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Then, with the help of a decision making "logic" included in a supervisory computer program, a position controller is generated to move the manipulator in the unconstrained directions and a force controller to push the manipulator against the environment with the desired contact force. Whitney [9] has developed a single-loop velocity control scheme with the net effect of controlling the contact forces. The schemes developed under this approach so far have not been proven to be stable and consequently limit the applicability of the schemes.

Yoshikawa [3] extended the work of Raibert and Craig [1] by including robot dynamics in the hybrid control scheme. He started with a description of the constraints through hypersurfaces and used the constraint information to generate a hybrid control law together with a servo compensator. It is unclear from [3] that stability could be achieved with the servo compensator. Similar work in this area has also been done by Khatib and Burdick [10] and An [5].

This paper is an in-depth generalization of Yoshikawa [3]. First, we assume that the contact between the end effector and the environment has been made and interaction forces in the tangent directions to the worksurface are negligible. Then, we develop in Section 2 a procedure for systematic description of the task constraints and use the developed constraints in Section 3.0 to define the constraint space. In Section 3.1 we project the robot dynamics into the constraint space to arrive at a joint force control law that realizes both the desired velocity/force trajectories preplanned in Section 2.3. In Section 3.2, we extend the control scheme to hybrid impedance/force control to include finite interaction forces in the tangent directions. In Section 3.3, we give several design examples and in Section 4 we collect some suggestions for future work in this area.

2. Description of Motion Constraints on the End Effector

2.1 Mathematical Preliminaries

Consider a rigid body B in R^3 as in Figure 2.1. Let X–Y–Z be an arbitrary inertial frame and x–y–z a body frame attached to the rigid body.

The instantaneous configuration of the rigid body can be described by the orientation and the position of the body frame with respect to the inertial frame. We define the configuration manifold M of the rigid body to be the space of configurations of the rigid body. Since three parameters are needed to specify a position in R^3 and three more parameters to specify an orientation, the configuration manifold M is six-dimensional. We use Euler angles (ϕ , θ , ψ) and the Cartesian coordinates (x, y, z) ([12]) to parametrize M. For this we let $m \in$ M denote a nominal configuration of the rigid body and U_m a neighborhood of m in M. Assume that at m the body

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ABSTRACT

We present a new stable hybrid velocity/force control scheme for a robot manipulator under constraint. We arrive at the joint force control law by first developing a systematic procedure for constraint description and then projecting the robot dynamics into the constraint space. We extend the scheme to hybrid impedance/force control to cover finite interaction forces in the tangent direction.

1. Introduction

A large class of manipulator tasks (e.g., Figures 1-3 of Appendix A) requires interaction of the manipulators with the task environment. In the process of executing the task, the manipulator trajectory is frequently modified by the contact forces occurring during the interaction. It has been well understood that either pure position or pure force control in this case is not adequate for completing the manipulation task. The necessity of more precise control of a manipulator is widely recognized.

To control a robot manipulator under constraint, Hogan [7] and Kazerooni et al [6] have developed the notion of mechanical impedance in the frequency domain as a parameterization of a rational set of performance specifications to generate the compliant motion while preserving stability in the presence of bounded model uncertainties. Salisbury [8] has also done similar work by defining a linear static function that relates interaction forces to the end-point position via a stiffness matrix in a Cartesian coordinate frame. Monitoring this relationship by means of a computer program ensures that the manipulator will be able to maneuver successfully in a constrained environment.

An alternative approach to controlling a robot manipulator under constraint has been pursued by Raibert and Craig [1], Mason [2], Paul and Shimano [14], and others. In this approach the manipulator motion is partioned into position- and force-control in a global Cartesian coordinate frame.

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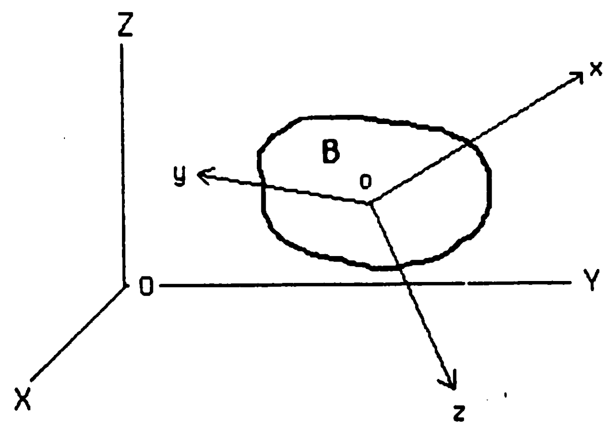

Figure 2.1. A Rigid Body in R^3-space.

frame coincides with the inertial frame. Then, there exists a natural coordinate map

$$\sigma : U_m \subset M \rightarrow R^6$$

given by

$$\sigma(m) = (0,0,....0) \text{ and } \sigma(p) = (x,y,z,\phi,\theta,\psi) \text{ for all } p \in U_m \tag{2.1-1}$$

where (x, y, z) and (ϕ, θ, ψ) are respectively the coordinate of the origin $\dot{o}$ and the Euler angles of the body frame at configuration p. If r(t) is a C^1 - curve in M representing the trajectory of the rigid body, then the generalized velocity of the rigid body is given by $\frac{dr(t)}{dt} \in T_{r(t)}M$, where T_mM is the tangent space to M at configuration m ([11]). In local coordinates the trajectory of r(t) can be written as

$$\tilde{r}(t) = \sigma(r(t)) = (x(t), y(t), z(t), \phi(t), \theta(t), \psi(t))$$

Assume that $r(t_0) = m$, the generalized velocity at $(t = t_0)$ is then given by

$$\frac{d\tilde{r}(t)}{dt}|_{t_0} = d\sigma_m \frac{dr(t)}{dt}|_{t_0} = (\dot{x}(t), \dot{y}(t), \dot{z}(t), \dot{\phi}(t), \dot{\theta}(t), \dot{\psi}(t))|_{t=t_0}$$

where $d\sigma_m$ is the tangent map of σ at m.

Following the convention of [13], we attach the body frame to the robot hand so that its origin is located centrally between the finger tips; the z vector lies in the direction from which the hand would approach the constraint surface and is known as the approach vector, a; the y vector known as the orientation vector o, is in the direction specifying the orientation of the hand, from finger to finger tip; the final vector x, known as the normal vector n, forms a right-handed set of vectors and is thus specified by the vector cross-product $n = o \times a$. In terms of the Euler angles, the set of orientation vectors (n, o, a) is given by

$$[n\ o\ a]=\begin{bmatrix} \cos\psi\cos\phi - \cos\theta\sin\phi\sin\psi & -\sin\psi\cos\phi - \cos\theta\sin\phi\cos\psi & \sin\theta\sin\phi \\ \cos\psi\sin\phi + \cos\theta\cos\phi\sin\psi & -\sin\psi\sin\phi + \cos\theta\cos\phi\cos\psi & -\sin\theta\cos\phi \\ \sin\theta\sin\psi & \sin\theta\cos\psi & \cos\theta \end{bmatrix} \quad (2.1\text{-}2)$$

2.2 Description of Motion Constraints on the End Effector

Following the notation of Section 2.1, we now use the examples of Figures 1 - 3 to demonstrate a procedure for the systematic description of the task constraints.

Example 2.1: First, we consider the planar manipulator of Figure 1, where the end effector is constrained to move horizontally. With the chosen inertial frame, a body frame is attached to the end effector. The configuration manifold M here is three dimensional and can be parametrized by the coordinates (x, y, θ). The constraint on the end effector can be described as *the zero set of the function*

$$C:\ M \to R, \quad C(r) = C((x, y, \theta)) = y - 1 \qquad (2.2\text{-}1)$$

Furthermore, the Jacobian $dC_r \neq 0$, for all $r \in M$; consequently the end effector is constrained to move within the submanifold $N = C^{-1}(0)$ of M ([11]).

Example 2.2: Consider the manipulator task shown in Figure 2, where the manipulator turns the crank. The constraints on the end effector are such that (1) it can move move only in the circle of radius 1 and height b and (2) the approach vector a is directed towards the Z-axis. The first constraint can be described as the zero set of the function

$$\tilde{C}: M \to R^2, \quad \tilde{C}(r) = \begin{bmatrix} c_1(r) \\ c_2(r) \end{bmatrix} = \begin{bmatrix} x^2 + y^2 - 1 \\ z - b \end{bmatrix} \qquad (2.2\text{-}2)$$

and the second constraint can be described in terms of the approach vector a defined in (2.1-2) by

$$\nabla c_1 \times a = 0 \qquad (2.2\text{-}3)$$

where "x" stands for the cross product and ∇ is the gradient operator. Expanding (2.2-3) we have

$$\begin{cases} y\cos\theta = 0 \\ x\cos\theta = 0 \\ -x\cos\phi\sin\theta - y\sin\phi\sin\theta = 0 \end{cases} \qquad (2.2\text{-}4)$$

It is easy to verify that only two of the above equations are independent and the entire set of constraints on the end effector can be described as *the zero set of the following function*,

$$C: M \rightarrow R^4, \quad C(r) = \begin{bmatrix} x^2 + y^2 - 1 \\ z - b \\ \cos\theta \\ x\cos\phi + y\sin\phi \end{bmatrix} \tag{2.2-5}$$

The Jacobian dC_r here is of full rank for all $r \in$ M and the manipulator is constrained to the submanifold N $\in C^{-1}(0)$ of M.

Example 2.3: Finally, consider the manipulator task in Figure 3, where the end effector is constrained to the sphere and the approach vector is directed towards the center of the sphere. The constraint is given by,

$$\begin{cases} c_1(r) = x^2 + y^2 + z^2 - 1 = 0 \\ \nabla c_1(r) \times a = 0 \end{cases} \tag{2.2-6}$$

where a is the approach vector defined in (2.1-2). Rewriting the second equation we obtain

$$\begin{cases} y\cos\phi - z\cos\phi\sin\theta = 0 \\ z\sin\phi\sin\theta - x\cos\theta = 0 \\ x\cos\phi\sin\theta + y\sin\phi\sin\theta = 0 \end{cases} \tag{2.2-7}$$

Since only two of the above equations are independent, the total constraints on the manipulator are given by the zero set of the function

$$C: M \rightarrow R^3, \quad C(r) = \begin{bmatrix} x^2 + y^2 + z^2 - 1 \\ y\cos\theta + z\cos\phi\sin\theta \\ z\sin\phi\sin\theta - x\cos\theta \end{bmatrix} \tag{2.2-8}$$

The Jacobian dC_r here is again of full rank for all $r \in$ M, and the end effector is constrained to the submanifold N $= C^{-1}(o)$ of M.

Generalizing from these examples, we conclude that the task constraints can be described as the zero set of a twice differentiable function

$$C: M \rightarrow R^m \tag{2.2-9}$$

for some integer $0 \leq m \leq 6$, and the Jacobian dC_r is of full rank for all $r \in$ M.

The submanifold N $= C^{-1}(0) \subset$ M, of dimension $(6-m)$, is called the constraint submanifold of M. It consists of the set of configurations of the end effector where the end effector is in contact with the environment. We say that a configuration $r \in M$ is a constrained configuration if $r \in N$. A twist motion of the end effector at a point $r \in N$, denoted by $\dot{r}$, must satisfy

$$dC_r \, \dot{r} = 0 \tag{2.2-10}$$

It is easy to see that the kernel of dC_r is just the tangent space to N at r, which we denote by T_rN. Since N is a $(6-m)$ dimensional submanifold of M, T_rN is a $(6-m)$ dimensional subspace of T_rM. We call T_rN is the velocity controlled subspace, or the constrained twist space of the end effector at configuration r. Any motion of the end effector at configuration r that conforms with the constraint must lie in the velocity controlled subspace. We define for the entire set N of constrained configurations the global set of constrained motions of the end effector by

$$TN = \{(r, v), r \in N, v \in T_rN\} \tag{2.2-11}$$

Here TN is just the union of all constrained twist spaces over the set of constrained configurations.

The set of forces that can be exerted on the end effector at configuration $r \in$ M, is denoted by by T_r^*M. T_r^*M, often called the wrench space of the end effector, is also identified as the dual space to T_rM, or the cotangent space to M at r. A wrench $f \in T_r^*M$ acting on a twist $v \in T_rM$ gives the work done per unit time on the end effector. For a manipulator under constraint, we define the set of wrenches that yield no work on all twist motion $v \in T_rN$ by

$$T_rN^c = \{f \in T_r^*M, \text{ such that } f(v) = 0, \text{ for all } v \in T_rN\} \tag{2.2-12}$$

T_rN^c is a m-dimensional subspace of the wrench space T_r^*M and is often called the force controlled subspace. A force f in T_rN^c is also called a normal force to the constrained manipulator, or simply a constrained wrench. It is clear from (2.2-10) that T_rN^c in local coordinates is given by the span of the rows of dC_r. Similarly, we define for the constrained submanifold N the global set of normal forces on the constrained manipulator by

$$TN^c = \{(r, f), r \in N, f \in T_rN^c\} \tag{2.2-13}$$

TN^c characterizes the set of all normal forces that can be exerted on the end effector without doing work on the global set of constrained motion TN.

2.3 Manipulator Velocity/Force Trajectory Planning

In this section, we study the problem of velocity/force trajectory planning. Our emphasis here is on velocity trajectories rather than on position trajectories because velocity and force form natural duals of each other. Moreover, if we can control the velocity of a manipulator, then we can control the position of the manipulator by appropriate choice of the desired velocity trajectory (see also Remark [3]). Thus, it suffices to study the velocity/force control of a robot manipulator.

Prior to performing the task, a set of desired velocity/ force trajectories must be specified. Only when the manipulator achieves these desired trajectories can the task be efficiently executed. For example, considering the manipulation task of Figure 1, where in order to the wipe the glass window, one must specify at each contact configuration the desired velocity and the desired contact force. In order to conform with the constraint, the desired velocity must be in velocity controlled subspace, and the desired contact force must be in the force controlled subspace. Otherwise, the manipulator may lose contact with the environment. To complete the manipulator task, the preplanned velocity/force at each contact point should match the task requirement as closely as possible. For example, in the instance when the robot hand is to grind a piece of metal, if the preplanned velocity is too large the grinding tool may stall, or break, and if the preplanned force is too small, the metal surface may not be effectively smoothed.

We now pursue the mathematical details of velocity/force trajectories planning for a manipulator. Let $r \in$ N denote a constrained configuration of the manipulator, the constrained motion is a vector in T_rN. *To plan a velocity trajectory, we need to specify at each $r \in N$, a desired velocity vector $v_{rd} \in T_rN$. Namely, we need to define a function*

$$Y_d : N \to TN, \quad \text{such that } Y_d(r) = v_{rd} \in T_rN \qquad (2.3\text{-}1)$$

The function Y_d assigns to each constrained configuration r a desired velocity vector in the velocity controlled subspace, and is called the desired velocity vector field on N. We assume that the assignment of Y_d is continuous in N. When following the desired velocity trajectory Y_d, the position trajectory $x_d(t)$ traced out by the end effector in the constrained space N, starting from an initial configuration $r_0 \in$ N at time t=0, is the solution of the following differential equation

$$\dot{x}_d(t) = Y_d(x_d(t)), \ \ x_d(0) = r_0 \qquad (2.3\text{-}2)$$

The assumption of $Y_d(\cdot)$ being continuous ensures the existence of an unique solution to the above differential equation. *We say that the manipulator achieves the desired velocity trajectory if and only if the actual velocity of the end effector converges to the desired velocity.* Thus, if we denote the actual position trajectory of the end effector by a curve r: $[0, \infty) \to N$, this simply means that

$$\|\dot{r}(t) - Y_d(r(t))\| \to 0, \ as \ t \to \infty \qquad (2.3\text{-}3)$$

where $\dot{r}(t)$ is the true velocity and $Y_d(r(t))$ the desired velocity at configuration $r(t)$. *We say that we can control the velocity of a manipulator if for every given desired velocity trajectory there exists a choice of joint torque input such that the true velocity converges to the desired velocity.*

At each contact point, the manipulator needs to exert forces on the workpiece. From Newton's third law, when the manipulator exerts a force f_{rd} on the workpiece, an equal but opposite force $-f_{rd}$ is also exerted on the manipulator. Since the workpiece is the only object in contact with the manipulator, the study of exerted forces on the workpiece is the same as the study of exerted forces on the manipulator. But, the sets of forces on the manipulator can be characterized by the wrench space T_r^*M and its subspaces. We will call the work done by the forces in T_r^*M on the twists in T_rM the work exchanged between the manipulator and the environment. To maintain a proper contact with the environment, the desired contact force at a constrained configuration r must be in the constrained wrench space T_rN^c. A force $f \in T_rN^c$ on the constrained twist space T_rN will yield no work exchange between the manipulator and the environment. *The problem of force trajectory planning is to specify at each constrained configuration $r \in N$, a desired contact force f_{rd} in the force controlled subspace T_rN^c, namely to define a function*

$$F_d: N \to TN^c, \text{ such that } F_d(r) = f_{rd} \in T_rN^c \tag{2.3-4}$$

We say that the manipulator achieves the desired force trajectory if the actual interaction force converges to the desired contact force given by F_d.

3. The Hybrid Velocity/Force Control Scheme

Given the desired velocity/force trajectories (2.3-1) and (2.3-4), we propose a stable hybrid control law that realizes both the trajectories simultaneously.

Consider the constraint map $C: M \to R^m$ in local coordinates $(r_1, r_2, \cdots r_6)$ given by $C(r) = (y_1(r), \cdots, y_m(r))$ and its tangent map

$$dC_r: T_rM \to R^m \tag{3.0-1}$$

given in the same coordinates by

$$E_f = \begin{bmatrix} \frac{\partial y_1}{\partial r_1} & \cdots & \frac{\partial y_1}{\partial r_6} \\ \vdots & \vdots & \vdots \\ \frac{\partial y_m}{\partial r_1} & \cdots & \frac{\partial y_m}{\partial r_6} \end{bmatrix} \tag{3.0-2}$$

The matrix $E_f \in R^{m \times 6}$, of rank m, is called the local representation of the tangent map and the constrained velocity $\dot{r}(t)$ satisfies

$$E_f \, \dot{r}(t) = 0 \tag{3.0-3}$$

Thus, the kernel of E_f is the constrained velocity subspace T_rN. The subspace in T_r^*M spanned by the rows of E_f is just the force controlled subspace T_rN^c. Related to E_f, we choose a set of $(6-m)$

linearly independent vectors $\{e_i\}_{i=1}^{6-m}$ such that

$$E_f \cdot e_i = 0 \quad \text{for } i=1, \cdots 6-m \tag{3.0-4}$$

and construct from $\{e_i\}_{i=1}^{6-m}$ a new matrix $E_p \in R^{(6-m)\times 6}$ by

$$E_p = \begin{bmatrix} e_i^t \\ \cdot \\ \cdot \\ \cdot \\ e_{6-m}^t \end{bmatrix} \quad \text{and} \quad E_f \cdot E_p^t = 0 \tag{3.0-5}$$

where e_i^t denotes the transpose of e_i, etc. E_p, of rank $(6-m)$, is used to project the end effector velocity into the velocity controlled subspace and is called the velocity projection matrix. We denote the projected velocity by $\dot{r}_p \in R^{6-m}$ and have that

$$\dot{r}_p = E_p \, \dot{r} \tag{3.0-6}$$

The i_{th} component of $\dot{r}_p$ is the projection of the total velocity in the e_i direction.

Remark (1): Since the kernel of E_f is just the velocity controlled subspace T_rN, aside from some physical considerations (see Section 3.3), we may choose for e_i the set of basis vectors that span T_rN.

Differentiating Equation (3.0-3) with respect to time t we get

$$E_f \, \ddot{r} + \dot{E}_f \, \dot{r} = 0 \tag{3.0-7}$$

where the second term may be expressed as

$$\dot{E}_f \, \dot{r} = \begin{bmatrix} \dot{r}^t d^2y_1 \dot{r} \\ \cdot \\ \cdot \\ \cdot \\ \dot{r}^t d^2y_m \dot{r} \end{bmatrix} \quad \text{and} \quad d^2y_i = \begin{bmatrix} \frac{\partial^2 y_i}{\partial r_1 \partial r_1} & \cdots & \frac{\partial^2 y_i}{\partial r_1 \partial r_6} \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \frac{\partial^2 y_i}{\partial r_1 \partial r_6} & \cdots & \frac{\partial^2 y_i}{\partial r_6 \partial r_6} \end{bmatrix} \quad \text{for } i = 1, \cdots m \tag{3.0-8}$$

namely, each component of $\dot{E}_f \, \dot{r}$ is the stiffness matrix in that direction operating on the velocity vectors [4].

Now, with respect to the chosen coordinate system that describes the constraint function, we identify the force controlled subspace with R^m and the velocity controlled subspace with R^{6-m}. Equations (3.0-3) and (3.0-7) can also be thought as the projection of the end effector velocity and acceleration to the force controlled subspace respectively. It is clear from (3.0-7) that the acceleration projected to the force controlled subspace depends on the velocity terms only.

Also, differentiating (3.0-6) with respect to time t yields the projected acceleration in the velocity controlled subspace.

$$\ddot{r}_p = E_p\ddot{r} + \dot{E}_p\dot{r} \qquad (3.0\text{-}9)$$

We define the constraint space (Mason [2] & Yoshikawa [4]) of the end effector at configuration r to be the direct sum of the velocity controlled subspace and the force controlled subspace, and denote it by $R^6 = R^{6-m} \oplus R^m$. From E_p and E_f we define a map E: $T_rM \rightarrow R^6$ by $E = \begin{bmatrix} E_p \\ E_f \end{bmatrix}$, that takes a velocity in T_rM into the constraint space. If $\dot{r}$ denotes the velocity of the end effector, we have from (3.0-3) and (3.0-6) that

$$E\,\dot{r} = \begin{bmatrix} E_p\,\dot{r} \\ E_f\,\dot{r} \end{bmatrix} = \begin{bmatrix} E_p\,\dot{r} \\ 0 \end{bmatrix} = \begin{bmatrix} \dot{r}_p \\ 0 \end{bmatrix} \qquad (3.0\text{-}10)$$

The map E projects the manipulator velocity $\dot{r}$ from the configuration space to the constraint space. It is easy to see that in the constraint space the velocity controlled subspace is orthogonal to the force controlled subspace.

Also, from Equation (3.0-7) and (3.0-9) we can project the end effector acceleration into the constraint space by

$$E\,\ddot{r} = \begin{bmatrix} E_p\,\ddot{r} \\ E_f\,\ddot{r} \end{bmatrix} = \begin{bmatrix} \ddot{r}_p - \dot{E}_p\,\dot{r} \\ -\dot{E}_f\,\dot{r} \end{bmatrix} = \begin{bmatrix} \ddot{r}_p \\ 0 \end{bmatrix} - \begin{bmatrix} \dot{E}_p\,\dot{r} \\ \dot{E}_f\,\dot{r} \end{bmatrix} \qquad (3.0\text{-}11)$$

3.1 Robot Dynamics in the Constraint Space and the Joint Force Control Law

In Section 3.0, we have used the constraint information to project the velocity and the acceleration of the end effector into the constraint space. We now want to incorporate the manipulator Jacobian information to project the robot dynamics from the joint space into the configuration space and then into the constraint space. The motivation for studying the robot dynamics in the constraint space stems from the fact that the velocity controlled subspace and the force controlled subspace in the constraint space are mutually orthogonal.

We consider here manipulators with six degrees of freedom only. The results are easily generalizable to manipulators of higher degrees of freedom. Denote the joint space of the manipulator by Θ , and a joint variable $\theta = (\theta_1, \theta_2, \cdots \theta_6) \in \Theta$ representing the joint angles of the manipulator joint actuators. The dynamic equation of the constrained manipulator in the Lagrangian form is given by

$$M(\theta)\,\ddot{\theta} + h(\theta, \dot{\theta}) + v_g(\theta) = \tau + \hat{\tau} \tag{3.1-1}$$

where $M(\theta)$ is the non-singular inertia matrix of the system, $h(\theta, \dot{\theta})$ the centrifugal and Coriolis forces, and $v_g(\theta)$ the gravitational force; $\hat{\tau}$ is the force at the joint resulting from the forces at the interaction between the end effector and the environment. τ is the joint driving force and is therefore the control input. The goal is to derive a set of inputs for τ so that the control objective may be achieved.

In order to project the robot dynamics into the constraint space, we first consider the forward kinematic equation of the manipulator

$$f\colon \Theta \to M,\ f(\theta) = r \tag{3.1-2}$$

and assume that the manipulator is operating in a region where the Jacobian of (3.1-2), given in (3.1-3), is nonsingular.

$$J\colon T_\theta\Theta \to T_{r(\theta)}M, \quad J\,\dot{\theta} = \dot{r} \tag{3.1-3}$$

The Jacobian J transforms a joint velocity into the configuration space. We differentiate the Jacobian equation with respect to time t to obtain

$$J\,\ddot{\theta} + \dot{J}\dot{\theta} = \ddot{r} \tag{3.1-4}$$

which relates $\ddot{r}(t)$ to the joint acceleration.

Furthermore, we can use (3.1-4) and (3.0-11) to transform the acceleration of the manipulator from the joint space into the constraint space. Denoting $\frac{d}{dt}(EJ)$ by $(EJ)'$ we have that

$$E\,J\,\ddot{\theta} \ = \ E(\,\ddot{r} - \dot{J}\,\dot{\theta}\,) = \begin{bmatrix} \ddot{r}_p \\ 0 \end{bmatrix} - \dot{E}\dot{r} - E\dot{J}\dot{\theta} = \begin{bmatrix} \ddot{r}_p \\ 0 \end{bmatrix} - (EJ)'\,\dot{\theta} \tag{3.1-5}$$

When the velocities are transformed forward from the joint space into the constraint space by (EJ), the interaction forces are transformed back from the constraint space into the joint space by $J^t E^t$. We consider here only the case where there is no work exchange between the manipulator and the environment and leave the other case to Section 3.2. The interaction force, denoted by $\hat{F}$, when yielding no work exchange with the environment is a normal force and belongs to the force controlled subspace R^m. $\hat{F}$ is transformed back into the joint space by

$$J^t E_f^t\colon R^m \to T_\theta^*\Theta,\ \ \hat{\tau} = J^t\,E_f^t\,\hat{F} \tag{3.1-6}$$

Rearranging the manipulator equation using (3.1-6) we have

$$\ddot{\theta} = M^{-1}(\theta)(\tau - h(\dot{\theta},\theta) - v_g(\theta) + J^t E_f^t \hat{F}) \tag{3.1-7}$$

We also assume that measurement of the joint angles θ, the joint velocity $\dot{\theta}$, and exact knowledge about the parameter values of the manipulator are available. Consequently, we can cancel the centrifugal, Coriolis and gravitational forces by choosing our first level control to be

$$\tau = h(\theta, \dot{\theta}) + v_g(\theta) + \tau_1 \tag{3.1-8}$$

where τ_1 is to be determined. Using this control in (3.1-7) and multiplying the results by EJ, we obtain from (3.1-5) and (3.1-7) that,

$$\begin{bmatrix} \ddot{r}_p \\ 0 \end{bmatrix} - (EJ)' \dot{\theta} = E\, J\, M^{-1}(\theta)\, \tau_1 + E\, J\, M^{-1}(\theta)\, J^t E_f^t \hat{F} \tag{3.1-9}$$

Since the time derivative term $(E\,J)'$ in the above equation depends on the velocity and the manipulator parameters only we can choose our second level control τ_1 to be

$$\tau_1 = -M(\theta)\, J^{-1}\, E^{-1}\, (EJ)'\, \dot{\theta} + \tau_2 \tag{3.1-10}$$

where τ_2 is to be determined. Use this control in (3.1-9) we progressively project the manipulator equation into the constraint space by

$$\begin{bmatrix} \ddot{r}_p \\ 0 \end{bmatrix} = \begin{bmatrix} E_p\, J\, M^{-1}(\theta)\, \tau_2 \\ E_f\, J\, M^{-1}(\theta)\, \tau_2 \end{bmatrix} + \begin{bmatrix} E_p\, J\, M^{-1}(\theta)\, J^t E_f^t \hat{F} \\ E_f\, J\, M^{-1}(\theta)\, J^t E_f^t \hat{F} \end{bmatrix} \tag{3.1-11}$$

The structures of the manipulator equation in the constraint space are now clear: *the top equation describes the dynamics in the velocity controlled subspace and the bottom equation describes the dynamics in the force controlled subspace; the force controlled loop is a pure algebraic loop and the interaction forces from the force controlled subspace is coupled to the velocity loop via* $E_p\, J\, M^{-1}(\theta)\, J^t\, E_f^t$.

Finally, we are now ready to complete the control input design. First, from the desired velocity trajectory $Y_d : N \rightarrow TN$ and the desired force trajectory $X_d : N \rightarrow TN^T$ we define

velocity error:

$$\dot{e}_p(t) \equiv \dot{r}_p(t) - Y_d(r_p(t)) = \dot{r}_p(t) - \dot{r}_{pd}(t)$$

and force error:

$$\hat{e}_f \equiv \hat{F} - F_d(r_p(t)) \equiv \hat{F} - \hat{F}_d$$

Theorem 3.1: Following the previous notation, the joint force control law given by (3.1-12) stabilizes simultaneously the force controlled loop and the velocity controlled loop. Furthermore, the convergence rates are determined by the velocity proportional gain ($K_v \in R^{(6-m)\times(6-m)}$), the force proportional gain ($K_{fp} \in R^{m\times m}$) and the force integral gain ($K_{fi} \in R^{m\times m}$) respectively.

$$\begin{aligned}\tau \;=\; & h(\theta,\dot{\theta}) + v_g(\theta) - M(\theta)\,J^{-1}\,E^{-1}\,(EJ)'\,\dot{\theta} \\ & + M(\theta)\,J^{-1}\,E_p^t\,(E_p\,E_p^t)^{-1}(\ddot{r}_{pd} - K_v\,\dot{e}_p) + J^t\,E_f^t\,(-\hat{F}_d + K_{fi}\int \hat{e}_f + K_{fp}\,\hat{e}_f)\end{aligned} \tag{3.1-12}$$

Remark (2): (i) The first term $(h(\theta,\dot{\theta}) + v_g(\theta))$ in (3.1-12) is used to cancel the nonlinearities in the robot dynamics caused by centrifugal and Coriolis forces, (ii) the second term $(-M(\theta)^{-1}\,J^{-1}\,E^{-1}\,(EJ)'\,\dot{\theta})$ is used to cancel the nonlinearities introduced by coordinate transformations, (iii) the term $M(\theta)J^{-1}\,E_p^t\,(E_pE_p^t)^{-1}(\ddot{r}_{pd} - K_v\,\dot{e}_p)$ is the compensation in the velocity loop, and (iv) $J^t\,E_f^t\,(-\hat{F}_d + K_{fi}\int \hat{e}_f + K_{fp}\,\hat{e}_f)$ is the compensation in the force loop.

(3): If we let $e_p(t) = \int_0^t \dot{e}_p\,dt$ be the position error, and replace the velocity loop compensator by $M(\theta)\,J^{-1}E_p^t(E_pE_p^t)^{-1}\,(\ddot{r}_{pd} - K_v\dot{e}_p - K_pe_p)$, where K_p is the position feedback gain, we obtain as in [1] a hybrid position/force control.

(4): The control law reduces to that of the computed torque method in the absence of constraints and to that of pure force control in the presence of complete constraints.

Proof. (of Theorem 3.1) Applying the first two terms of the control input to the manipulator equation (3.1-1) and projecting the resulting equation into the constraint space we obtain the reduced manipulator equation (3.1-11). With the remaining control in (3.1-11) we have that

$$\begin{cases} \ddot{e}_p + K_v\,\dot{e}_p = E_p\,J\,M^{-1}(\theta)\,J^t\,E_f^t\,(\hat{e}_f + K_{fi}\int \hat{e}_f + K_{fp}\,\hat{e}_f) \\ 0 = E_f\,J\,M^{-1}(\theta)\,J^t\,E_f^t\,(\hat{e}_f + K_{fi}\int \hat{e}_f + K_{fp}\,\hat{e}_f) \end{cases} \tag{3.1-13}$$

Since E_f, J and $M(\theta)$ are all of full rank, the product $E_f\,J\,M^{-1}(\theta)\,J^t\,E_f^t$ is nonsingular. In particular, the force loop equation implies that

$$\hat{e}_f + K_{fi}\int \hat{e}_f + K_{fp}\,\hat{e}_f \;=\; 0 \tag{3.1-14}$$

and using this in the velocity loop we have that

$$\ddot{e}_p + K_v\,\dot{e}_p \;=\; 0 \tag{3.2-6}$$

We can choose the gain matrices (K_v, K_{fi}, K_{fp}) appropriately so that the two uncoupled equations will be exponentially stable. Furthermore, the convergence rate in each loop is determined by K_v

and (K_{fi}, K_{fp}) respectively.

3.2 An Extension to Hybrid Impedance/Force Control

When deriving the control law in Section 3.1, we have assumed that there is no exchange of work between the manipulator and the environment. Consequently, the interaction forces are in the directions normal to the constraint surface. For manipulator tasks such as polishing a glass window, pushing a chip along the surface of a smooth worktable, or inserting a peg into a hole, this assumption is reasonable. The small interaction force in the normal directions as compared to the large interaction force in the tangent directions is a common feature to this class of constrained manipulator tasks.

However, for manipulator tasks such as grinding, metal cutting, drilling, or polishing a bumpy surface, this assumption no longer holds; the influence of the interaction forces from the tangent directions are significant to the robot dynamics. Under these conditions, pure velocity control in the tangent directions may produce excessive interaction forces and subsequently stall the grinding tools (if not break them) or destroy the drills. To cope with this problem the interaction forces from the tangent directions should be "properly " accommodated rather than resisted ([6] & [7]). Previous researchers ([6]~[8]) have proposed a so called *impedance control methodology* for the robot system so that the interaction forces can be stabilized while commanding the position inputs. Here we generalize the results of [6] & [7] to impose *impedance control in the tangent directions* so that the manipulator task can be executed.

To accommodate the interaction forces in the tangent directions, the design objective of impedance control must provide a stabilizing dynamic compensator for the system so that (1) the ratio of the position of the closed-loop system to the interaction force is constant within a given operating frequency range, (2) the closed-loop system is stable, and (3) the closed-loop system is robust under model uncertainties. The above statement can be mathematically represented by

$$(s^2 I + K_v s + K_p)e_p(s) = \hat{F}_v(s), \quad s = j\omega, \text{ for all } 0 < \omega < \omega_0 \tag{3.2-1}$$
with $(s^2 I + K_v s + K_p)$ representing the *impedance*

Here $\hat{F}_v(s) \in C^{6-m}$ is the vector of deviation of the tangent direction interaction forces from an equilibrium value in the constraint space; $e_p(s) \in C^{6-m}$ is the vector of deviation of the tangent direction position from an equilibrium point in the constraint space; $K_p \in R^{(6-m)\times(6-m)}$ is the desired stiffness matrix; K_v and I $\in R^{(6-m)\times(6-m)}$ are respectively the damping and the identity matrices; and ω_0 is the bandwidth of operation.

The stiffness matrix K_p is the designer's choice and, depending on the application, contains different values for each of the tangent directions. By specifying K_p, the designer governs the behavior of the system in constrained maneuvers. Large entries in the K_p matrix imply large interaction forces. Small entries in K_p allow for a considerable amount of motion in the system in response to interaction forces. The choice of the damping matrix K_v assures the achievement of ω_0 and the stability of the system. A complete discussion on the choice of K_v to achieve a certain performance specification and a certain stability robustness specification is given by Kazerooni et al in [6]. We will assume that the desired closed-loop behavior in the tangent direction is given by the specification of (3.2-1). Our second step design objective is to develop a joint force control law so that (3.2-1) is realized in the tangent direction.

On the other hand, it is adequate for most constrained manipulator tasks to impose *pure force control in normal directions.* Only the interaction forces in the normal directions need to be regulated. *Since the inertia forces in the normal directions are considerably less than the tangent direction counterpart* the manipulator equation in the force controlled subspace remains an algebraic equation (see Section 3.1).

In summary, our strategy to control a constrained manipulator task with finite interaction forces in the tangent directions is to impose *impedance control in the tangent directions* and *force control in the normal directions.* Such a control is called *a hybrid impedance/force control.*

The tangent direction interaction force $\hat{F}_v \in R^{6-m}$ is transformed back into the joint space by

$$J^t E_p^t : R^{6-m} \rightarrow T_\theta^* \Theta, \ \ \hat{\tau}_v = J^t E_p^t \hat{F}_v \tag{3.2-2}$$

Adding this term to the manipulator equation, we obtain

$$M(\theta)\ddot{\theta} + h(\theta,\dot{\theta}) + v_g(\theta) = \tau + \hat{\tau} + \hat{\tau}_v \tag{3.2-3}$$

The manipulator Equation (3.1-11) now reads

$$\begin{bmatrix} \ddot{r}_p \\ 0 \end{bmatrix} = \begin{bmatrix} E_p J M^{-1}(\theta)\, \tau_2 \\ E_f J M^{-1}(\theta)\, \tau_2 \end{bmatrix} + \begin{bmatrix} E_p J M^{-1}(\theta)\, J^t\, (E_f^t \hat{F} + E_p^t \hat{F}_v) \\ E_f J M^{-1}(\theta)\, J^t\, (E_f^t \hat{F} + E_p^t \hat{F}_v) \end{bmatrix} \tag{3.2-4}$$

The vector of deviation of the tangent direction position e_p is given by Remark (3) of Section 3.1 as,

$$e_p(t) = \int_0^t \dot{e}(\tau)\, d\tau \tag{3.2-5}$$

We claim that the control inputs of (3.2-6) will achieve the desired control objective, if the constant matrix $K_0 \in R^{(6-m)\times(6-m)}$ is chosen appropriately.

$$\tau = h(\theta,\dot\theta) + v_g(\theta) - M(\theta)\, J^{-1}\, E^{-1}(E\, J)'\, \dot\theta$$
$$+ M(\theta)\, J^{-1}\, E_p^t\, (E_p\, E_p^t)^{-1}\, \{\ddot r_{pd} - K_v\, \dot e_p - K_p\, e_p + K_0\, \hat F_v\,\} \qquad (3.2\text{-}6)$$
$$+ J^t\, E_f^t\, \{-\hat F_d + K_{fi} \textstyle\int \hat e_f + K_{fp}\, \hat e_f\,\}$$

To show our claim, we see that the first three components of (3.2-6) are used to reduce the robot dynamics to the form (3.2-4). Applying the remaining control to (3.2-4), we have

$$\begin{cases} \ddot e_p + K_v\, \dot e_p + K_p\, e_p = K_0\, \hat F_v + E_p\, J\, M^{-1}\, J^t\, \{E_f^t\,(\,(I + K_{fp}\,)\, \hat e_f + K_{fi} \int \hat e_f\,) + E_p^t\, \hat F_v\} \\ \\ 0 = E_f\, J\, M^{-1}\, J^t\{E_f^t\,(\,(I + K_{fp}\,)\, \hat e_f + K_{fi} \int \hat e_f\,) + E_p^t\, \hat F_v\} \end{cases} \qquad (3.2\text{-}7)$$

To simplify the above equation, we define

$$M_f = E_f\, J\, M^{-1}\, J^t\, E_f^t \;\; \in R^{(6-m)\times(6-m)}$$

$$M_{fp} = E_f\, J\, M^{-1}\, J^t\, E_p^t \;\; \in R^{(6-m)\times m} \qquad (3.2\text{-}8)$$

$$M_p = E_p\, J\, M^{-1}\, J^t\, E_p^t \;\; \in R^{m\times m}$$

Here M_f and M_p are nonsingular matrices because E_f and E_p are of full rank. We obtain from the second equation of (3.2-7) that

$$(I + K_{fp}\,)\, \hat e_f + K_{fi} \int \hat e_f = -\, M_f^{-1}\, M_{fp}\, \hat F_v \qquad (3.2\text{-}9)$$

Using this in the first equation of (3.2-7), we have

$$\ddot e_p + K_v\, \dot e_p + K_p\, e_p = (K_0 - M_{fp}^t\, M_f^{-1}\, M_{fp} + M_p\,)\, \hat F_v \qquad (3.2\text{-}10)$$

If we choose K_0 to be $(I + M_{fp}\, M_f^{-1}\, M_{fp} - M_p)$ the closed loop equation becomes

$$\begin{cases} \ddot e_p + K_v\, \dot e_p + K_p\, e_p = \hat F_v \\ (I + K_{fp})\, \hat e_f + K_{fi} \int \hat e_f = -M_f^{-1}\, M_{fp}\, \hat F_v \end{cases} \qquad (3.2\text{-}11)$$

While the top quation achieves the desired impedance specification in the tangent directions, the bottom equation is bounded input and bounded output stable when the gain matrices K_{fi} and K_{fp} are properly chosen. This completes the proof.

Remark (5): It is easy to see that the hybrid impedance/force control recovers the hybrid velocity/force control in the absence of interaction forces from the tangent directions. When the interaction forces are small the hybrid velocity/force control is a good approximation to the hybrid impedance/force control scheme.

(6): In deriving the joint force control input (3.2-6), we did not assume slow operation of the manipulator as did in Kazerooni et al [6], consequently the scheme developed here is applicable to a larger class of manipulator tasks. On the other hand, the need to measure the interaction forces in both the tangent and the normal directions may be a source of trouble in the implementation of the scheme.

3.3 Several Design Examples

In the previous sections we have relied on the existence of a velocity projection matrix E_p to derive the control inputs. Here we want to study the selection of the velocity projection matrix for task examples shown in Figures 1-3. Since it is easy to find a basis set for the velocity controlled subspace and E_p is also orthogonal to E_f, we can choose for the rows of E_p the basis vectors of $T_r N$. In general, because the basis vectors have different physical units from that of the velocity projection matrix E_p (it should be unitless), we need to make some corrections to justify for the physical units. This is also demonstrated on the following examples.

Example 3-1: Consider the planar manipulator of Figure 1, where the constraint function is given by (2.2-1), and the associated force projection matrix is $E_f = [0\ 1\ 0]$. Clearly, we can choose the following velocity projection matrix,

$$E_p = \begin{bmatrix} 1\,0\,0 \\ 0\,0\,1 \end{bmatrix} \tag{3.3-1}$$

and $\dot{r}_p = E_p \dot{r} = \binom{\dot{x}}{\dot{\theta}}$ defines the constrained velocity. Notice that the basis vectors for the velocity controlled subspace are constant over the entire submanifold N.

Example 3-2. Considering the manipulator task of Figure 2, the constraint equation is given by (2.2-5) which yields the following force projection matrix

$$E_f = \begin{bmatrix} x & y & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\sin\phi & 0 \\ \cos\phi & \sin\phi & 0 & -x\sin\phi + y\cos\phi & 0 & 0 \end{bmatrix} = \begin{bmatrix} e_3^t \\ e_4^t \\ e_5^t \\ e_6^t \end{bmatrix} \tag{3.3-2}$$

We let $\begin{bmatrix} e_1^t \\ e_2^t \end{bmatrix} = E_p$ be the basis vectors that span the two dimensional velocity controlled subspace. $e_1 = (0\ 0\ 0\ 0\ 0\ 1)^t$ clearly satisfies the requirement $E_f\, e_1 = 0$. To furnish a design for e_2 we assume that it is of the form

$$e_2 = (\alpha_1, \alpha_2, 0, \beta_1, \beta_2, 0)^t$$

where $(\alpha_1, \alpha_2, 0) \in T_r S^1$ is a vector tangent to the circle at configuration r. Since a tangent vector in S^1 is perpendicular to the normal vector, we have that

$$\begin{aligned} &\nabla c_1 (x, y, z) \cdot (\alpha_1, \alpha_2, 0)^t = (x, y, 0) \cdot (\alpha_1, \alpha_2, 0)^t = 0 \\ &c_1(x, y, z) = x^2 + y^2 - 1 \end{aligned} \tag{3.3-3}$$

A solution to the above equation is $(\alpha_1, \alpha_2, 0) = (-y, x, 0)$. We complete the choice of e_2 by requiring that

$$\begin{bmatrix} e_5^t \\ e_6^t \end{bmatrix} e_2 = 0 \tag{3.3-4}$$

Expanding the above equation, we have

$$\begin{cases} -\beta_2 \sin\phi = 0 \\ -y \cos\phi + x \sin\phi + (y \cos\phi - x \sin\phi)\beta_1 = 0 \end{cases} \tag{3.3-5}$$

Solving the equation incorporating the constraint yields $(\beta_1 = 1, \beta_2 = 0)$ and the corresponding velocity projection matrix E_p is

$$E_p = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ -y & x & 0 & 1 & 0 & 0 \end{bmatrix} \tag{3.3-6}$$

The set of basis for the velocity controlled subspace are now dependent on the configuration of the manipulator. Expressing the manipulator velocity in terms of the chosen basis we have

$$\dot{r}_p = E_p \, \dot{r} = \begin{bmatrix} \dot{\psi} \\ -y\,\dot{x} + x\,\dot{y} + \dot{\phi} \end{bmatrix} \tag{3.3-7}$$

One can now discover a pitfall in the second component of $\dot{r}_p$, where the angular velocity $\dot{\phi}$ in units of radian/sec is added to the linear velocity $-y\dot{x} + x\dot{y}$ in units of meter^2/sec. This has no physical significances. $\dot{\phi}$ is introduced because we need a nonzero β_1 to render e_2 orthogonal to E_f. But, recall from Figure 2 that the actual velocity really should be the component that is tangent to the circle, i.e., the term $(-y\dot{x} + x\dot{y})$, and the rotational velocity $\dot{\psi}$. When we plan the desired velocity trajectory, we assign to each configuration r a desired velocity vector that in the local basis is given by $(\dot{\psi}_d, -y\dot{x}_d + x\dot{y}_d)^t \equiv \dot{\bar{r}}_{pd}$. We call $\dot{\bar{r}}_{pd}$ the true desired velocity and the values of $\dot{\psi}_d, \dot{x}_d, \dot{y}_d$ at r are predetermined by the task requirement. *In the control, we use the measured $\dot{\phi}$ to construct a desired pseudo-velocity vector $\dot{r}_{pd}$ by*

$$\dot{r}_{pd} = \begin{bmatrix} \dot{\psi}_d \\ -y\,\dot{x}_d + x\,\dot{y}_d \end{bmatrix} + \begin{bmatrix} 0 \\ \dot{\phi} \end{bmatrix} \tag{3.3-8}$$

It is easy to verify that the true velocity converges if and only if the pseudo velocity $\dot{r}_p$ ($= E_p\,\dot{r}$) converges to the desired pseudo-velocity $\dot{r}_{pd}$. Namely,

$$\begin{aligned} \dot{e}_p \equiv \dot{r}_p - \dot{r}_{pd} &= \begin{bmatrix} e_1^t \\ e_2^t \end{bmatrix}\dot{r} - \begin{bmatrix} \dot{\psi}_d \\ -y\dot{x}_d + x\dot{y}_d \end{bmatrix} + \begin{bmatrix} 0 \\ \dot{\phi} \end{bmatrix} \\ &= \begin{bmatrix} \dot{\psi} \\ -y\dot{x} + x\dot{y} \end{bmatrix} - \begin{bmatrix} \dot{\psi}_d \\ -y\dot{x}_d + x\dot{y}_d \end{bmatrix} = \dot{\tilde{e}}_p \end{aligned} \tag{3.3-9}$$

where $\dot{\tilde{e}}_p$ is the true velocity error and $\dot{e}_p$ the pseudo-velocity error.

Example 3-3. Finally, we consider the manipulator task of Figure 3. This example is similar to Example 3.2 except for the additional degree of freedom for the motion of the end effector. The force projection matrix associated with the constraint (2.2-8) is given by

$$E_f = \begin{bmatrix} x & y & z & 0 & 0 & 0 \\ 0 & \cos\theta & \cos\phi\,\sin\theta & -z\,\sin\phi\,\sin\theta & -y\,\sin\theta + z\,\cos\phi\,\sin\theta & 0 \\ -\cos\theta & 0 & \sin\phi\,\sin\theta & z\,\cos\phi\,\sin\theta & z\,\sin\phi\,\cos\theta + x\,\sin\theta & 0 \end{bmatrix} \tag{3.3-10}$$

To find a set of E_f-orthogonal basis for the velocity controlled subspace, we let $f_1 = x^2 + y^2 + z^2 - 1$ denote the equation of the sphere S^2, the gradient ∇f_1 is normal to the sphere; we choose at each configuration two tangent vectors $t_1, t_2 \in T_{(x,y,z)}S^2$, where they satisfy $t_i \cdot \nabla f_1 = 0$, for $i = 1,2$ and $t_2 = t_1 \times \nabla f_1$, and write $t_1 = (\alpha_1, \alpha_2, \alpha_3)$, $t_2 = (\alpha'_1, \alpha'_2, \alpha'_3)$. Assuming that the velocity projection matrix is of the form

$$E_p = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ \alpha_1 & \alpha_2 & \alpha_3 & \beta_1 & \beta_2 & 0 \\ \alpha'_1 & \alpha'_2 & \alpha'_3 & \beta'_1 & \beta'_2 & 0 \end{bmatrix} = \begin{bmatrix} e_1^t \\ e_2^t \\ e_3^t \end{bmatrix} \tag{3.3-11}$$

where (β_1, β_2) and (β'_1, β'_2) are chosen to satisfy

$$E_f \cdot e_2 = 0 \quad \text{and} \quad E_f \cdot e_3 = 0 \tag{3.3-12}$$

Expanding (3.3-12) and solving for (β_1, β_2) we have

$$\begin{bmatrix} \beta_1 \\ \beta_2 \end{bmatrix} = A^{-1} \begin{bmatrix} -\alpha_2\cos\theta - \alpha_3\cos\phi\,\sin\theta \\ \alpha_1\cos\theta + \alpha_3\sin\phi\,\sin\theta \end{bmatrix} \tag{3.4-13a}$$

where

$$A = \begin{bmatrix} -z \sin\phi \sin\theta & -y \sin\theta + z \cos\phi \cos\theta \\ z \cos\phi \sin\theta & z \sin\phi \cos\theta + x \sin\theta \end{bmatrix} \tag{3.4-13b}$$

is generically nonsingular. Similarly, replacing t_1 by t_2 in the above equation we can solve for (β'_1, β'_2). We have thus completed the selection of E_p when A is nonsingular. The case when A is singular is treated separately in Appendix B.

In general, when a manipulator is constrained to a two dimensional surface, the first three components of e_2 and e_3 are two independent vectors tangent to the constraint surface. We append to the last three entries of the basis vectors terms that render E_p orthogonal to E_f. Consequently, we run into the same pitfalls as in Example 3.2. For instance, looking at the second component of $\dot{r}_p$ given by (3.3-11), where the angular velocity $\beta_1\dot{\phi} + \beta_2\dot{\theta}$ in unit radian /sec is added to the linear velocity $\alpha_1\dot{x} + \alpha_2\dot{x}\dot{y} + \alpha_3\dot{z}$ in units of *meter*^{2}*/sec.* On the other hand, the actual velocity of the end effector (recall from Figure 3) should be a vector tangent to the sphere and the rotational velocity $\dot{\psi}$. Since a tangent vector to the sphere can be expressed as a linear combination of the basis vectors t_1 and t_2, the actual velocity may be expressed as

$$\dot{\tilde{r}}_p = \begin{bmatrix} \dot{\psi} \\ \alpha_1\dot{x} + \alpha_2\dot{y} + \alpha_3\dot{z} \\ \alpha'_1\dot{x} + \alpha'_2\dot{y} + \alpha'_3\dot{z} \end{bmatrix} \tag{3.3-14}$$

and the true desired velocity $\dot{\tilde{r}}_{pd}$ as

$$\dot{\tilde{r}}_{pd} = \begin{bmatrix} \dot{\psi}_d \\ \alpha_1\dot{x}_d + \alpha_2\dot{y}_d + \alpha_3\dot{z}_d \\ \alpha'_1\dot{x}_d + \alpha'_2\dot{y}_d + \alpha'_3\dot{z}_d \end{bmatrix} \tag{3.3-15}$$

In the control we construct from measured $\dot{\phi}$ and $\dot{\theta}$ the pseudo desired velocity $\dot{r}_{pd}$ by

$$\dot{r}_{pd} = \dot{\tilde{r}}_{pd} + \begin{bmatrix} 0 \\ \beta_1\dot{\phi} + \beta_2\dot{\theta} \\ \beta'_1\dot{\phi} + \beta'_2\theta \end{bmatrix} \tag{3.3-16}$$

and use $\dot{r}_{pd}$ in the control scheme. It is easy to verify that the true velocity converges if and only if the pseudo velocity converges.

This procedure of developing the velocity projection matrix E_p can be generalized to manipulator tasks with arbitrary constraints. As long as the constraint surface has well defined normals, we can always choose the tangent vectors to construct the E_p matrix as described in these examples.

4. Suggestions for Future Work

Since the hybrid velocity/force control theory is developed with respect to a chosen coordinate system and there is no natural way to specify a coordinate system, it is important to further investigate the consequences of changing the coordinates on the performance of the control schemes ([4]). The robustness of the control scheme with respect to modeling uncertainties in the constraints and in the parameter values needs to be formulated and carefully studied. It is also important to realize that implementation of the scheme requires measurement of the interaction forces and the inversion of the Jacobian matrix. Part of the future work should be to modify the scheme so that its implementation will be less computationaly intensive.

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Appendix A. List of Figures

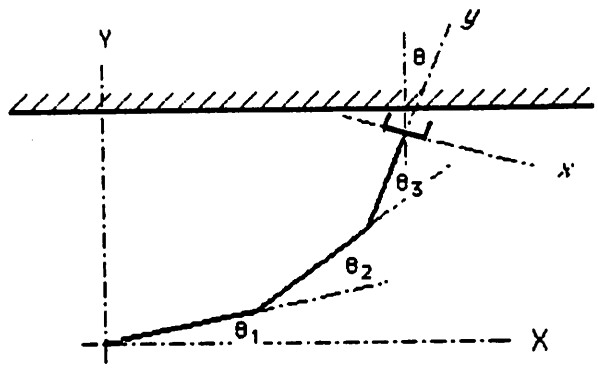

Figure 1. A Robot Manipulator Washing a Glass Window

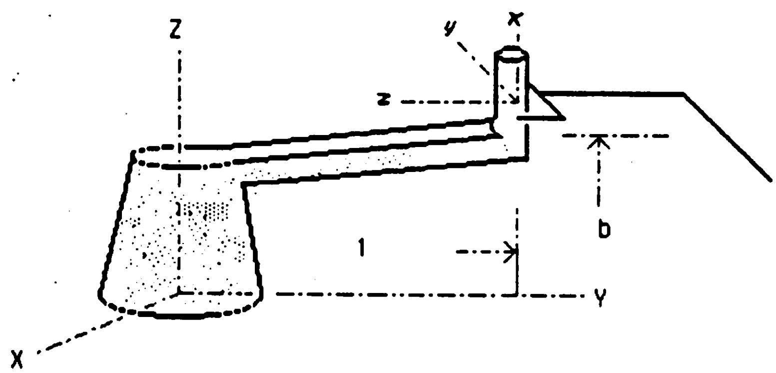

Figure 2. A Robot Manipulator Turning a Crank

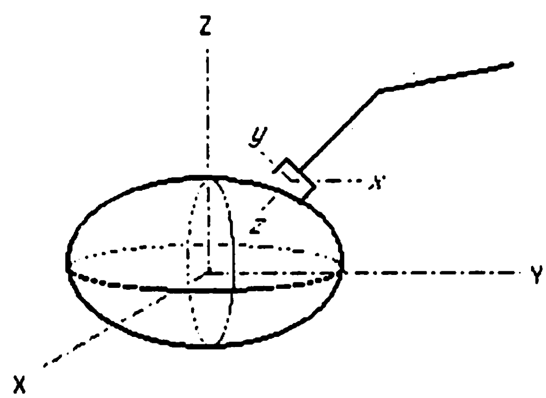

Figure 3. A Robot Manipulator Polishing a Sphere Surface

Appendix B

We show in this appendix the selection of a E_f-orthogonal basis for the velocity controlled subspace when the matrix A of (3.3-13b) is singular. From (3.3-13b)

$$A = \begin{bmatrix} -z \sin\phi \sin\theta, & -y \sin\theta + z \cos\theta \cos\phi \\ z \cos\phi \sin\theta, & z \sin\phi \cos\theta + x \sin\theta \end{bmatrix} \tag{B-1}$$

we obtain

$$-\det(A) = z \sin\theta\,(z \cos\theta + x \sin\theta \sin\phi - y \cos\phi \sin\theta) \tag{B-2}$$

and det(A) = 0 if either (i) $\sin\theta = 0$, (ii) $z = 0$ or (iii) $z \cos\theta + x \sin\theta \sin\phi - y \cos\phi \sin\theta = 0$.

We treat the design for each case separately.

Case (i): If $\sin\theta = 0$, then (2.2-7) implies that $x = 0$ and

$$E_f = \begin{bmatrix} 0 & y & z & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & y & 0 \\ -1 & 0 & 0 & 0 & z\sin\phi & 0 \end{bmatrix}$$

Let $e_2 = (\alpha_1 \;\; -z \;\; y \;\; \beta_1 \;\; \beta_2 \;\; 0)$ and (3.3-12) requires that

$$\begin{cases} -z + \beta_2 y = 0 \\ -\alpha_1 + z \sin\phi\, \beta_2 = 0 \end{cases} \tag{B-3}$$

Thus, $\beta_2 = z/y$, $\alpha_1 = z^2 \sin\phi / y$ and $\beta_1 = 0$ is a satisfactory design for e_2. Consequently, we can choose $e_3 = (\alpha_1',\ z,\ -y,\ \beta_1',\ \beta_2',\ 0)$, where $\beta_2' = -z/y$, $\alpha_1' = z^2 \sin\phi / y$ and β_1' is any value that renders e_3 independent from e_2.

Case (ii): $z = 0$ is treated similarly as in case (i).

Case (iii): We may assume without loss of generality that $z \neq 0$, $\sin\theta \neq 0$, and $\cos\phi \neq 0$, and rewrite E_f here for convenience,

$$E_f = \begin{bmatrix} x & y & z & 0 & 0 & 0 \\ 0 & \cos\theta & \cos\phi\sin\theta & -z\sin\phi\sin\theta & -y\sin\theta + z\cos\phi\sin\theta & 0 \\ -\cos\theta & 0 & \sin\phi\sin\theta & z\cos\phi\sin\theta & z\sin\phi\cos\theta + x\sin\theta & 0 \end{bmatrix} \equiv \begin{bmatrix} e_4^f \\ e_5^f \\ e_6^f \end{bmatrix}$$

Multiplying e_5^f by $\cos\phi$, e_6^f by $\sin\phi$ and adding the results we have

$$\cos\phi\, e_5^f + \sin\phi\, e_6^f = (-\cos\theta\sin\phi \;\; \cos\theta\cos\phi \;\; \sin\theta \;\; 0 \;\; z\cos\phi + x\sin\theta\sin\phi - y\sin\theta\cos\phi \;\; 0)$$

Consequently, the constraint equation and the requirement $(\cos\phi\, e_5^f + \sin\phi\, e_6^f) \cdot e_2 = 0$ together implies that

$$\begin{cases} -\alpha_1 \cos\theta \sin\phi + \alpha_2 \cos\theta \cos\phi + \alpha_3 \sin\theta = 0 \\ \alpha_1 x + \alpha_2 y + \alpha_3 z = 0 \end{cases} \tag{B-4}$$

which immediately yields

$$(\alpha'_1 \;\; \alpha'_2 \;\; \alpha'_3) = (\alpha_1 \;\; \alpha_2 \;\; \alpha_3) = (x \;\; y \;\; z) \times (-\cos\theta \sin\phi \;\; \cos\theta \cos\phi \;\; \sin\theta) \tag{B-5}$$

On the other hand, the requirement $e_6^t \cdot e_2 = 0$ (or $e_6^t \cdot e_3) = 0$ implies that

$$\beta_1 \, z \cos\phi \sin\theta + \beta_2(z \sin\phi \cos\theta + x\sin\theta) = \alpha_1 \cos\theta - \alpha_3 \sin\phi \sin\theta \tag{B-6}$$

By assumption, there exist two linearly independent solutions (β_1, β_2) to the above equation, and this completes the construction of E_p.

implies that

$$\begin{cases} -\alpha_1 \cos\theta \sin\phi + \alpha_2 \cos\theta \cos\phi + \alpha_3 \sin\theta = 0 \\ \\ \alpha_1 x + \alpha_2 y + \alpha_3 z = 0 \end{cases} \tag{B-4}$$

which immediately yields

$$(\alpha'_1 \ \ \alpha'_2 \ \ \alpha'_3) = (\alpha_1 \ \ \alpha_2 \ \ \alpha_3) = (x \ \ y \ \ z) \times (-\cos\theta \sin\phi \ \ \cos\theta \cos\phi \ \ \sin\theta) \tag{B-5}$$

On the other hand, the requirement $e_6^t \cdot e_2 = 0$ (or $e_6^t \cdot e_3) = 0$ implies that

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By assumption, there exist two linearly independent solutions (β_1, β_2) to the above equation, and this completes the construction of E_p.