DYNAMICS OF A COLLISIONAL, CAPACITIVE R.F. SHEATH

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M. A. LIEBERMAN

Department of Electrical Engineering and Computer Sciences

and the Electronics Research Laboratory

University of California, Berkeley, CA 94720

ABSTRACT

A self-consistent solution for the dynamics of a high voltage, capacitive r.f. sheath driven by a sinusoidal current source is obtained, under the assumptions of time-independent, collisional ion motion and inertialess electrons. Results are: (1) the ion current density is $1.68\,\varepsilon_0(2e/M)^{1/2}\overline{V}^{3/2}\lambda_i^{1/2}/s_m^{5/2}$, where $\overline{V}$ is the d.c. self-bias voltage, s_m is the sheath thickness, e/M is the ion charge-to-mass ratio, and ε_0 is the free space permittivity; (2) the sheath capacitance per unit area for the fundamental voltage harmonic is $1.52\,\varepsilon_0/s_m$; (3) the ratio of the d.c. to the peak value of the oscillating voltage is 0.40; (4) the second and third voltage harmonics are respectively 19.3% and 5.3% of the fundamental; and (5) the conductance per unit area for stochastic heating by the oscillating sheath is $2.17\,(e^2n_0/mu_e)\,\lambda_D^{2/3}/(\lambda_i s_m)^{1/3}$, where n_0 is the ion density and λ_D is the Debye length at the plasma-sheath edge, and $u_e = (8eT_e/\pi m)^{1/2}$ is the mean electron speed.

I. INTRODUCTION

Low pressure capacitive, radio frequency (r.f.) discharges are widely used for materials processing in the electronics industry. Typical discharge parameters are pressure $p \approx$ 10-300 mTorr, r.f. frequency $\omega/2\pi \approx$ 13.56 MHz, and r.f. voltage $V_{rf} \approx$ 50-500 volts. Almost all the applied voltage is dropped across capacitive r.f. sheaths at the discharge electrodes. In order to develop adequate models for these discharges, it is important to determine the dynamics and current-voltage characteristics of the sheaths. The sheath dynamics are strongly nonlinear. Godyak and collaborators have developed a homogeneous model of a collisionless sheath.[1,2] Other authors have used a Child-Langmuir law for the ions within the sheath to model the sheath dynamics.[3-5] An approximate model of the effect of the time-average electron density on the ion dynamics within the sheath has been developed.[6] The self-consistent voltages for a single sheath and for a symmetrically driven discharge having two sheaths 180° out of phase have been given [7]. While numerical solutions of the self-consistent dynamics can be obtained,[8] they are not particularly illuminating.

In a previous study[9], the solution for a collisionless sheath driven by a sinusoidal r.f. current source was found. The ion response to the average electric field $\bar{E}$ was assumed to be collisionless; i.e., the force equation was taken to be $M du_i/dt = e\bar{E}$, where e, M, and u_i are respectively the ion charge, mass and velocity. To estimate the pressure regime where this is valid, we can compare the ion mean free path λ_i to the ion sheath thickness s_m. For example, in an argon discharge, $\lambda_i \approx (300p)^{-1}$ cm. For s_m of order 1 cm, the sheath is collisionless ($\lambda_i \geq s_m$) for $p \leq 3$ mTorr. Therefore, the collisionless theory is not valid for typical materials processing pressures.

In this work we give an analytical, self-consistent solution for a collisional sheath driven by a sinusoidal, r.f. current source. We obtain the time-average electric field and potential within the sheath, the nonlinear oscillation motion of the electron sheath boundary and the nonlinear oscillating sheath voltage. Finally, we determine the effective sheath capacitance and conductance.

The assumptions of the analysis are:

(1) The ion motion is collisional with λ_i a constant within the sheath. The ions respond only to the time-average electric field. The ion sheath-plasma boundary is stationary, and ions enter the

sheath with a Bohm presheath velocity $u_B = (eT_e/M)^{1/2}$, where T_e is the electron temperature (in volts).

(2) The electrons are inertialess and respond to the instantaneous electric field. The electron Debye length λ_D everywhere within the sheath is assumed to be much smaller than s_m. This holds provided $V_{rf} \gg T_e$. Since $\lambda_D \ll s_m$, the electron density falls sharply (within a few Debye lengths) from $n_e \approx n_i$ at the plasma side of the electron sheath boundary to $n_e \approx 0$ at the electrode side. The electron sheath oscillates between a maximum thickness of s_m and a minimum thickness of a few Debye lengths from the electrode surface.

II. BASIC EQUATIONS

The structure of the r.f. sheath is shown in Fig. 1. Ions crossing the ion sheath boundary at $x = 0$ accelerate within the sheath and strike the electrode with 50-500 volt energies. Since the ion flux $n_i u_i$ is conserved and u_i increases as ions transit the sheath, n_i drops. This is sketched as the heavy solid line in Fig. 1. The ion particle and momentum conservation equations are respectively

$$n_i u_i = n_0 u_B \tag{1}$$

$$u_i = \mu_i \bar{E} = \frac{2e\lambda_i}{\pi M u_i} \bar{E} \,, \tag{2}$$

where n_0 is the plasma density at $x = 0$ and the mobility μ_i is itself a function of u_i; $\bar{E}$, n_i and u_i are functions of x. The factor $2/\pi$ in (2) accounts for the time averaging of the ion velocity over the distribution of mean free paths to obtain the mean velocity u_i. The Maxwell equation for the instantaneous electric field $E(x,t)$ within the sheath is

$$\begin{aligned} \frac{\partial E}{\partial x} &= \frac{e}{\varepsilon_0} n_i(x), \quad s(t) < x; \\ &= 0, \quad s(t) > x. \end{aligned} \tag{3}$$

Here, $s(t)$ is the distance from the ion sheath boundary at $x = 0$ to the electron sheath edge; the electron sheath thickness is $s_m - s(t)$. The instantaneous potential $\Phi(x,t)$ is determined from the equation

$$\frac{\partial \Phi}{\partial x} = -E. \tag{4}$$

Time-averaging (3) and (4) over an r.f. cycle, we obtain the equations for the time-average electric field $\overline{E}(x)$ and potential $\overline{\Phi}(x)$:

$$\frac{d\overline{E}}{dx} = \frac{e}{\varepsilon_0}(n_i(x) - \overline{n}_e(x)), \tag{5}$$

$$\frac{d\overline{\Phi}}{dx} = -\overline{E}, \tag{6}$$

where $\overline{n}_e(x)$ is the time-average electron density within the sheath. We can determine $\overline{E}$, $\overline{\Phi}$ and $\overline{n}_e$ from $s(t)$. For example, we note that $n_e(x,t) = 0$ during the part of the r.f. cycle when $s(t) < x$; otherwise, $n_e(x,t) = n_i(x)$. We therefore have

$$\overline{n}_e(x) = \left[1 - \frac{2\phi}{2\pi}\right] n_i(x) \tag{7}$$

where $2\phi(x)$ is the phase interval during which $s(t) < x$. Qualitatively, we sketch $\overline{n}_e$ as the dashed line in Fig. 1. For x near zero, $s(t) < x$ during only a small part of the r.f. cycle; therefore $2\phi \approx 0$ and $\overline{n}_e \approx n_i(x)$. For x near s_m, $s(t) < x$ during most of the r.f. cycle; therefore $2\phi \approx 2\pi$ and $\overline{n}_e \approx 0$.

To determine the time averages quantitatively, we assume that a sinusoidal r.f. current density passes through the sheath:

$$J_{rf}(t) = -\tilde{J}_0 \sin \omega t. \tag{8}$$

Equating this displacement current to the conduction current at the electron sheath boundary, we obtain the equation for the electron sheath motion:

$$-en_i(s)\frac{ds}{dt} = -\tilde{J}_0 \sin \omega t. \tag{9}$$

III. SOLUTION

We integrate (3) to obtain

$$E = \frac{e}{\varepsilon_0} \int_s^x n_i(\zeta) d\zeta \,, \quad s(t) < x,$$
$$= 0 \,, \quad s(t) > x. \tag{10}$$

We integrate (9) to obtain

$$\frac{e}{\varepsilon_0} \int_0^s n_i(\zeta) d\zeta = \frac{\tilde{J}_0}{\varepsilon_0 \omega}(1 - \cos \omega t), \tag{11}$$

where we have chosen the integration constant so that $s(t) = 0$ at $\omega t = 0$. From (10) and (11), we obtain

$$E(x, \omega t) = \frac{e}{\varepsilon_0} \int_0^x n_i(\zeta) d\zeta - \frac{\tilde{J}_0}{\varepsilon_0 \omega}(1 - \cos \omega t), \quad s(t) < x;$$
$$= 0, \quad s(t) > x. \tag{12}$$

We must time average (12) to obtain $\bar{E}$. Figure 2 shows a sketch of $s(t)$ vs ωt. We note that $s(t) = x$ for $\omega t = \pm\phi$, and that $s(t) < x$ for $-\phi < \omega t < \phi$. The time average is then

$$\bar{E} = \frac{1}{2\pi} \int_{-\phi}^{\phi} E(x, \omega t) d(\omega t). \tag{13}$$

Inserting (12) into (13), we find

$$\bar{E}(x) = \frac{e}{\varepsilon_0} \frac{\phi}{\pi} \int_0^x n_i(\zeta) d\zeta + \frac{\tilde{J}_0}{\varepsilon_0 \omega \pi}(\sin \phi - \phi). \tag{14}$$

Inserting (11) with $s = x$, $\omega t = \phi$ into (14) we obtain

$$\bar{E}(x) = \frac{\tilde{J}_0}{\varepsilon_0 \omega \pi}(\sin \phi - \phi \cos \phi). \tag{15}$$

Using (6),

$$\frac{d\bar{\Phi}}{dx} = -\frac{\tilde{J}_0}{\varepsilon_0 \omega \pi}(\sin \phi - \phi \cos \phi). \tag{16}$$

Solving (1) and (2) for n_i, we obtain

$$n_i = n_0 u_B (2e\lambda_i \bar{E}/\pi M)^{-1/2} . \quad (17)$$

Inserting (17) into (9) with $s = x$, $\omega t = \phi$, we obtain

$$\frac{d\phi}{dx} = \frac{u_B}{\tilde{s}_0} \left[\frac{\pi M}{2e\lambda_i}\right]^{1/2} \frac{1}{\bar{E}^{1/2} \sin\phi} , \quad (18)$$

where

$$\tilde{s}_0 = \tilde{J}_0/(e\omega n_0) \quad (19)$$

is an effective oscillation amplitude.

Equations (15) and (18) are the fundamental equations of the self-consistent r.f. sheath. Inserting (15) into (18) and integrating, we obtain

$$x/\tilde{s}_0 = H \int_0^{\phi} (\sin\zeta - \zeta\cos\zeta)^{1/2} \sin\zeta \, d\zeta , \quad (20)$$

where

$$H = \left[\frac{2\lambda_i \tilde{s}_0}{\pi^2 \lambda_D^2}\right]^{1/2} \quad (21)$$

and $\lambda_D = (\varepsilon_0 T_e/en_0)^{1/2}$ is the electron Debye length at $x = 0$. In (20), we have used the boundary condition that $x = 0$ at $\phi = 0$. Setting $x = s(t)$ and $\phi = \omega t$ in (20), we obtain the nonlinear oscillation motion of the electron sheath, which is shown in Fig. 3. Setting $s = s_m$ at $\phi = \pi$ in (20), we obtain the ion sheath thickness

$$s_m = 1.95 H\tilde{s}_0 . \quad (22)$$

Using (21) in (22) and solving for $\tilde{s}_0$, we obtain

$$\tilde{s}_0 = 1.09 \lambda_D^{2/3} s_m^{2/3}/\lambda_i^{1/3} . \quad (23)$$

The time-average potential is found by integrating (16), which yields

$$\bar{\Phi} = -\frac{\tilde{J}_0}{\pi\varepsilon_0\omega} \int_0^{\phi} (\sin\zeta - \zeta\cos\zeta) \frac{dx}{d\zeta} d\zeta . \quad (24)$$

Using (18) and (19) in (24), we obtain

$$\frac{\overline{\Phi}}{T_e} = -\frac{H}{\pi}\frac{\overline{s}_0^2}{\lambda_D^2}\int_0^{\phi}(\sin\zeta - \zeta\cos\zeta)^{3/2}\sin\zeta\, d\zeta\,. \tag{25}$$

The total d.c. voltage across the sheath is related to the d.c. ion current and the ion sheath thickness by:

$$J_i = K\,\varepsilon_0\left[\frac{2e}{M}\right]^{1/2}\frac{\overline{V}^{3/2}}{s_m^2}, \tag{26}$$

where $J_i = en_0u_B$ is the d.c. ion current and $\overline{V} = -\overline{\Phi}(\phi = \pi)$ is the voltage across the sheath. Setting $\phi = \pi$ and $\Phi = -\overline{V}$ in (25) and evaluating the integral, we obtain

$$\frac{\overline{V}}{T_e} = 3.15\frac{H}{\pi}\frac{\overline{s}_0^2}{\lambda_D^2}\,. \tag{27}$$

Using (21) and (23) in (27) and the definitions for λ_D and J_i, we obtain (26) with

$$K_c = 1.68\,(\lambda_i/s_m)^{1/2}\,. \tag{28}$$

In contrast, the self-consistent result is $K_f = 0.82$ for collisionless ion motion in the sheath[9]. We see that the current density scales as the inverse 5/2 power of the sheath thickness, in contrast to the (collisionless) Child law scaling as the inverse square power. We also note that for a fixed voltage and current, ion collisions in the sheath lead to a reduction in the sheath thickness.

IV. SHEATH CAPACITANCE

The instantaneous electric field within the sheath is given by (12). Substituting (11) with $s = x$ and $\omega t = \phi$ into (12), we obtain

$$E(x,t) = \frac{\tilde{J}_0}{\varepsilon_0\omega}(\cos\omega t - \cos\phi), \quad s(t) < x,$$
$$= 0, \quad s(t) > x. \tag{29}$$

Integrating with respect to x, we obtain the instantaneous voltage from the plasma to the electrode across the sheath

$$V(t) = \int_s^{s_m} E(x,t)dx. \tag{30}$$

Changing variables from x to ϕ and using (29), we obtain

$$V(t) = \frac{\tilde{J}_0}{\varepsilon_0 \omega} \int_{\omega t}^{\pi} (\cos \omega t - \cos \phi) \frac{dx}{d\phi} \, d\phi. \tag{31}$$

Using (15) and (18) to evaluate $dx/d\phi$ in (31) we obtain, for $0 < \omega t < \pi$,

$$V(t) = (en_0 \tilde{s}_0^2/\varepsilon_0) H \int_{\omega t}^{\pi} (\cos \omega t - \cos \phi)(\sin \phi - \phi \cos \phi)^{1/2} \sin \phi \, d\phi \, . \tag{32}$$

$V(t)$ is an even, periodic function of ωt with period 2π. For $-\pi < \omega t < 0$, we find that $V(t)$ is given by the right hand side of (32) with ωt replaced by $-\omega t$. A plot of V versus ωt is given in Fig. 4. The peak value of $V(t)$ occurs at $\omega t = 0$:

$$V(0) = 2.50 H (en_0 \tilde{s}_0^2/\varepsilon_0) \, . \tag{33}$$

Expanding $V(t)$ in a Fourier series

$$V(t) = \sum_{k=0}^{\infty} V_k \cos (k \omega t),$$

we obtain

$$\begin{aligned} V_0 = \overline{V} &= 1.00 H (en_0 \tilde{s}_0^2/\varepsilon_0) \, , \\ V_1 &= 1.28 H (en_0 \tilde{s}_0^2/\varepsilon_0) \, , \\ V_2 &= 0.25 H (en_0 \tilde{s}_0^2/\varepsilon_0) \, , \\ V_3 &= -.034 H (en_0 \tilde{s}_0^2/\varepsilon_0) \, . \end{aligned} \tag{34}$$

The second harmonic is 19.3% of the fundamental, and the third harmonic is 5.3% of the fundamental. The ratio of the d.c. value to the peak value of the voltage is $\overline{V}/V(0) = 0.40$. Defining the effective capacitance per unit area from the relation

$$-\tilde{J}_0 \sin \omega t = C_s' \frac{d}{dt}(V_1 \cos \omega t),$$

we obtain

$$C_s' = 1.52 \, \varepsilon_0/s_m \, , \tag{35}$$

where s_m is the ion sheath thickness given by (22). In contrast, the coefficient in (35) is 1.23 for collisionless ion motion in the sheath[9].

For a symmetrically-driven, parallel plate r.f. discharge (equal area plates) there are two r.f. sheaths in series. We let $V_{ap}(t)$ be the voltage on plate a with respect to the plasma and $V_{bp}(t)$ be the voltage on plate b with respect to the plasma. By symmetry, $V_{bp}(\omega t) = V_{ap}(\omega t - \pi)$. The series voltage across both sheaths is $V_{ab} = V_{ap} - V_{bp}$. Using (32), we obtain, for $0 < \omega t < \pi$,

$$V_{ab} = (en_0\bar{s}_0^2/\varepsilon_0)\left[\int_{\omega t}^{\pi}(\cos\omega t - \cos\phi)(\sin\phi - \phi\cos\phi)^{1/2}\sin\phi\, d\phi + \right.$$

$$\left. + \int_{\pi-\omega t}^{\pi}(\cos\omega t + \cos\phi)(\sin\phi - \phi\cos\phi)^{1/2}\sin\phi\, d\phi\right].$$

The peak-to-peak value of V_{ab} is $2V(0)$, with $V(0)$ given in (33). Expanding V_{ab} in a Fourier series, we obtain $V_{ab1} = -2V_1$ and $V_{ab3} = -2V_3$. All even harmonics, including the d.c. value, are zero, as expected for a symmetrically-driven discharge. The third harmonic is 5.3% of the fundamental, and the higher harmonics are much smaller. Thus, to a very good approximation, a sinusoidal sheath current leads to a linear response; i.e., a sinusoidal voltage across the discharge. Defining the effective capacitance per unit area of the series combination of the two sheaths from the relation

$$J_{ab}(t) = C_{sym}' \frac{d}{dt} V_{ab1}(t),$$

we obtain $C_{sym}' = 0.76\ \varepsilon_0/s_m$.

V. SHEATH CONDUCTANCE

The r.f. conductance of the sheath is due to stochastic heating of the electrons by the oscillating sheath. An electron that is reflected from a moving sheath experiences a change of energy. If the sheath moves toward the electron, then the energy increases; if the sheath moves away, then the energy decreases. For an oscillating sheath, some electrons gain energy and others lose energy. However, averaging over an oscillation period, the net effect is an energy gain, corresponding to a *dissipation* in the sheath. This mechanism also has been called "Fermi acceleration" [10–14] or "wave riding".[4–5,15]

If u is the parallel velocity (along z) of an incident electron at the electron sheath edge $s(t)$ and $u_s(t)$ is the sheath velocity, then the reflected electron has a velocity $u_r = -u + 2u_s$. We let $f_s(u,t)$ be the electron velocity distribution at s, normalized so that

$$\int_{-\infty}^{\infty} f_s(u,t)\,du = n_i(s(t)) = n_s(t)\ .$$

The electron flux Γ_s incident on the sheath is

$$\Gamma_s = \int_0^{\infty} u f_s(u,t)\,du\ . \tag{36}$$

To determine the power transferred to the electrons, we note that in a time interval dt and for a speed interval du, the number of electrons per unit area that collide with the sheath is given by $(u - u_s) f_s(u,t)\,du\,dt$. This results in a power transfer dS per unit area

$$dS = \frac{1}{2}\,m(u_r^2 - u^2)(u - u_s) f_s(u,t)\,du. \tag{37}$$

Using $u_r = -u + 2u_s$ and integrating over all incident velocities, we obtain

$$S = -2m \int_{u_s}^{\infty} u_s (u - u_s)^2 f_s(u,t)\,du\ . \tag{38}$$

To determine f_s, we first note that the sheath is oscillating because the electrons in the plasma are oscillating in response to a time-varying electric field. If the velocity distribution function within the plasma in the absence of the electric field is a Maxwellian $g_0(u)$, then the distribution within the plasma is $f_0(u,t) = g_0(u - u_0)$, where $u_0(t)$ is the time-varying oscillation velocity of the plasma electrons. Because $n_s < n_0$, not all electrons having $u > 0$ at $x = 0$ collide with the sheath at s. Many electrons are reflected within the region $0 < x < s$ where the ion density drops from n_0 to n_s. This reflection is produced by a weak electric field whose value is such that $n_e = n_i$ at all times. The transformation of f_0 across this region to obtain f_s is complicated. However, the essential features to determine the stochastic heating are seen if we approximate

$$f_s = \frac{n_s}{n_0}\, g_0(u - u_0), \qquad u > 0\ . \tag{39}$$

Inserting (39) into (38) and transforming to a new variable $u' = u - u_0$, we obtain

$$S(t) = -2m \int_{u_s - u_0}^{\infty} u_s n_s [u'^2 - 2u'(u_s - u_0) + (u_s - u_0)^2] \, g_0(u') \, du' \, . \qquad (40)$$

Assuming that $|u_s - u_0|$ is much less than the characteristic electron thermal velocity, we can take the lower limit of the integral in (40) to be zero. From (9) we note that

$$n_s u_s = \tilde{u}_0 n_0 \sin\phi \, , \qquad (41)$$

and differentiating (20) with respect to $\phi = \omega t$, we obtain

$$u_s = \tilde{u}_0 H (\sin\phi - \phi\cos\phi)^{1/2} \sin\phi \, . \qquad (42)$$

Averaging (40) over $\phi = \omega t$ and noting that (41) and (42) are odd functions of ϕ, the first and third terms in (40) average to zero and we obtain

$$\bar{S} = 4m\Gamma_s n_0^{-1} \langle (u_s - u_0) u_s n_s \rangle_\phi \, . \qquad (43)$$

Noting that $|u_0| \ll |u_s|$ for $H \gg 1$, inserting (41) and (42) into (43) and averaging, we obtain

$$\bar{S} = 0.49 H \, m n_0 u_e \tilde{u}_0^2 \, , \qquad (44)$$

where for a Maxwellian distribution g_0, the incident flux is

$$\Gamma_s = \frac{1}{4} n_0 u_e , \qquad (45)$$

and

$$u_e = \left[\frac{8eT_e}{\pi m} \right]^{1/2} \qquad (46)$$

is the mean electron speed.

The sheath conductance G_s' per unit area is defined through the relation

$$\bar{S} = \frac{1}{2} \frac{\tilde{J}_0^2}{G_s'} , \qquad (47)$$

where $\tilde{J}_0 = en_0\tilde{u}_0$. Equating (44) and (47), we obtain

$$G_s' = \frac{1.02}{H} \left[\frac{e^2 n_0}{m u_e} \right] . \qquad (48)$$

We note using (21) and (23) that

$$H = 0.47 \left[\frac{\lambda_i}{s_m} \right]^{1/3} \left[\frac{s_m}{\lambda_D} \right]^{2/3} . \tag{49}$$

We then obtain

$$G_s{}' = 2.17 \left[\frac{s_m}{\lambda_i} \right]^{1/3} \left[\frac{e^2 n_0}{m u_e} \right] \left[\frac{\lambda_D}{s_m} \right]^{2/3} . \tag{50}$$

This effective surface conductance per unit area represents a powerful electron heating mechanism in a capacitive r.f. discharge. The quantity $2.17\ (s_m/\lambda_i)^{1/3}$ in (50) is replaced by the coefficient 2.98 for collisionless ion motion in the sheath[9].

As an example, we choose $\bar{V} = 400$ V, $p = 47$ mTorr, $T_e = 3$ eV, $J_i = 0.5$ mA/cm^2, $\omega = 2\pi \times 13.56$ MHz, and $M = 40$ amu (i.e., argon). Then we obtain $\lambda_i = .070$ cm, $u_B = 2.7\times 10^5$ cm/s, $n_0 = 1.2\times 10^{10}$ cm^{-3}, $H = 4.1$, $\tilde{J}_0 = 10.8$ mA/cm^2, $\tilde{s}_0 = 6.8\times 10^{-2}$ cm, $\lambda_D = 1.2\times 10^{-2}$ cm, $s_m = 0.54$ cm, $C_s{}' = 0.25$ pF/cm^2, $\tilde{u}_0 = 5.8\times 10^6$ cm/s, $u_e = 1.2\times 10^8$ cm/s, $\bar{S} = 8.3\times 10^{-3}$ W/cm^2, and $G_s{}' = 7.1\times 10^{-3}$ S/cm^2. The d.c. ion power flux incident on the electrode is $S_i = J_i \bar{V} = 0.2$ W/cm^2.

For a homogeneous (uniform ion density) sheath, $n_s = n_0$ and $u_s = u_0$. Then the integral in (40) vanishes and there is no stochastic heating; $G_s{}' \to \infty$. We can understand this physically as follows: In the accelerated frame moving with the plasma, the electron sheath edge at $s(t)$ is stationary; therefore, no energy is transferred to electrons that collide with the sheath. Thus the non-homogeneous nature of the self-consistent ion density within the r.f. sheath is an essential feature of the stochastic heating mechanism.

This work was supported by National Science Foundation Grant ECS-8517363 and Department of Energy Grant DE-FG03-87ER13727. Helpful discussions with A. J. Lichtenberg and S.E. Savas are gratefully acknowledged.

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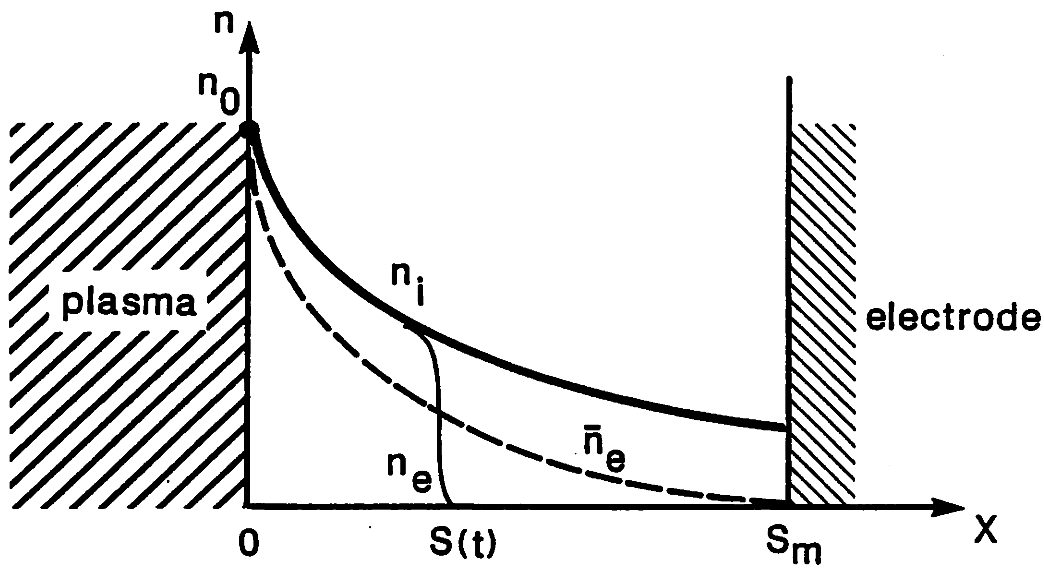

Fig. 1. Structure of high voltage, capacitive r.f. sheath.

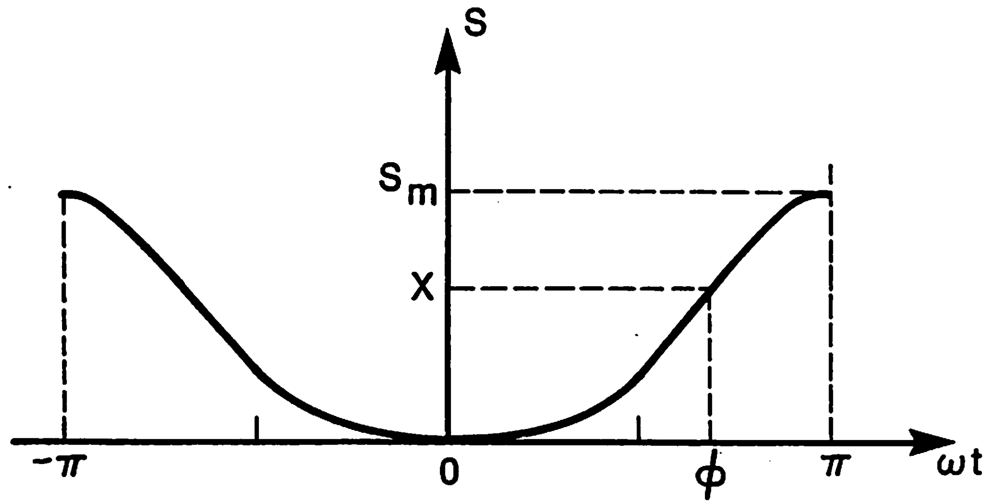

Fig. 2. Sketch of $s(t)$ versus ωt, showing the definition of the phase $\phi(x)$.

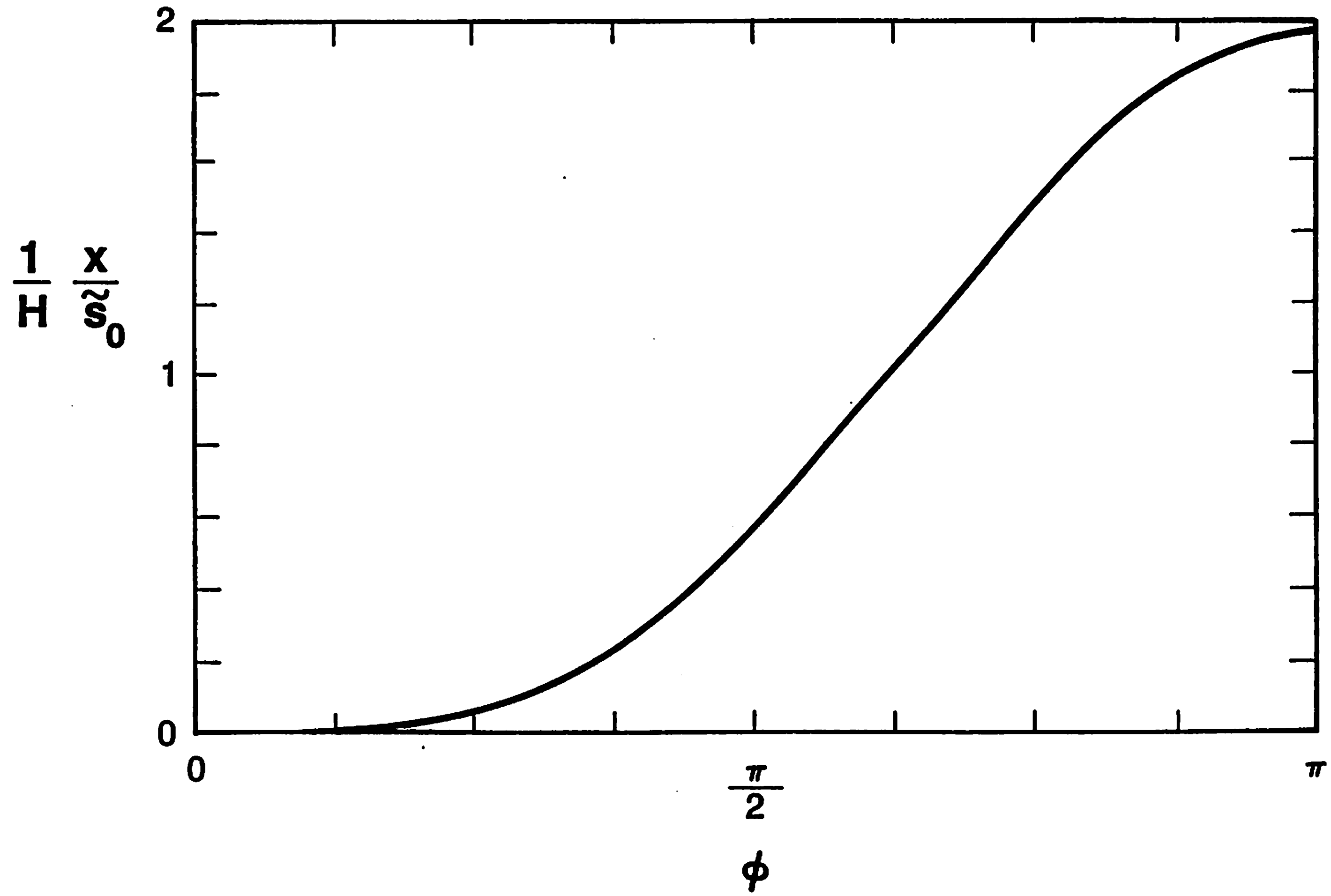

Fig. 3. Normalized position versus phase for the self-consistent r.f. sheath.

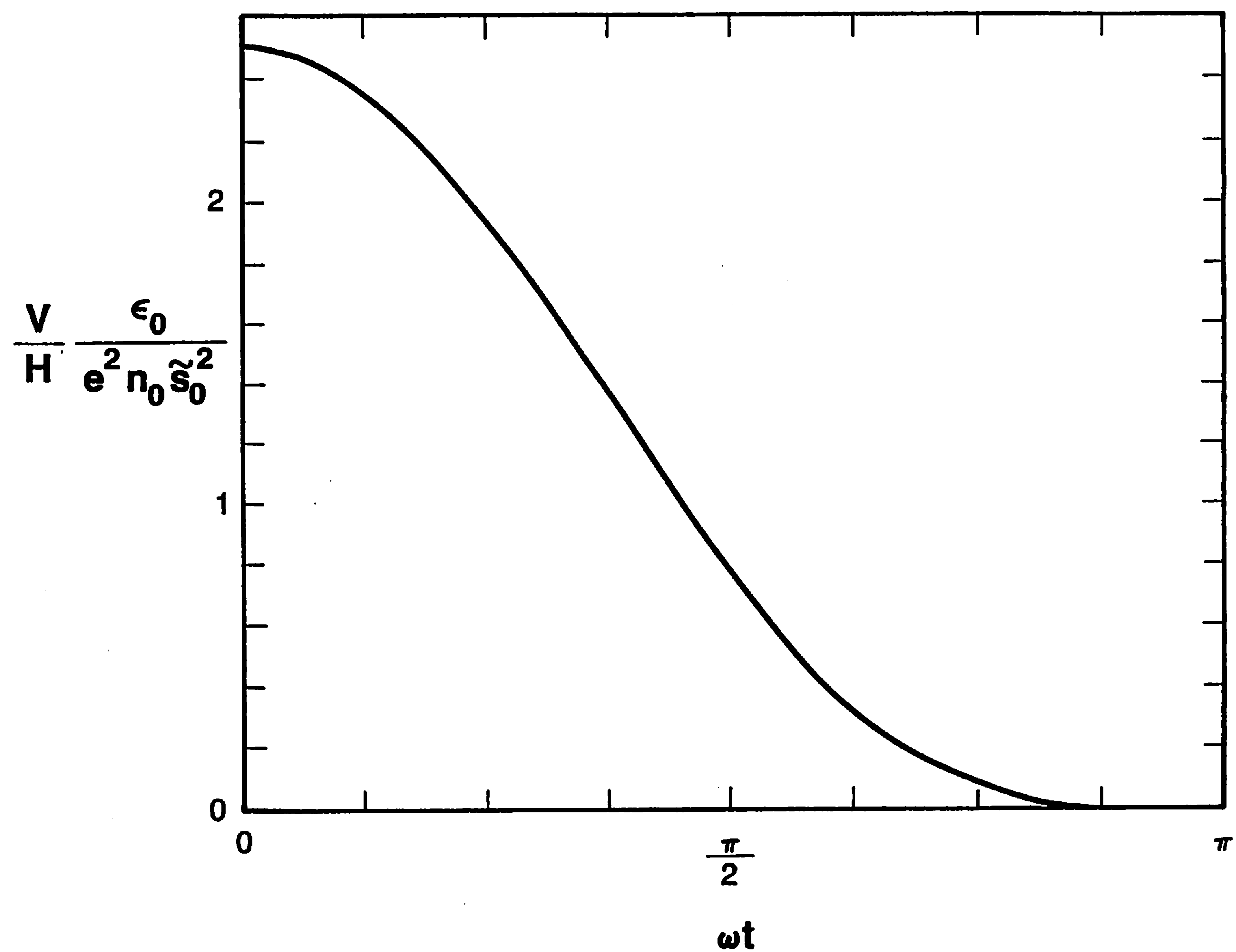

Fig. 4. Normalized time-varying sheath voltage versus ωt.