COORDINATED CONTROL OF MULTI-MANIPULATOR SYSTEMS

by

Ping Hsu

Memorandum No. UCB/ERL M88/51

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COVER PAGE

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Coordinated Control of Multi-manipulator Systems †

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ABSTRACT

A multi-manipulator system provides greater lifting and manipulation capability and higher flexibility in automated manufacturing. The main problem in controlling such a system is to coordinate all manipulators such that they work in a cooperative way. In this paper, we propose a coordinated control law for a multi-manipulator system performing parts-matching tasks. This control law enables the manipulators to perform the pre-planned parts-matching maneuver while the entire parts-matching system is driven to follow a desired path. When the parts-matching system consists of only a single object, the control law degenerates to an expression that will drive a group of manipulators transporting a single object. The proposed control law also includes internal force control and a load distribution mechanism. The load sharing scheme minimizes the weighted norm of the force applied to the object. In this way, a heavily weighted direction tends to get less load distribution. This scheme does not require a force sensor. We also discuss issues of choosing the weighting factor and show that the proposed control law can be implemented in a decentralized fashion.

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1. Introduction

A multi-manipulator system provides greater lifting and manipulation capability and higher flexibility in automated manufacturing. The main problem in controlling such a system is to coordinate all manipulators such that they work in a cooperative way. More specifically, the control problem includes system formulation, internal force control, and load distribution. These problems have been studied by many researchers [1-12]. The following is a brief review of recent literature.

In the master-slave control scheme proposed by Alford and Belyen [1], one of the manipulators is assigned to be the 'master' arm and the other, the 'slave' arm. The master arm is position controlled to follow a given trajectory while the slave arm is servoed to follow the master arm with a fixed relative position/orientation to accommodate the object. This scheme was generalized to a multi-manipulator system by Arimoto [2]. Nakamura et al. [5] suggested the position control of each manipulator to follow a trajectory that precisely matches the trajectory of the object and at the same time, manipulators are force controlled to exert forces on the object so that the desired body trajectory is achieved. Hayati [3] derived the dynamic equation of a multi-manipulator system using the position/orientation of the object as the generalized coordinate. This approach yielded a 6-dimensional, 6-variable differential equation describing the system dynamics. Based on this equation, Hayati extended Mason's force control scheme [14] to the multi-manipulator case. Tarn et al. [7] linearized each manipulator by a non-linear feedback. A master-slave scheme was then developed based on this linearized model. Zheng and Luh derived an force expression [9] that drives two manipulators to follow two trajectories satisfying a set of motion constraint equations imposed by an object. This scheme also has a master-slave structure. Zheng and Luh [10] proposed a load distribution scheme for a two-manipulator system. This system was set up to minimize either the energy consumption or the exerted force on the object.

Pittelkau suggested in [6] the use of a load sharing force controller to distribute control forces between two linearized manipulators. An adaptive algorithm was included in his scheme to search for the optimum load sharing ratio.

In this paper, we study the control strategy of a multi-manipulator system performing parts matching tasks. When the parts-matching system consisting only a single object, the system degenerates to a group of manipulators transporting an object. We combine the Lagrange equation of the parts-matching system and that of a group of manipulators to form a complete multi-manipulator system equation. Based on this equation, a load distribution scheme and internal force control are studied. The system equation is then transformed to the generalized coordinate space of the parts-matching system by a projection map which removes all internal forces from the equation. A computed-torque-like controller is then added to this reduced order equation to ensure trajectory tracking. The proposed load distribution scheme minimizes the weighted norm of the force applied to the object. In this way, a heavily weighted direction tends to get a smaller share of the load. This scheme does not require a force sensor. The following four considerations are discussed in the choice of load sharing weighting factors: (1) Ratio of maximum output torque of each joint (2) Configuration of each manipulator (3) Contact positions on the object and (4) Type of contact between the manipulator and the object.

When manipulators are not rigidly attached to the object or the object consists of loosely fitted parts, it is desirable to squeeze the object slightly when manipulating it. For example, in the case of a multi-fingered hand, a large 'squeezing' force is essential to guarantee that the contact forces lie in a friction cone [15] so as to prevent slippage of the object relative to the fingers. This squeezing force can be generated by applying a large internal force to the object and this internal force control term is included in the proposed control law. We will also show that the proposed control law can be implemented in a decentralized fashion.

The following is an outline of this paper. In section 2, we develop a system dynamic equation. In section 3, we introduce a coordinated control law. A brief summary is given in section 4.

2. System Dynamic Equation Formulation

In the process of matching parts (e.g. screwing a nut onto a bolt), the position constraints on the parts imposed by their geometries are often holonomic. The problem concerned with the engaging of parts due to the uncertainty of the system configuration is not treated in this paper. This problem (known as the peg-in-hole problem) has been a popular research area [16] in recent years. The dynamic equation of a mechanical system with holonomic constraints can be derived by way of Lagrange formulation. This formulation yields an equation of the form:

$$M(q)\ddot{q} + N(q,\dot{q}) = J(q)^T F \tag{1}$$

where $M(q)$ is usually referred to as the inertia matrix, $N(q,\dot{q})$ contains Coriolis, centrifugal, and gravitational forces, F contains all the externally applied forces (except the gravitational force), and $J(q)^T$ is the matrix relating the force F to its equivalent generalized force. In this section, we combine the Lagrange equations of a system consisting a number of matching parts and those of a group of manipulators to form a complete multi-manipulator system equation which will then be used to develop a coordinated control law in section 3.

2.1 Dynamic Equation of Parts-matching Systems

For a system consisting a number of matching parts, we may chose the position of one of the parts together with the relative position between this part and all other parts as the generalized coordinates. For example, we may chose

$$x \triangleq [\,a, b, c, \phi, \psi, \theta, \alpha, \beta, \gamma\,]^T \tag{2}$$

as the generalized coordinate of the system shown in Figure 1 where $[a, b, c]^T$ is the position of the bolt, $[\phi, \psi, \theta]^T$ is a local parametrization of the orientation of the bolt, γ is the arc-length of the bolt thread between the nut and the bolt head, and α, β are the relative position between the nut and the bolt.

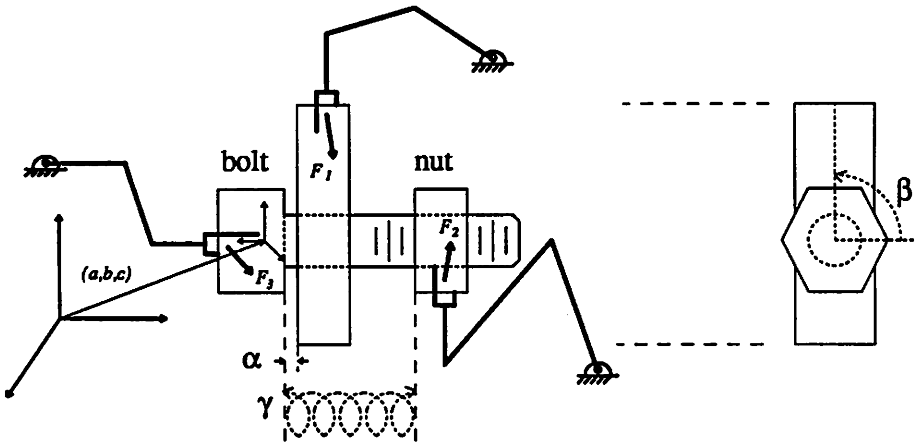

Figure 1

The dynamic equation generated based on the generalized coordinate x is represented by the following expression

$$M_o(x)\ddot{x} + N_o(x,\dot{x}) = J_o(x)^T F_o \tag{3}$$

where $F_o \triangleq [F_1^T, \cdots, F_k^T]^T$ and where F_i is the force that the i th manipulator applies to the parts (as shown in Figure 1).

In general, the matrix $J_o(x)$ is not square (i.e, not one-to-one). Therefore, there is a set of F's that satisfy equation (3) with a given trajectory $x(t)$. It can be verified [see Appendix] that these F's have a general expression of the form:

$$F_o = P^{-1}J_o^T(J_oP^{-1}J_o^T)^{-1}\left\{ M_o(x)\ddot{x} + N_o(x,\dot{x})\right\} + F_I \triangleq J_{oP}^{+}\left\{ M_o(x)\ddot{x} + N_o(x,\dot{x})\right\} + F_I \tag{4}$$

where F_I can be any vector in the null space of J_o and P is a positive definite 'weighting' matrix. The purpose of this weighting matrix will be clear in the next section. Note that the term $J_oP^{-1}J_o^T$ is invertible only if (1)J_o is full rank and (2)P is positive definite. Intuitively, the full rank condition on J_o means that, through the force F, we can directly control all degrees of freedom of the system. In a parts-matching system, this condition implies that each part in the system has at least one manipulator attached to it. We assume that this condition is satisfied through out this paper.

As mentioned earlier, $J_o(x)$ is not square in general. We call an applied force which lies in the null space of $J_o(x)$, an internal force. Note that an internal force produces zero generalized force and, hence does not affect the motion of the system.

2.2 Dynamic Equation of the Manipulators

The most convenient choice of generalized coordinate for a manipulator is its joint positions, i.e.,

$$\theta \triangleq [\theta_1, \theta_2, \cdots, \theta_6]^T. \tag{5}$$

With this set of generalized coordinate, the Lagrange equation of a manipulator takes the form [17] of

$$M_i(\theta_i)\ddot{\theta}_i + N_i(\theta_i,\dot{\theta}_i) = \tau_i + J_i^T(\theta_i)F_i \tag{6}$$

where the subscript i denotes the assigned number of this manipulator, F_i is the interaction force between the i th manipulator and the object, τ is the torque generated by the joint motor, and J_i is the Jacobian matrix of the forward kinematic function. A dynamic equation of a multi-manipulator system is obtained by aggregating all such equations:

$$\begin{bmatrix} M_1 & 0 & . & 0 \\ 0 & M_2 & . & . \\ . & . & . & . \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & . & M_k \end{bmatrix} \begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \\ . \\ \vdots \\ \ddot{\theta}_k \end{bmatrix} + \begin{bmatrix} N_1 \\ N_2 \\ . \\ \vdots \\ N_k \end{bmatrix} = \begin{bmatrix} \tau_1 \\ \tau_2 \\ . \\ \vdots \\ \tau_n \end{bmatrix} + \begin{bmatrix} J_1^T & 0 & . & 0 \\ 0 & J_2^T & . & . \\ . & . & . & . \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & . & J_k^T \end{bmatrix} \begin{bmatrix} F_1 \\ F_2 \\ . \\ \vdots \\ F_k \end{bmatrix} \tag{7}$$

or simply

$$M_m \ddot{\Theta} + N_m = \tau + J_m^T F_m \tag{8}$$

where

$$M_m(\Theta) \triangleq \begin{bmatrix} M_1(\theta_1) & 0 & \cdots & 0 \\ 0 & M_2(\theta_2) & \cdots & . \\ . & . & \cdots & . \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & M_k(\theta_k) \end{bmatrix}, \quad N_m(\Theta,\dot{\Theta}) \triangleq \begin{bmatrix} N_1(\theta_1,\dot{\theta}_1) \\ . \\ \vdots \\ N_k(\theta_k,\dot{\theta}_k) \end{bmatrix} \quad \text{and} \quad \tau \triangleq \begin{bmatrix} \tau_1 \\ . \\ \vdots \\ \tau_k \end{bmatrix}$$

2.3 Complete System Equation

Since manipulators' grippers are rigidly attached to the parts, the velocity of the reference frame fixed at contacts (v_{cont}) can be derived from either the joint velocity ($\dot{\Theta}$) or the generalized velocity ($\dot{x}$). From the the duality of the force space and the velocity space, we know that these three velocities are related by

$$v_{cont} = J_m \dot{\Theta} = J_o \dot{x}. \tag{9}$$

By differentiating this equation, we obtain the following acceleration equation:

$$J_m \ddot{\Theta} + \dot{J}_m \dot{\Theta} = \dot{J}_o \dot{x} + J_o \ddot{x}. \tag{10}$$

In the case that J_m is non-singular, we can express $\ddot{\Theta}$ as

$$\ddot{\Theta} = J_m^{-1}\left[\dot{J}_o \dot{x} + J_o \ddot{x} - \dot{J}_m \dot{\Theta}\right]. \tag{11}$$

Furthermore, from the fact that the reaction force equals the negative of the action force, we have

$$F_m = -F_o \tag{12}$$

where F_m and F_o are defined in (3) and (8). Now, combining (4), (8), (11), and (12) we get

$$M_m(\Theta)J_m^{-1}\left[\dot{J}_o \dot{x} + J_o \ddot{x} - \dot{J}_m \dot{\Theta}\right] + N_m(\Theta,\dot{\Theta}) = \tau - J_m^T J_{oP}^{+}\left\{M_o(x)\ddot{x} + N_o(x,\dot{x})\right\} - J_m^T F_I \tag{13}$$

We abbreviate this equation in the following form:

$$M_h \ddot{x} + N_h = \tau - J_m^T F_I. \tag{14}$$

where M_h and N_h are defined as

$$M_h \triangleq M_m(\Theta)J_m^{-1}J_o + J_m^T J_{oP}^{+} M_o(x) \tag{15}$$

and

$$N_h \triangleq M_m(\Theta)J_m^{-1}\left[\dot{J}_o \dot{x} - \dot{J}_m \dot{\Theta}\right] - J_m^T J_{oP}^{+} N_o(x,\dot{x}) + N_m(\Theta,\dot{\Theta}) \tag{16}$$

3. The Control Algorithm

The main control objective is to specify a set of joint torque inputs τ so that the desired parts-matching maneuver and the desired trajectory of the overall parts-matching system are achieved; or, in short, the desired trajectory $x_d(t)$ in the generalized coordinated space is realized. There are two secondary control objectives: (1) load distribution and (2) internal force control. In this section, we introduce a dynamic control law which guarantees motion tracking of the parts-matching system. This control law also contains two vector variables which can be adjusted (independently from the motion control loop) to optimize the secondary control objectives. We then show that the proposed control law can be implemented in a decentralized fashion.

3.1 Coordinated Control Law

In this section, we propose a coordinated control law for a multi-manipulator system performing parts matching tasks. When the parts-matching system consists of only a single object, the control law degenerates to an expression that drives a group of manipulators transporting an object.

Proposition (Coordinated control law):

Assume that

(1) J_m (as defined in (8)) is non-singular, i.e., the manipulators do not go through any singular configurations,

(2) the part-matching system does not go through a parameterization singularity (i.e., the generalized coordinate remains valid) and

(3) the matrix J_o is non-singular, i.e., each part in the parts-matching system has at least one manipulator attached to it.

Define the position error $e \in R^6$ to be $e = x - x_d$ where x is the generalized coordinate of the parts-matching system and x_d is the desired parts-matching system trajectory. Then, the control law specified by (17) guarantees that (I) x converge to x_d exponentially and (II)the internal force term F_I (as defined in (4)) equals the commanded internal force F_{Ic} in the control law.

$$\tau = M_h \left[\ddot{x}_d - K_v \dot{e} - K_p e \right] + N_h + J_m^T F_{Ic} \tag{17}$$

where M_h and N_h are defined in (15) and (16).

Remark:

The first term is the position loop error compensation. The second term in (17) is used for cancellation of Coriolis, gravitational and centrifugal forces; this term behaves exactly like the nonlinearity cancellation terms in the computed torque control for a single manipulator. F_{Ic} is the internal force feedforward term.

Proof:

First, we substitute (17) into (14) to get

$$M_h \ddot{x} + N_h = M_h \left[\ddot{x}_d - K_v \dot{e} - K_p e \right] + N_h + J_m^T F_{Ic} - J_m^T F_I . \tag{18}$$

This equation can be simplified to

$$M_h \left[\ddot{e}_d + K_v \dot{e} + K_p e \right] = J_m^T (F_{Ic} - F_I). \tag{19}$$

Multiply this equation by $J_o [J_m^T]^{-1}$, we obtain the following equation.

$$J_o [J_m^T]^{-1} M_h \left[\ddot{e} + K_v \dot{e} + K_p e \right] = J_o (F_{Ic} - F_I) = 0 \tag{20}$$

where we have used the fact that both F_I and F_{Ic} lie in the null space of J_o, i.e.,

$$J_o (F_{Ic} - F_I) = 0. \tag{21}$$

Since $J_o [J_m^T]^{-1} M_h = J_o [J_m^T]^{-1} M(\Theta) J_m^{-1} J_o^T + M_o$ is positive definite, (20) implies that

$$\ddot{e} + K_v \dot{e} + K_p e = 0 \tag{22}$$

Thus, we have shown that the tracking error e can be driven to zero with proper choice of the feedback gain matrices K_v and K_p. If we substitute (22) into (19) and notice that J_m is non-singular by assumption, we get the following relation

$$F_{Ic} = F_I \tag{23}$$

which says that the component F_I in the actual interaction force between the object and the manipulator equals to the force term F_{Ic} in the control law. This relation is used later in the Internal Force Control section.

Q.E.D.

Note that the dynamics of the trajectory tracking control loop is independent of the choice of P and is decoupled from the internal force feedforward term F_{Ic}.

3.2 Load Distribution

We will first show that the forces applied to the object by control law (17) is distributed between the manipulators according to the weighting matrix P. We will then discuss several considerations in the choice of P.

Consider the control law (17) without the internal force term, i.e.,

$$\tau = \left[M_m(\Theta) J_m^{-1} J_o + J_m^T J_{oP}^{+} M_o(x) \right] (\ddot{x}_d + K_v \dot{e} + K_p e) + M_m(\Theta) J_m^{-1} \left[\dot{J}_o \dot{x} - \dot{J}_m \dot{\Theta} \right] \quad (24)$$

$$- J_m^T J_{oP}^{+} N_o(x, \dot{x}) + N_m(\Theta, \dot{\Theta})$$

This control law can be decomposed into two terms τ_m and τ_o:

$$\tau_m \triangleq N_m(\Theta, \dot{\Theta}) + M_m(\Theta) J_m^{-1} \left[J_o \left[\ddot{x}_d + K_v \dot{e} + K_p e \right] + \dot{J}_o \dot{x} - \dot{J}_m \dot{\Theta} \right] \quad (25)$$

$$\tau_o \triangleq J_m^T J_{oP}^{+} \left[N_o + M_o \left[\ddot{x}_d - K_v \dot{e} - K_p e \right] \right]. \quad (26)$$

Note that τ_m interacts with the manipulator body and τ_o provides the necessary forces to realize the desired parts-matching system motion. This partition of control force is illustrated in the following figure which gives a graphic representation of the control law (17).

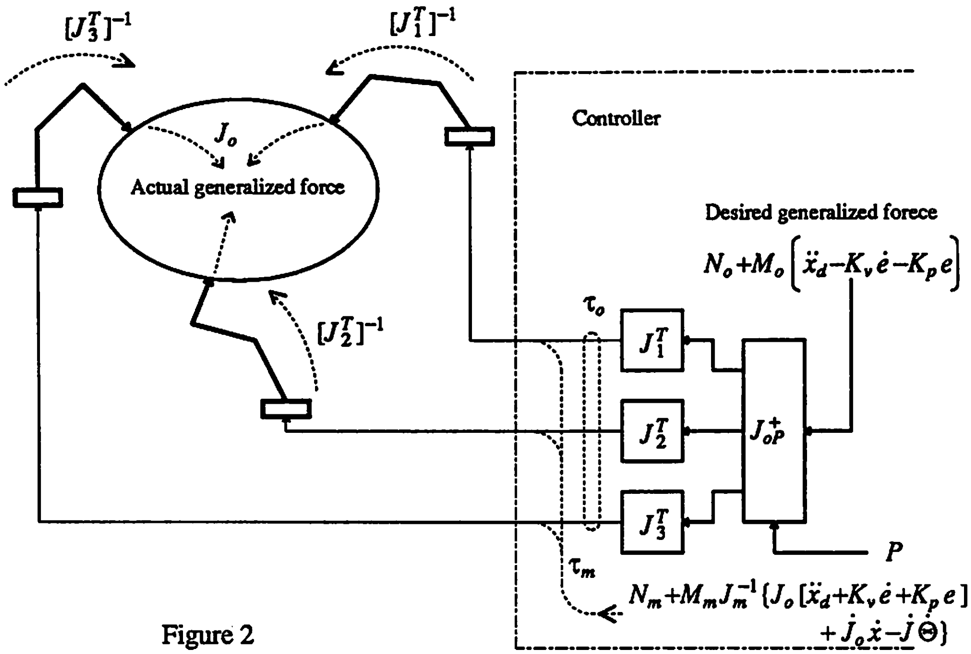

Figure 2

τ_m cancels the manipulator dynamics so that τ_o and the actual generalized force are related by a algebraic relation as will be shown later. This point can be seen from combining equation (11) and (22) and realizing that τ_m, in fact, equals to

$$\tau_m = N_m(\Theta,\dot{\Theta}) + M_m(\Theta)\ddot{\Theta} \tag{27}$$

Similar to the computed torque control law for a manipulator, the desired input generalized force to the parts-matching system is

$$N_o + M_o\left[\ddot{x}_d - K_v\dot{e} - K_pe\right]. \tag{28}$$

From Figure 2, we see that this desired generalized force is distributed to three branches (for three manipulators). To generate the control force component τ_o, we multiply each of the force branches by the matrix J_i^T which undoes the transformation $[J_i^T]^{-1}$ that occurs when the control force propagates through the manipulator body. Note that the transfer function between the desired generalized force (input) and the actual generalized force (output) is simply an identity function, i.e.,

$$J_o[J_m^T]^{-1}J_m^TJ_{oP}^+ = J_oJ_{oP}^+ = I \tag{29}$$

and the effect of the weighting matrix P is completely transparent to this input-output pair. This is the reason that the P was not used in the proof of tracking property of the control law. Now let us concentrate on the force distribution operator J_{oP}^{+}. It is easy to verify [see Appendix] that J_{oP}^{+} (as defined in (4)) distributes a given desired generalized force (f_o) to each force/torque direction at each contact (between a manipulator and the parts-matching system) according to the weighting matrix P. More precisely,

$$F^* \triangleq J_{oP}^{+} f_o \tag{30}$$

minimizes the cost function $C \triangleq F^T P F$ over all solutions, F's, that satisfy the equation $f_d = J_o F$. Consider the case that $P \triangleq \text{diag}\{p_1, \cdots, p_s\}$ where s is the dimension of the force vector F. In this case, the cost function is

$$C = p_1 f_1^2 + \cdots + p_s f_s^2. \tag{31}$$

Suppose p_1 is chosen to be much greater than all other entries, since F^* minimizes C, it must contain a relatively small f_1^*. Note that only the relative magnitude of each diagonal entry of P affects the load distribution and the smaller the value of the weighting the greater the load share. The off-diagonal terms of P do not have any known physical meaning and, are therefore set to zero.

We consider the following four questions when determining the load distribution: (i) Which manipulator is capable of exerting greater force? (ii) Which manipulator is in a better posture (manipulator configuration) to exert the required force? (iii) Which contact position is structurely more suitable to apply the required force? and (iv) Which direction of a contact can stand higher stress without breaking the contact?

The following formulation deals with the first two considerations. The magnitude of the maximum force that the ith manipulator can exert on an object along the x direction in frame C_i is

$$\alpha_x^i = \max_{\beta} \left\{ \beta \, | (J_i^T \hat{x})_j | < \tau_{ij_{max}} - (\tau_{im})_j , \text{ for all } j \right\} \tag{32}$$

where $\hat{x}$ is the unit vector in the x direction in C_i. $(J_i^T \hat{x})_j$ is the jth element of the vector $J_i^T \hat{x}$ and $\tau_{ij_{max}}$ is the torque limit of the actuator at jth joint of the ith manipulator. τ_{im} is the ith segment of the composite torque vector τ_m defined in (26), i.e., the part of the joint torque that interacts with the manipulator itself. Loosely speaking, $\tau_{ij_{max}} - (\tau_{im})_j$ is the 'left over' joint torque.

In a two-manipulator case, the weighting matrix is constructed as follows:

$$P = \mathrm{diag}\left\{ \frac{1}{\alpha_x^1}, \frac{1}{\alpha_y^1}, \frac{1}{\alpha_z^1}, \frac{1}{\alpha_{\bar{x}}^1}, \frac{1}{\alpha_{\bar{y}}^1}, \frac{1}{\alpha_{\bar{z}}^1}, \frac{1}{\alpha_x^2}, \frac{1}{\alpha_y^2}, \frac{1}{\alpha_z^2}, \frac{1}{\alpha_{\bar{x}}^2}, \frac{1}{\alpha_{\bar{y}}^2}, \frac{1}{\alpha_{\bar{z}}^2} \right\} \tag{33}$$

where $\bar{x}$ is the moment about the direction of x.

Remarks:

(1) α in (33) is a function of both joint position and velocity since τ_{im} contains velocity terms. In practice, if the joint velocity is small, we can ignore the 'velocity square' terms, i.e., centrifugal and Coriolis force. With this simplification, τ_{im} contains only the torque that is required to counter balance the manipulator's own weight and, hence, α is a function of joint positions.

(2) Since the percentage of load sharing is inversely related to the relative magnitude of diagonal elements of P, an alternative way of constructing P is to take the complement of $\alpha's$ with respect to a fixed large number, e.g., $P_{11} = (q - \alpha_x^1)$ where q is a number greater then all α's.

The questions (iii) and (iv) concern the structure of the object, positions of contacts and the type of contacts. These considerations, though intuitively clear, are difficult to formulate. A case-by-case study is probably the best approach. Let us consider the following examples:

Example (1)

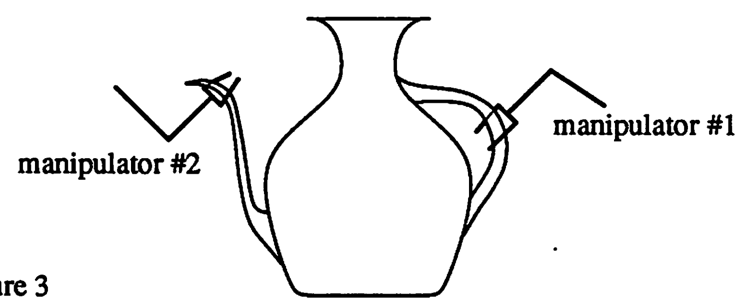

Figure 3

Figure (3) shows two manipulators holding a pitcher. Clearly, we should assign greater share of the load to manipulator #1 since the pitcher handle is structured to carry the pitcher body. Manipulator #2, although it can only apply very limited forces to the pitcher for obvious reasons, greatly helps with the control of the orientation of the pitcher. One other example of structure dependent load sharing is that, in the case the object consists of two parts rigidly connected by a joint. In order not to put too much stress on the joint, the manipulator attached to the heavier part should carry higher load. In fact, the stress on the joint will be minimized if the load sharing ratio between the two manipulators equals the ratio of the masses of two parts.

Example (2)

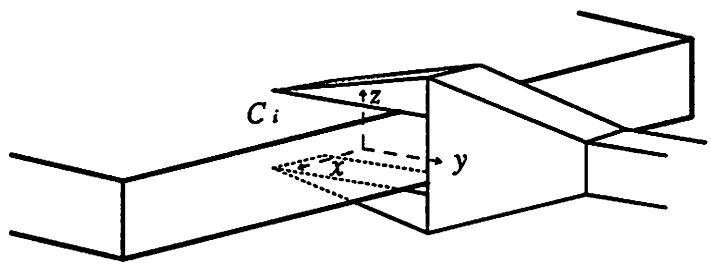

Figure 4

In this example, it is easy to see that the manipulator can apply a higher force in the z direction than in the x and y directions because the interaction force in these two directions are mainly due to friction. Also the manipulator can apply a higher moment about the x direction than the y direction because of the shape of the contact area. The total maximum applicable force of a contact depends on the contact area. With a larger contact area, the manipulator can apply a higher force to the object while keeping the force/area ratio low. It is certainly possible to derive a precise ratio of maximum applicable force in all directions by a careful analysis but, in practice a rough estimate should be sufficient.

Thus the final P is determined based on the weighting factors from all four of the aforementioned considerations. If all weighting factors are based on the same unit (e.g. maximum applicable force), they can be combined in a precise way such as a linear combination or product of all the factors. However, as mentioned earlier when considering the object structure and contact type, a rough estimate of weighting factor is usually sufficient. In this case, integrating all the weighting factors may be based on experience and personal judgement.

Remark:

The load distribution mechanism described in this section can be easily modified to distribute control forces in the joint force/torque space. Instead of using J_o^+ as the force distributor, we use the operator defined by

$$J_{moP}^{+} \triangleq P^{-1}\left[J_o[J_m^T]^{-1}\right]^T\left\{J_o[J_m^T]^{-1}P^{-1}\left[J_o[J_m^T]^{-1}\right]^T\right\}^{-1}. \tag{34}$$

With this new force distributor, the modified τ_o becomes

$$\tau_o \triangleq J_{moP}^{+}\left[N_o + M_o\left[\ddot{x}_d - K_v\dot{e} - K_p e\right]\right]. \tag{35}$$

3.3 Internal Force Control

When manipulators are not rigidly attached to the object or the object consists of loosely fitted parts, it is desirable to squeeze the object slightly when manipulating it. The objective of internal force control is to produce such a 'squeezing' force. For example, in the case of a multi-fingered hand, an sufficiently large 'squeezing' force is essential to guarantee that the contact forces lie in the friction cone [15] so as to prevent slippage of the object relative to the fingers. In the case that the object consists of loosely fitted parts, we can eliminate relative motions by pressing, pulling, or twisting them against each other, and thus prevent undesirable collisions between parts and excessive stresses on the joints. In the example given in Figure (5), we might want to push the plate against the bolt head while manipulator A screws on the nut. This squeezing force eliminates the relative motion between the bolt and the plate and therefore, these two objects should be considered as a rigid body when formulating the control law.

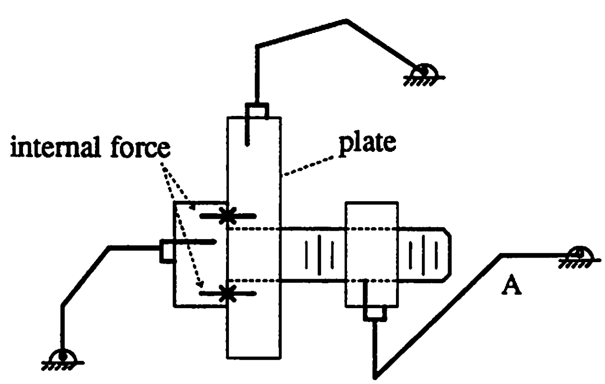

Figure 5

In general, a multi-robot system cannot achieve both desired load distribution and desired internal force if the motion tracking task is to be accomplished first. This point can be easily seen when the desired object velocity is zero and a large squeezing force is desired. In this case, in order to maintain a static equilibrium, two manipulators must exert equal and opposite forces on the object (i.e., an even distribution of control force between two manipulators) regardless of the desired load distribution. This is the reason that the internal force term was ignored when discussing the load distribution issue in section 3.2.

In this section we first show a method of determining the required contact force to produce a desired squeezing force under equilibrium conditions. We then discuss how motion and gravity affects the actual squeezing force.

In the following figure, we show a two-manipulator system holding an object consisting of two loosely attached parts.

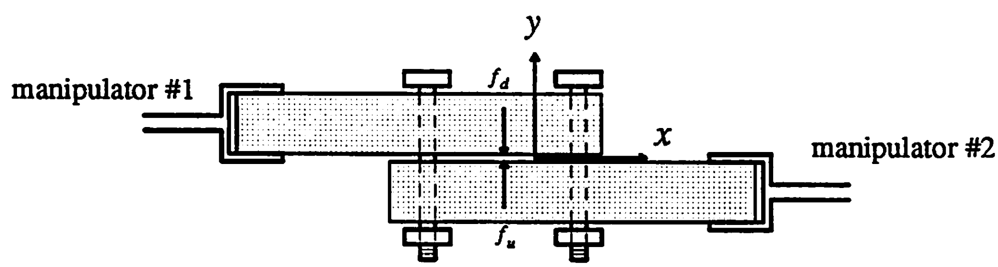

Figure 6

In this example, the desired squeezing force f_I consists of two opposing forces (as shown in the figure), i.e.,

$$f_I = \left\{ f_u , f_d \right\} \quad \text{where} \quad f_u + f_d = 0. \tag{36}$$

It is clear that f_d should be exerted by manipulator 1, so the applied force is given by

$$f_1 = W_1^{-1} f_d \tag{37}$$

and similarly, manipulator #2 should apply the following force

$$f_2 = W_2^{-1} f_u \tag{38}$$

where W_1 and W_2 are the matrices relating the applied forces to their equivalent forces at the junction. It is clear that the force $[f_1^T, f_2^T]^T$ is an internal force of the entire system. Now, we set the commanded internal force F_{Ic} in the control law (17) to be

$$F_{Ic} = [f_1^T, f_2^T]^T \tag{39}$$

As we just proved, the control law guarantees that the internal force term F_I (in (4)) equals $[f_1^T, f_2^T]^T$. But does this mean that the desired squeezing force is produced? The answer is no, unless F_{Ic} is the only force acting on the object. This point can be seen from the following example.

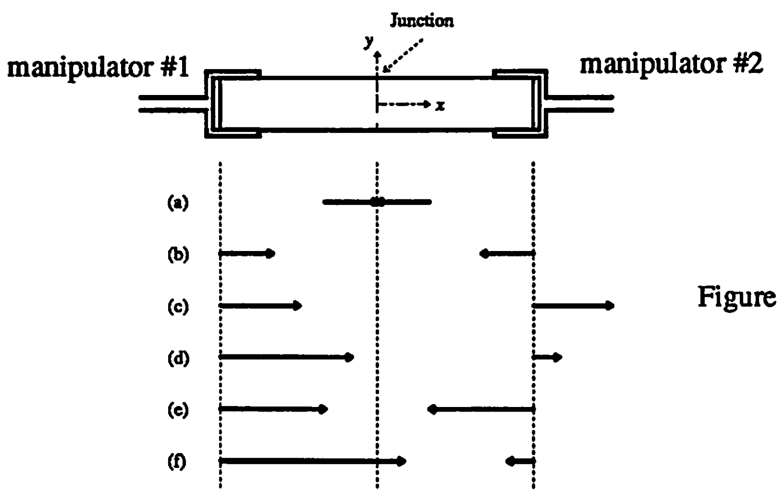

Figure 7.

In the above figure, the desired squeezing force (marked (a)) is a compression force in the x direction. This force can be achieved by applying the internal force (b) when the object is at rest. When the object is accelerated, the total contact force is the sum of the internal force (b) and the force (c) that is required to accelerate the object. As shown in (d), the total applied force does not 'squeeze' the object. In this case, the actual force acting on the junction depends also on the masses of both parts. In the case that the part on the right is much lighter than the other, the junction tends to be pulled apart. We have the following three solutions to this problem:

(1) If the actual force at the junction can be measured, we can adjust the magnitude of the applied internal force as needed.

(2) If the inertia properties of both objects are known, we can derive the actual force acting on the junction and then adjust the internal force term accordingly.

(3) We can simply apply a large enough internal force that will overwhelm the maximum inertia force that can occur and thus a squeezing force at the junction is guaranteed. For example, in figure (7), if the internal force is increased from (b) to (e), then the total applied force (f) will produce a squeezing force at the junction.

Approaches (1) and (2) are not very practical since approach (1) requires the measurement of the force at the junction, and approach (2) requires a complete knowledge of the inertia property of each of the parts plus extensive computation. The third alternative is a feedforward open loop approach. Since the magnitude of the desired squeezing force is usually not critical, this approach appears to be the most practical one provided the object is not fragile.

3.4 Decentralized Control Architecture

Control law (17) may look simple in the symbolic form, the actual computation involved is, in fact very complex. In order to minimize the effect of sampling, when implementing such a continuous control law, we must keep the execution cycle time to a minimum. A solution to this problem is to divide the program into several modules so that each of them is executed by a less powerful processor in parallel. This partitioning of the task not only speeds up the computation time but also simplifies the program structure. But for this parallel computation scheme to work effectively, the control law itself must have a 'nice' structure. As we will show later in this section, control law (17) has a natural structure as shown in figure 8. It can be divided into a coordinator level and manipulator module level. At the manipulator module level, each module is closely associated with a manipulator. Each manipulator module can be further divided into submodules [19]. In this section, we will concentrate only on the partition of tasks between the coordinator level and manipulator module level.

The basic idea of dividing a large computation task is to minimize communication between modules and to evenly distribute the computation load to each module. With this guideline in mind, we first divide the control law τ into two parts τ_m and $J^T F_s$ (i.e., $\tau = \tau_m + J_m^T F_s$) where τ_m is defined as follows (same as the definition given in (26))

$$\tau_m = N_m(\Theta,\dot{\Theta}) + M_m(\Theta) J_m^{-1}\left[J_o\left[\ddot{x}_d - K_v\dot{e} - K_p e\right] + \dot{J}_o\dot{x} - \dot{J}_m\dot{\Theta}\right] \tag{40}$$

and F_s is defined as

$$F_s \triangleq J_{oP}^{+}\left[N_o + M_o\left[\ddot{x}_d - K_v\dot{e} - K_p e\right]\right] + F_{Ic}. \tag{41}$$

Note that, except for the first two terms in the bracket, each segment of τ_m is only a function of the state variables of the corresponding manipulator. For example, the ith segment of τ_m (i.e., the torque vector of the i th manipulator) has the following form.

$$\tau_i = N_i(\theta_i,\dot{\theta}_i) + M_i(\theta_i)J_i^{-1}\left[\psi_i - \dot{J}_i\dot{\theta}_i\right] \tag{42}$$

where ψ_i is the i th segment of

$$J_o\left[\ddot{x}_d + K_v\dot{e} + K_p e\right] + \dot{J}_o\dot{x} \tag{43}$$

On the other hand, F_s contains only the variables associated with the parts-matching system. Now, it is clear that the coordinator module should evaluate F_s and each manipulator module should carry out the following computation

$$\tau_i = N_i(\theta_i,\dot{\theta}_i) + M_i(\theta_i)J_i^{-1}\left[\psi_i - \dot{J}\dot{\theta}_i\right] + J_i^T F_{s_i} \tag{44}$$

where F_{s_i} is the i th segment of F_s, and ψ_i and F_{s_i} are supplied by the coordinator. The following diagram shows the decentralization of the control law and the signal flow between modules.

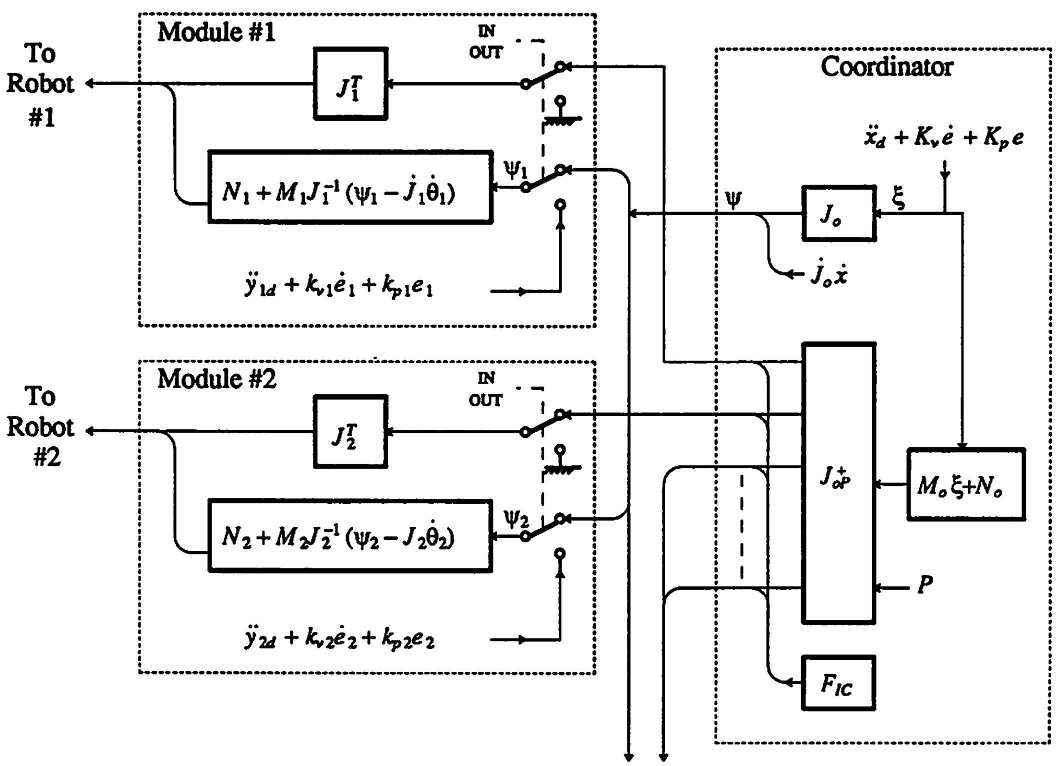

Figure 8 To other manipulator modules

Note that manipulator modules do not require direct access of the state of the parts-matching system and the coordinator module does not require access to the state of the manipulators except for the load distribution purpose for which timing is not critical.

In a parts-matching process cycle, manipulators do not always work in the cooperative mode. When a manipulator is not interacting with other manipulators, it should be controlled as a individual system. This change of control mode of a manipulator module can be easily carried out by doing the following substitutions:

$$F_{s_i} \Longleftrightarrow 0, \quad \text{and} \quad \psi_i \Longleftrightarrow \ddot{y}_d + k_{v_i}\dot{e}_{y_i} + k_{p_i}e_{y_i} \tag{45}$$

where y is the work space coordinate of the end effector of the i th manipulator. Note that after these substitutions each manipulator module has the form of

$$\tau_i = N_i(\theta_i,\theta_i) + M_i(\theta_i)J_i^{-1}\left\{\ddot{y}_d + k_{v_i}\dot{e}_{y_i} + k_{p_i}e_{y_i} - \dot{J}\theta_i\right\} \tag{46}$$

which is precisely the 'resolved acceleration control law' [18]. Any combination of manipulators can be switched in or out of the coordinated control mode without changing the basic structure of the control algorithm. This simple transition from coordinated control to individual control makes this controller structure a *decentralized control architecture* rather than a parallel processing architecture.

4. Summary

We have proposed a cooperative control law for a multi-manipulator system. The feedback loop in this controller is closed around the generalized coordinate of the part-matching system. Manipulators are essentially treated as six degree of freedom actuators with some non-linear dynamics which exert a set of contact forces on the object so that trajectory tracking is achieved and desired internal force is realized. We have shown that this control law can be implemented in a decentralized fashion. The proposed control law contains a weighting matrix P which is used to determine load sharing among the manipulators. We then discussed several issues on choosing the weighting factors in P. We have also shown a way of determining the required contact forces for generating the desired squeezing forces.

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Appendix

Claim:

Assume J_o is full rank. For a given f_o, any F that satisfies the equation

$$f_o = J_o F \tag{A1}$$

can be expressed in the following form

$$F = P^{-1}J_o^T(J_o P^{-1} J_o^T)^{-1} f_o \triangleq J_{oP}^{+} f_o + F_I \tag{A2}$$

where F_I belongs to the null space of J_o.

Proof:

First, we must show that $J_o P^{-1} J_o^T$ is invertible. In case $P=I$, $J_o J_o^T$ is clearly full rank because the range of J_o^T (denoted by $R(J_o^T)$) 'misses' the null space of J_o (denoted by $N(J_o)$). In fact, these two subspaces form an orthogonal decomposition of the domain of J_o. To prove $J_o P^{-1} J_o^T$ is also full rank, we must show that $R(P^{-1}J_o^T)$ does not intersect $N(J_o)$ except at 0. Suppose $u \in R(P^{-1}J_o^T) \cap N(J_o)$ and $u \neq 0$. This implies that u and Pu are orthogonal (i.e., $u^T P u = 0$) since $u \in N(J_o)$ and $Pu \in R(J_o^T)$. This contradicts the fact that P is positive definite.

It is easy to verify that the expression of F given by (A2) satisfies (A1). We simply substitute (A2) into (A1) to obtain

$$J_o F = J_o J_{oP}^{+} f_o + J_o F_I = J_{oP}^{-1} J_o^T (J_o P^{-1} J_o^T)^{-1} f_o = f_o \tag{18}$$

where we use the fact that F_I belongs to the null space of J_o, i.e., $J_o F_I$=0.

Since these two sets are hyper-planes in a linear space and we have shown that one contains the other, to prove they are in fact the same plane we just need to verify that they have the same dimension. It is clear that the dimension of the plane defined by (A2) has the same dimension as N(J_o) and the dimension of the solution set of equation (A2) is also the dimension of N(J_o). This completes the proof.

Q.E.D.

Claim (2):

For a given f_o, the solution of the equation $f_o = J_o F$ that minimizes the cost function $C \triangleq \frac{1}{2} F^T P F$ is

$$F^* = P^{-1} J_o^T (J_o P^{-1} J_o^T)^{-1} f_o, = J_{oP}^+ f_o.$$

Proof:

The Lagrangian of this optimization problem is

$$L(F,\lambda) = \frac{1}{2} F^T P F - \lambda (J_o F - f_o)$$

where λ is the vector of Lagrangian multiplier. The minimizing F^* must satisfies the following two equations:

$$\frac{\partial L}{\partial F} = PF - J_o^T \lambda = 0$$

and

$$\frac{\partial L}{\partial \lambda} = J_o F - f_o = 0.$$

Solving these two equations simultaneously, we get

$$F^* = P^{-1} J_o^T (J_o P^{-1} J_o^T)^{-1} f_o = J_{oP}^+ f_o.$$

Q.E.D.