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EXPERIMENTAL CONFIRMATION OF THE PERIOD-ADDING ROUTE TO CHAOS IN A NONLINEAR CIRCUIT†

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ABSTRACT

Experimental confirmation has been made on a negative-resistance oscillator circuit which exhibits the new period-adding route to chaos recently reported by Kaneko on a discrete map. The nonlinear element in the circuit is a negative-resistance device synthesized by using two bipolar transistors and four positive linear resistors. This circuit exhibits a new route to chaos; namely, through period-adding where the periods of two successive periodic waveforms belonging to each period-adding sequence differ by one; i.e., from period n to n-1 or n+1. The transition to chaos (after one or more periodic states) resulted directly from a loss of stability of a periodic state at a bifurcation point. Several general features of the periodic-chaotic transition sequence have been observed and presented.

1. INTRODUCTION

In recent years more and more studies on the chaotic phenomena in nonlinear dynamical systems have appeared. Chaotic behavior arises primarily from the nonlinear nature of the system rather than from external stochastic fluctuations and can be described exactly by deterministic equations. Several transition sequences leading to chaos have been observed [1] in which the period-doubling, U-sequence, intermittency and frequency-locking, etc., are well known and have been observed in several physical systems.

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In our previous studies on an RL-varactor oscillator all of the above sequences were obtained [2-5]. However, the period-adding phenomena and the alternating periodic-chaotic transition sequence are relatively new discoveries and have so far been observed only from few physical systems. The period-adding phenomena were first reported by Kaneko [6] on a 1-dimensional iterated map. This new route to chaos has also been observed in 2-dimensional iterated maps and in experiments on the Belousov-Zhabotinskii reaction [7-8], which, in its original form, is a nonlinear "distributed" system (i.e., an accurate model involves nonlinear partial differential equations). To the best of our knowledge, no "lumped" physical systems (i.e., actual systems which are realistically modelled by ordinary differential equations) exhibiting this phenomenon have so far been reported.

In this paper we will present a simple experimental circuit with a negative-resistance device that displays the period-adding phenomena through an alternating periodic-chaotic transition sequence. This circuit was investigated by applying a sinusoidal input signal to an autonomous circuit operating in dc equilibrium, i.e., the circuit does not oscillate when the input signal is set to zero.

By selecting an appropriate dc operating point on the negative-resistance v-i characteristic, we observed numerous complicated dynamical behaviors. The highest distinguishable period in the period-adding sequence reaches as high as period 21. Based on our experiments we identify several features of this bifurcation sequence.

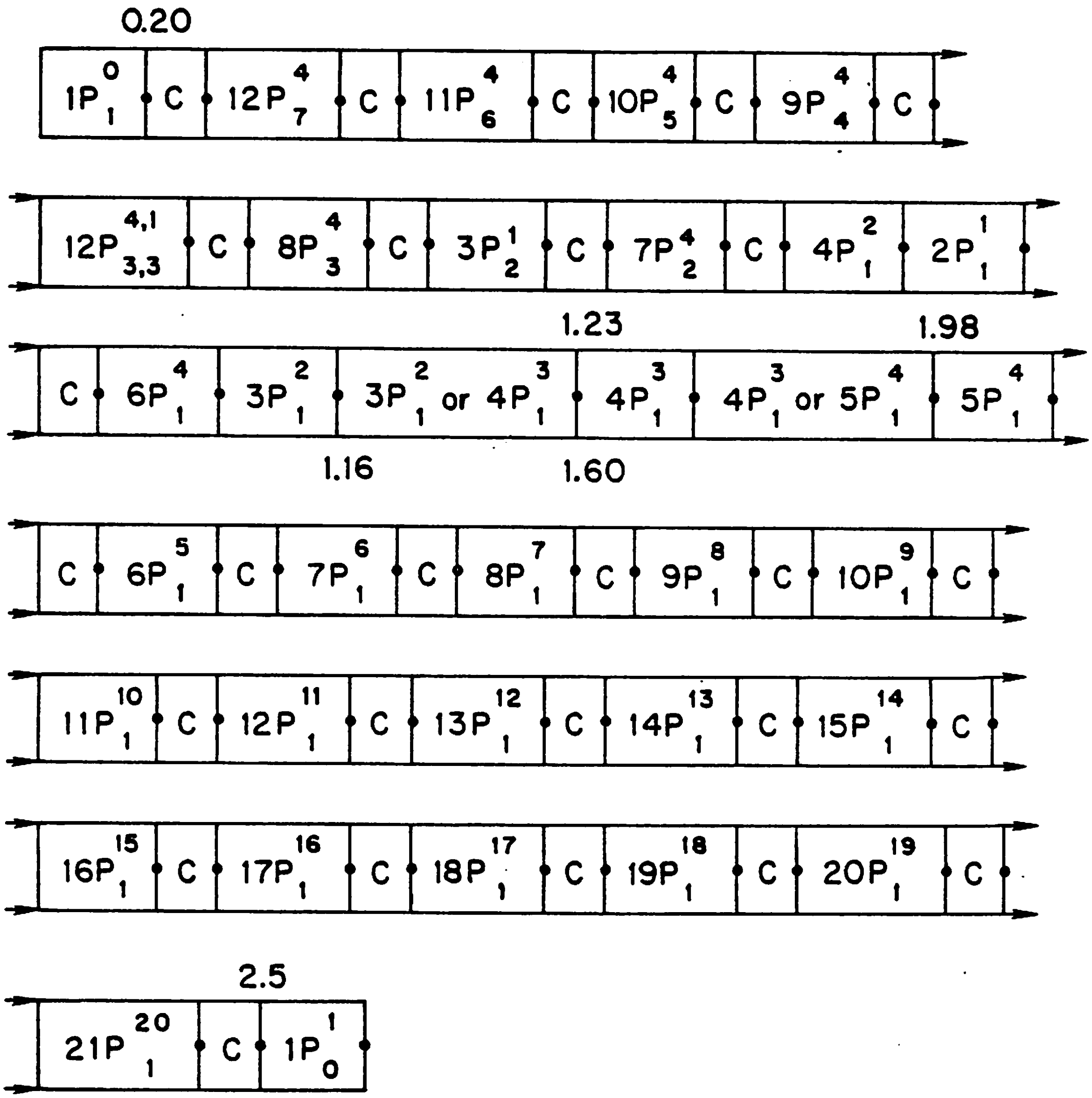
2. EXPERIMENTAL CIRCUIT

The circuit we consider is an RLC oscillator with a negative-resistance device, as shown in Fig.1(a). Here, $V_i = V\cos 2\pi ft$ is the driving voltage source, V and f are the driving amplitude and frequency, respectively, and L=10mH, C=6800pF, R=51Ω. Here N denotes an N-type negative-resistance device [9-10] made of two bipolar transistors and four positive linear resistors, the realization of which is given in Fig.1(b). The elements L' and E in Fig.1(a) form the dc bias circuit for N, where L'=20 mH in our experiment. The natural (unforced) frequency was measured as f_0=19KHz. Figure 2 shows the measured v-i characteristic of the negative-resistance device. Note that the dynamic range of the negative slope is rather wide — in this case it is about 3.5 volts. This is one reason why we choose a synthesized [9-10] rather than intrinsic negative-resistance device. Furthermore, by selecting different parameter values for the linear resistors we can obtain different characteristic curves. Hence, the synthesized negative-resistance device can be easily "tuned" to obtain a wide range of negative resistance characteristics [11].

3. EXPERIMENTAL RESULTS

We fixed the driving frequency at f=24.19KHz, which is close to the natural frequency of the unforced circuit (when V=0). We adjusted the bias voltage and choose the value E=6.134 volts such that the unforced circuit did not oscillate. By increasing V from zero to 2.50 volts, we observed a sequence of distinct states of the system as shown in table 1. Some bifurcation values of the neighboring states are also specified in this table.

TABLE 1


0.20
$1P_1^0$ C $12P_7^4$ C $11P_6^4$ C $10P_5^4$ C $9P_4^4$ C
$12P_{3,3}^{4,1}$ C $8P_3^4$ C $3P_2^1$ C $7P_2^4$ C $4P_1^2$ $2P_1^1$
1.23
1.98
C $6P_1^4$ $3P_1^2$ $3P_1^2$ or $4P_1^3$ $4P_1^3$ $4P_1^3$ or $5P_1^4$ $5P_1^4$
1.16
1.60
C $6P_1^5$ C $7P_1^6$ C $8P_1^7$ C $9P_1^8$ C $10P_1^9$ C
$11P_1^{10}$ C $12P_1^{11}$ C $13P_1^{12}$ C $14P_1^{13}$ C $15P_1^{14}$ C
$16P_1^{15}$ C $17P_1^{16}$ C $18P_1^{17}$ C $19P_1^{18}$ C $20P_1^{19}$ C
2.5
$21P_1^{20}$ C $1P_0^1$

As the parameters were carefully tuned, the system displayed a sequence of rich dynamical behaviors. For small amplitude V we first observed a small-amplitude period-1 oscillation (denoted by $1P_1^0$ in table 1) which bifurcates directly (with no detectable intermediate states) into chaos.[1] Then there is a reverse period-adding sequence of periodic states (denoted by nP_{n-5}^4 for n=12, 11, $\cdots$, 7 in table 1) separated by one or more chaotic state C and/or other periodic states (e.g., periodic windows). In particular, at least one chaotic state is observed between successive periods. Moreover, periodic windows (such as period $12P_{3,3}^{4,1}$ and $5P_2^1$) are imbedded within some chaotic regimes.

After the reverse period-adding sequence we observed another period-adding squence[2] of periodic states (denoted by nP_1^{n-1} for n=2,3,$\cdots$,21). This period-adding sequence begins with the period-2 waveform $2P_1^1$ which resulted from a reverse period-doubling route to chaos in table 1, i.e., from $2P_1^1$ to $4P_2^1$ and then to C in the reverse direction. Immediately following $2P_1^1$, a similar reverse period-doubling process (from $3P_1^2$ to $6P_1^4$ to C in table 1) leads to a period-3 waveform; namely $3P_1^2$. A typical hysteresis phenomenon is observed during the transitions from $3P_1^2$ to 4 P_1^3 and from $4P_1^3$ to $5P_1^4$: We observed from table 1 that the state changes from $3P_1^2$ to $4P_1^3$ at 1.23 volts, but if we reverse the voltage V, the change does not occur at the same point. The same phenomenon was seen in the transition from $4P_1^3$ to $5P_1^4$. The period-adding process continues up to some relatively large number N (in our case N=21) before it bifurcates into a chaotic regime and then reverts back to a periodic waveform $1P_0^1$ of period 1 but with a large-amplitude oscillation.

It appears that even higher periods are present in the above experiment but we were unable to observe it so far. The higher the period, the narrower the window, so the behavior becomes more unstable and more difficult to observe. Between all the successive periodic states there are chaotic transitions except in the transitions from $3P_1^2$ to $4P_1^3$ and from $4P_1^3$ to $5P_1^4$ where we conjecture that there exist additional chaotic transitions but we were not able to observe them because of their increasingly narrower voltage ranges. Hence, we call the sequence of bifurcation phenomena in table 1 an alternating periodic-chaotic transition sequence. There may also exist additional periodic windows in all of the chaotic regions in table 1, but since we are limited by their increasingly narrower voltage ranges and by the precision of our measuring instruments, we have only observed two such periodic windows so far.

For convenience, we have introduced a symbol to denote the different periodic states. For example, although we have observed three period-12 states, their waveforms clearly suggest that they belong to different bifurcation sequences. From table 1, we can see that the first period-12 waveform $12P_7^4$ has 4 large-amplitude oscillations and 7 small-amplitude oscillations in each period. Hence, it belongs to the reverse period-adding sequence nP_{n-5}^4, where n=12 in this case. The second period-12 waveform[3] $12P_{3,3}^{4,1}$ has two groups of relative oscillation amplitudes per period: the first group has 4 large-amplitude oscillations and 3 small-amplitude oscillations; the second has 1 large-amplitude oscillation and 3 small-amplitude oscillations. Hence, it is a periodic window. Similarly, the third period-12 waveform $12P_1^{11}$ belongs to yet another period-adding sequence; namely, nP_1^{n-1}, where n is the period number.

In comparison we show the time waveforms for several periodic states in Fig.3(a)-(f). In Fig.3(a) and (b) are the two reverse period-adding states $11P_6^4$ and $9P_4^4$. In Fig.3(c) and (d) are the two period-adding states $15P_1^{14}$ and $17P_1^{16}$. Figure 3(e) and (f) shows two periodic windows $12P_{3,3}^{4,1}$ and $3P_2^1$ whose waveforms and Lissajous figures are clearly distinct from the other periodic states and hence they belong neither to the reverse period-adding family, nor the period-adding family.

The analysis of the Lissajous figure in the V_c-I plane is a useful method especially in identifying its corresponding bifurcation sequence. In Fig.4(a)-(f) we give several Lissajous figures associated with the two periodic waveforms $V_c(t)$ and I(t) taken from an oscilloscope. Spectral analysis is another useful method for distinguishing periodic or chaotic dynamical behaviors. In Fig. 5(a) and (b) we give the comparison between a periodic spectrum and a chaotic spectrum. A periodic waveform has a discrete spectrum whereas a chaotic waveform displays a continuous broadband spectrum.

We also show the time waveforms and the V_c-I relationships for several chaotic states in Fig.6. From this figure we can see that some chaotic states contain component waveforms belonging to nearby periodic states, but of course they appear stochastically and unrepeatedly. For example, between $6P_1^5$ and $7P_1^6$ we obtain the chaotic transition in Fig.6(c) using a single-sweep time base. Note that this chaotic waveform begins with an oscillation contained in the periodic waveform $6P_1^5$ and then changes to an oscillation contained in the periodic waveform $7P_1^6$. The succeeding waveform is again different so that the complete waveform containing Fig. 6(c) is chaotic. The V_c-I Lissajous figures in Fig.6 show that the corresponding waveforms represent a deterministic chaos rather than a stochastic noise as a cursory glance of the spectrum might at first suggest.

Note that the above experiment was carried out under the condition that no oscillation exists when the input amplitude V=0. Much more complicated dynamical behaviors are observed when the input is applied with the circuit exhibiting a self oscillation.

By applying other values of f and E, we observe many additional but distinct alternating periodic-chaotic sequences but we will not describe them here due to the lack of space.

4. DISCUSSION

The alternating periodic-chaotic bifurcation sequence and the forward period-adding phenomenon observed in our circuit also appeared in several other systems. They all share the following common features: the successive period-adding states are related by a definite law; namely, each period-n waveform contains one small-amplitude oscillation and n-1 large-amplitude oscillations during each period and that their sum is equal to n. For chaotic states, some portion of the waveform contains periodic oscillations from nearby periodic states. Moreover, our circuit was found to exhibit the following additional dynamical behaviors which have not been reported before. (1) As we increase the parameter V just beyond a small-amplitude period-1 oscillation, we observed a direct transition into chaos with no discernible intermediate states, and then followed by an onset of a reverse period-adding sequence. Further increase in V eventually leads into a distinct period-adding sequence before it bifurcates into chaos again. Immediately after chaos, we observed a large-amplitude period-1 oscillation. (2) For each periodic state in the reverse period-adding sequence there are four large-amplitude oscillations in each period. The sum of the large- and small-amplitude oscillations is found to be equal to n-1, where n is the period number.

Note that the preceding phenomena are all observed when the unforced circuit does not oscillate. However, if we change the circuit parameters such that the unforced circuit operates as an oscillator, then even more complicated phenomena have been observed when the input signal is applied.

Since the phenomena described in this paper have been observed in similar circuits but using different transistors, it follows that the circuit in Fig.1 is quite robust in displaying the periodic-chaotic sequence and the period-adding phenomena. In fact, similar phenomena using a unijunction transistor as the negative-resistance device has also been observed recently.[4] We conjecture therefore that similar as well as new phenomena could also be observed using the numerous negative-resistance devices recently reported in [9-12] .

Finally, we remark that computer simulation of the circuit in Fig.1 is presently being carried out in order to make a more detailed analysis of this circuit. Our ultimate goal is of course to develop a theory which explains the observed phenomena.

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FOOTNOTES

1. Each chaotic state is denoted by the symbol C in table 1.
2. The <u>reverse</u> period-adding periodic states and the period-adding periodic states represent two <u>unrelated</u> families of periodic waveforms. We can easily identify members from each family because they have similar waveforms and their Lissajous figures share similar characteristic features. In particular, the sum of the subscript and superscript in the <u>reverse</u> period-adding states nP_{n-5}^{4} is equal to (n-5)+4=n-1, whereas the sum of the subscript and superscript in the period-adding states nP_{1}^{n-1} is equal to (n-1)+1=n.
3. This periodic waveform is denoted by 2 subscripts and 2 superscripts in order to emphasize two distinct features in the waveform.
4. Private communication with Xu Yun from Gui Zhou Institute of Technology, Gui Zhou, China.

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FIGURE AND TABLE CAPTIONS

Fig.1, (a) Negative-resistance oscillator circuit.

(b) Two transistor negative-resistance device. The transistors used in this circuit are type 3DG6 (China).

Fig.2, The measured v-i characteristic of the negative-resistance device. Scale for the vertical axis I_1 is 1mA/div; scale for the horizontal axis V_I is 1 volt/div .

Fig.3, Time waveforms associated with the periodic states. The upper waveform corresponds to V_i. The bottom corresponds to V_c. (a) $11P_6^4$, (b) $9P_4^4$, (c) $15P_1^{14}$, (d) $17P_1^{16}$, (e) $12P_{3,3}^{4,1}$, (f) $3P_2^1$.

Fig.4, V_c-I relationships (Lissajous figures) associated with the periodic states of the period-adding sequence. (a) $2P_1^1$, (b) $3P_1^2$, (c) $4P_1^3$, (d) $5P_1^4$, (e) $6P_1^5$, (f) $7P_1^6$.

Fig.5, Spectra of waveforms corresponding to (a) $15P_1^{14}$ and (b) chaos.

Fig.6, Time waveforms and the V_c-I relationships associated with the chaotic states. Left: time waveform obtained by a single sweep. Right: V_c-I relationships. (a) The last chaotic state in table 1; (b) Chaos between $10P_5^4$ and $9P_4^4$; (c) Chaos between $6P_1^5$ and $7P_1^6$.

Table 1, The alternating periodic-chaotic sequence obtained from experiments, where P denotes periodic, C denotes chaotic. The superscript (subscript) denotes the number of large (small)-amplitude oscillations per period. The values above the neighboring states indicate the bifurcation thresholds in the forward direction, the values below indicate the bifurcation thresholds in the reverse direction.

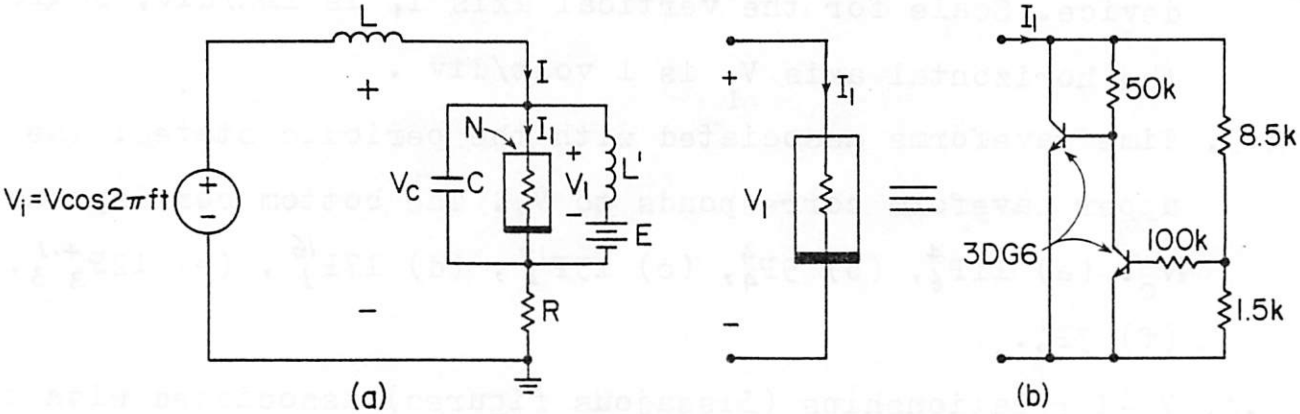


Figure 1

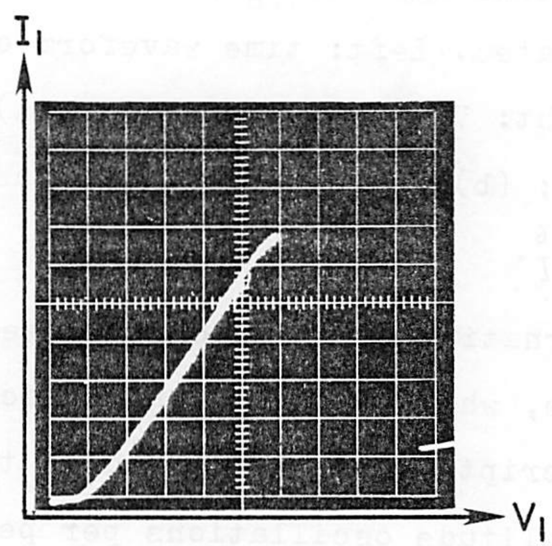


Figure 2

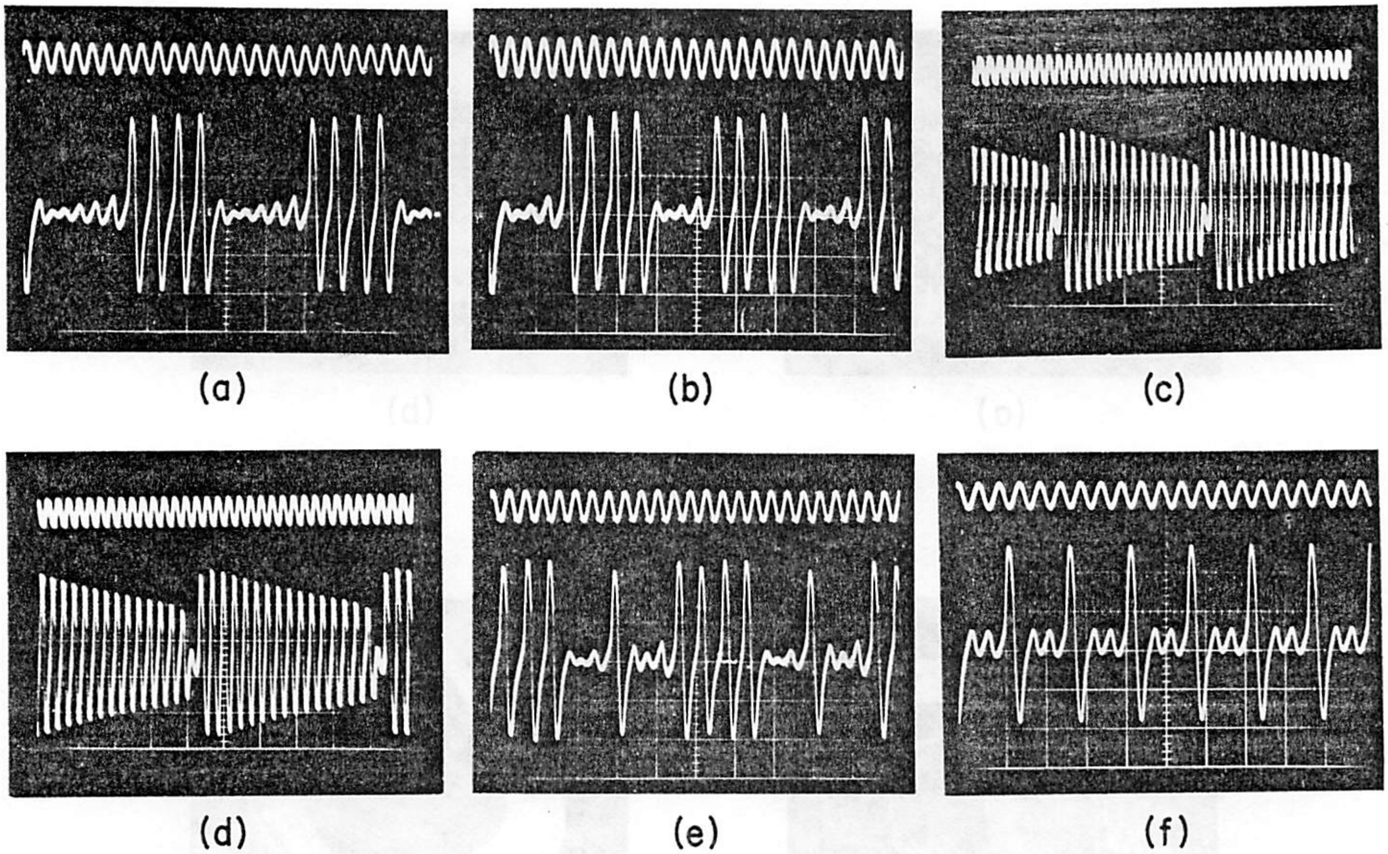


Figure 3

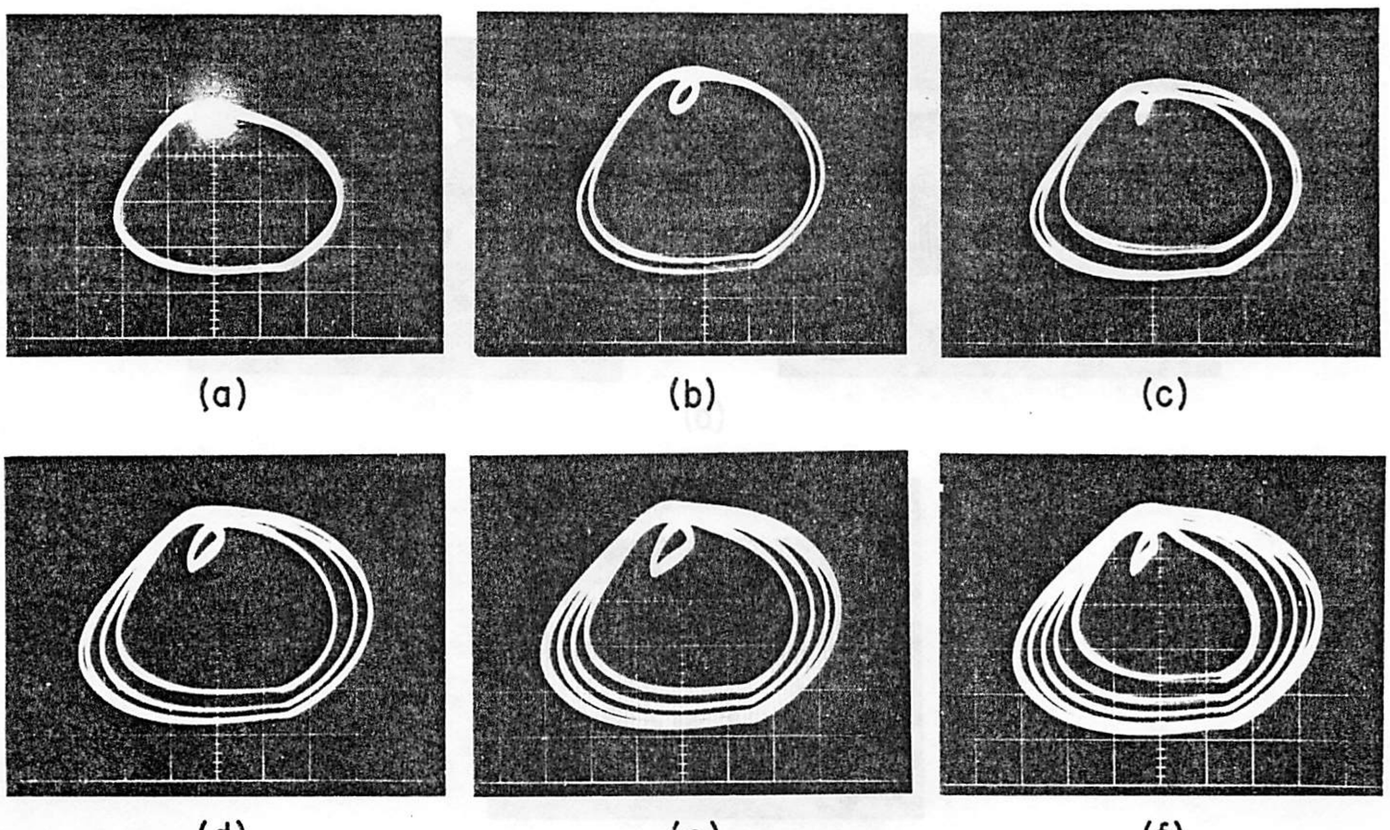
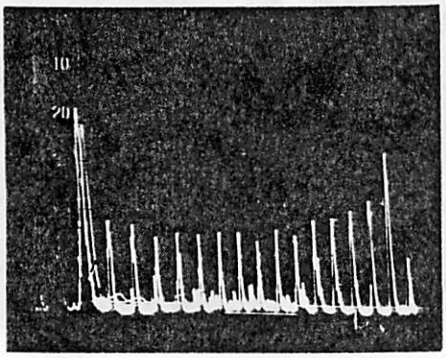
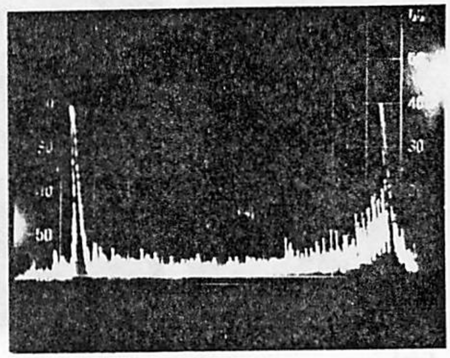


Figure 4

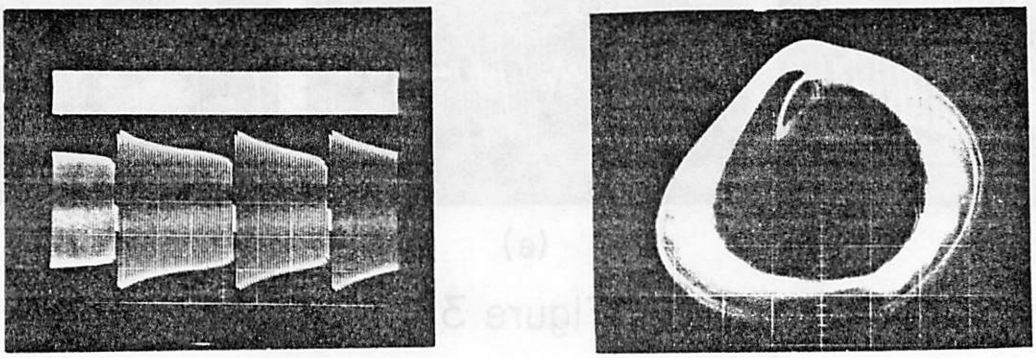


(a)

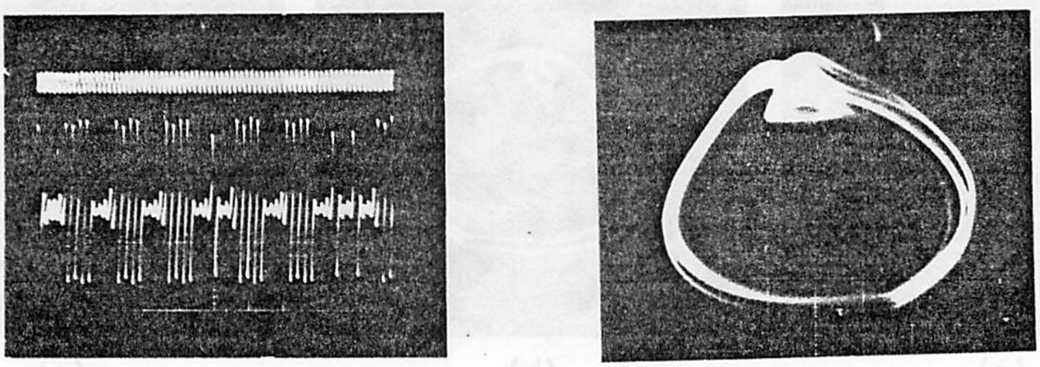


(b)

Figure 5



(a)



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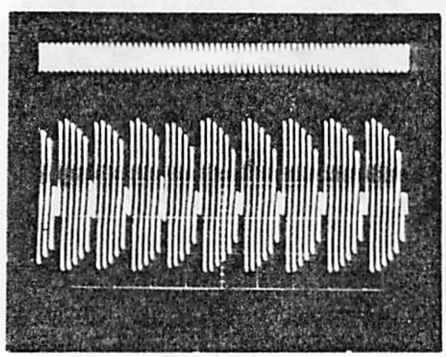

(c)

Figure 6