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Hybrid Control Schemes for Robot Manipulators and Hands under Constrained Motion *

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Abstract

In this paper, we propose a new hybrid control scheme for robot manipulators under constrained motion. This control law provides simultaneous tracking of orthogonal position and force trajectories on a frictionless constraint surface. It is shown how the proposed scheme may be applied to manipulators performing constrained tasks. In the second part of this work, the analysis is extended to consider the case of multifingered hands. We present a control scheme which allows a fixed point on a grasped object, to follow a prespecified desired position/force trajectory on a constraint surface. Examples are given to illustrate the task descriptions.

1 Introduction

As more flexibility is demanded of robot manipulators in terms of the variety of tasks they must perform, the problem of robot control for constrained tasks becomes a more pressing problem requiring solution. For tasks of constrained motion, it is usually the case that neither pure force control nor pure position control is sufficient or effective.

Constraints on robot motion may be imposed by a requirement that the robot end effector maintain physical contact with the environment along

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some surface, as in contour following or scribing. Constraints may also arise from the task description by considering interactions of the manipulator with the environment, as occurs in peg-in-hole insertion or turning a crank. In these cases, the task determines an imaginary constraint surface as discussed in Mason [1]. One may thus directly specify those directions in space which are to be position controlled and those which are to be force controlled. In the literature, the problem of robot control under constraint has been studied by two main approaches. The first is to model the environment as a mechanical impedance so as to produce compliant motion, see Kazerooni et al. [9] and Whitney [10]. The second approach is by the use of hybrid control schemes as discussed in Raibert and Craig [5], Vukobratovic and Vujic [2], McClamroch and Wang [4], Mills and Goldenberg [7].

This paper describes a hybrid control scheme for robot manipulators and grasped objects constrained to follow some constraint surface which is rigid and frictionless. This work is divided into two main parts. Part One, covered in sections two to four, provides the control scheme for a single manipulator, while sections five to seven extend the concepts developed and provides a control scheme for the case of a multifingered hand. The control schemes which we propose achieve simultaneous, independent control of both position and force of the manipulator end effector. The analysis in the first part of this work initially parallels, but then provides a generalization to the work of [12] and [11]. The description of the constraint space for the task is specified by a slightly different approach and the end result is the same. The analysis of [4] requires a trajectory specified in terms of the joint coordinates of the manipulator and the constraint force. The specification of a task in terms of those parameters is not obvious and the simplified structure of the dynamic equation obtained here, is not obtained, thus the analysis given is more involved. The second part of this work dealing with multifingered hands is completely new, and it has not till now been covered in the literature. The content of this paper is outlined as follows.

In Section two, the class of tasks to which our control scheme applies is defined. This is defined by a physical (or imaginary) constraint surface for the manipulator end-effector to follow. It is necessary that we have a perfect model of the constraint surface. The six-dimensional tangent space of the manipulator workspace is partitioned into two orthogonal subspaces, one of which is velocity-controlled and the other is dually force-controlled. Since locally this six-dimensional tangent space looks like $\Re^6$, we can describe each subspace as the range of a linear map with a corresponding set of independent parameters.

In Section three, the dynamic equation for a single manipulator under constrained motion is formulated in terms of the parameters described in Section two. These parameters are used to specify the desired motion of the end-effector on the surface. Contact between the manipulator and constraint surface is maintained by the application of a specified force normal to the constraint surface. A control scheme for a single manipulator is proposed, which provides asymptotic trajectory tracking along the constraint surface together with independent force control normal to the surface. In Section four the decomposition which produces the task specification described in Section two, is applied to a number of different examples in order to define the relevant constraint space matrices for the control scheme. The simplicity of defining our control objectives and applying the control scheme is also illustrated by an example in Section four.

Section five is devoted to applying the previously developed concepts to the situation of simultaneous position and force control at a point on an object grasped by a multifingered hand. The kinematic constraints on the object are examined and described fully, together with the corresponding dual force relations which apply. In Section six the dynamic equations of the total constrained system are determined, and a hybrid control scheme for a grasped tool under constraint is proposed.

2 Description of a Constraint Surface

Of interest to us are two classes of tasks which define constraints on the motion of a robot manipulator. The first class of tasks involve a manipulator constrained to move so that its end-effector remains in contact with a physical constraint surface. The second class are those tasks which can be modelled by the C-surface description of Mason [1]. The term end-effector applies also to an object or tool which is rigidly attached to the gripper of the manipulator. Our analysis covers the second class of tasks as a special case of the first, provided the correct interpretation is made for the relevant terms. In this section we describe a method for obtaining a decomposition and exact description of that subspace which is to be position-controlled versus that which is to be force-controlled by the hybrid scheme to follow.

Consider a single articulated arm of n links in contact with a rigid, frictionless surface, for which we locally have a perfect parametric model. Let the end-effector of the manipulator contact the surface at a point which changes in response to manipulator motion. Denote by C_b a fixed inertial frame of reference. Fix a coordinate frame C_s to the end-effector such that at the point of contact with the constraint surface, the z-axis is in the direction of the inward surface normal, and the x and y axes span the tangent space at the contact point such that x-y-z forms a right-handed coordinate system. We use the coordinate frame C_s to define the constraints on the manipulator end-effector. Constrained motion is generally achieved through point contacts between the end-effector and surface, and this imposes the requirement that the contact at the end-effector and that of the contact point on the surface are identical in space. For point contacts the orientation variables may be free if the manipulator has sufficient degrees of freedom, so these must be included as part of the control variables.

In general, the orientation of a coordinate frame is specified in terms of three local parameters of $SO(3)$, such as Euler angles (or roll, pitch, yaw), denote this by $\gamma \in \Re^3$. Thus if orientation of the frame C_s relative to the base frame C_b is specified by $\gamma \in \Re^3$, then the rotational velocity of the frame C_s is $\omega \in \Re^3$, where there exists some linear transformation $\tilde{P}(\gamma)$ such that $\omega = \tilde{P}(\gamma)\dot{\gamma}$. Denote the position of the origin of the frame C_s relative to C_b by $x(t) \in \Re^3$, then the position/orientation of the manipulator end effector is given by $X \triangleq [x^T \ \gamma^T]^T \in \Re^6$. If the translational velocity of the frame C_s is denoted by $v \in \Re^3$, with $v = \dot{x}(t)$, then the vector $Y = [\ v^T \ \ \omega^T]^T \in \Re^6$ specifies the velocity of C_s relative to C_b. This vector is related to $\dot{X}$ by a non-singular linear transformation, P, such that $Y = P\dot{X}$.

Suppose the end-effector is constrained to follow a physical surface. Let the constraint surface have dimension r, where $0 < r < 6$, and be parameterized by $u = [u_1, \ldots, u_r]^T$. Suppose there exists some function $\phi : \Re^r \rightarrow \Re^6$ determining the position, X, of the end-effector by the equation

$$X = \phi(u). \tag{1}$$

We may obtain a constraint on the acceleration of the end-effector in terms of the surface parameters u, by differentiating the above equation 1 twice to obtain

$$\ddot{X} = J_r \ddot{u} + \dot{J}_r \dot{u} \tag{2}$$

where we define

$$J_r \triangleq \frac{\partial \phi}{\partial X_r} \in \Re^{6 \times r}$$

to be the Jacobian matrix for the surface. This matrix has full column rank r since the parameterization of the surface is minimal, and the columns of the matrix span the tangent space of the surface at the contact point. The position of the robot end-effector on the constraint surface may thus be specified in terms of the parameters u. We ensure that contact between the end-effector and the constraint surface is maintained by the application of an appropriate contact force normal to the surface.

Let the generalized (or joint) coordinates for the manipulator be denoted by the vector $q \in \Re^n$. The kinematics of the manipulator determines a map $f : \Re^n \rightarrow \Re^6$ which provides a relation between the position of the end-effector, $X \in \Re^6$, and the joint coordinates for the manipulator $q \in \Re^n$ given by

$$X = f(q). \tag{3}$$

Obtain a constraint equation between the acceleration of the end effector and that of the manipulator joints by differentiating equation 3 twice, to obtain

$$\ddot{X} = J\ddot{q} + \dot{J}\dot{q}, \tag{4}$$

where we define

$$J \triangleq \frac{\partial f}{\partial q} \in \Re^{6 \times n}$$

to be the manipulator Jacobian. In order that we may consider the invertibility of the manipulator Jacobian, J, in the analysis to follow we would like J to be a square matrix. We thus consider a manipulator with six joints (i.e. $n = 6$). Eliminating the acceleration variable $\ddot{X}$ between equations 2 and 4, we may relate the joint accelerations and velocities to those of the surface parameter, u, to obtain

$$J\ddot{q} + \dot{J}\dot{q} = J_r\ddot{u} + \dot{J}_r\dot{u}. \tag{5}$$

If the manipulator is not in a singular configuration, then the matrix J is invertible, and the joint acceleration vector is given by

$$\ddot{q} = J^{-1}(J_r\ddot{u} + \dot{J}_r\dot{u} - \dot{J}\dot{q}). \tag{6}$$

Now for motion along a frictionless constraint surface, the contact force is in a direction normal to the surface. This force is to be controlled simultaneously with velocity (or equivalently position) on the constraint surface. Typically a physical constraint surface is five-dimensional, i.e. $r = 5$, so force need only be controlled in the one-dimensional normal direction.

As mentioned previously, the tangent space of the constraint surface is spanned by the r columns of the matrix $J_r \in \Re^{6 \times r}$. Define a rank $6 - r$

matrix $B \in \Re^{6\times(6-r)}$ whose columns span the null space of $J_r{}^T$. Thus B satisfies the relation

$$J_r{}^T B = 0, \tag{7}$$

and $Range(B) = Null\ space(J_r{}^T)$. The columns of the matrix B span the space orthogonal to the tangent space of the constraint surface, so we say B specifies the normal direction of the constraint surface and the applied contact force f_c is of the form

$$f_c = B\lambda \tag{8}$$

where $\lambda \in \Re^{6-r}$. Normalize the columns of B to unit length then $\lambda \in \Re^{6-r}$ are just scaling parameters for the applied force.

By duality of the force and velocity spaces from the principle of virtual work, the above interpretation is in fact reasonable. The range of B, $Range(B)$ provides those forces (or wrenches) which are reciprocal to the velocities (infinitesmal motions or twists) in the range of J_r^T. That is, those forces which produce no work when they act on a twist in $Range(J_r{}^T)$ are the set of constraint forces which cause the manipulator to maintain contact with the environment. We will refer to the space spanned by the columns of B as *the normal direction of the constraint surface*, while the space spanned by the columns of J_r is *the tangent space of the surface.*

For the manipulator at any given position $X \in \Re^6$, the tangent space may be identified with $\Re^6$. Thus provided equation 7 holds, it is possible to decompose $\Re^6$ into two orthogonal subspaces and it may be written as the direct sum

$$\Re^6 = Range(J_r) \oplus Range(B). \tag{9}$$

This is the decomposition is the basis for determining the two constraint subspaces, since any $x \in \Re^6$ can be written as

$$x = J_r x_r + B x_f\ , \tag{10}$$

for some unique $x_r \in \Re^r$ and $x_f \in \Re^{6-r}$. Using this decomposition, the $Range(J_r)$ defines the velocity controlled subspace, while the $Range(B)$ is the force controlled subspace.

Constraints on the motion of the manipulator may arise from tasks other than just those for which the end-effector must maintain contact with a physical surface. In the case of constraints which arise from the mechanical and geometric characteristics of a specified task, Mason's task description framework provides us with a method of partitioning the six-dimensional space into two orthogonal subspaces, one of which is to be velocity (or position)-controlled and the other force-controlled. Position constrained directions are directions in which arbitrary force may be applied so force is controlled here, while velocity is controlled tangentially to produce the desired motion.

Consider the task of manipulating a peg in a hole, where the peg fits snugly in the hole. The motion of the peg is constrained by the hole. It can rotate about its own axis and translate up and down along this axis (zero contact force in these directions), but the peg has no freedom to move in any of the other four degrees of freedom so these directions will be force controlled.

Recall from the problem formulation that with motion constraints in effect, the position/orientation, X, of the manipulator end-effector is defined by that of a constraint frame aligned with the normal and tangential directions of the constraint surface. Comparing the formulation of this section with that of Mason [1], the *Range*(J_r) is the force constrained subspace S_f, while the *Range*(B) is the velocity constrained subspace S_v.

3 Dynamics and Control of a Single Manipulator

Consider a six degree-of-freedom manipulator in contact with a rigid, frictionless constraint surface of dimension r. Suppose contact with the environment is made at a point. The control objective is to provide a set of joint input torques so that the manipulator follows some desired position/force trajectory on a constraint surface specified by (u_d, λ_d), such that position along the constraint surface is controlled, and a specified desired force is exerted normal to the constraint surface.

Since the surface is frictionless, the contact force is normal to the surface, and the dynamic equation of the constrained manipulator is given by

$$M(q)\ddot{q} + N(q,\dot{q}) = \tau - J^T B\lambda \tag{11}$$

where $M(q) \in \Re^{6\times 6}$ is a positive definite inertia matrix for the manipulator, $N(q,\dot{q}) \in \Re^6$ is a vector of coriolis, gravity and frictional terms, τ is the vector of input joint torques. The matrices $J \in \Re^{6\times 6}$ is the manipulator Jacobian. The matrix $B \in \Re^{6\times 6-r}$ specifies those directions along which the manipulator may apply force, and $\lambda \in \Re^{6-r}$ provides the free parameters of force.

Suppose the manipulator is not in a singular configuration, so that the Jacobian matrix J is invertible. We may rewrite the system dynamic equation in terms of the surface parameters $u \in \Re^r$ by substituting equation 6 into equation 11 above, to obtain

$$M(q)J^{-1}J_r\ddot{u} + M(q)J^{-1}(\dot{J}_r\dot{u} - \dot{J}\dot{q}) + N(q,\dot{q}) = \tau - J^T B\lambda. \qquad (12)$$

Define

$$\mathcal{N}(\dot{u},\dot{q},q) \triangleq M(q)J^{-1}(\dot{J}_r\dot{u} - \dot{J}\dot{q}) + N(q,\dot{q}) \quad \in \Re^6 \qquad (13)$$

to be a vector of non-linear terms, and define the the matrix H by

$$H \triangleq J^{-1}J_r \in \Re^{6\times r} \qquad (14)$$

then the dynamic equation for the system simplifies to the form of

$$M(q)H\ddot{u} + \mathcal{N}(\dot{u},q,\dot{q}) = \tau - J^T B\lambda. \qquad (15)$$

The following theorem provides position tracking, provided we have complete knowledge of the non-linear terms, $N(\dot{q},q)$, in the manipulator dynamics. This is used as part of the feedforward component of the control law.

Theorem 1 (Position tracking, unspecified force) *Consider a six degree-of-freedom manipulator operating in the six-dimensional position/orientation space. Suppose the tip of the manipulator is required to move along a rigid, frictionless constraint surface described by $X = \phi(u)$, where this surface be parameterized by the variables $u \in \Re^r$, for $r < 6$ the dimension of the constaint surface. Specify the desired trajectory of the manipulator end effector on the surface by the parameter values $u_d \in \Re^r$. If the manipulator does not go through a singular configuration, the following control law will achieve asymptotic tracking of the desired trajectory on the constraint surface by the manipulator end effector.*

$$\tau = M(q)H(\ddot{u}_d + k_v\dot{e}_r + k_p e_r) + \mathcal{N}(\ddot{u},\dot{q},q) + \tau_n \qquad (16)$$

where $\mathcal{N}$ is defined by equation 13, H is defined by equation 14, $e_r = u_d - u$ and $\dot{e}_r = \dot{u}_d - \dot{u}$, and $\tau_n \in \Re^6$ is a torque which lies in the null space of H^T. The scalars k_v, k_p are chosen such that the polynomial $s^2 + k_v s + k_p$ is Hurwitz.

Proof: Note the matrix H is full rank, since J is non-singular and J_r has full column rank. Thus the linear map defined by the matrix $H^T : \Re^6 \rightarrow \Re^r$ provides a projection from the six-dimensional space of joint coordinates onto the r-dimensional constraint surface. Substitute the control law 16 into the system equation 15 to obtain

$$M(q)H(\ddot{e}_r + k_v\dot{e}_r + k_pe_r) = \tau_n - J^TB\lambda. \tag{17}$$

Then project this error equation onto the r-dimensional constraint space by premultiplying 17 by H^T to obtain

$$\mathcal{M}(q)(\ddot{e}_r + k_v\dot{e}_r + k_pe_r) = H^T\tau_n - {J_r}^TB\lambda, \tag{18}$$

where we define

$$\mathcal{M}(q) \triangleq H^TM(q)H \quad \in \Re^{r\times r},$$

a positive definite matrix.

Note that RHS of equation 18 is zero, since τ_n lies in the null space of H^T, and by equation 7 the second term is also zero. By the positive definiteness of $\mathcal{M}(q)$, equation 18 implies

$$\ddot{e}_r + k_v\dot{e}_r + k_pe_r = 0 \tag{19}$$

It follows that both $e_r, \dot{e}_r \to 0$ as $t \to \infty$, by choice of the scalars k_p, k_v.

In Theorem 1, the vector of joint torques $\tau_n \in \Re^6$ is not specified, it is only required that this set of torques lie in the null space of H^T. Note by the proof of Theorem 1 that a torque in the null space of H^T will not affect position tracking and we have an independent choice of τ_n, subject to the restriction $\tau_n \in$ Null space (H^T).

Since the null space of H^T is of dimension $6 - r$, it seems feasible that τ_n should be determined by using the $6 - r$ free variables specified in the parameter $\lambda_d \in \Re^{6-r}$, corresponding to the desired normal contact force. This is in fact the case and the following theorem provides us with a method of choosing the vector of input torques which will independently realize the desired force, specified by λ_d.

Theorem 2 (Force control normal to constraint surface)
Consider a six-degree of freedom manipulator constrained to move along a rigid, frictionless surface under the conditions described in Theorem 1. Suppose the manipulator is required to exert a specified contact force trajectory f_d, which lies in the subspace which is the orthogonal complement to a space of position controlled variables - that is, conditions (7) - (9) hold, then the desired contact force $f_d(= B\lambda_d)$ is uniquely specified by $\lambda_d \in \Re^{6-r}$.

Then the followng input joint torque will realize the contact force f_d without affecting position tracking.

$$\tau_n = J^T B(\lambda_d + k_f \int e_f) \quad \in \Re^6 \tag{20}$$

where $e_f = \lambda_d - \lambda$, is the error in the force parameter and $k_f > 0$.

Proof: Note that τ_n lies in the null space of H^T so it will not affect position tracking, also $J^T B \lambda_d$ is that torque which realizes the force f_d $(= B\lambda_d)$. Thus any force $J^T B \lambda$ can be realized independently of position tracking if

$$Range(J^T B) \subseteq Null\ space(H^T). \tag{21}$$

This condition is in fact true, since $H^T(J^T B\lambda) = J_r^T B \lambda = 0$.

Now given position tracking so that equation 19 holds, we now show that the actual contact force (specified by λ) tracks the desired contact force (specified by λ_d), we substitute equation 19 into equation 17 with the expression for τ_n to obtain

$$J^T B(e_f + k_f \int e_f) = 0\ . \tag{22}$$

Since J is non-singular this implies that $(e_f + k_f \int e_f) \in Null\ space(B)$. But $B \in \Re^{6 \times m}$ has full column rank $m(< 6)$ so the $Null\ space(B) = 0$ and we obtain

$$e_f + k_f \int e_f = 0. \tag{23}$$

Since $k_f > 0$ it follows that $\lambda \to \lambda_d$, as $t \to \infty$, or equivalently the actual contact force tracks the desired force, f_d.

Thus the torque τ_n of 20 will track the force f_d, independently of position tracking in the orthogonal subspace.

Theorem 3 (Simultaneous Position and Force Control) *Consider a six degree-of-freedom manipulator in contact with a rigid, frictionless surface. Suppose the manipulator end-effector is to be constrained to move along an r-dimensional constraint surface as described in Theorem 1, while simultaneously exerting a specified desired contact force $f_d = B\lambda_d$ normal to the constraint surface. Specify the desired trajectory of the manipulator end-effector on the surface by the r surface parameters, $u_d \in \Re^r$. Then the following control law will simultaneously provide both position and force tracking of a desired trajectory (u_d, λ_d) asymptotically.*

$$\tau = M(q)H(\ddot{u}_d + k_v \dot{e}_r + k_p e_r) + \mathcal{N}(\dot{u}, \dot{q}, q) + J^T B(\lambda_d + k_f \int e_f) \tag{24}$$

Proof: This result follows directly from Theorems 1 and 2.

4 Examples of task descriptions

In applying the control scheme developed in the previous section, the task specification is given by the matrices B and J_r which define the constraint surface, together with the parameters values of u and λ. We now consider a number of different tasks, and specify the task constraint matrices for each of these cases. Note that all other terms in the control law of theorem 3 are dependent on the kinematics of the particular manipulator.

Example 1 : Peg-in-Hole insertion.

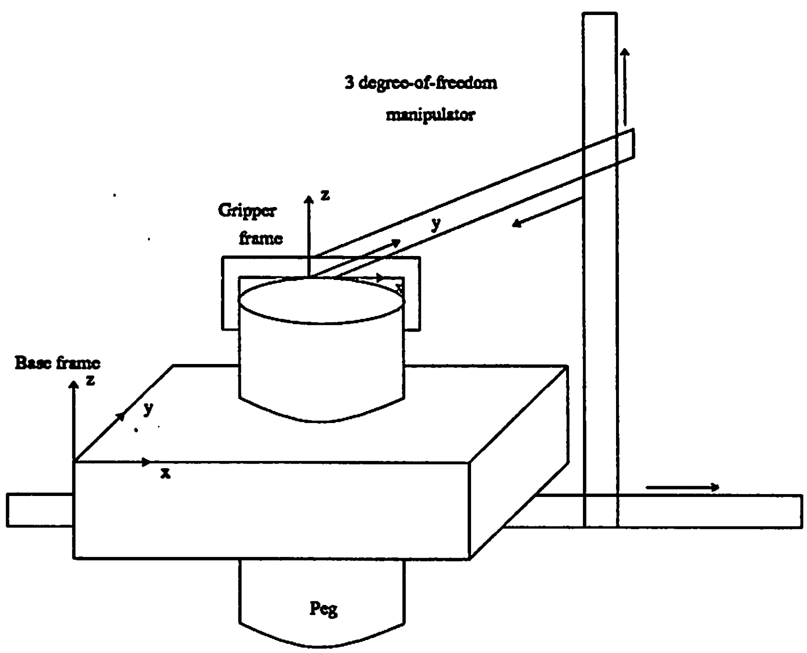

Figure 1: Peg-in-hole insertion.

Consider the task of manipulating a peg in a hole, where the peg fits snugly in the hole. In order to keep the analysis simple we will only consider control of the translational degrees of freedom in the task. Thus consider a three degree-of-freedom cartesian manipulator with the gripper holding the peg is *locked* in the correct orientation for insertion, so that three degrees of freedom are sufficient for the task. Assume no friction on the surfaces between the peg and hole. Let the axis of the peg define the z-axis of the constraint frame. The peg can translate up and down along this axis (zero contact force in this direction), but the peg has no freedom to move in directions perpendicular to this axis, i.e. in the x-y plane, so these directions will be force controlled. The position controlled space, specified by the parameter u, is defined by the matrix J_r satisfying $\dot{X} = J_r \dot{u}$, where

$$J_r = [0\ 0\ 1]^T.$$

The force-controlled space, specified by λ, is defined by the matrix B, where

$$B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}^T.$$

The contact force is given by $f_c = B\lambda$, for $\lambda \in \Re^2$. A desired trajectory may involve the manipulator moving the peg up and down sinusoidally along the z-axis, while applying a unit force in the x direction. The desired trajectory is of the form

$$\begin{aligned} \lambda_d(t) &= [1\ 0]^T \\ u_d(t) &= z_0 cos(t) + offset. \end{aligned}$$

The kinematics for a cartesian manipulator may be simply written as

$$[x\ y\ z]^T = [q_1\ q_2\ q_3]^T + constant.$$

Thus the matrices $M(q) = J = I_3$, $\mathcal{N}(\dot{u}, \dot{q}, q) = 0$, $H = J_r$ and the joint torques determined by the control law of Theorem 3 which will realize the above trajectory, is

$$\tau = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} [\ddot{u}_d + k_1(\dot{u}_d - \dot{u}) + k_2(u_d - u)] + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

This is exactly what one would expect. Note that our control objectives can be very simply specified in terms of the force and surface parameters, and the control scheme is very simply formulated.

Example 2 : Polishing a sphere.

Consider the task of polishing a spherical surface using the manipulator. In this case, the end-effector of the manipulator is constrained to move on the sphere with its orientation predetermined in two dimensions by the tangent space at the point of contact. We use Euler angles to describe the orientation. With respect to an inertial frame fixed at the center of the sphere, the constraint surface is given by the equation

$$\begin{bmatrix} x \\ y \\ z \\ \alpha \\ \theta \\ \psi \end{bmatrix} = \begin{bmatrix} r\cos\theta\sin\alpha \\ r\cos\theta\cos\alpha \\ r\sin\theta \\ \alpha \\ \theta \\ \psi \end{bmatrix} = \phi(\alpha,\theta,\psi).$$

Thus the velocity-controlled directions are defined by the matrix J_r given by

$$J_r = \frac{\partial\phi}{\partial u} = \begin{bmatrix} r\cos\theta\cos\alpha & -r\sin\theta\sin\alpha & 0 \\ -r\cos\theta\sin\alpha & -r\sin\theta\cos\alpha & 0 \\ 0 & r\cos\theta & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

and the parameter $u = (\alpha,\theta,\psi)$. The matrix B which defines the force-controlled directions is given by

$$B = \begin{bmatrix} r\cos\theta\sin\alpha & \cos\alpha/(r\cos\theta) & 0 \\ r\cos\theta\cos\alpha & \sin\alpha/(r\cos\theta) & 0 \\ \sin\theta & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} r & 0 & 0 \\ 0 & \sqrt{1+r^2\cos\theta} & 0 \\ 0 & 0 & \sqrt{1+r^2\cos\theta} \end{bmatrix}^{-1}.$$

with parameter $\lambda \in \Re^3$. Note that $J_r^T B = 0$ holds.

The matrices B and J_r are defined in terms of the constraint frame, C_s. In the global inertial frame the surface parameterization is defined by $u = (\alpha,\theta,\psi)$, and ∂u specifies respectively the x, y and rotational ω_z directions of the constraint frame along which position is to be controlled, while the force controlled directions correspond to the rotational ω_x ,ω_y and z directions of the constraint frame.

We now specify the control objective of moving around the sphere in its x-y plane, while exerting a normal force of 2 units at the contact point by the desired trajectory $u_d = [\ ct \quad 0 \quad 0\ ]^T$, where c is a constant and

$\lambda_d = [\ 0\ \ 0\ \ 2\ ]^T$. Taking into account the kinematics of the manipulator, the control law of Theorem 3 may be applied.

Example 3 : A Scribing Task.

Consider the task of tracing out a *figure 8* on a frictionless planar surface, by using a pointed tool. The plane is a two-dimensional surface, and since contact with the plane is made at a point, the three orientational degrees-of-freedom are free to be specified and controlled. The general equation of a plane is

$$c_1x + c_2y + c_3z + c_4 \ = \ 0.$$

If, for example, $c_2 \neq 0$, then this may be written

$$y = ax + bz - c_4/c_2 = \phi(x, z, \alpha, \theta, \psi)$$

where $a = -c_1/c_2$ and $=-c_3/c_2$. The matrix B specifying the force controlled direction is given as follows

$$B = \frac{1}{\sqrt{1+a^2+b^2}}[\ a\ \ 1\ \ b\ \ 0\ \ 0\ \ 0\ ]^T$$

$\lambda \in \Re$ is a scaling factor.

The position controlled directions are specified by the matrix J_r given by

$$J_r = \frac{\partial \phi}{\partial u} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ -a & -b & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Suppose the orientation of the tool is specified to be fixed at $(\alpha_d, \theta_d, \psi_d)$ then a *figure 8* on the plane with a desired normal force of unity, is achieved by the desired trajectory

$$\begin{aligned} u_{1d}(t) &= \cos(\omega t)/2 \\ u_{2d}(t) &= \sin(\omega t) \\ u_{3d}(t) &= \alpha_d \\ u_{4d}(t) &= \theta_d \\ u_{5d}(t) &= \psi_d \\ \lambda_d &= 1, \end{aligned}$$

The above specification of the task can now be used in the control law.

Other examples of task specifications can be found in [12] and [11].

5 Extension of theory to multifingered hands

The development of the previous sections applies to a single manipulator. Thus it can only be applied to tasks for which a tool is rigidly attached to the end-effector with no freedom between the manipulator tip and the tool in the end-effector. Now there are some tasks such as the scribing task of section 4, which require fine motion of the tool. So these tasks may be best be performed if there are extra degrees of freedom between the manipulator and tool. This redundancy may be provided by attaching a multifingered hand to the tip of the manipulator. In this section and the rest of the paper, we apply the previously developed concepts to the case of an object grasped by a multifingered hand but constrained to move along a rigid, frictionless constraint surface. We provide an analogous control scheme for the hand which causes the object to track the specified desired trajectory. We use the description of the constraint surfaces provided in section two, and in the discussion to follow, we refer to the tool as the *object*.

In analysing the system under consideration, we note that it consists of three parts: the Hand, the Tool or object, and the Constraint surface. The object makes contact with the constraint surface (at a fixed on its surface) and this point is required to move along the constraint surface following a prespecified trajectory. Motion of the object is achieved by its manipulation within the hand which grasps it. The hand comprises a set of articulated links of which we have control through actuators at the joints. Thus the objective of the analysis to follow, is to give a precise determination of what joint torques are required at the finger actuators to cause the object to follow a prespecified trajectory on the constraint surface while simultaneously exerting a specified force.

Now the object forms the link between the hand and the constraint surface, and this provides additional kinematic constraints on the system. We examine these more closely in the section to follow. By duality, or a quasi-static analysis, we determine the relations between the different forces acting on the system. Finally we bring this together to derive the dynamic equation for the system in terms of the constraint parameters.

5.1 Kinematic Constraints on a grasped tool

For tasks involving fine motion of a grasped object along a physical surface, we will assume that the contact point on the object does not change. Call this contact point p_o. Let C_b be a fixed inertial base frame. Fix a coordinate frame C_s to the object at the point p_o, aligning its z-axis with the direction of the contact normal, and x and y axes with the tangential direction. If the object must maintain contact with the surface during motion, the point p_o on the object surface must follow the constraint surface. The position and orientation of any coordinate frame can be specified by a vector in $\Re^6$. Note that physical constraint surfaces in space are, in general, two dimensional. But for the purpose of formulating the control problem, it may also be necessary to consider the orientation at the contacting surface. Thus the specification of the constraint surface used here will include the orientation variables, and will be of dimension r.

For a constraint surface of dimension r, we use the variable $u \in \Re^r$ to parameterize the surface. Let the function $\phi : \Re^r \rightarrow \Re^6$ determine the position/orientation of the frame C_s relative to the base frame C_b by the equation

$$X_s = \phi(u) \tag{25}$$

where $X_s \in \Re^6$ and $u \in \Re^r$.

The condition of contact between the point p_o on the object and constraint surface determines the origin of the frame C_s must follow this surface $\phi(u)$. The dimension, r, of the constraint surface is defined depending on the type of contact which occurs between the object and physical surface. We consider two cases.

- If the contact point on the object is at a differentaibly smooth point of the object surface, the contact normal on the object surface is aligned with that on the constraint surface and their tangent spaces are identical. The orientation of the object is thus predetermined, and a complete specification of the object position and orientation may be provided by two free variables (for a soft finger contact type), or three variables (for a frictional point contact). So in this case, $r = 2$ *or* 3.

- If the contact point on the object surface is at a point of singularity, such as at the tip of a pencil, then the orientation of the object is free to vary and the set of parameters u must include the three parameters of orientation of the object. In this case, five variables are required to completely specify the position/orientation of the constraint frame, and the value of r is 5.

Let us now consider the kinematic constraint on the grasped object. Fix a coordinate frame C_o to the object at its centre of mass. Let the position of the frame C_o relative to the base frame be given by $x_o \in \Re^3$. Specify

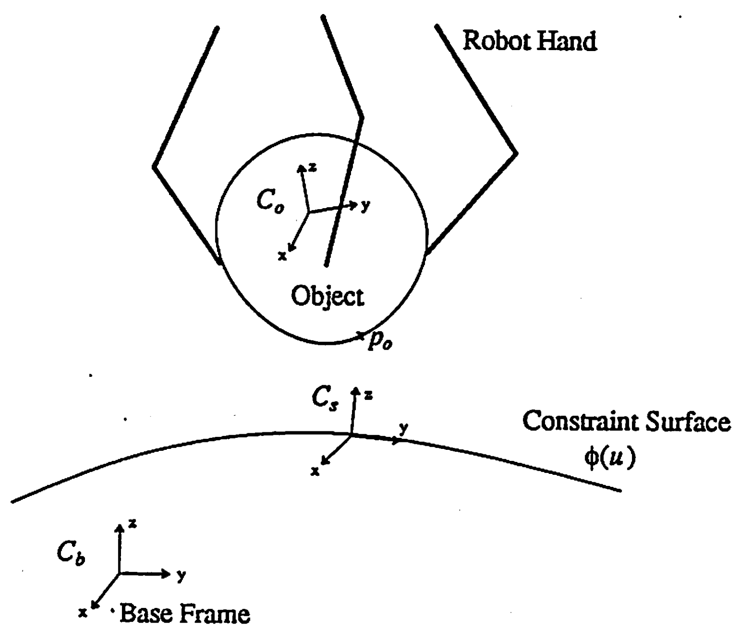

Figure 2: Object constrained to follow a surface by a hand.

its orientation by $R_o(t) \in SO(3)$, with Euler angles $\gamma_o \in \Re^3$. Suppose the position of the point, p_o, relative to the object frame C_o is specified by the vector $c_o \in \Re^3$, then the position of frame C_s relative to the base frame is given by the expression $x_o(t) + R_o(t)c_o$. Let the translational variable of the vector X_s in equation 25 be denoted by X_{sp}, and its orientation variable be denoted by X_{so}, then we may write $X_s = [X_{sp}^T \; X_{so}^T]^T$. The condition of contact between the point p_o on the object and constraint surface is given by the equation

$$X_{sp} = x_o(t) + R_o(t)c_o. \tag{26}$$

The expression relating the surface parameter $u \in \Re^r$ to the position and orientation of the object, is given as follows

$$\phi(u) = \begin{bmatrix} x_o(t) + R_o(t)c_o \\ \gamma_o + constant \end{bmatrix}. \tag{27}$$

Define the object velocity by $Y \triangleq [v_o{}^T \; \omega_o{}^T]^T$, where $v_o = \dot{x}_o(t)$ and $\omega_o = \dot{\gamma}_o$ or $\omega_o \times R_o = \dot{R}_o$. Also note that there exists a matrix $P_1()$ such that $\omega_o = P_1(\gamma_o)\dot{\gamma} = P_1(X_{so})\dot{X}_{so}$. Thus for $X_o = [x_o^T \; \gamma_o^T]^T$ there exist some matrix P satisfying $Y = P\dot{X}_o$. Differentiate equation 27 to obtain the following constraint equation between $\dot{u}$ and Y,

$$J_r\dot{u} = UY \tag{28}$$

where we define

$$U \triangleq \begin{bmatrix} I & -(R_o(t)c_o)\times \\ 0 & I \end{bmatrix} \in \Re^{6\times 6} \;\; and \;\; J_r \triangleq \frac{\partial \phi}{\partial u} \in \Re^{6\times 5}.$$

Note that U is a non-singular square matrix.

We now consider contact between the robot hand and the object. Consider a hand of m fingers, each finger making contact with the object through point contacts. Fix a coordinate frame C_{fi} at the contact point of finger i on the object. Let its orientation relative to the object frame C_o be specified by $R_i(t) \in SO(3)$. Let these m contact points have coordinates $c_1 \ldots c_m$ respectively relative to the object frame C_o. The velocity v_i of the contact point i, is given in terms of the object velocity Y by

$$v_i = U_i \begin{bmatrix} v_o \\ \omega_o \end{bmatrix}, \;\; for \;\; i = 1 \ldots m \tag{29}$$

where $U_i = [\ I \quad -(R_i(t)c_i)\times\ ]$. Stacking these contact velocities we obtain the following matrix equation.

$$\begin{bmatrix} v_1 \\ \vdots \\ v_m \end{bmatrix} = \begin{bmatrix} U_1 \\ \vdots \\ U_m \end{bmatrix} Y = G^T Y \tag{30}$$

where $G \triangleq [U_1^T \ \dots \ U_m^T] \in \Re^{6\times 3m}$ is the Grasp matrix. At each contact point between a fingertip and the object, a friction cone is defined for the finger. Define the friction cone, FC, for the hand to be the cartesian product of all individual friction cones at the contact points. We say that a grasp is *force closure* if G maps FC *onto* $\Re^6$. See [13] and [14].

Each finger is a manipulator with a specific forward kinematic map. Suppose finger i has n_i joints, with generalized coordinates denoted by $q_i \in \Re^{n_i}$. For each finger i we can define a Jacobian matrix $J_i \in \Re^{3\times n_i}$. This matrix relates the joint velocities ($\dot{q}_i \in \Re^{n_i}$) to the translational velocity of the fingertip ($v_i \in \Re^3$). Since finger i maintains contact with the object at the point c_i , it follows that its fingertip velocity is equal to v_i, defined in equation 29. Thus

$$v_i = J_i \dot{q}_i, \qquad for\ i = 1 \dots m, \tag{31}$$

and substituting each of these m equations into equation 30, we obtain an expression relating the joint velocities to the object velocity, of the form

$$J\dot{q} = G^T Y \tag{32}$$

where $J = Block\ diag[J_i \ \dots \ J_m] \in \Re^{3m\times mn_i}$ and $\dot{q} = [q_1^T \ \dots \ q_m^T]^T$. We will use fingers of three joints each (i.e. $n_i = 3$) so that each finger Jacobian is a square matrix.

5.2 System Force Relations

Force relations exist which are dual to the velocity relations defined in the previous section. We specify these relation in this section, using the following notation. Let $f_c \in \Re^{2m}$ be the vector of forces applied to the object by the m fingers of the hand at their respective contact points. Let $\tau = [\tau_1{}^T \dots \tau_m{}^T]^T \in \Re^{2m}$, where τ_i is the vector of input torques at the joints of finger i. Denote by $F_o \in \Re^6$ the resultant force/moment at the object centre of mass due to the application of forces on its surface. Denote the contact force between the object and constraint surface by $f_o \in \Re^6$. The following force relations may be obtained by duality.

Contact between Hand and Object

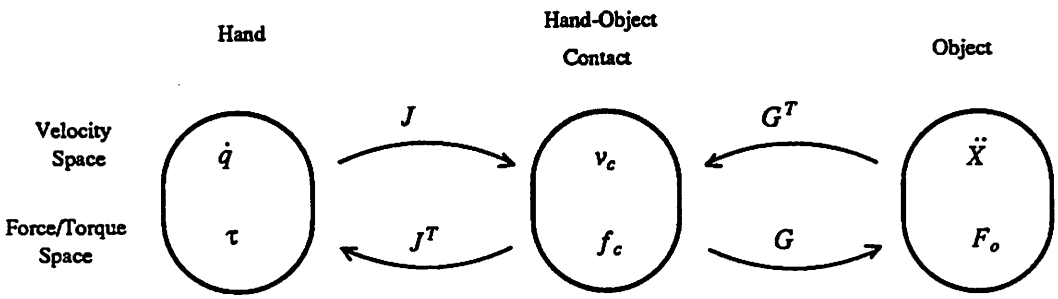

Figure 3: Velocity/Force Duality relations between the Object and Hand.

- The set of torques $\tau \in \Re^{2m}$, at the finger joints which produce the set of contact forces $f_c \in \Re^{2m}$ is given by

$$\tau = J^T f_c \tag{33}$$

- The resultant force/moment at the object centre of mass, $F_o \in \Re^6$, due to the contact forces $f_c \in \Re^{2m}$ is given by

$$F_o = G f_c \tag{34}$$

Contact between Constraint surface and Object

- The force $f_o \in \Re^6$ at the contact point between the object and constraint surface will produce a force/moment, $F_o \in \Re^6$, at the object centre of mass given by

$$F_o = U^T f_o \tag{35}$$

- Since the constraint surface is frictionless the constraint force f_o is normal to the constraint surface, and we can write

$$f_o = B\lambda \tag{36}$$

  where $B \in \Re^{6\times(6-r)}$ for $J_r^T B = 0$ and $\lambda \in \Re^{6-r}$, as described in section two.

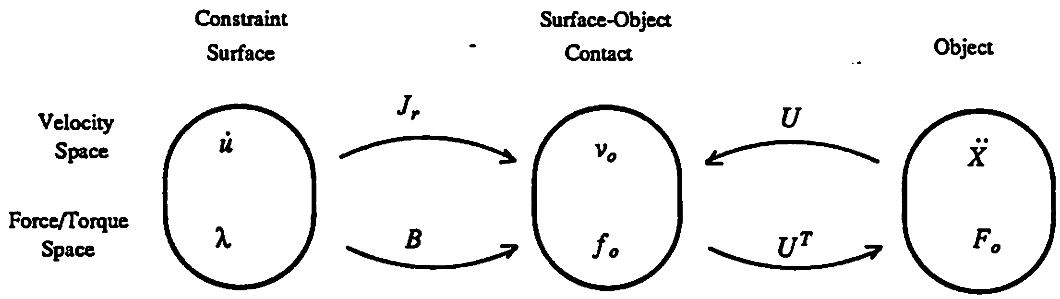

Figure 4: Velocity/Force Duality between the Object and Constraint surface.

Combining equations 35 and 36 the resultant force/moment at the object centre of mass due to the application of a constraint force of magnitude λ, is given by

$$F_o = U^T B\lambda \tag{37}$$

Now this force is to be produced by the application of appropriate contact forces f_c by the fingers at their respective contact points. Thus eliminating the variable F_o between equations 34 and 37, we obtain

$$Gf_c = U^T B\lambda, \tag{38}$$

and the finger contact force which will produce a constraint force of magnitude λ is given by

$$f_c = G^+ U^T B\lambda + f_I \tag{39}$$

where $f_I \in Null\ space(G)$ is called the Internal Force (see [14]).

6 Constrained Dynamics and Hybrid Control

We may now derive the dynamic equations for the full system in terms of the parameters of the constraint surface described in section two. The object moves along the constraint surface in response to manipulation by a multifingered hand. It has been shown in [13] and [14] that the dynamic equation of the hand is of the form

$$M(q)\ddot{q} + N(q,\dot{q}) = \tau - J^T f_c \tag{40}$$

where $M(q) = Block\ diag[M_1(q_1)\ \ldots\ M_m(q_m)] \in \Re^{2m\times 2m}$, and $N(q,\dot{q}) = [N_1(q_1,\dot{q}_1)^T\ \ldots\ N_m(q_m,\dot{q}_m)^T]^T$, for $M_i(q_i) \in \Re^{n_i\times n_i}$ is the positive definite inertia matrix for finger i, and $N_i(q_i,\dot{q}_i) \in \Re^{n_i}$ is the corresponding vector of coriolis and gravity terms. $\tau \in \Re^{2m}$ is the set of input joint torques with which the actuators provide control, $f_c \in \Re^{2m}$ is the contact force between the hand and the object, determined by equation 39 above.

We would like to express this dynamic equation in terms of the surface parameters (u,λ). The kinematic constraint equations 28 and 32 provide a means of achieving this. Since constraints are required on the joint accelerations, we differentiate each of these equations to obtain respectively

$$J_r\ddot{u} + \dot{J}_r\dot{u} = U\dot{Y} \tag{41}$$

and

$$J\ddot{q} + \dot{J}\dot{q} = G^T\dot{Y}. \tag{42}$$

If no finger is in a singular configuration so that the hand Jacobian matrix is non-singular, we may eliminate the variable $\dot{Y}$ between these two equations to obtain the acceleration constraint equation

$$\ddot{q} = J^{-1}[G^TU^{-1}(J_r\ddot{u} + \dot{J}_r\dot{u}) - \dot{J}\dot{q}] \tag{43}$$

The dynamic equation for the full hand-object-constraint surface system is given in terms of the constraint space parameters (u,λ) as follows. Substitute the expression for $\ddot{q}$ given in 43 and the expression for the contact force given in equation 39, into the hand dynamic equation 40 to obtain the dynamic equation in the constraint parameters

$$\begin{aligned} M(q)J^{-1}G^TU^{-1}J_r\ddot{u} &= \tau - M(q)J^{-1}[G^TU^{-1}\dot{J}_r\dot{u} - \dot{J}\dot{q}] - N(q,\dot{q}) \\ &\quad - J^T(G^+U^TB\lambda + f_I) \end{aligned} \tag{44}$$

Note the decoupled manner in which the constraint parameters u and λ, enter into the system dynamic equation.

The system dynamic equation is now in the general form which allows the application of a generalized computed torque control. Note that the equation consists of an acceleration term which appears on the left hand side of equation 44, a set of non-linear terms in the variables $\dot{q}$ and $\dot{u}$. Also all torque terms which involve the forces or force parameters, lie in the null space of a matrix H^T. It is thus possible (by theorem 2) to provide independent force control at the fingertips and on the constraint surface.

The control objective is to provide a set of input joint torques, to the fingers of the hand which grasps the object, so that the point p_o on the object surface maintains contact with a physical surface, and tracks a trajectory on this surface while applying a specified contact force. Thus we may specify the desired constraint trajectory by the parameters (u_d, λ_d). If contact between the object and constraint surfaces allows freedom in the orientation of the object, such as occurs when the tip of a pencil makes contact with a surface, then the desired orientation of the object is included in the control variable, $u \in \Re^r$.

The control law provided below is based on the generalization of the computed torque method of control for multifingered hands. It provides tracking of a desired position/force trajectory on a constraint surface by a specific point p_o on the object held by a multifingered hand, while simultaneously maintaining fixed point contacts between the fingers of the hand and the object, with no slipping. Note the structure of the control law in term of the three parts of the system dynamics described above. It provides proportional-plus-derivative feedback of the position error, integral feedback control of the force error with an additional force term for no slip. Also there is feedforward cancellation of all non-linear terms.

Theorem 4 (Position and Force Control at a point of a grasped object) *Consider an object grasped by a hand of m fingers. Each finger having three degrees of freedom. Let a point p_o on the object be constrained to move along a rigid frictionless constraint surface $X = \phi(u)$, of dimension r, having minimal parameterization $u \in \Re^r$ (where $r = 2, 3$ or 5), while simultaneously maintaining a contact force f_d (determined by the parameter λ_d) normal to the constraint surface. Let the desired trajectory of the point p_o on the constraint surface is specified by u_d. Suppose no finger goes through a singularity over the trajectory and the grasp maintains force closure over the trajectory. Consider the following control law:*

$$\tau = M(q)\tilde{H}(\ddot{u}_d + k_v\dot{e}_r + k_p e_r) + M(q)J^{-1}(G^T U^{-1}\dot{J}_r\dot{u} - \dot{J}\dot{q}) + N(\dot{q}, q) + \tau_n + J^T f_N \quad (45)$$

where $\tilde{H} \triangleq J^{-1}G^T U^{-1} J_r$, $\tau_n = J^T G^+ U^T B(\lambda_d + k_f \int e_f)$, $f_N \in Null\ space(G)$, $k_f > 0$ and $e_r = u_d - u$, $e_f = \lambda_d - \lambda$. Note that $\tau_n \in Null\ space(\tilde{H}^T)$. k_v, k_p are chosen such that $s^2 + k_v s + k_p$ is Hurwitz. This control law achieves simultaneous tracking of both the desired position *trajectory (u_d), and force trajectory (specified by λ_d), and by appropriate choice of f_N, there is no slipping at the contacts between the fingers and object.*

Proof: Substitute the control law 45 into the constraint space dynamic equation 44, to obtain

$$M(q)\tilde{H}(\ddot{e}_r + k_v\dot{e}_r + k_p e_r) = J^T(f_I - f_N) - J^T G^+ U^T B(e_f + k_f \int e_f) \quad (46)$$

Note that the matrix $\tilde{H}^T = J_r^T U^{-T} G J^{-T}$ which maps $\Re^{3m} \to \Re^r$ is onto, since J and U are non-singular, G is onto since the grasp is force closure, and $J_r^T \in \Re^{r\times 6}$ is onto since the parameterization is minimal. Thus let us premultiply the above equation by $\tilde{H}^T$ to obtain

$$\tilde{H}^T M(q)\tilde{H}(\ddot{e}_r + k_v\dot{e}_r + k_p e_r) = 0. \quad (47)$$

The right hand side is zero since $\tilde{H}^T J^T G^+ U^T B = J_r^T B = 0$ and $f_I - f_N \in$ *Null space*(G). Note that $\tilde{H}^T M(q)\tilde{H}$ is a positive definite matrix, so the above equation implies that

$$\ddot{e}_r + k_v\dot{e}_r + k_p e_r = 0 \quad (48)$$

and $e_r, \dot{e}_r \to 0$ by appropriate choice of k_p , k_v.

To show tracking of the force trajectory λ_d, substitute this error equation into equation 46 above to obtain

$$J^T G^+ U^T B(e_f + k_f \int e_f) = J^T(f_I - f_N)\,. \quad (49)$$

Premultiply by GJ^{-T} to obtain

$$U^T B(e_f + k_f \int e_f) = 0 \quad (50)$$

since $f_I - f_N \in$ *Null space*(G).

Since U^T is non-singular and the matrix $B \in \Re^{6\times m}$ has full rank m so the Null space of B is zero and we have the force error equation

$$e_f + k_f \int e_f = 0, \quad (51)$$

which implies that $e_f \to 0$ also, since $k_f > 0$. Equivalently the actual contact force tracks the desired contact force.

As with the control schemes of [13] and [14] for multifingered hands, the internal force f_N should be chosen so as to keep the contact forces within the friction cones at each point of contact. It has been shown that this can be done if the prehensility condition (see [14]) or if the force closure condition is satisfied.

7 Conclusion

In this paper, a new hybrid control scheme has been presented for robot manipulators under constrained motion. This control law is based on the computed torque method of control and provides simultaneous tracking of orthogonal position and force trajectories for a frictionless constraint surface. It has shown how the proposed scheme works for manipulators performing constrained tasks. The analysis has been extended to cover the case of a multifingered hand. The control scheme which makes a fixed point on a grasped object to follow a prespecified desired position/force trajectory on a constraint surface.

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