ***most***:

$$\tau = {}_{\mu}MOST[Proportion = \rho] \ . \tag{4.25}$$

This test score is the desired overall test score for the proposition under consideration, and the test procedure which computes τ represents the meaning of p.

5. Vector test scores

In the examples considered so far, the overall test score is a scalar. In the case of the following example (Zadeh, 1981):

$$p \triangleq \textit{By far the richest man in France is bald} \ , \tag{5.1}$$

[5]

the overall test score assumes the form of an ordered pair,

$$\tau \triangleq (\tau_0, \tau_p) \ ,$$

in which τ_0 is the test score associated with the fuzzy presupposition

$$p^* \triangleq \textit{There exists by far the richest man in France} \ , \tag{5.2}$$

and τ_p is the degree to which the by far the richest man in France is bald. The presupposition expressed by (5.2) is fuzzy by virtue of the fuzziness of the predicate ***by far the richest man in France***. What this implies is that the existence of ***by far the richest man in France*** is a matter of degree.

To represent the meaning of p (5.1), we choose the following EDF:

$$EDF \triangleq POPULATION\,[Name;\ Wealth; \mu Bald] + \tag{5.3}$$
$$BY.FAR.RICHEST\,[Wealth1;\ Wealth2; \mu] \ .$$

In the first relation in (5.3), *Wealth* is interpreted as the net worth of *Name* and *μBald* is the degree to which *Name* is bald. In the second relation, *Wealth*1 is the wealth of the richest man, *Wealth*2 is the wealth of the second richest man, and μ is the degree to which *Wealth*1 and *Wealth*2 qualify the richest man in France (who is assumed to be unique) to be regarded as by far the richest man in France.

To compute the compatibility of p with the explanatory database, perform the following test.

1. Sort POPULATION in descending order of Wealth. Denote the result by POPULATION ↓ and let Name_i be the ith name in POPULATION ↓.

2. Determine the degree to which the richest man in France is bald:

$$\tau_p = {}_{\mu Bald}POPULATION{\downarrow}[Name = Name_1] \ . \tag{5.4}$$

3. Determine the wealth of the richest and second richest men in France:

$$w_1 = {}_{Wealth}POPULATION{\downarrow}[Name = Name_1]$$

$$w_2 = {}_{Wealth}POPULATION{\downarrow}[Name = Name_2] \ .$$

4. Determine the degree to which the richest man in France is by far the richest man in France:

$$\tau_o = {}_{\mu}BY.FAR.RICHEST[Wealth1 = w_1; Wealth2 = w_2] \ . \tag{5.5}$$

5. The overall test score is taken to be the ordered pair

$$\tau = (\tau_o \, , \tau_p) \ . \tag{5.6}$$

Thus, instead of aggregating τ_1 and τ_2 into a single test score, we maintain their separate identities in the overall test score. We do this because the aggregated test score

$$\tau = \tau_o \wedge \tau_p$$

would be creating a misleading impression when τ_o is small, that is, when the test score for the constraint on the existence of *by far the richest man in France* is low.

More generally, the need for a vector test score arises when the degree of summarization which is implicit in a scalar test score may be excessive in relation to the purpose of meaning-representation in a particular context. As an illustration, consider the proposition

$$p \triangleq \textit{Berkeley has a temperate climate} \ , \tag{5.7}$$

and suppose that the intended meaning of p is represented by the following three propositions:

$p_1 \triangleq$ *The average temperature in Berkeley is around* 70° .

$p_2 \triangleq$ *The fraction of hot days is small* .

$p_3 \triangleq$ *The fraction of cold days is small* .

The EDF for p will be assumed to be the union of the EDF's for p_1, p_2 and p_3, which are:

$EDF_1 \triangleq AROUND[T_1;\ T_2;\ \mu]$.

$EDF_2 \triangleq HOT[T;\ \mu] + SMALL[Proportion\ ;\ \mu]$.

$EDF_3 \triangleq COLD[T;\ \mu] + SMALL[Proportion\ ;\ \mu]$.

In $AROUND$, μ is the degree to which a temperature, T_1, is *around* T_2; in HOT, μ is the degree to which T is *hot*; in $COLD$, μ is the degree to which T is *cold*; and in $SMALL$, μ is the degree to which *Proportion* is *small*. Note that all of these relations serve to calibrate the meanings of the predicates which they represent.

Now suppose that the overall test scores for p_1, p_2 and p_3 are τ_1, τ_2 and τ_3, respectively. We can compute the overall test score by aggregating these test scores, yielding

$$\tau = \tau_1 \wedge \tau_2 \wedge \tau_3 \ . \tag{5.8}$$

On the other hand, for some purposes, the single test score represented by (5.8) might be insufficiently informative. Then, a vector test score represented by the triple

$$\tau = (\tau_1,\ \tau_2,\ \tau_3) \tag{5.9}$$

might be more appropriate.

6. Representation of the Meaning of Dispositions

A concept which plays an important role in natural languages is that of a *disposition*. Informally, a disposition is a proposition which is preponderantly, but not necessarily always, true. For example, *snow is white* is a disposition, as are the propositions *trees are green*, *small cars are unsafe*, *John is always drunk*, *overeating causes obesity*, etc. Technically, a disposition may be defined as a proposition with implicit fuzzy quantifiers (Zadeh, 1983bc). This definition is a *dispositional definition* in the sense that it, too, is a disposition. It should be noted that what is usually called *commonsense knowledge* may be viewed as a collection of dispositions (Zadeh, 1983d).

Usually, a disposition may be derived from a dispositional proposition by a suppression of one or more fuzzy quantifiers. For example, by suppressing the fuzzy temporal quantifier *usually* in the dispositional proposition

$$dp \triangleq \textit{Usually snow is white} \ , \tag{6.1}$$

we obtain the disposition

$$d \triangleq \textit{Snow is white} \ . \tag{6.2}$$

Similarly, by suppressing the fuzzy quantifier *most of* in the dispositional proposition

$$dp \triangleq \textit{Most of those who overeat are obese} \ , \tag{6.3}$$

we arrive at the disposition

$$d \triangleq \textit{Those who overeat are obese} \ , \tag{6.4}$$

which, in an associational sense of causality (Suppes, 1970) may be expressed as

$$d \triangleq \textit{Overeating causes obesity} \ . \tag{6.5}$$

As a further illustration, by suppressing the fuzzy quantifiers *most* and *mostly* in the dispositional proposition

$$dp \triangleq \textit{Most young men like mostly young women} \ , \tag{6.6}$$

we obtain the disposition

$$d \triangleq \textit{Young men like young women} \ . \tag{6.7}$$

Frequently, a proposition with nonfuzzy quantifiers is intended to be understood as a dispositional proposition. For example,

$$p \triangleq \textit{John is always drunk} \ , \tag{6.8}$$

would usually be understood as

$$dp \triangleq \textit{John is frequently drunk} \ , \tag{6.9}$$

in which the fuzzy temporal quantifier *frequently* conveys the intended meaning of *always*. (Note, however, that the fuzzy quantifier *frequently* in (6.9) cannot be suppressed without distorting the intended meaning of p.) The replacement of a nonfuzzy quantifier with a fuzzy quantifier is an instance of *fuzzification*.

More generally, the transformation of a proposition with implicit fuzzy quantifiers into one in which the fuzzy quantifiers are explicit may be viewed as an instance of *explicitation*. The process of explicitation constitutes the first step in representing the meaning of a disposition.

In general, explicitation is an interpretation-dependent process in the sense that the restoration of suppressed fuzzy quantifiers and/or the fuzzification of nonfuzzy quantifiers depends on the intended meaning of the disposition. As an illustration, consider the disposition

$$d \triangleq \textit{Overeating causes obesity} \ . \tag{6.10}$$

The intended meaning of this disposition may be conveyed by the restored dispositional proposition

$$dp \triangleq \textit{Most of those who overeat are obese} \ . \tag{6.11}$$

On the other hand, the intended meaning of the disposition

$$d \triangleq \textit{Heavy smoking causes lung cancer} \ , \tag{6.12}$$

which is similiar in form to (6.10), may be conveyed by the dispositional proposition

$$dp \triangleq \textit{The proportion of cases of lung cancer among heavy smokers is much higher than among nonsmokers.}$$

Similarly,

$$d \triangleq \textit{Young men like young women} \tag{6.13}$$

has a number of different inerpretations, among them:

$$(a) \quad dp \triangleq \textit{Most young men like most young women} \ , \tag{6.14}$$

and, what appears to be much more reasonable:

$$(b) \quad dp \triangleq \textit{Most young men like mostly young women} \ . \tag{6.15}$$

At a later point in this section, we shall use (6.15) to illustrate the representation of the meaning of a disposition through the use of test-score semantics.

The concept of a dispositional proposition opens the door to the construction of a number of other concepts with a *dispositional* flavor, e.g., *dispositional predicate*, *dispositional containment* (in the sense of set inclusion), *dispositional command*, *dispositional preference relation*, etc. For example, *smokes* in *Virginia smokes cigarettes*; *like* in *Young men like young women*; *like* in *Frenchmen are like Spaniards*; *loves* in *Ann loves men* are dispositional predicates. *Keep under refrigeration*, *Avoid overeating*, *Stay away from bald men* are dispositional commands; and *Gentlemen prefer blondes*, *Mike is a better tennis player than Claudine*, *Tokyo is much safer than New York*, are examples of dispositional preference or ordering relations.

As an illustration of how the meaning of a dispositional predicate may be defined, consider the proposition

$$d \triangleq \textit{Virginia smokes cigarettes} \ , \tag{6.16}$$

which is a disposition by virtue of containing the dispositional predicate *smokes*.

Asume that the intended meaning of d is conveyed by the proposition

$$p \triangleq \textit{On the average Virginia smokes at least a few cigarettes a day} \ , \tag{6.17}$$

in which *smokes* is used in its literal (nondispositional) sense. At this point, we can employ test-score semantics to represent the meaning of p. Specifically, let the following relations be the constituents of EDF:

$$EDF \triangleq RECORD[Day; Number] + FEW[Number; \mu] \ , \tag{6.18}$$

in which *RECORD* is a daily record of the number of cigarettes smoked by Virginia during a representative period, say a month; and *FEW* is a fuzzy relation which calibrates the meaning of the fuzzy number *few*.

The steps in the test procedure which represents the meaning of p may be described as follows:

1. Let Day_i denote the ith day in $RECORD$, $i = 1, \ldots, 30$. Determine the number of cigarettes smoked by Virginia on Day_i:

$$Number(Day_i) = {}_{Number}RECORD[Day = Day_i] \ . \tag{6.19}$$

2. Compute the average number of cigarettes smoked by Virginia during the period under consideration:

$$\rho = \frac{1}{30} \Sigma_i Number(Day_i) \ . \tag{6.20}$$

3. Test the constraint induced by the fuzzy quantifier ***at least a few***:

$$\tau = {}_{\mu}{\geq} FEW[Number = \rho] \ , \tag{6.21}$$

in which $\geq FEW$ represents the relation *at least a few*, and τ is the overall test score.

The ability to represent the meaning of a dispositional predicate is of use in representing the meaning of complex dispositions. This aspect of the representation of the meaning of dispositions is illustrated by the following example (Zadeh, 1983b):

$$d \triangleq \textit{Young men like young women} \ , \tag{6.22}$$

which will be interpreted as the proposition

$$p \triangleq \textit{Most young men like mostly young women} \ . \tag{6.23}$$

Now, in d the predicate

$$D \triangleq \textit{likes young women}$$

may be viewed as a dispositional predicate, so that (6.23) may be rewritten as

$$p \triangleq \textit{Most young men are } D \ . \tag{6.24}$$

In this way, representation of the meaning of d (6.22) may be accomplished in two steps: first, we construct a test procedure to represent the meaning of D, and second, a test procedure is constructed to represent the meaning of p (6.24).

We shall assume that the EDF for p consists of the following relations:

$$\begin{aligned} EDF \triangleq\ & POPULATION\,[Name\,;\, Age\,;\, Sex\,] + \\ & LIKE\,[Name\,1;\ Name\,2;\ \mu] + \\ & YOUNG\,[Age\,;\,\mu] + \\ & MUCH.HIGHER\,[Proportion\,1;\ Proportion\,2;\ \mu] + \\ & MOST\,[Proportion;\,\mu]\ . \end{aligned} \tag{6.25}$$

In *LIKE*, μ is the degree to which *Name* 1 likes *Name* 2; and in *MUCH.HIGHER*, μ is the degree to which *Proportion* 1 is much higher than *Proportion* 2.

The meaning of D may be represented by the following test procedure:

1. Divide *POPULATION* into the population of males, *M.POPULATION*, and population of females, *F.POPULATION*:

$$\begin{aligned} M.POPULATION &= {}_{Name\,,Age}POPULATION\ [Sex\ =\ Male\,] \\ F.POPULATION &= {}_{Name\,,Age}POPULATION\ [Sex\ =\ Female\,]\ , \end{aligned} \tag{6.26}$$

   where ${}_{Name\,,Age}$ *POPULATION* denotes the projection of *POPULATION* on the attributes *Name* and *Age*.

2. For each $Name_j$,$j = 1, \ldots, K$, in *F.POPULATION*, find the age of $Name_j$:

$$A_j = {}_{Age}\ F.POPULATION[Name\ =\ Name_j]\ . \tag{6.27}$$

3. For each $Name_j$, find the degree to which $Name_j$ is young:

$$\alpha_i = {}_{\mu}\ YOUNG[Age\ =\ A_j]\ , \tag{6.28}$$

where α_i may be interpreted as the grade of membership of $Name_j$ in the fuzzy set, YW, of young women.

4. For each $Name_i$,$i = 1, \ldots, L$, in $M.POPULATION$, find the age of $Name_i$:

$$B_i = {}_{Age}\, M.POPULATION[Name = Name_i] \ . \tag{6.29}$$

5. For each $Name_i$, find the degree to which $Name_i$ is young:

$$\delta_i = {}_{\mu}\, YOUNG[Age = B_i] \ , \tag{6.30}$$

where δ_i may be interpreted as the grade of membership of $Name_i$ in the fuzzy set, YM, of young men.

6. For each $Name_j$, find the degree to which $Name_i$ likes $Name_j$:

$$\beta_{ij} = {}_{\mu}\, LIKE[Name\,1 = Name_i \,; Name\,2 = Name_j] \ , \tag{6.31}$$

with the understanding that β_{ij} may be interpreted as the grade of membership of $Name_j$ in the fuzzy set, WL_i, of women whom $Name_i$ likes.

7. For each $Name_j$, find the degree to which $Name_i$ likes $Name_j$ and $Name_j$ is young:

$$\gamma_{ij} = \alpha_j \wedge \beta_{ij} \ . \tag{6.32}$$

Note: As in previous examples, we employ the aggregation operator min ($\wedge$) to represent the meaning of conjunction. In effect, γ_{ij} is the grade of membership of $Name_j$ in the intersection of the fuzzy sets WL_i and YW.

8. Compute the relative sigma-count of young women among the women whom $Name_i$ likes:

$$\begin{aligned} \rho_i &= \Sigma Count\,(YW / WL_i) \\ &= \frac{\Sigma Count\,(YW \cap WL_i)}{\Sigma Count\,(WL_i)} \\ &= \frac{\Sigma_j\, \gamma_{ij}}{\Sigma_j\, \beta_{ij}} \\ &= \frac{\Sigma_j\, \alpha_j \wedge \beta_{ij}}{\Sigma_j\, \beta_{ij}} \ . \end{aligned} \tag{6.33}$$

9. Compute the relative sigma-count of (not young) women among the women whom $Name_i$ likes:

$$\gamma_i = \frac{\Sigma_j\, (1-\alpha_j) \wedge \beta_{ij}}{\Sigma_j\, \beta_{ij}} \ . \tag{6.34}$$

10. Compute the degree to which ρ_i and γ_i satisfy the constraint induced by *MUCH.HIGHER*:

$$\tau_i = {}_{\mu} MUCH.HIGHER[Proportion1 = \rho_i ; Proportion2 = \gamma_i] \ . \tag{6.35}$$

This test score represents the overall test score for the proposition "*$Name_i$ is D*," and may be interpreted as the grade of membership of $Name_i$ in the fuzzy set, *D*, of men who have property *D*.

We are now in a position to compute the overall test score for *p*. Thus, on continuing the test, we have

11. Compute the relative sigma-count of men who have property *D* among young men:

$$\begin{aligned} \rho &= \Sigma Count(D / YM) \\ &= \frac{\Sigma Count(D \cap YM)}{\Sigma Count(YM)} \\ &= \frac{\Sigma_i \tau_i \wedge \delta_i}{\Sigma_i \delta_i} \ . \end{aligned} \tag{6.36}$$

12. Test the constraint induced by *MOST*:

$$\tau = {}_{\mu} MOST[Proportion = \rho] \ . \tag{6.37}$$

The test score expressed by (6.37) represents the overall test score for the disposition $d \triangleq$ *Young men like young women*.

As the final example in this Section, we shall consider the dispositional command:

$$c \triangleq \textit{Stay away from bald men} \ . \tag{6.38}$$

In general, to represent the meaning of a command, *c*, it is necessary to associate with *c* its *compliance criterion*, cc, which may then be viewed as the definition of *c*. In the case of *c* (6.28), the compliance criterion will be assumed to be represented by the proposition

$$cc \triangleq \textit{Staying away from most bald men} \ . \tag{6.39}$$

To represent the meaning of cc (6.29), we shall employ the following EDF:

$$\begin{aligned} EDF \triangleq & RECORD[Name ; \mu Bald ; Action] + \\ & MOST[Proportion ; \mu] \ . \end{aligned} \tag{6.40}$$

In this EDF, the relation *RECORD* is the record of successive actions which constitute the execution of c over a representative period of time; $\mu Bald$ is the degree to which *Name* is bald; and *Action* is a variable which takes the value 1 if *Name* is stayed away from and 0 otherwise.

The steps in the test procedure are the following:

1. For each $Name_i$ in *RECORD*, $i = 1, \ldots, n$, find (a) the degree to which $Name_i$ is bald, and (b) the action taken:

$$(a)\ \mu Bald\,(Name_i) = {}_{\mu}RECORD[Name\ = Name_i] \tag{6.41}$$

$$(b)\ Action\,(Name_i) = {}_{Action}RECORD[Name\ = Name_i]\ .$$

2. Using the relative sigma-count, find the proportion of cases in which the command is complied with:

$$\rho = \frac{\Sigma_i\ Action(Name_i)\mu Bald(Name_i)}{\Sigma_i\ \mu Bald(Name_i)}\ . \tag{6.42}$$

3. Test the constraint induced by the fuzzy quantifier ***most***:

$$\tau = {}_{\mu}MOST[Proportion\ = \rho]\ . \tag{6.43}$$

The computed value of τ represents the degree of compliance over the period under consideration; and the test procedure which yields τ represents the meaning of the dispositional command (6.38).

7. The Concept of a Canonical Form and its Application to the Representation of Meaning

When the meaning of a proposition, p, is represented as a test procedure, it may be hard to discern in the description of the procedure the underlying structure of the process through which the meaning of p is constructed from the meanings of the composants of p.

A concept which makes it easier to perceive the logical structure of p and thus to develop a better understanding of the meaning representation process, is that of a canonical form of p, abbreviated as $cf\,(p)$ (Zadeh, 1981).

The concept of a canonical form relates to the basic idea which underlies test-score semantics, namely, that any semantic entity -- and, in particular, a proposition -- may be viewed as a system of elastic constraints whose domain is a collection of relations in the explanatory database. Equivalently, let $X_1, \ldots, X_n$ be a collection of variables which are constrained by p. Then, the canonical form of p

may be expressed as

$$cf(p) \triangleq X \text{ is } F \ , \tag{7.1}$$

where $X = (X_1, \ldots, X_n)$ is the constrained variable which is usually implicit in p, and F is a fuzzy relation, likewise implicit in p, which plays the role of an elastic (or fuzzy) constraint on X. The relation between p and its canonical form will be expressed as

$$p \rightarrow X \text{ is } F \ , \tag{7.2}$$

signifying that the canonical form may be viewed as a representation of the meaning of p.

In general, the constrained variable X in $cf(p)$ is not uniquely determined by p, and is dependent on the focus of attention in the meaning-representation process. To place this in evidence, we shall refer to X as the *focal* variable.

As a simple illustration, consider the proposition

$$p \triangleq \textit{Janet has blue eyes} \ . \tag{7.3}$$

In this case, the focal variable may be expressed as

$$X \triangleq Color\,(Eyes\,(Janet)) \ ,$$

and the elastic constraint is represented by the fuzzy relation *BLUE*. Thus, we can write

$$p \rightarrow Color\,(Eyes\,(Janet)) \text{ is } BLUE \ . \tag{7.4}$$

As an additional illustration, consider the proposition

$$p \triangleq \textit{Dick is much taller than Nina.} \tag{7.5}$$

Here, the focal variable has two components, $X = (X_1, X_2)$, where

$X_1 = Height\,(Dick)$

$X_2 = Height\,(Nina)$;

and the elastic constraint is characterized by the fuzzy relation *MUCH.TALLER* [*Height* 1; *Height* 2; μ], in which μ is the degree to which *Height* 1 is *much taller* than *Height* 2. In this case, we have

$$p \rightarrow (Height\,(Dick),\ Height\,(Nina)) \text{ is } MUCH.TALLER \ . \tag{7.6}$$

To make the meaning of a canonical form more precise, it is necessary to introduce the concept of a possibility distribution (Zadeh, 1978a). Specifically, let

X be an n-ary variable $(X_1, \ldots, X_n)$ in which $X_i, i = 1, \ldots, n$, takes values in a universe of discourse U_i, implying that X takes values in $U = U_1 \times U_2 \times \cdots \times U_n$. Informally, the *possibility distribution* of X, expressed as Π_X, is the fuzzy set of possible values of X, with the understanding that the possibility that X may take a value $u \in U$, written as $Poss\{X = u\}$, is a number in the interval $[0,1]$.

In terms of the possibility distribution of X, the canonical form of p may be interpreted as the assignment of F to Π_X. Thus, we may write

$$p \rightarrow X \text{ is } F \rightarrow \Pi_X = F \ , \tag{7.7}$$

in which the equation

$$\Pi_X = F \tag{7.8}$$

is termed the *possibility assignment equation* (Zadeh 1978b). In effect, this equation signifies that the canonical form $cf\,(p) \triangleq X \text{ is } F$ implies that

$$Poss\{X = u\} = \mu_F(u) \ , \quad u \in U \ , \tag{7.9}$$

where μ_F is the membership function of F. It is in this sense that F, acting as an elastic constraint on X, restricts the possible values which X can take in U. An important implication of this observation is that a proposition, p, may be interpreted as an implict assignment statement which characterizes the possibility distribution of the focal variable in p.

As an illustration, consider the disposition

$$d \triangleq \textit{Overeating causes obesity} \ , \tag{7.10}$$

which upon restoration becomes

$$p \triangleq \textit{Most of those who overeat are obese} \ . \tag{7.11}$$

If the focal variable in this case is chosen to be the relative sigma-count of those who are obese among those who overeat, the canonical form of p (7.11) becomes

$$\Sigma Count\ (OBESE/\ OVEREAT) \text{ is } MOST \ , \tag{7.12}$$

which in virtue of (7.9) implies that

$$Poss\{\Sigma Count\ (OBESE/\ OVEREAT) = u\} = \mu_{MOST}(u) \ , \tag{7.13}$$

where μ_{MOST} is the membership function of *MOST*. What is important to note is that (7.13) is equivalent to the assertion that the overall test score for p is expressed by

$$\tau = \mu_{MOST}(\Sigma Count(OBESE / OVEREAT)) \ , \tag{7.14}$$

in which *OBESE*, *OVEREAT* and *MOST* play the roles of composants of p.

It is of interest to observe that the notion of a semantic network may be viewed as a special case of the concept of a canonical form. As a simple illustration, consider the proposition

$$p \triangleq \textit{Ron gave Shelley a red pin} \ . \tag{7.15}$$

As a semantic network, this proposition may be represented in the standard form:

$$\begin{aligned} &Agent\ (GIVE) = Ron \\ &Recipient\ (GIVE) = Shelley \\ &Time\ (GIVE) = Past \\ &Object\ (GIVE) = Pin \\ &Color\ (Pin) = Red \ . \end{aligned} \tag{7.16}$$

Now, if we identify X_1 with *Agent* (*GIVE*), X_2 with *Recipient* (*GIVE*), etc., the semantic network representation (7.16) may be regarded as a canonical form in which $X = (X_1, \ldots, X_5)$, and

$$\begin{aligned} &X_1 = Ron \\ &X_2 = Shelley \\ &X_3 = Past \\ &X_4 = Pin \\ &X_5 = Red \ . \end{aligned} \tag{7.17}$$

More generally, since any semantic network may be expressed as a collection of triples of the form (Object, Attribute, Attribute Value), it can be transformed at once into a canonical form. However, since a canonical form has a much greater expressive power than a semantic netowrk, it may be difficult to transform a canonical form into a semantic network. A simple example of a proposition for which this is true is

$$p \triangleq \textit{Over the past several years the combined income of Patricia's close friends was about half a million dollars.} \tag{7.18}$$

Concluding Remark

We have described some of the basic ideas underlying test-score semantics and illustrated its application to the representation of the meaning of propositions, predicates, dispositions and commands. These ideas are simple in nature and, with a little practice, it is easy to learn how to use test-score semantics to represent the meaning of almost any semantic entity through the construction of an explanatory database and a test procedure. What is much more difficult, however, is to write a program which could construct an explanatory database and a test procedure without human assistance. This is a longer range problem whose complete solution must await the development of a substantially better understanding of natural languages and knowledge representation than we have at this juncture.

Notes

1. It should be noted that a quantifier may be viewed as a second order predicate.
2. As will be seen presently, this mode of counting yields the so-called *sigma-count* of large balls. An alternative way of counting the number of large balls is described in Zadeh (1981).
3. Standardized rules for aggregation and combination may be likened to ready-made clothing.
4. A more detailed discussion of various types of counts may be found in Zadeh (1981, 1983a) and Dubois (1982). Note that the power or the sigma-count of a fuzzy set may be viewed as a special case of its *measure* (Zadeh, 1968).
5. This proposition is a fuzzy version of the familiar example "*The King of France is bald,*" which is associated with the presupposition "*There exists the King of France.*"

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