VORTEX DYNAMICS AND TRANSPORT TO THE WALL IN A CROSSED-FIELD PLASMA SHEATH

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Abstract

Results of numerical simulations of the time-dependent behavior of a transversely magnetized plasma-wall sheath are presented. These simulations have been conducted with the aim of modelling plasma behavior in the vicinity of the limiters and walls of a fusion device. The two-dimensional, bounded particle simulation code "ES2" has been used as a tool for the investigation of these edge effects, in an idealized geometry which retains, however, the essential features of the physics of the edge plasma. The simulations have revealed that the bounded plasma is subject to the so-called "Kelvin-Helmholtz" instability[1] an instability maintained by the non-uniform electric field which is induced by the presence of the material walls. This instability is seen to saturate into large and stable vortices, with $e\phi/T_i \sim 1$, which exist in the vicinity of the walls, and drift parallel to their surfaces. An important feature of these structures is that they continuously convect particles to the walls, at an "anomalous" rate much greater than that induced by collisional diffusion, a feature which seems tied to the mutual interaction of the vortices. In the code "ES2", volume ionization of neutrals has been modelled by a uniform electron-ion pair creation in the simulation region, and this results in a steady state, in which the linear edge instability, the nonlinear fluid dynamics of the vortices, and the nonlinear dynamics of the particles scattered by the vortices all balance each other. This steady-state but non-equilibrium configuration, which is a first model of the edge behavior induced by the boundaries, is conceptually analogous to Rayleigh-Bénard convection.

[1] S. Chandrasekhar, *Hydrodynamic and Hydromagnetic Stability*, Oxford, at the Clarendon Press, 1961.

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Vortex Dynamics and Transport to the Wall in a Crossed-Field Plasma Sheath – K. Theilhaber and C.K. Birdsall.

1 Introduction

The following Memorandum is a preliminary report on the results of our numerical simulations of the magnetized plasma-wall sheath. Specifically, we have been studying transport across the magnetic field in a plasma bounded by conducting walls, with the aim of modelling plasma behavior in the vicinity of the limiters and the walls of a fusion device. Our approach has been to use a two-dimensional, bounded particle simulation code, "ES2", as a tool for the investigation of edge effects, in an idealized geometry which retains, however, the essential features of the physics of the edge plasma.

Thus far, our simulations have revealed that the bounded plasma is subject to the so-called "Kelvin-Helmholtz" (or "Diocotron") instability[1,2,3,4,5], an instability maintained by the sheared electric field which is induced by the presence of the material walls. It is important to emphasize that this electric field is not imposed *a priori*, but is a natural consequence of the loss of charged particles to the boundaries. The instability is seen to saturate into large and stable vortices, which remain in the vicinity of the walls, and drift parallel to their surfaces. An important feature of these vortices is that they continuously convect particles to the walls, at a rate greater than that expected from collisional diffusion. This "anomalous" transport seems tied to the mutual interaction of the vortices, which induces stochastic particle orbits. In the code ES2, we have also modelled volume ionization of neutrals by a uniform electron-ion pair creation in the simulation region, and this results in a fully self-consistent steady-state, in which the linear edge instability, the nonlinear fluid dynamics of the vortices, and the nonlinear dynamics of the particles scattered by the vortices all balance each other.

We can describe the steady states reached in our simulations in terms of non-equilibrium thermodynamics (see Fig.(1)). The vortices, which behave as convection cells, are "dissipative structures", which are driven by the free energy available in the system, and which the plasma has spontaneously formed so as to maximize heat and particle fluxes to the boundaries. These structures are of course not arbitrary, but have formed subject to the constraints inherent in the system: energy and momentum conservation, and the physical length and time scales available to the plasma. The source of free energy which maintains these structures resides in the temperature difference between the reservoir of hot ions and electrons, which are continuously fed into the system by the uniform ionization of neutrals, and the walls, which are decreed to be at absolute zero, insofar as they are perfect absorbers of incoming particles. Our system is reminiscent of Rayleigh-Bénard convection, where convection cells and enhanced transport are also driven by a temperature gradient. However, the detailed dynamics of each system, and the physics of the media considered, are of course very different.

In what follows, we shall rather briefly outline our simulation results, leaving to the next Memorandum, to be published in the near future, a more systematic presentation of results. In particular, a detailed numerical study of scaling laws for anomalous transport will be deferred to this second Memorandum. Similarly, we shall wait to present complete analytic models which describe and couple together the various processes alluded to in Fig.(1).

2 Simulation Model

Our model for edge effects is incorporated in the computer program "ES2". This is a two-dimensional, electrostatic "explicit" particle-in-cell simulation code, with full electron and ion dynamics. The spatial motion of the particles is confined to a single (x-y) plane in which they are subject to an external magnetic field B_0, rigorously perpendicular to the plane of motion, and to their self-consistent electrostatic fields. The simulation region is rectangular in shape, with periodic boundary conditions on the particles and fields in one direction (y), and finite bounding surfaces in the other, perpendicular direction(x). The bounding surfaces, which model the walls or limiters of the Tokamak, are perfect conductors, and are perfectly "cold", in that they absorb all impacting particles, with no reflection or reemission. When a particle hits a boundary, it is lost from the inner plasma, and its charge is immediately dispersed as surface charge. In solving Poisson's equation, the code keeps track of the total surface charge accumulated on the walls, and also automatically accounts for all image charges induced on the walls by the particles remaining in the plasma. In the simulations discussed below, the walls are kept "floating", in that they are not connected to any external circuit. Thus, all charge flow must occur between each wall and its surrounding plasma.

The choice in the simulation model of two finite boundaries is dictated by the finite extent of the simulation region; each boundary is meant to independently model a material wall in contact with the plasma. Thus, the two simulation boundaries, taken together, are certainly not meant to represent opposite walls in a minor cross-section of the Tokamak.

In what follows, we shall consider a representative simulation, to be called "Simulation 1", with the following parameters. In the normalized units of ES2, the mass ratio is $m_i/m_e = 40$ and the magnetic field is $B_0 = 1$, with an initial particle density of $\overline{n} = 5$ particles per unit area. Because ES2 assumes $m_e = e$, and $\epsilon_0 = 1$, this implies that $\omega_{ce} = 1$, $\omega_{pe} = 0.3$, $\omega_{pi} = 0.047$, and $\omega_{pi} = 0.025$. We have initially Maxwellian electrons and ions, with $T_e = T_i$, and the thermal velocities are $v_{te} = 1$, $v_{ti} = 0.158$. From these we obtain the length scales $\rho_e = 1$, $\lambda_{de} = \lambda_{di} = 3.3$, and $\rho_i = 6.32$, so that we have the ordering $\rho_e \ll \lambda_{de} = \lambda di \ll \rho_i$. The frequency ratios are then: $\omega_{pe}/\omega_{ce} = 0.3$, $\omega_{pi}/\omega_{ci} = 1.96$. The initial number of ions and electrons in the system is for each species $N = 40960$, and the system size is $L_x = 64 \sim 10\rho_i$, and $L_y = 128 \sim 20\rho_i$.

3 Overview of a Simulation

In Figs.(2a,b,c), we show an overview of the time-evolution of the electrostatic potentials in the system, by displaying three contour plots of $\phi(x, y)$, at times $\omega_{ci}t = 5$, 55 and 305. Fig.(2a) shows the essentially y-uniform sheath which forms at the beginning of the evolution of the system, after one rotation of the ions, $\omega_{ci}t \sim 2\pi$. This sheath is due to an initial loss of ions which have impacted into the walls. This results in a layer of depletion of positive charge over a depth in x of about $2\rho_i \sim 13$, and a corresponding potential drop from the wall into the plasma. This is in sharp contrast to the situation in the unmagnetized sheath, which is dominated by electron flow to the walls, and in which there is a potential rise from the wall into the plasma. In Fig.(2a), the total potential drop from the wall to the "valley" floor is $e\Delta\phi/T_i = 1.6$. It should be noted that the contours in Fig.(2a) are not perfectly y-uniform, as the potential contains small fluctuations, which can be ascribed at this early phase in the evolution of the system to thermal noise. These fluctuations are particularly visible near the "valley" floor, where the horizontal excursions of the contours are amplified by the flatness of the potential in this region.

In our simulations, the creation of the inital sheath is accompanied by the generation of two perpendicular waves, which propagate from the edges inward, at a velocity comparable to the ion-acoustic speed. These waves are uniform in y, and persist for several transits across the system, being reflected at each boundary. This phenomenon is a transient which lasts up to $\omega_{ci}t \sim 20$.

The sheathes which are shown in Fig.(2a) contain strongly nonuniform electric fields $E_x(x)$ pointing inward from the walls. These fields in turn give rise to highly sheared $\mathbf{E} \times \mathbf{B}$ drifts of electrons and ions in the y direction. As sheared fluid flow is in general unstable, it might be expected that the initial structure shown in Fig.(2a) will not persist, with the flow vulnerable to the Kelvin-Helmholtz instability. This is precisely what is observed in the subsequent evolution of the potential: y-dependent ripples develop in the potential contours, and are both amplified and convected by the $\mathbf{E}\times\mathbf{B}$ flow, over a time-scale of order $\omega_{ci}t \sim 20$. A first saturated state of the instability is shown in Fig.(2b). This state consists of a set of well-defined circular potential contours which drift parallel to the walls, in accordance with the local $\mathbf{E} \times \mathbf{B}$ drift (downwards on the left, upward on the right). These contours correspond to sizeable perturbations of the potential, with $\delta\phi/T_i \sim 0.5$. To the extent that the particle motion is determined by the $\mathbf{E}\times\mathbf{B}$ drift, the electrostatic potential is in fact a stream function for the particles, and the circular structures are quite simply large-amplitude vortices, in close analogy to the evolution of the Kelvin-Helmholtz instability in fluid flow.

The state shown in Fig.(2b) is only an approximate steady-state. Over a longer time scale, of the four vortices shown in Fig.(2b) only two larger vortices with $e\delta\phi/T_i \sim 1$ survive by $\omega_{ci}t = 305$, as shown in Fig.(2c). The process which leads in time from Fig.(2b) to Fig.(2c) occurs as follows. The vortices induce a steady ambipolar transport of particles from the inner plasma to the walls which gradually depletes the density profiles. Simultaneously the density profiles change by broadening and by taking on a triangular shape, and as the profiles broaden in x, the vortices grow in extent, in both x and y. This leads to a competition between the vortices filing parallel to each wall, the outcome being that in each line of vortices, the largest vortex, which is also the fastest, overtakes and absorbs the others in what might be considered completely inelastic collisions. On the other hand, the counterstreaming vortices have only highly "elastic" encounters, and periodically overlap with no subsequent effect. The final state shown in Fig.(2c) thus consists of two large and apparently stable counterstreaming vortices, which continue to transport particles outward. The density profile in this configuration is approximately triangular and quasineutral ($n_e(x) \approx n_i(x)$), with the exception of small trapped-electron populations in the center of each vortex.

The configuration of Fig.(2c) is only a quasi-steady-state, in that when pursued over a long time the continued convection of particles greatly decreases the particle densities and eventually quenches the activity of the vortices. We were able however to create a true steady-state, very similar to that depicted in Fig.(2c), by running simulations identical to Simulation 1, but with a steady and uniform volume injection of electron-ion pairs. This process was meant to simulate the outgassing and subsequent ionization of neutrals from the walls. The injection rate was set to be identical to the loss rate observed in Simulation 1 at $\omega_{ci}t = 305$. An important result of these new simulations was that the final steady-states obtained were found to be independent of the initial conditions imposed on the system: whether the simulation region was initially full or empty, the final steady-state ressembled that shown in Fig.(2c), with two large counterstreaming vortices, with similar amplitudes, and with very similar particle density profiles.

4 Observations of Anomalous Transport

In Figs.(3a,b), we show the time evolution of the total number of electrons and ions, $N_e(t)$ and $N_i(t)$ for Simulation 1 and the evolution of the loss rates for each species, $d\log N/dt$. From these figures, we can distinguish four phases in the history of the outward transport of particles; (i) an initial loss of electrons, in a very short time-scale $2\pi/\omega_{ce}$, (ii) an initial and larger loss of ions, occuring over the longer but still short time-scale

$2\pi/\omega_{ci}$, (iii) a time-scale of order $60/\omega_{ci}$ over which occurs the formation and saturation of the vortices, and (iv) a quasi steady-state, which persists for the remainder of the time-evolution of the system, in which transport is ambipolar in that the electron and ion fluxes are very nearly equal. The steady state is only approximate, and as a result of the slow outflux, both the total numbers of particles and their rate of loss decrease with time. The net charge density on the walls remains, however, very nearly constant throughout the evolution of the system.

An essential question is to determine to what extent the outward transport of particles observed in the simulations is truly a consequence of collective effects, in other words of the large-scale vortex motion, and not the result of numerical collisionality, the latter being due to the scattering of particles by thermal fluctuations. In all generality, we should assume that the waves in the thermal spectrum, which are generated by particle discreteness, can act independently of or in concert with the vortex motions of the system. We shall, however, ignore such coupling effects, and obtain a qualitative estimate of the collisional transport by assuming that it occurs in an equilibrium plasma.

The first step is to evaluate the collisional diffusion coefficient for the two-dimensional strongly magnetized plasma of Simulation 1. We use the results of Okuda and Dawson[6] and find that for both electrons and ions diffusion is dominated by the effects of the thermally random $\mathbf{E} \times \mathbf{B}$ drifts. The perpendicular diffusion coefficient is found to be:

$$D_{\perp} = \frac{c_s}{\sqrt{n}} \left[\frac{1}{2\pi} \log \frac{L_y}{2\Delta y} \right]^{1/2}, \qquad (1)$$

where c_s is the ion acoustic speed, n the average number of particles per unit area, L_y the total system length in the direction perpendicular to the density gradients, and Δy is the grid spacing, which determines the highest Fourier mode allowed in the system, with $k_{y,max} = \pi/\Delta y$.

Using Eq.(1), we can make a very qualitative prediction for the collisional transport of electrons. It should be noted that insofar as the system is far from thermal equilibrium, with large fluctuations and steep gradients, fitting a diffusion-like transport to such a situation is not a truly valid approach. However, we believe we will obtain the basically correct scaling law for the collisional transport. We simplify the situation by assuming a linear density profile over half the width of the system, with a maximum density $n = n_{max}$ at the center. We then estimate the particle flux per unit length to either wall as:

$$\Gamma_x = D_\perp \frac{dn(x)}{dx} \sim D_\perp n_{max}/(L_x/2), \tag{2}$$

With Eq.(1), we find that the total collisonal loss to both walls is roughly:

$$\frac{1}{N}\frac{dN}{dt} \sim \frac{4}{L_x}\left(\frac{L_y}{L_x}\right)^{1/2}\left[\frac{1}{2\pi}\log\frac{L_y}{2\Delta y}\right]^{1/2}\frac{c_s}{\sqrt{N}}, \tag{3}$$

where N is the total number of particles in the system. For the parameters of Simulation 1, we have the result:

$$\frac{1}{N}\frac{dN}{dt} \sim \frac{10^{-2}}{\sqrt{N}}, \tag{4}$$

In a series of numerical experiments, we preserved the physical parameters of Simulation 1, but varied the number of numerical particles per Debye square. The results are shown in Fig.(4), where we compare values of $d\log N/dt$ at $\omega_{ci}t = 305$. It can be seen that Eq.(4) predicts a flux which is both smaller than the observed outflux, and which strongly scales with the number of particles per Debye length, in opposition to the observed outflux, which is weakly dependent on this number.

5 Conclusion

Our simulations of the magnetized plasma-wall sheath have revealed the existence of a dynamic steady-state, in which a large, anomalous rate of transport to the wall is a maintained by a set of long-lived vortices, which drift parallel to the boundary. This steady-state is fully self-consistent and is maintained by a continuous injection of electron-ion pairs into the system, simulating volume ionization of neutrals. It can be regarded as a balance between a linear instability, the nonlinear fluid dynamics of the vortices, and the nonlinear dynamics of the particles scattered by the vortices into the walls. In a future Memorandum, we shall present systematic results for the scaling of the transport as a function of plasma parameters, and offer complete analytic models for the processes involved.

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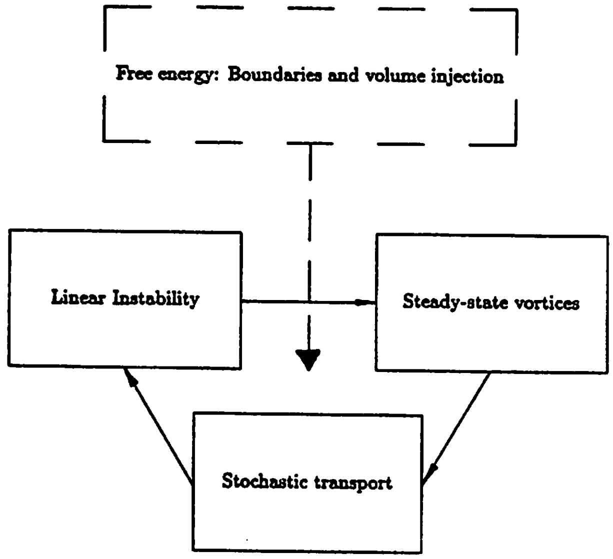

Figure 1: Schematic representation of the processes operating in the crossed-field sheath.

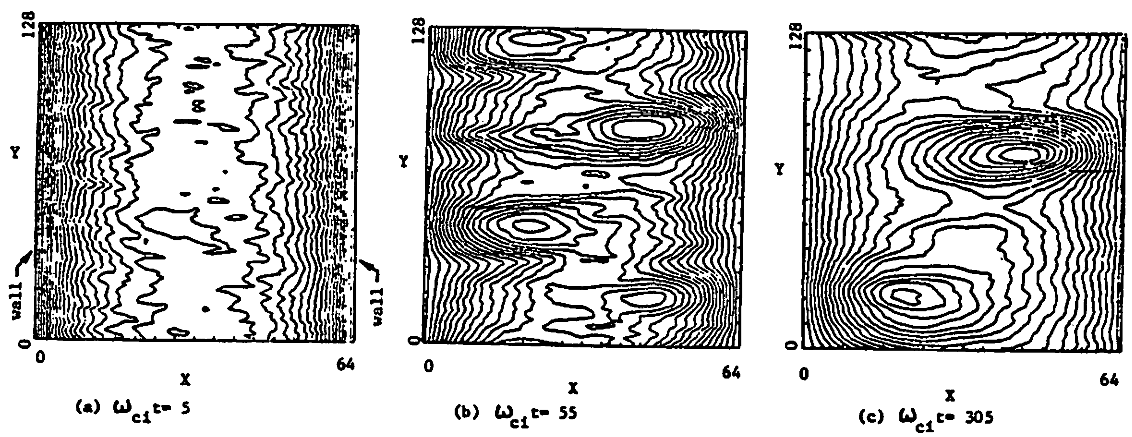

Figure 2: Equipotential contours at three different times.

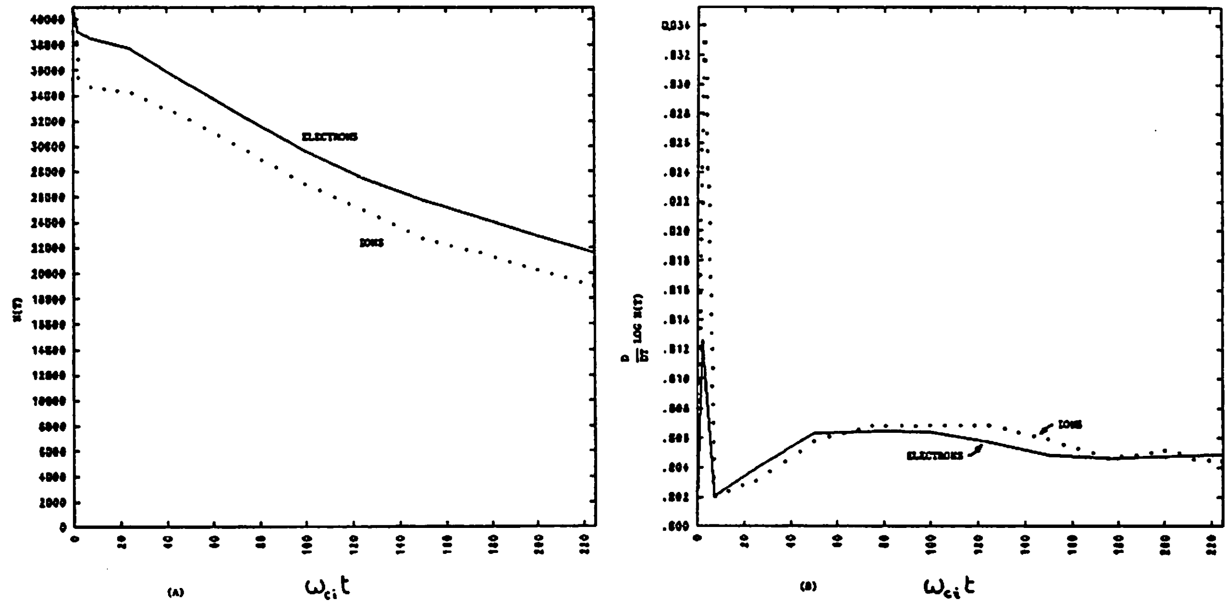

Figure 3: (a)Time evolution of the total number of electrons and ions in Simulation 1; (b) time evolution of the total loss rates of particles in Simulation 1, $d \log N/dt$.

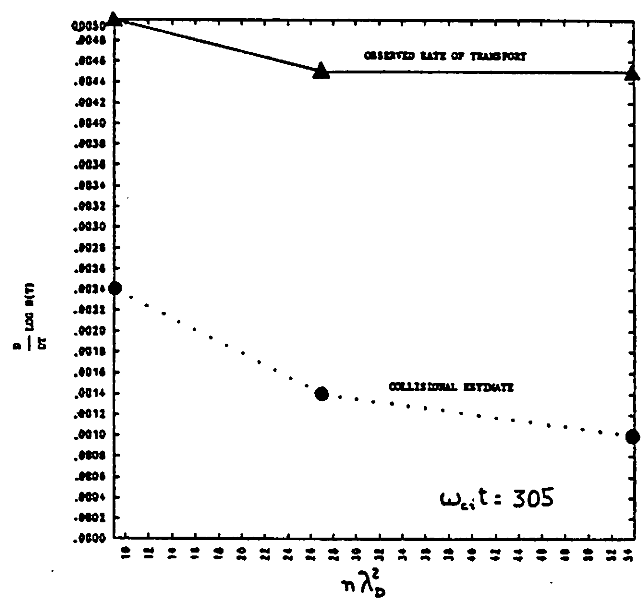

Figure 4: Comparison of particle transport in variations of Simulation 1, with the same continuum plasma parameters, and varying numbers of particles per Debye square.