PERIODICITY AND CHAOS IN CHUA'S CIRCUIT

by

G. Q. Zhong, and F. Ayrom

Memorandum No. UCB/ERL M84/81

19 September 1984

ELECTRONICS RESEARCH LABORATORY

College of Engineering
University of California, Berkeley
94720

PERIODICITY AND CHAOS IN CHUA'S CIRCUIT†

G. Q. Zhong†† and F. Ayrom
Department of Electrical Engineering and Computer Sciences
and the Electronics Research Laboratory
University of California, Berkeley, CA 94720

ABSTRACT

This letter reports a period-doubling route to chaos as observed from a laboratory model of the simplest possible chaotic autonomous circuit: it is made of two linear capacitors, one linear inductor, one linear resistor, and only one nonlinear 2-terminal resistor characterized by a 5-segment piecewise-linear v-i characteristic.

† Research sponsored by the Office of Naval Research Contract N00014-76-C-0572, and the Semiconductor Research Corporation SRC-82-11-008.

†† Guo-Qun Zhong was a visiting scholar at the University of California, Berkeley. He is now with Guangzhou Research Institute of Electronic Technology, Academia Sincia, Guangzhou, People's Republic of China.

The circuit shown in Fig. 1(a) with the nonlinear resistor v-i characteristic shown in Fig. 1(b) was recently conceived and proposed by Chua to be the simplest 3rd-order, autonomous (no ac sources), and reciprocal† circuit that could give rise to complicated chaotic dynamics. Several chaotic attractors had since been observed by Matsumoto via computer simulation of this circuit over a rather robust parameter range [1]. Our objective in this letter is to report some bifurcation and chaotic phenomena as measured from an actual circuit. Since the nonlinear resistor in Chua's circuit is not available as an "off-the shelf" device, our first task was to design and build such a device. The final circuit shown in Fig. 2(a) uses two operational amplifiers (op amp) and 6 linear resistors (in addition to the standard power supply for the op amp). The v-i characteristics in Figs. 2(b) and (c) are traced from this circuit with two different sets of resistor values (see Fig. caption). Our experiments reported in this letter are based on measurements obtained by connecting each of these two set-ups in place of the 2-terminal nonlinear resistor in Fig. 1(a). Note that we have exploited and made full use of the intrinsic saturation characteristic of the op amp (normally shunned in standard op amp circuit design) to realize the 5-segment piecewise-linear v-i characteristic in Fig. 1(a); no other nonlinear device is used. From a circuit-theoretic point of view the op amp circuit in Fig. 2(a) should be enclosed by a box with only two terminals (a) - (b) accessible for external connection, and is therefore to be classified as a 2-terminal device. Indeed, the circuit can be easily fabricated as an integrated circuit (IC) and encapsulated as any other standard device.

Compared to the two other autonomous chaotic circuits reported in the literature [2], [3], Chua's circuit is the simplest possible in the sense that chaos can not occur in an autonomous circuit with fewer than 3 energy storage elements (capacitors and inductors) and that at least one nonlinear element is needed even for oscillation to be possible.

We have built the circuit in Fig. 1(a) and observed a great variety of bifurcation and chaotic phenomena from different combinations of circuit parameters (R, L, C_1 and C_2) as well as different choices of the nonlinear v-i characteristics (breakpoints coordinates and slopes). In this letter, we summarize the phenomena observed from two set-ups corresponding to the two v-i characteristics shown in Figs. 2(b) and (c), respectively.

---

†From a circuit theory viewpoint, a reciprocal circuit is one made of only two-terminal elements such as resitors, inductors, capacitors, batteries, diodes, etc. Circuits containing transistors are therefore non-reciprocal and are generally considered to be more complicated.

Connecting terminals (a) - (b) of the circuit in Fig. 2(a) in place of the nonlinear resistor in Fig. 1(a), various Lissajous figures and spectrums are measured and shown in Figs. 3 and 4, respectively. The circuit parameters and scales used in these oscilloscope tracings are given in the figure captions.

Figures 3(a), (b), and (c) give 2 perspectives of the same chaotic attractor. It consists of two rings joined at the upper and the lower edge by a thin sheet of ribbon made of trajectories. A computer simulation of this circuit (not shown) gives rise to a virtually identical attractor and reveals that each trajectory winds around each ring in a counterclockwise direction and grow in size until it hits some boundary set near the outer edge of the ring, whereupon it exits rapidly along a thin ribbon and lands near the center of the other ring, thereby repeating the above phenomenon. The exit points and times are observed to be random, thereby accounting for the "double-ring" limiting set S. Since trajectories originating from nearby points outside of S are quickly attracted to S, it is natural to call S a chaotic attractor. The spectrums corresponding to the voltage waveform $V_{C1}(t)$ across capacitor C_1, the voltage waveform $V_{C2}(t)$ across capacitor C_2 and the current waveform $i_1(t)$ through the inductor L as shown in Figs. 3(d), (e), and (f), respectively, are seen to resemble a broad-spectrum noise.

Figures 4(a)-4(e) display the period-doubling route to chaos in Chua's circuit using the nonlinearity of Fig. 2(c). The single-loop limit cycle in Fig. 4(a) represents a well-defined periodic waveform spawned by a Hopf bifurcation process when a pair of complex-conjugate eigenvalues associated with an equilibrium point of this circuit crosses the imaginary axis and enters the right-half plane. A slight decrease in the value of R leads to the sequence of Lissajous figures shown in Figs. 4(b)-4(e). Figures 4(a)-4(d) shows three successive period-doublings giving rise to a 2, 4, and 8-loop limit cycles as we continue to decrease R in small amounts. A further small decrease in R leads to the chaotic attractor in Fig. 4(e). The structure between the two rings in this attractor is clearer than that shown in Figs. 3(a)-3(c). It appears also to be quite a bit more complicated than is apparent from the earlier thin ribbon paths. Since the range of R between Figs. 4(a) and 4(e) is very narrow (1.7 KΩ to 1.5 KΩ), as is typical of the convergence property of the period-doubling bifurcation parameter, we were unable to observe periodic waveforms with order higher than 8. Our results, however, suggest strongly that the chaotic attractor in Fig. 4(e) is spawned by a period-doubling mechanism.

REFERENCES

1. T. Matsumoto, "A chaotic attractor from Chua's circuit," IEEE Trans. on Circuits and Systems, vol. CAS-32, no. 1, to appear January 1985.
2. J. P. Gollub, T. O. Brunner, and B. G. Danly, "Periodicity and chaos in coupled nonlinear oscillators," Science, vol. 200, no. 7, pp. 48 -50, April 1978.
3. E. Freire, L. G. Franguelo, and J. Aracil, "Periodicity and chaos in an autonomous electronic system," IEEE Trans. on Circuit and Systems, vol. CAS-31, no. 3, pp. 237-247, March 1984.

Figure Captions

Fig. 1. (a) Chua's circuit

(b) v-i characteristic of nonlinear resistor

Fig. 2. (a) By adjusting the resistor values, any prescribed 5-segment piece-wise linear v-i characteristic similar to Fig. 1(b) can be realized with this circuit (op amp: National/8035 741LN).

(b) v-i characteristic obtained with $V_{cc} = 18V$, $R_1 = 376\Omega$, $R_2 = 78\Omega$ $R_3 = 5.98K\Omega$, $R_4 = 312\Omega$, $R_5 = 1.91K\Omega$ and $R_6 = 52\Omega$.

(c) v-i characteristic obtained with $V_{cc} = 15V$, $R_1 = 3.67K\Omega$, $R_2 = 1.09K\Omega$, $R_3 = 5.43K\Omega$, $R_4 = 104\Omega$, $R_5 = 5.36K\Omega$, and $R_6 = 128\Omega$.

Fig. 3. Chaotic attractors measured from Chua's circuit using the nonlinear device of Fig. 2(b) and with $R = 1.03K\Omega$, $C_1 = .005\mu F$, $C_2 = .1\mu F$ and $L = 7.6mH$.

(a) chaotic attractor in the $V_{C1} - V_{C2}$ plane: scale: $V_{C1} = 4V/div$, $V_{C2} = 2V/div$.

(b) chaotic attractor in the V_{C1}-i_L plane: scale: $i_L = 6ma/div$, $V_{C1} = 4V/div$.

(c) chaotic attractor in the V_{C2}-i_L plane: scale: $i_L = 6ma/div$, $V_{C2} = 2V/div$.

(d) spectrum of $V_{C1}(t)$.

(e) spectrum of $V_{C2}(t)$.

(f) spectrum of $i_L(t)$.

Fig. 4. Waveforms (left side) and spectrums (right side) showing the period doubling route to chaos in Chua's circuit using the nonlinear device of Fig. 2(c) and with $C_1 = .005\mu F$, $C_2 = .05\mu F$ and $L = 72mH$.

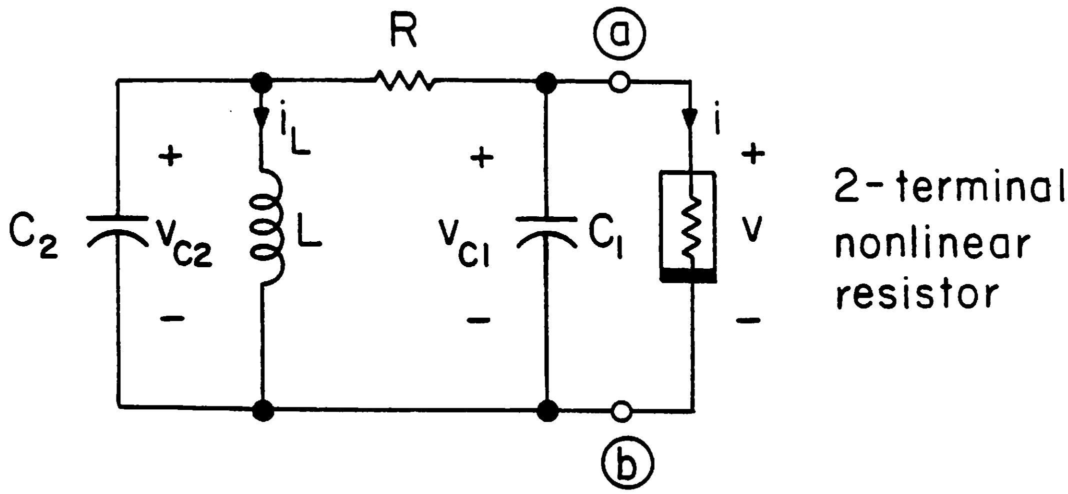
(a) single-loop cycle. Scale: $V_{C1} = 5V/div$, $V_{C2} = .4V/div$.

(b) double-loop cycle. Scale: $V_{C1} = 5V/dib$, $V_{C2} = .4V/div$.

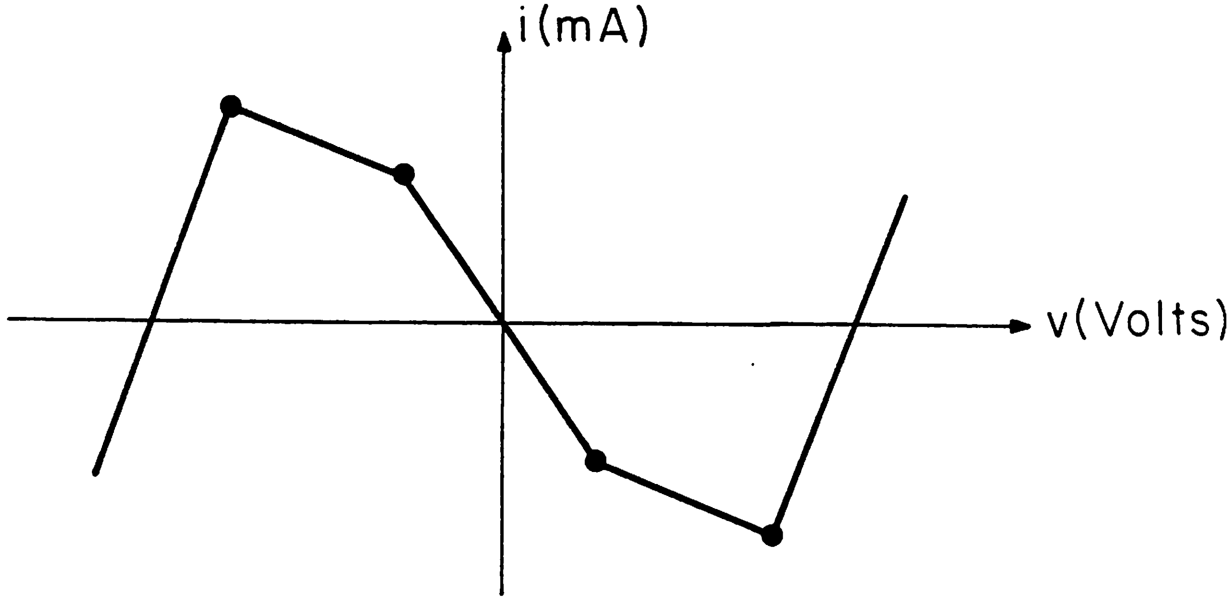
(c) 4-loop cycle. Scale: $V_{C1} = 5V/div$, $V_{C2} = .4V/div$.

(d) 8-loop cycle. Scale: $V_{C1} = 2V/div$, $V_{C2} = .4V/div$.

(e) chaotic attractor in the V_{C2}-V_{C1}. Scale: $V_{C1} = 5V/div$, $V_{C2} = .4V/div$.


R
a
i_L
i
+
+
+
2-terminal
nonlinear
resistor
C_2
v_{C2}
L
v_{C1}
C_1
v
−
−
−
b

(a)


i(mA)
v(Volts)

(b)

Fig. 1

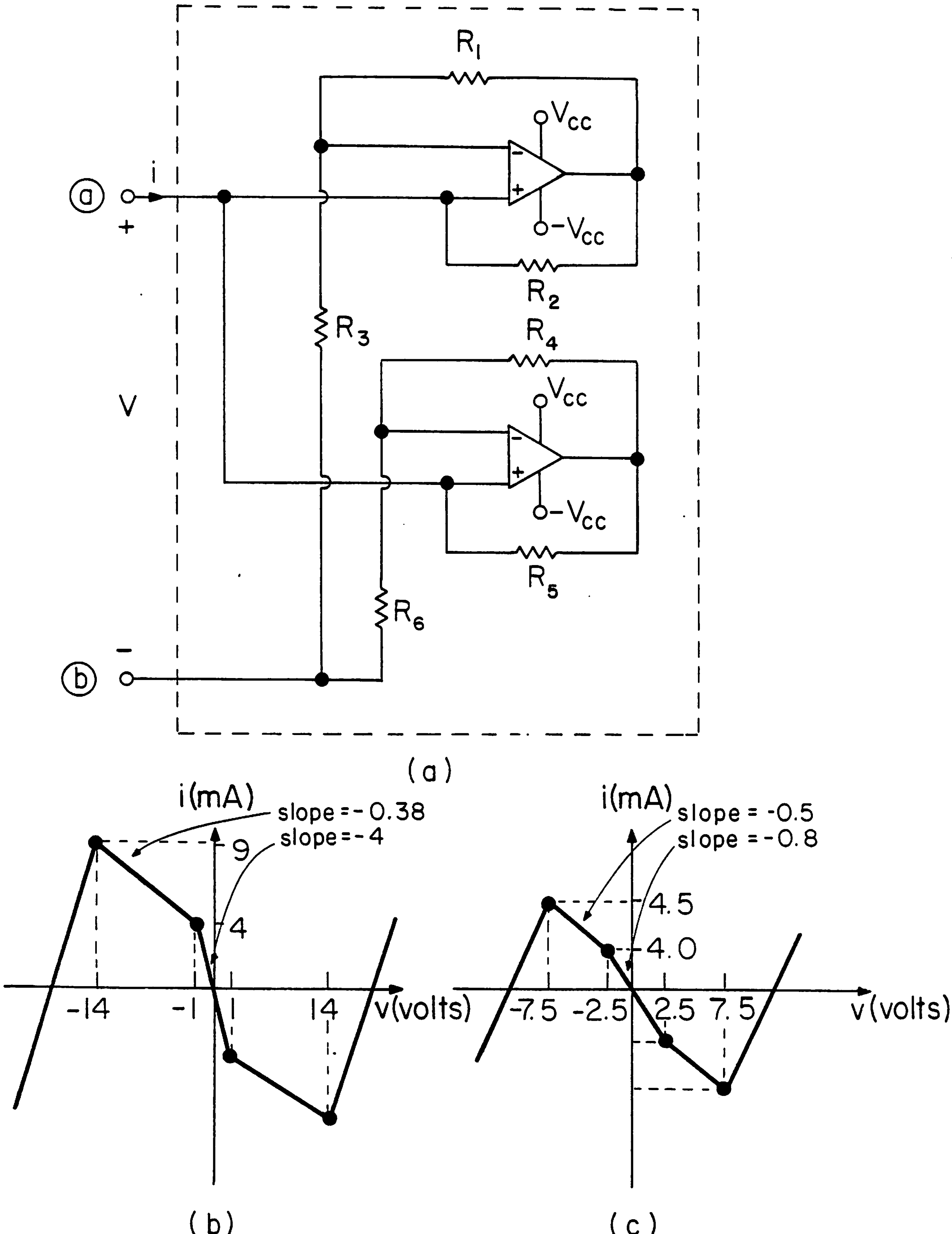

R1
Vcc
-Vcc
R2
R3
R4
Vcc
-Vcc
R5
R6
a
b
i
+
-
V
(a)
i(mA)
slope=-0.38
slope=-4
9
4
-14
-1
1
14
v(volts)
(b)
i(mA)
slope = -0.5
slope = -0.8
4.5
4.0
-7.5
-2.5
2.5
7.5
v(volts)
(c)

Fig. 2

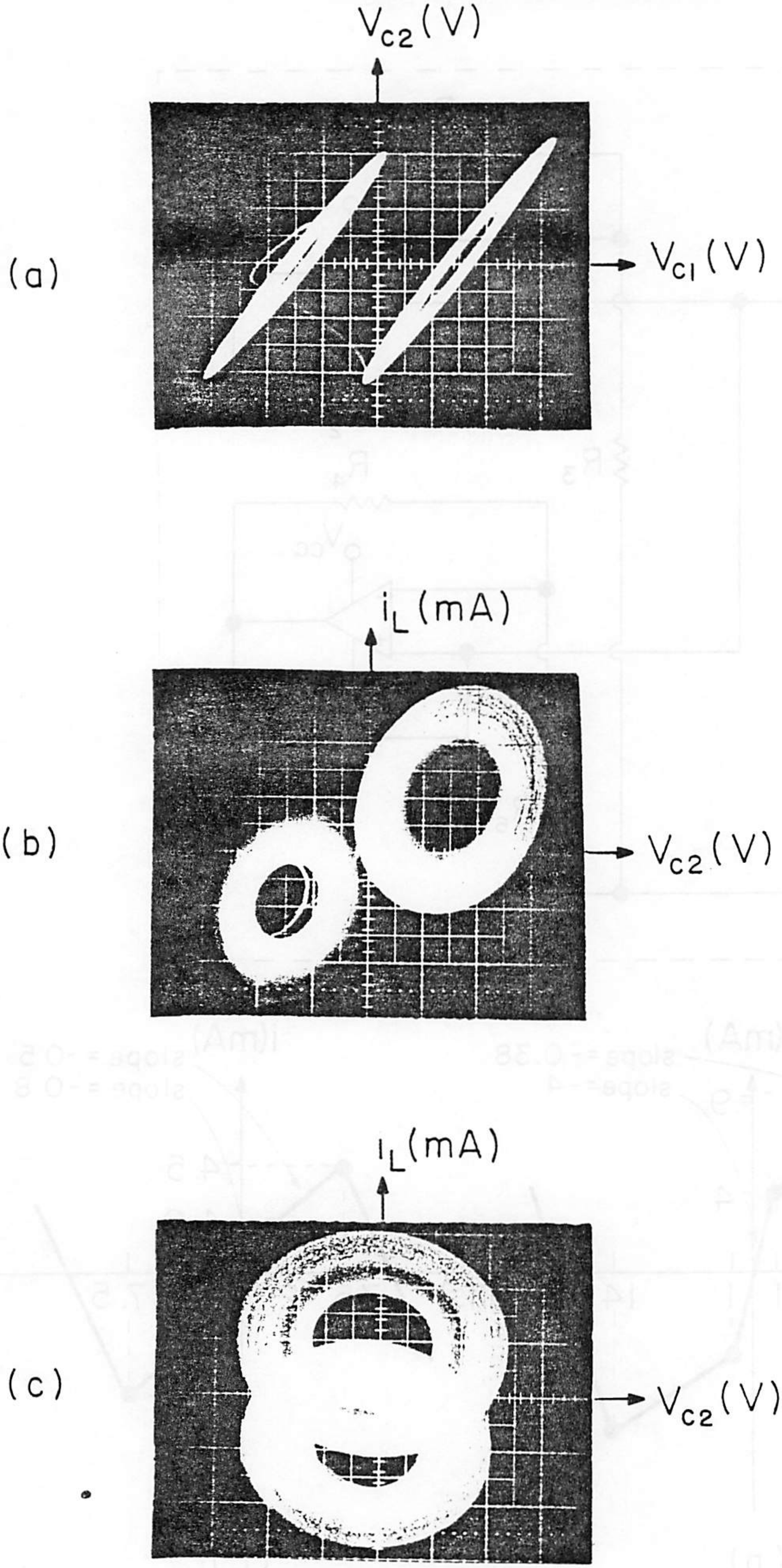
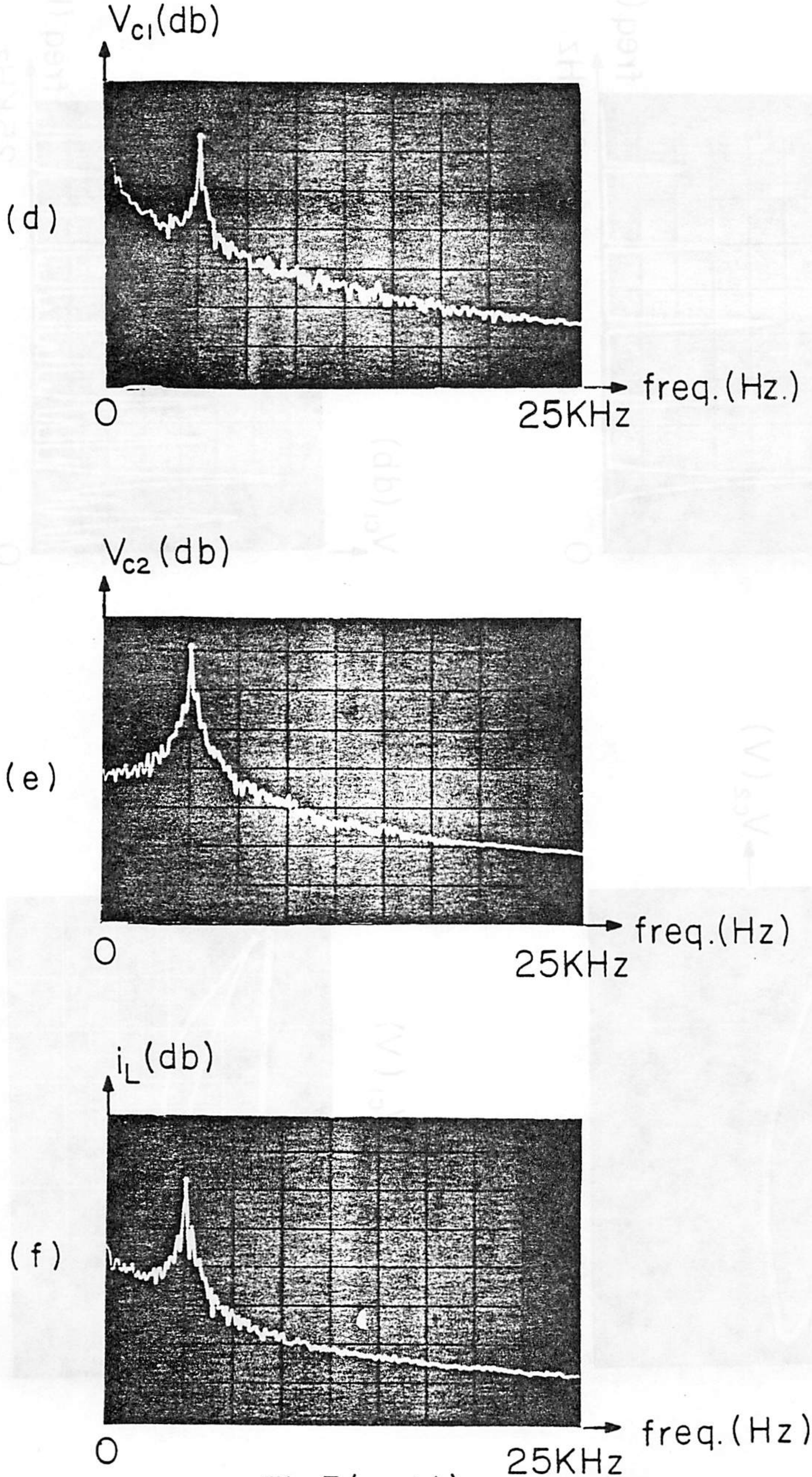
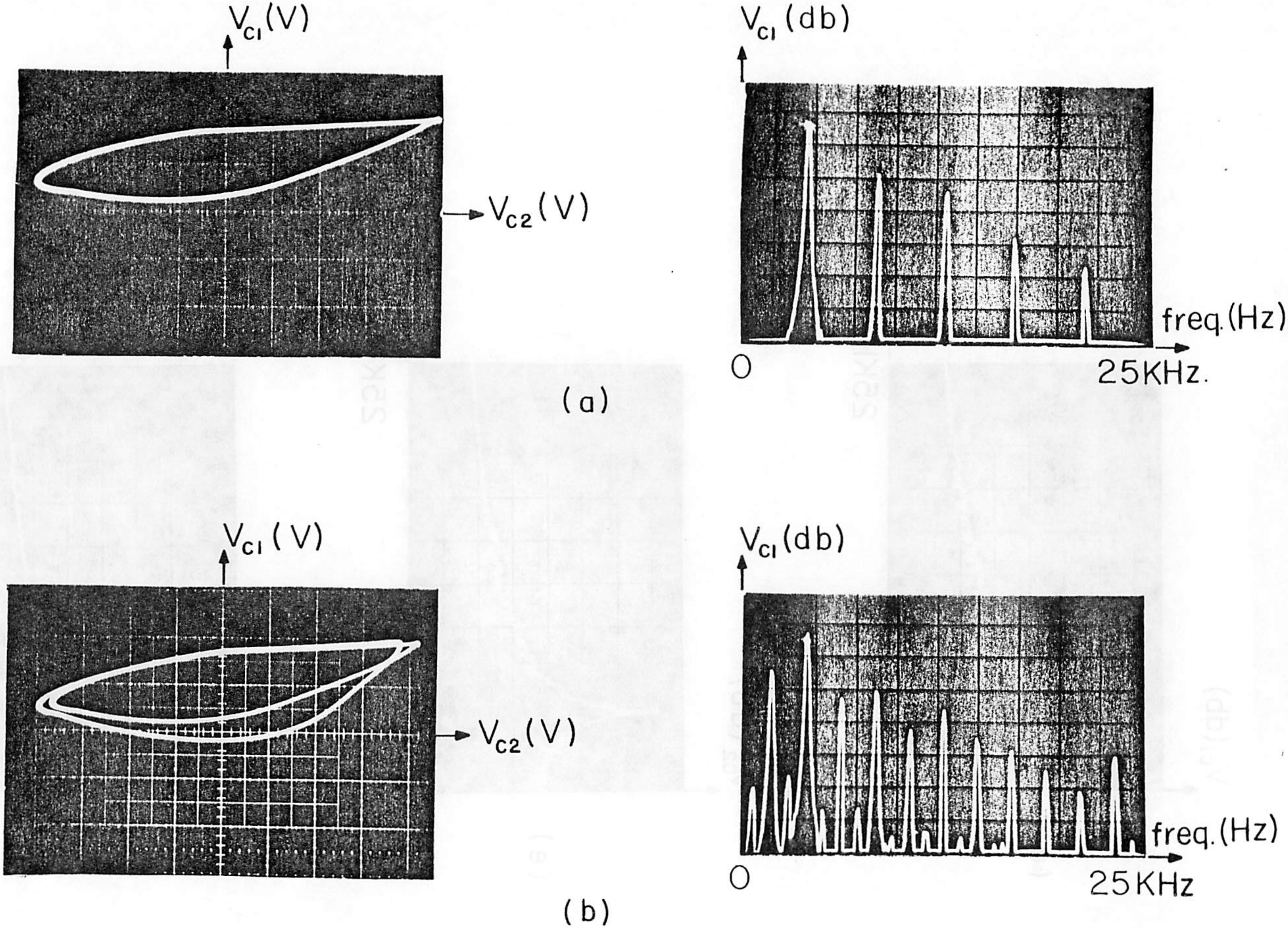


Fig. 3



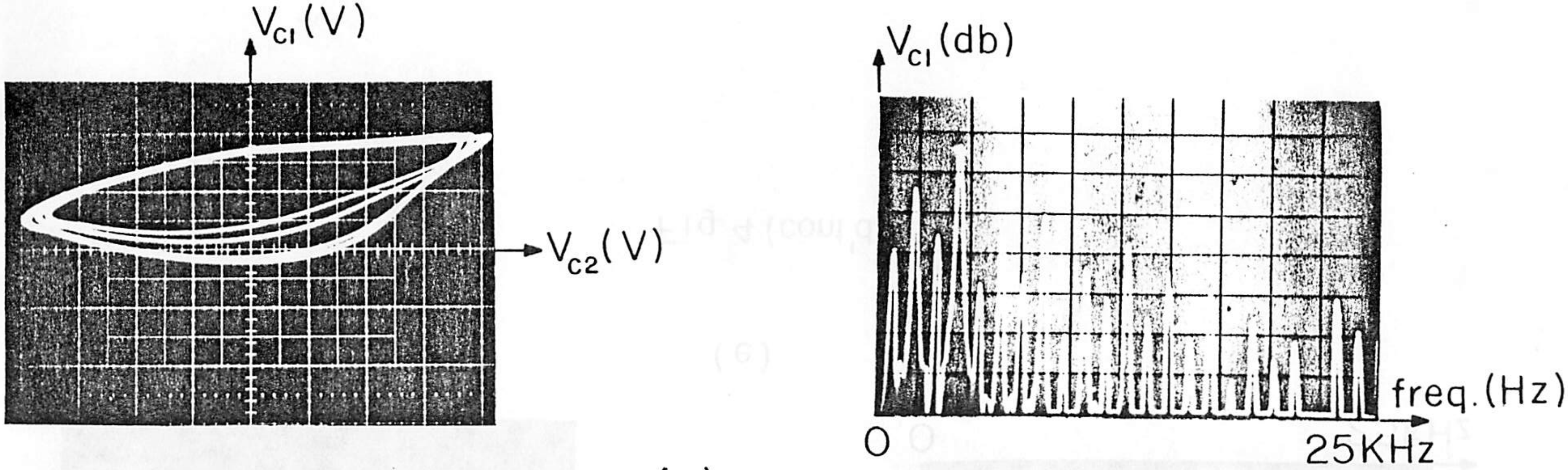
V_c1(db)
(d)
0
25KHz
freq.(Hz.)
V_c2(db)
(e)
0
25KHz
freq.(Hz)
i_L(db)
(f)
0
25KHz
freq.(Hz)

Fig.3(contd.)

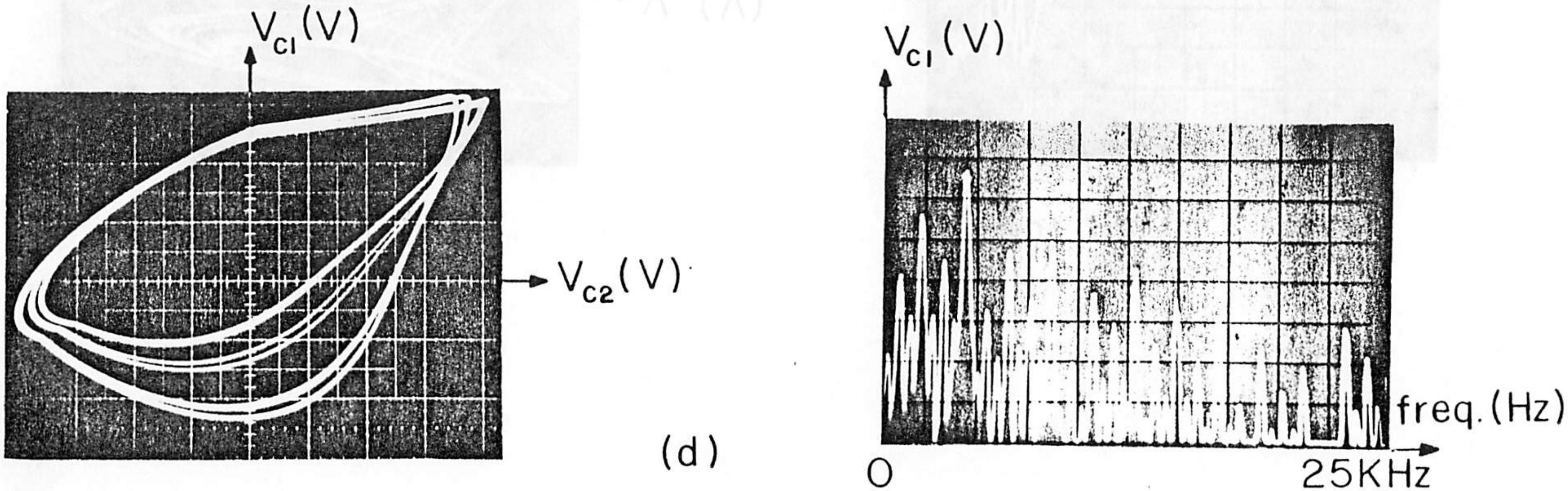


$V_{c1}(V)$
$V_{c2}(V)$
$V_{c1}(db)$
freq.(Hz)
0
25KHz.
(a)
$V_{c1}(V)$
$V_{c2}(V)$
$V_{c1}(db)$
freq.(Hz)
0
25KHz
(b)

Fig. 4

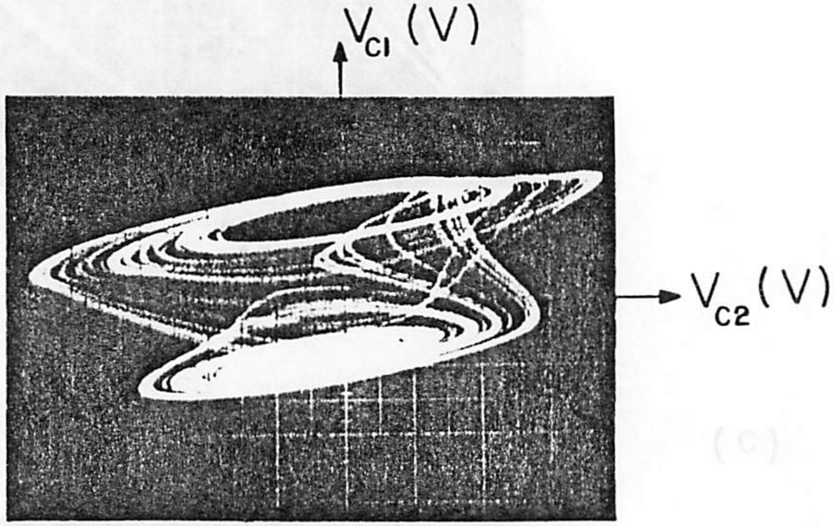


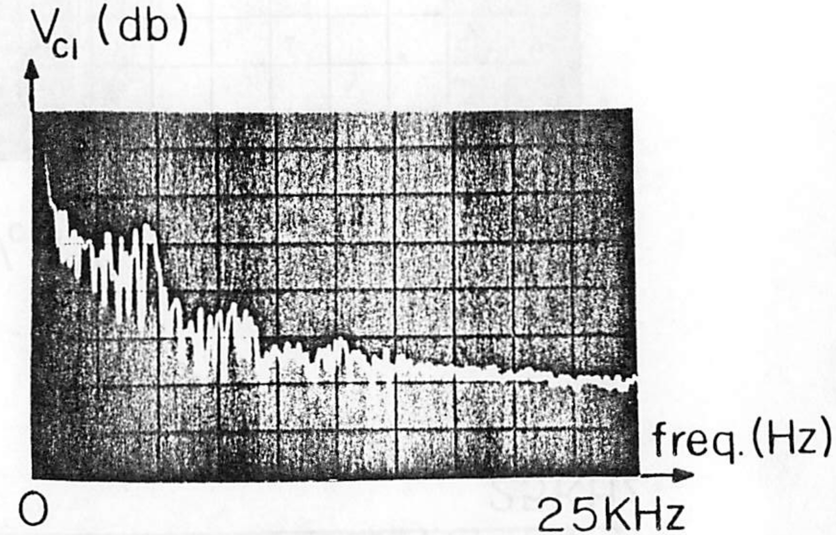
(c)



(d)

Fig. 4 (Cont'd.)





(e)

Fig. 4 (cont'd.)