A UNIFIED APPROACH FOR THE CONTROL OF MULTIFINGERED ROBOT HANDS

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Abstract

In the study of multifingered robot hands, the process of manipulating an object from one *grasp* configuration to another is called dexterous manipulation. In this paper, we study controls for dexterous manipulation by a multifingered robot hand. We use the basic building blocks developed by previous investigators to formulate the kinematics of a multifingered robot hand system. For finger contact with an object, we classify three useful types of constraints: fixed point of contact, rolling contact and sliding contact. Then, we propose control laws for dexterous manipulation of the object under these contact constraints. We show that the control laws realize both the desired position trajectory and the desired grasp force simultaneously. We also provide simulation results based on a planar manipulation system.

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1 Introduction

A new avenue of progress in the area of robotics is the use of a multifingered robot hand for fine motion manipulation. The versatility of robot hands accrues from the fact that fine motion manipulation can be accomplished through relatively fast and small motions of the fingers and from the fact that they can be used on a wide variety of different objects (obviating the need for a large stockpile of custom end effectors). Several articulated hands such as the JPL/Stanford hand (Salisbury 1982 [12]), the Utah/MIT hand (Jacobsen et. al. 1985 [5]) have recently been developed to explore problems relating to manipulation of objects. It is of interest to note that the coordinated action of multiple robots in a single manufacturing cell may be treated in the same framework as a multifingered hand.

Manipulation of objects by a multifingered robot hand is more complicated than the manipulation of an object rigidly attached to the end of a six-axis robotic arm for two reasons: the kinematic relations between the finger joint motion and the object motion are complicated, and the hand has to firmly grasp the object during its motion.

The majority of the literature in multifingered hands has dealt with kinematic design of hands and the automatic generation of stable grasping configurations (see for example Salisbury 1982 [12], Kerr 1985 [6], Li and Sastry 1988 [8]). A few control schemes for the coordination of a multifingered robot hand or a multiple robotic system have been proposed by Nakamura et. al. [11], Zheng and Luh [13], Arimoto [1] and Hayati [4]. The most developed scheme is the master-slave methodology ([13] and [1]) for a two-manipulator system. The schemes developed so far all suffer from the drawback that they either assume rigid attachment of the fingertips to the object or are open loop. The schemes do not account for an appropriate contact model between the fingertips and the object.

In this paper, we study control laws for coordinated manipulation by a multifingered robot hand under the following contact constraints: *(1) Fixed points of contact and (2) rolling contacts.* In [7], a robot hand manipulating an object with fixed points of contact is called coordinated manipulation, and is called rolling motion with rolling contacts.

A brief outline of the paper is as follows: In Section 2, we review some basic concepts concerning rigid body motion and kinematics of contact. In Section 3, we formulate the kinematics of a multifingered robot hand system, and develop the force/velocity transformation relations. In Section 4, we propose the corresponding coordinated control laws. We

also present simulation results on a planar example. In Section 5, we conclude this paper with several important remarks.

2. Preliminaries

In this section, we discuss concepts concerning rigid body motion and kinematics of contact. For further treatment of this subject see (Li, Hsu and Sastry [9]) and (Montana [10]).

Notation 1 *Let C_i and C_j be two coordinate frames of $\Re^3$, where i, j are arbitrary subscripts. Then, $r_{i,j}$ and $R_{i,j}$ denote the position and orientation of C_i relative to C_j. Furthermore, $g_{i,j} \triangleq (r_{i,j}, R_{i,j}) \in SE(3)$ denotes the configuration of C_i relative to C_j, where the Euclidean group, $SE(3)$, denotes the configuration space of an object.*

Definition 1 *The velocity of C_i relative to C_j is defined using left translation by*

$$\begin{bmatrix} v_{i,j} \\ w_{i,j} \end{bmatrix} = \begin{bmatrix} R_{i,j}^t \dot{r}_{i,j} \\ \mathcal{S}^{-1}(R_{i,j}^t \dot{R}_{i,j}) \end{bmatrix} \tag{1}$$

where the map $\mathcal{S}$

$$\mathcal{S} : \Re^3 \longrightarrow so(3) : \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix} \longmapsto \begin{bmatrix} 0 & -w_3 & w_2 \\ w_3 & 0 & w_1 \\ -w_2 & w_1 & 0 \end{bmatrix}$$

identifies $\Re^3$ with $so(3)$, the space of 3 by 3 skey-symmetric matrices (or the Lie algebra of SO(3)).

Proposition 1 *Consider three coordinate frames C_1, C_2 and C_3. The following relation exists between their relative velocities:*

$$\begin{bmatrix} v_{3,1} \\ w_{3,1} \end{bmatrix} = Ad_{g_{3,2}^{-1}} \begin{bmatrix} v_{2,1} \\ w_{2,1} \end{bmatrix} + \begin{bmatrix} v_{3,2} \\ w_{3,2} \end{bmatrix} \tag{2}$$

where $Ad_{g_{3,2}^{-1}}$, the Adjoint map of SE(3), is a similarity transformation given by

$$Ad_{g_{3,2}^{-1}} = \begin{bmatrix} R_{3,2}^t & -R_{3,2}^t \mathcal{S}(r_{3,2}) \\ 0 & R_{3,2}^t \end{bmatrix}.$$

The proof is rather straightforward, see for example [9].

Corollary 1 *Consider three coordinate frames C_1, C_2 and C_3. Suppose that C_3 is fixed relative to C_2. Then, the velocity of C_3 relative to C_1 is related to that of C_2 by a constant transformation, given by*

$$\begin{bmatrix} v_{3,1} \\ w_{3,1} \end{bmatrix} = \begin{bmatrix} R_{3,2}^t & -R_{3,2}^t \mathcal{S}(r_{3,2}) \\ 0 & R_{3,2}^t \end{bmatrix} \begin{bmatrix} v_{2,1} \\ w_{2,1} \end{bmatrix} \tag{3}$$

In this paper, we will assume that the object in consideration is smooth and convex. This assumption also applies to the fingers of a robot hand.

Definition 2 *The boundary of a smooth rigid object is an embeded 2-dimensional manifold $S \subset \Re^3$, which can be expressed as the union of finitely many open sets $\{S\}_i$, such that S_i is the image of a diffeomorphism*

$$\varphi : U \subset \Re^2 \longrightarrow S_i \subset \Re^3.$$

The pair (φ, U) is called a coordinate system of S. The coordinates of a point $s \in S_i$ are $\mathbf{u} = (u, v) = \varphi^{-1}(s)$.

We assume that an object in consideration has an orthogonal coordinate system, in the sense that

$$\varphi_u(\mathbf{u}) \cdot \varphi_v(\mathbf{u}) = 0, \ \ \forall \mathbf{u} \in U,$$

where $\varphi_u(\mathbf{u})$ denotes the partial derivative of φ with respect to u. When (φ, U) is orthogonal, we define the Gauss frame at a point $\mathbf{u} \in U$ as the coordinate frame with origin at $\varphi(\mathbf{u})$ and coordinate axes

$$\mathbf{x}(\mathbf{u}) = \varphi_u(\mathbf{u})/\|\varphi_u(\mathbf{u})\|, \ \mathbf{y}(\mathbf{u}) = \varphi_v(\mathbf{u})/\|\varphi_v(\mathbf{u})\| \text{ and } \mathbf{z}(\mathbf{u}) = \mathbf{x}(\mathbf{u}) \times \mathbf{y}(\mathbf{u}).$$

Definition 3 *Consider a manifold S with an orthogonal coordinate system (φ, U). At a point $s \in S_i$, the curvature form K is defined as the 2×2 matrix*

$$K = [\mathbf{x}(\mathbf{u}), \mathbf{y}(\mathbf{u})]^t [\mathbf{z}_u(\mathbf{u})/\|\varphi_u(\mathbf{u})\|, \mathbf{z}_v(\mathbf{u})/\|\varphi_v(\mathbf{u})\|],$$

the torsion form T as the 1×2 matrix

$$T = \mathbf{y}(\mathbf{u})^t [\mathbf{x}_u(\mathbf{u}) \backslash \|\varphi_u(\mathbf{u})\|, \mathbf{x}_v(\mathbf{u}) \backslash \|\varphi_v(\mathbf{u})\|],$$

and the metric as the 2×2 diagonal matrix

$$M = diag(\|\varphi_u(\mathbf{u})\|, \|\varphi_v(\mathbf{u})\|).$$

Example 1. Consider the sphere of radius R, with the following coordinate system

$$U = \{(u, v) \in \Re^2 | -\pi/2 < u < \pi/2, -\pi < v < \pi\}$$

and the map

$$\varphi : U \longrightarrow \Re^3 : (u, v) \longmapsto (R \cos u \cos v, -R \cos u \sin v, R \sin u).$$

The coordinates u and v are known as the latitude and longitude, respectively. The Gauss frame is defined by

$$\mathbf{x}(\mathbf{u}) = \begin{bmatrix} -\sin u \cos v \\ \sin u \sin v \\ \cos u \end{bmatrix}, \mathbf{y}(\mathbf{u}) = \begin{bmatrix} -\sin v \\ -\cos v \\ 0 \end{bmatrix}, \mathbf{z}(\mathbf{u}) = \begin{bmatrix} \cos u \cos v \\ -\cos u \sin v \\ \sin u \end{bmatrix}.$$

The curvature form, torsion form and metric are, respectively,

$$K = \begin{bmatrix} 1/R & 0 \\ 0 & 1/R \end{bmatrix}, T = [0, \ -\tan u/R], M = \begin{bmatrix} R & 0 \\ 0 & R\cos u \end{bmatrix}.$$

We now consider two objects, called obj1 and obj2, that move while maintaining contact with each other (Figure 1). By the earlier convexity assumption the objects will make contact over isolated points. We wish to describe here motion of the contact points in response to a relative motion of the objects.

Let C_{r1} and C_{r2} be the coordinate frames fixed relative to obj1 and obj2, respectively. Let S_1 and S_2 be the embeded 2-manifolds representing the boundary of obj1 and obj2. S_1 and S_2 can be expressed as the union of open sets, $S_1 = \bigcup_j S_1^j$, and $S_2 = \bigcup S_2^j$, where S_1^j has orthogonal coordinate system (φ_1^j, U_1^j) and S_2^j has orthogonal coordinate system (φ_2^j, U_2^j). Let $c_1(t) \in S_1$, and $c_2(t) \in S_2$ be the positions at time t of the point of contact relative to C_{r1} and C_{r2}, respectively. We will restrict our attention to an interval I so that $c_i(t)$ belong to a single coordinate system of $S_i, i = 1, 2$.

The coordinate system $(\varphi_i^j, U_i^j), i = 1, 2$, induces a Gauss frame at all points of S_i, which will be denoted by $C_{ci}, i = 1, 2$. We also define a continuous family of coordinate frames, two for each $t \in I$, as follows. Let the local frames at time t, $C_{l1}(t)$ and $C_{l2}(t)$ be the coordinate frames fixed relative to C_{r1} and C_{r2}, respectively, that coincide at time t with the Gauss frames at $c_1(t)$ and $c_2(t)$.

The five parameters that describe the 5 degrees of freedom for the motion of the points of contact are: the coordinates of the point of contact relative to the coordinate system (φ_1^j, U_1^j) and (φ_2^j, U_2^j), given by, respectively,

$$\mathbf{u}_1(t) = (\varphi_1^j)^{-1}(c_1(t)) \in U_1^j, \text{and } \mathbf{u}_2(t) = (\varphi_2^j)^{-1}(c_2(t)) \in U_2^j.$$

and the angle of contact, $\phi(t)$, defined as the angle between the x-axes of C_{c1} and C_{c2}. We chose the sign of ϕ so that a rotation of C_{c1} through angle $-\phi$ around its z-axis aligns the x-axes.

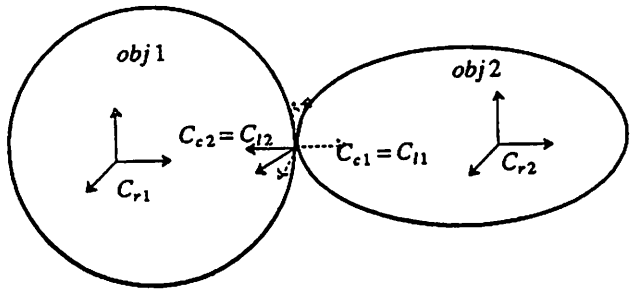

Figure 1: Coordinate frames for two objects which are in contact.

We describe the motion of obj1 relative to obj2 at time t, using the local coordinate frames $C_{l1}(t)$ and $C_{l2}(t)$. Let (v_x, v_y, v_z) and (w_x, w_y, w_z) be the translational and rotational velocities, respectively, of $C_{l1}(t)$ relative to $C_{l2}(t)$. These provide the 6 degrees of freedom for the relative motion between the objects.

The symbols K_1, T_1 and M_1 represent, respectively, the curvature form, torsion form and metric at time t at the point $c_1(t)$ relative to the coordinate system (φ_1^j, U_1^j). We can analogously define K_2, T_2 and M_2. We also let

$$R_\phi = \begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ -\sin\phi & -\cos\phi & 0 \\ 0 & 0 & -1 \end{bmatrix} \triangleq \begin{bmatrix} \hat{R}_\phi & \begin{matrix} 0 \\ 0 \end{matrix} \\ 0\ 0 & -1 \end{bmatrix}, \quad \tilde{K}_2 = \hat{R}_\phi K_2 \hat{R}_\phi.$$

Note that R_ϕ is the orientation matrix of C_{c1} relative to C_{c2}. Hence, $\tilde{K}_2$ is the curvature of obj2 seen from obj1. By the convexity assumption, the relative curvature form $K_1 + \tilde{K}_2$ is invertible. The following equations that relate motion of the points of contact to the relative velocity of the objects are due to Montana ([10]).

Theorem 1 *The point of contact and the angle of contact evolve according to*

$$\begin{cases} \dot{\mathrm{u}}_1 = M_1^{-1}(K_1 + \tilde{K}_2)^{-1}\left(\begin{bmatrix} -w_y \\ w_x \end{bmatrix} - \tilde{K}_2 \begin{bmatrix} v_x \\ v_y \end{bmatrix}\right), \\ \dot{\mathrm{u}}_2 = M_2^{-1}\hat{R}_\phi(K_1 + \tilde{K}_2)^{-1}\left(\begin{bmatrix} -w_y \\ w_x \end{bmatrix} + K_1 \begin{bmatrix} v_x \\ v_y \end{bmatrix}\right), \\ \dot{\phi} = w_z + T_1 M_1 \dot{\mathrm{u}}_1 + T_2 M_2 \dot{\mathrm{u}}_2, \\ 0 = v_z. \end{cases} \tag{4}$$

Montana ([10])calls the first three equations of (4) the kinematic equations of contact, and the last equation the constraint equation.

We define three special modes of contact in terms of the relative velocity components (v_x, v_y, v_z) and (w_x, w_y, w_z) by

(1) Fixed point of contact:

$$\begin{bmatrix} v_x \\ v_z \end{bmatrix} = 0, \ and \begin{bmatrix} w_x \\ w_y \end{bmatrix} = 0; \tag{5}$$

(2) Rolling contact:

$$\begin{bmatrix} v_x \\ v_y \end{bmatrix} = 0, \ \ and \ \ w_z = 0; \tag{6}$$

(3) Sliding contact:

$$\begin{bmatrix} w_x \\ w_y \\ w_z \end{bmatrix} = 0. \tag{7}$$

We have from (4) that

Corollary 2 *The kinematic equations of contact correspond to each of the contact modes are*

$$\begin{cases} \dot{\mathbf{u}}_1 = 0, \\ \dot{\mathbf{u}}_2 = 0, \\ \dot{\phi} = w_z, \end{cases} \tag{8}$$

for fixed point of contact,

$$\begin{cases} \dot{\mathbf{u}}_1 = M_1^{-1}(K_1 + \tilde{K}_2)^{-1} \begin{bmatrix} -w_y \\ w_x \end{bmatrix}, \\ \dot{\mathbf{u}}_2 = M_2^{-1} \hat{R}_\phi (K_1 + \tilde{K}_2)^{-1} \begin{bmatrix} -w_y \\ w_x \end{bmatrix}, \\ \dot{\phi} = T_1 M_1 \dot{\mathbf{u}}_1 + T_2 M_2 \dot{\mathbf{u}}_2. \end{cases} \tag{9}$$

for rolling contact, and

$$\begin{cases} \dot{\mathbf{u}}_1 = -M_1^{-1}(K_1 + \tilde{K}_2)^{-1} \tilde{K}_2 \begin{bmatrix} v_x \\ v_y \end{bmatrix}, \\ \dot{\mathbf{u}}_2 = M_2^{-1} \hat{R}_\phi (K_1 + \tilde{K}_2)^{-1} K_1 \begin{bmatrix} v_x \\ v_y \end{bmatrix}, \\ \dot{\phi} = T_1 M_1 \dot{\mathbf{u}}_1 + T_2 M_2 \dot{\mathbf{u}}_2. \end{cases} \tag{10}$$

for sliding contact.

When a robot hand manipulates an object with its fingers contacting the object by one of the above contact modes, it defines, respectively, coordinated manipulation, rolling motion and sliding motion ([7]).

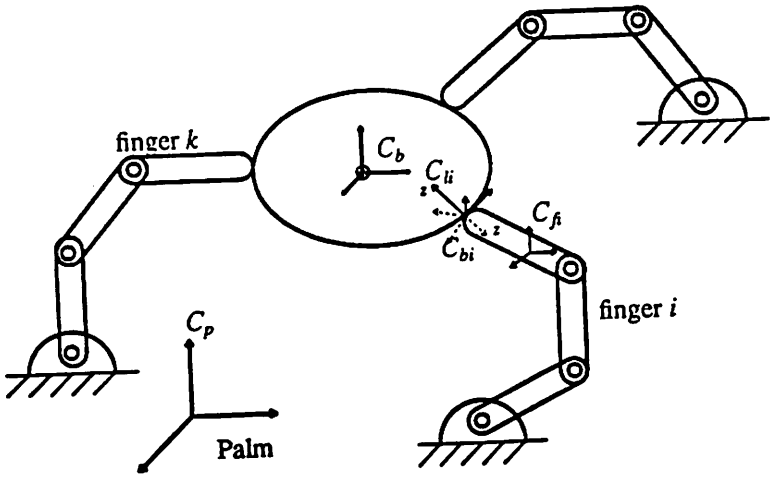

Figure 2: A hand manipulation system

Kinematics of a Multifingered Robot Hand

In this section, we derive the kinematics of a multifingered robot hand system and formulate the the velocity/force transformation relations so that control schemes for coordinated manipulation and for rolling motion can be studied easily in the section that follows. See [3] for the control of sliding motion.

Consider the hand manipulation system shown in Figure 2, which consists of an object and a $k-$fingered robot hand.

We denote by $m_i, i = 1, ...k$, the number of joints of finger i, and by $\theta_i, \tau_i \in \Re^{m_i}$, the vector of joint angles and the vector of joint torque, respectively, of finger i. We define a set of coordinate frames as follows. The reference frame of the system is C_p, which is fixed to the hand palm. The body coordinate frame is C_b, which is fixed to the mass center of the object. At time t the local frame of the object at the point of contact with finger i is $C_{bi}(t)$, which by our earlier convention is fixed relative to C_b. The reference frame of finger i is C_{fi}, which is fixed to the last link of finger i, and the local frame of finger i at time t is $C_{li}(t)$, which is fixed relative to C_{fi} and has origin at the point of contact with the object.

Let $c_{oi}(t) \in S_o$ and $c_{fi}(t) \in S_i$ be the positions at time t of the point of contact of the object with finger i relative to C_b and C_{fi}, respectively. Here, S_o denotes the boundary of the object and S_i the boundary of the last link of finger i. We assume that the contact point relative to finger i occurs only over the last link. We will restrict our attentions to a time interval I so that $c_{oi}(t)$ belongs to a single coordinate system (φ_o^j, U_o^j) of S_o, and $c_{fi}(t)$

belongs to a single coordinate system (φ_i^j, U_i^j) of S_i, $i = 1, ...k$. The coordinates of $c_{oi}(t)$ relative to (φ_o^j, U_o^j) will be denoted by $\mathbf{u}_{oi}(t) \in \Re^2$, and the coordinates of $c_{fi}(t)$ relative to (φ_i^j, U_i^j) by $\mathbf{u}_{fi}(t) \in \Re^2$. Let $\phi_i(t)$ be the angle of contact of the object at time t with respect to finger i.

Let (v_x^i, v_y^i, v_z^i) be the components of translational velocity of $C_{bi}(t)$ relative to $C_{li}(t)$, and (w_x^i, w_y^i, w_z^i) the components of rotational velocity (to apply the results of the previous section, let the object be obj1, and finger i be obj2.). Since the local frames $C_{bi}(t)$ and $C_{li}(t)$ share a common origin (i.e., $r_{bi,li} = 0$), we obtain from Proposition 1 the following velocity constraint relation:

$$\begin{bmatrix} v_{bi,p} \\ \omega_{bi,p} \end{bmatrix} = \begin{bmatrix} R_{\phi_i} & 0 \\ 0 & R_{\phi_i} \end{bmatrix} \begin{bmatrix} v_{li,p} \\ \omega_{li,p} \end{bmatrix} + \begin{bmatrix} v_x^i \\ v_y^i \\ v_z^i \\ w_x^i \\ w_y^i \\ w_z^i \end{bmatrix} \tag{11}$$

where

$$R_{\phi_i} = \begin{bmatrix} \cos\phi_i & \sin\phi_i & 0 \\ -\sin\phi_i & -\cos\phi_i & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

is the orientation matrix of C_{bi} relative to C_{li}. Note that by the constraint equation of (4), we have in (11) that $v_z^i = 0$.

On the other hand, by Corollary 1, the velocity of C_{bi} is related to the velocity of C_b by a constant transformation

$$\begin{bmatrix} v_{bi,p} \\ w_{bi,p} \end{bmatrix} = \begin{bmatrix} R_{bi,b}^t & -R_{bi,b}^t S(r_{bi,b}) \\ 0 & R_{bi,b}^t \end{bmatrix} \begin{bmatrix} v_{b,p} \\ w_{b,p} \end{bmatrix} = Ad_{g_{bi,b}^{-1}} \begin{bmatrix} v_{b,p} \\ w_{b,p} \end{bmatrix}, \tag{12}$$

and similarly for finger i one has

$$\begin{bmatrix} v_{li,p} \\ w_{li,p} \end{bmatrix} = \begin{bmatrix} R_{li,fi}^t & -R_{li,fi}^t S(r_{li,fi}) \\ 0 & R_{li,fi}^t \end{bmatrix} \begin{bmatrix} v_{fi,p} \\ w_{fi,p} \end{bmatrix} = Ad_{g_{li,fi}^{-1}} \begin{bmatrix} v_{fi,p} \\ w_{fi,p} \end{bmatrix}. \tag{13}$$

Moreover, the velocity of the finger reference frame, C_{fi}, is related to the velocity of the finger joints, $\dot{\theta}_i$, by the finger Jacobian,

$$\begin{bmatrix} v_{fi,p} \\ w_{fi,p} \end{bmatrix} = J_i(\theta_i)\dot{\theta}_i. \tag{14}$$

Substituting (12), (13) and (14) into (11) yields the constraint equation relating

velocity of the object to the joint velocity of the fingers.

$$Ad_{g_{bi,b}^{-1}} \begin{bmatrix} v_{b,p} \\ w_{b,p} \end{bmatrix} = J_{fi}\dot{\theta}_i + \begin{bmatrix} v_x^i \\ v_y^i \\ 0 \\ w_x^i \\ w_y^i \\ w_z^i \end{bmatrix}, \tag{15}$$

where

$$J_{fi} \triangleq \begin{bmatrix} R_{\phi_i} & 0 \\ 0 & R_{\phi_i} \end{bmatrix} \cdot Ad_{g_{li,fi}^{-1}} \cdot J_i(\theta_i).$$

For a point contact with friction, a finger can exert linear forces upon the object about the point of contact. Thus, only contact constraints in the directions of the translational velocities can be re-enforced. When finger i contacts the object with a fixed point of contact, substituting (5) into (15) yields the following specialized constraint equation

$$B_i^t Ad_{g_{bi,b}^{-1}} \begin{bmatrix} v_{b,p} \\ w_{b,p} \end{bmatrix} = B_i^t J_{fi}\dot{\theta}_i, \tag{16}$$

where

$$B_i^t = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}.$$

Note that if finger i is a soft finger as in (Salisbury [12]), which enables the finger to exert an additional torque upon the object about the contact normal, or if finger i is rigidly attached to the object as in (Zheng [13]), which enables the finger to exert all six components of forces and torques upon the object, B_i^t can be modified accordingly as in (Li, Hsu and Sastry [9]), and (16) still holds.

When finger i contacts the object with rolling constraint, the corresponding constraint equation remains the same form as in (16), except that the contact coordinates $\mathbf{u}_{oi}(t)$ and $\mathbf{u}_{fi}(t)$ are no longer stationary and evolve according to (9).

When finger i slides across the object, the corresponding constraint equation becomes

$$B_i^t Ad_{g_{bi,b}^{-1}} \begin{bmatrix} v_{b,p} \\ w_{b,p} \end{bmatrix} = B_i^t J_{fi}\dot{\theta}_i + \begin{bmatrix} v_x^i \\ v_y^i \\ 0 \end{bmatrix}, \tag{17}$$

and the contact coordinates evolve according to (10).

We now examine briefly contact constraints in terms of contact wrenches. For a contact model, let n_i denote the total number of independent contact wrenches that finger i can apply to the object. For a point contact with friction, $n_i = 3$ (i.e., a force in the

normal direction and two components of frictional forces in the tangent directions) but for a soft finger $n_i = 4$ (i.e., in addition to the three contact wrenches of a frictional point contact, a torque about the contact normal.). The resulting body wrench from applied contact wrenches of finger i can be expressed as

$$\begin{bmatrix} f_b \\ m_b \end{bmatrix} = Ad^t_{g^{-1}_{bi,b}} B_i x_i \tag{18}$$

where $f_b \in \Re^3$ is a linear force and $m_b \in \Re^3$ is a torque about the origin of C_b, and $x_i \in \Re^{n_i}$ is the magnitude vector of applied contact wrenches along the basis directions of B_i. For a frictional point contact, x_i is constrained to lie in the frictional cone K_i specified by

$$K_i = \left\{ x_i \in \Re^{n_i}, x_{i,3} \leq 0, x^2_{i,1} + x^2_{i,2} \leq \mu^2 x^2_{i,3} \right\}$$

where μ is the coefficient of static Coulomb friction. Note that when finger i slides across the object, contact wrenches are restricted to the boundary ∂K_i of the friction cone, given by

$$\partial K_i = \left\{ x_i \in \Re^{n_i}, x_{i,3} \leq 0, x^2_{i,1} + x^2_{i,2} = \mu^2 x^2_{i,3} \right\}.$$

By the *Principle of Virtual Work*, the joint torque required for maintaining static equilibrium in the presence of contact wrench $x_i \in \Re^{n_i}$, is given by

$$\tau_i = J^t_{fi} B_i x_i. \tag{19}$$

Finally, for the hand manipulation system, we define by $m = \sum_{i=1}^k m_i$, the total number of joints; $n = \sum_{i=1}^k n_i$, the total number of constraints; $\theta = (\theta^t_1, ...\theta^t_k)^t$, $\tau = (\tau^t_1, ...\tau^t_k)^t \in \Re^m$, respectively, the hand joint variable and the hand joint torque vectors; $B = \text{diag}(B_1, ...B_k)$ the basis matrix; $x = (x^t_1, ...x^t_k)^t \in \Re^n$ the magnitude vector of contact wrenches along the directions of B, and $K = K_1 \oplus ... \oplus K_k$ the force cone. Then, the contact constraint equation (16) for both fixed points of contact, and rolling contacts can be concatenated for $i = 1, ...k$ to give,

$$G^t \begin{bmatrix} v_{b,p} \\ w_{b,p} \end{bmatrix} = J_h \dot{\theta}, \tag{20}$$

where

$$G = [Ad^t_{g^{-1}_{bi,b}}, ...Ad^t_{g^{-1}_{bi,b}}]B \text{ and } J_h = B^t \text{diag}\{J_{f1}, ...J_{fk}\}$$

is called the grip Jacobian and the hand Jacobian, respectively.

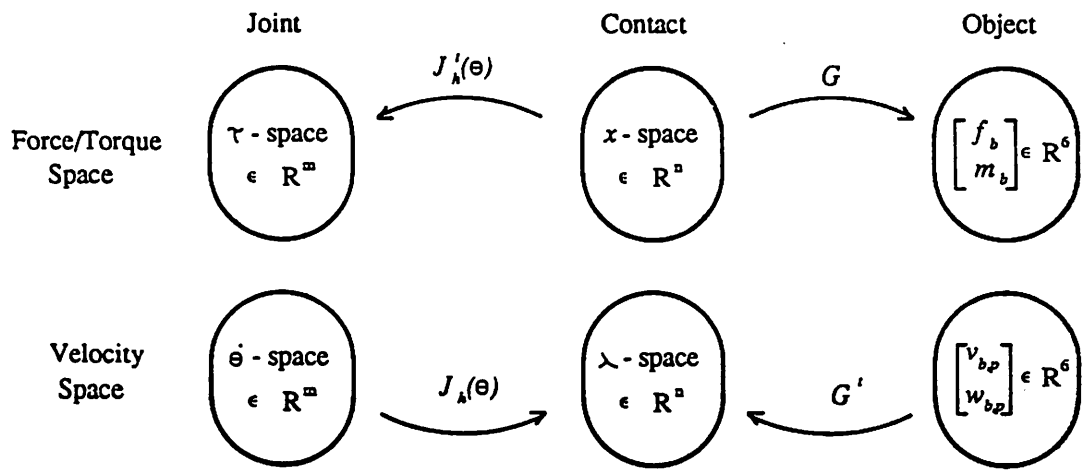

Figure 3: Force/velocity transformation for a hand manipulation system.

It is important to observe that *(1) for fixed points of contact G is constant, but J_h is not necessary constant unless $w_z^i = 0$. (2) For rolling contact both G and J_h depend on the contact coordinates, which are not stationary.*

The equation that relates the resulting body wrench to the applied contact wrenches is

$$\begin{bmatrix} f_b \\ m_b \end{bmatrix} = Gx \tag{21}$$

and the equation that relates contact wrenches to the required joint torques for maintaining static equilibrium is

$$\tau = J_h^t x. \tag{22}$$

The relations have been summarized in Table 1, while Figure 3 illustrates transformation of forces and motion in a hand manipulation system. Note that the vector $\lambda \in \Re^n$ is called the contact velocity.

	Force Torque Relations	Velocity Relations
Body to Fingertip	$\begin{bmatrix} f_b \\ m_b \end{bmatrix} = Gx$	$\lambda = G^t \begin{bmatrix} v_{b,p} \\ w_{b,p} \end{bmatrix}$
Fingertip to Joints	$\tau = J_h^t(\theta)x$	$J_h(\theta)\dot{\theta} = \lambda$

Table 1. Force/velocity transformation for a robot hand system.

Remarks: *(1) The null space of the grip Jacobian G, denoted as $\eta(G)$, is called the space of internal grasping forces (Kerr [6]). Any applied finger forces in $\eta(G)$ do not contribute to the motion of the object. However, during the course of manipulation a set of nonzero internal grasping forces is needed to assure that the grasp is maintained. Both Kerr and Roth ([6]); Nakamura et. al. ([11]) have presented detailed discussions on the optimal selection of internal grasping forces.*

The following dual definitions are now intuitive.

Definition 4 *(Stability and Manipulability of a Grasp) Define a grasp by a multifingered hand by $\Omega \triangleq (G, K, J_h)$ (see Figure 3). Then, for $K = R^n$ we have:*

1. *The grasp Ω is said to be stable if, for every body wrench $(f_b^t, m_b^t)^t$, there exists a choice of joint torque τ to balance it.*
2. *The grasp Ω is said to be manipulable if, for every body motion $(v_{b\ p}^t, w_{b.p})^t$, there exists a choice of joint velocity $\dot{\theta}$ to accommodate this motion without breaking contact.*

Remarks: *(1) A stable grasp has been called a force-closure grasp by Salisbury (1982). It is important to note that stability is not to be understood in the sense of Lyapunov since we are not discussing stability of a differential equation. (2) A manipulable grasp is called a grasp with full mobility by Salisbury (1982).*

Grasp stability and manipulability are now easily characterized for a given position of the fingers by

Proposition 2 *(1) A grasp is stable if and only if G is onto, i.e. the range space of G is the entire $\Re^6$. (2) A grasp is manipulable if and only if $R(\ J_h) \supset R(G^t)$, where $R(\ \cdot)$ denotes the range space.*

We remark that the conditions (1) and (2) superficially appear to be distinct, but they are related. In particular, a stable grasp which requires zero joint torque to balance a non-zero body wrench will be non-manipulable. Conversely, a manipulable grasp which requires zero joint motion to accommodate a non-zero body motion will be non-stable. Figure 4 (a) shows a planar two-fingered grasp, where each finger is one-jointed and contacts the object with a point contact with friction. Clearly the grasp is stable and a force f_y can be resisted with no joint torques. But the grasp is not manipulable, since a y-direction velocity on the body cannot be accommodated. Figure 4(b) shows a grasp of a body in

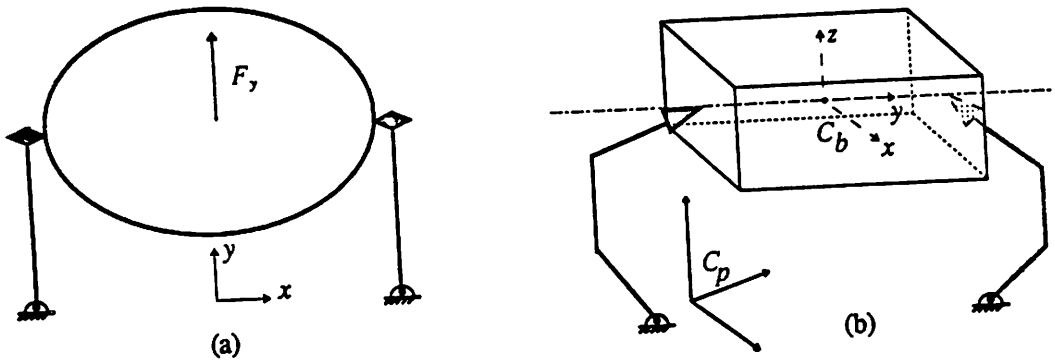

Figure 4: (a) A stable but not manipulable grasp, (b) a manipulable but not stable grasp.

$\Re^3$ by two three jointed fingers. The contacts are point contacts with friction. The grasp is manipulable, though the object can spin around the y-axis with zero joint velocities $\dot{\theta}$. However the grasp is not stable since a body torque τ_n about the y-axis cannot be resisted by any combination of joint torques.

In view of the preceding remarks, we will require a grasp to be both manipulable and stable, i.e.,

$$R(G) = \Re^6, \text{ and } R(J_h) \supset R(G^t). \tag{23}$$

Condition (1) suffers from the drawback that the force domain is left completely unconstrained. As we have seen earlier that the forces are constrained to lie in a convex cone K, taking into account the unidirectionality of the contact forces and finite friction al forces, in which case the image of $K \cap R(J_h)$ under G should cover all of $\Re^6$. Thus, we have

Corollary 3 *A grasp under unisense and finite frictional forces is both stable and manipulable if and only if*

$$G(K \cap R(J_h)) = \Re^6, \textit{ and } R(J_h) \supset R(G^t). \tag{24}$$

4. Control Algorithms for Dexterous Manipulation

In this section, we develop control algorithms for dexterous manipulation. The objectives for each of the manipulation modes are:

1. *Coordinated Manipulation:* Control the fingers, *with fixed points of contact*, so that the object can be manipulated along a prescribed trajectory in $SE(3)$ while exerting

possibly a set of desired contact forces on the environment.

2. *Rolling Motion*: Control the fingers, *with rolling contacts*, so that the object can be manipulated along a prescribed trajectory in $SE(3)$, while exerting a set of desired contact forces on the environment.

We will first propose the control scheme for coordinated manipulation, and then show that, after minor modifications, it also applies to rolling motion.

4.2 A Control Algorithm for Coordinated Manipulation

We assume that the desired trajectory of the object is

$$g_{b,p}^d(t) = (r_{b,p}^d(t), A_{b,p}^d(t)) \in SE(3), t \in [t_0, t_f]. \tag{25}$$

Since $SO(3)$ is a three dimensional manifold, we may choose pitch-roll-yaw variables, or the exponential coordinates to parameterize it. Let $\phi_{b,p} = (\phi_1, \phi_2, \phi_3)^t$ be one of these locally nonsingular parameterization. Then, we can express a trajectory of the object in terms of the parameterization variables

$$g_{b,p}(t) = (r_{b,p}(t), A_{b,p}(\phi_{b,p}(t))) \in SE(3),$$

and the body velocity in terms of the derivatives of the parameterization variables

$$\begin{bmatrix} v_{b,p} \\ w_{b,p} \end{bmatrix} = U(r_{b,p}, \phi_{b,p}) \begin{bmatrix} \dot{r}_{b,p} \\ \dot{\phi}_{b,p} \end{bmatrix} \tag{26}$$

Clearly, $U(r_{b,p}, \phi_{b,p})$ is a nonsingular 6×6 matrix. Differentiating (26) with respect to time t, yields the acceleration relation

$$\begin{bmatrix} \dot{v}_{b,p} \\ \dot{w}_{b,p} \end{bmatrix} = U(r_{b,p}, \phi_{b,p}) \begin{bmatrix} \ddot{r}_{b,p} \\ \ddot{\phi}_{b,p} \end{bmatrix} + \dot{U}(r_{b,p}, \phi_{b,p}) \begin{bmatrix} \dot{r}_{b,p} \\ \dot{\phi}_{b,p} \end{bmatrix}. \tag{27}$$

Note that to realize a desired trajectory of the object it is important that contact constraints be retained. On the other hand, validity of the contact constraints depends on if a proper set of internal grasp forces can be maintained. Thus, the second goal of the control algorithm is to regulate the internal grasp force to the following desired value

$$x_o^d(t) \in \eta(G), t \in [t_0, t_f], \tag{28}$$

where $\eta(G)$ is the null space of G. For fixed points of contact G is time independent, and a set of constant internal grasp forces suffices to guarantee contact constraints. Nevertheless, one may choose $x_o^d(t)$ to realize any other criteria.

We now proceed to formulate the control algorithms with (25) and (28) as our objectives.

From Section 3, the finger joint velocity $\dot{\theta}$ and the object velocity $(v_{b,p}^t, w_{b,p}^t)^t$ are related by the following constraint equation

$$J_h(\theta)\dot{\theta} = G^t \begin{bmatrix} v_{b,p} \\ w_{b,p} \end{bmatrix} \tag{29}$$

(29) is valid regardless of rolling or fixed points of contact. Differentiating (29) with respect to time t, yields the following acceleration constraint equation

$$J_h(\theta)\ddot{\theta} + J_h(\theta)\dot{\theta} = G^t \begin{bmatrix} \dot{v}_{b,p} \\ \dot{w}_{b,p} \end{bmatrix} + \dot{G}^t \begin{bmatrix} v_{b,p} \\ w_{b,p} \end{bmatrix}. \tag{30}$$

Note that for fixed points of contact G is constant and the second term to the right hand side of (30) vanishes.

We will need the following assumption about the grasp.

A1. The grasp is both stable and manipulable.

Generation of object/finger trajectories so that assumption (A1) can be satisfied is discussed in (Li, Canny and Sastry [7]).

By Assumption (A1), we have that $R(J_h) \supset R(G^t)$ and we may express the joint acceleration $\ddot{\theta}$ in terms of the object acceleration by

$$\ddot{\theta} = J_h^+ G^t \begin{bmatrix} \dot{v}_{b,p} \\ \dot{w}_{b,p} \end{bmatrix} + J_h^+ \dot{G}^t \begin{bmatrix} v_{b,p} \\ w_{b,p} \end{bmatrix} - J_h^+ \dot{J}_h \dot{\theta} + \ddot{\theta}_o. \tag{31}$$

Here $J_h^+ = J_h^t (J_h J_h^t)^{-1}$ is the generalized inverse of J_h, and $\ddot{\theta}_0 \in \eta(J_h)$ is the internal motion of redundant joints not affecting the object motion.

<u>Remarks:</u> *(1) Using (31) will lead to a control algorithm in the task space. But if we express the object acceleration in terms of $\ddot{\theta}$ by*

$$\begin{bmatrix} \dot{v}_{b,p} \\ \dot{w}_{b,p} \end{bmatrix} = (GG^t)^{-1} G \left(J_h \ddot{\theta} + \dot{J}_h \dot{\theta} - \dot{G}^t \begin{bmatrix} v_{b,p} \\ w_{b,p} \end{bmatrix} \right)$$

a control algorithm in the joint space of the fingers can be developed. (2) When J_h is square, its generalized inverse J_h^+ is just the usual inverse, and $\ddot{\theta}_o$ disappears from (31). This also implies that the joint motion is determined uniquely by the motion of the object.

The dynamics of the object are given by the Newton-Euler equations

$$\begin{bmatrix} \hat{m} & 0 \\ 0 & \mathcal{I} \end{bmatrix} \begin{bmatrix} \dot{v}_{b,p} \\ \dot{w}_{b,p} \end{bmatrix} + \begin{bmatrix} w_{b,p} \times \hat{m} v_{b,p} \\ w_{b,p} \times \mathcal{I} w_{b,p} \end{bmatrix} = \begin{bmatrix} f_b \\ m_b \end{bmatrix}, \tag{32}$$

where $\hat{m} \in \Re^{3\times3}$ is the diagonal matrix with the object mass in the diagonal, $\mathcal{I} \in \Re^{3\times3}$ is the object inertia matrix with respect to the body coordinates, and $[f_b^t, m_b^t]^t$ is the applied body wrench in the body coordinates which is also related to the applied finger wrench $x \in \Re^n$ through

$$Gx = \begin{bmatrix} f_b \\ m_b \end{bmatrix}. \tag{33}$$

Notice that gravity and interaction forces from the environment can always be added to the right hand side of (32), and corresponding contact wrenches will be generated to counteract them.

Assumption A1 also implies that G is onto, and we can solve (33) as

$$x = G^+ \begin{bmatrix} f_b \\ m_b \end{bmatrix} + x_o, \tag{34}$$

where $G^+ = G^t(GG^t)^{-1}$ is the left inverse of G, and $x_o \in \eta(G)$ is the internal grasping force. The second control goal is to steer the internal grasping force x_o to the desired value $x_o^d(t) \in \eta(G)$.

Combining (32) and (34) yields

$$x = G^+ \left\{ \begin{bmatrix} \hat{m} & 0 \\ 0 & \mathcal{I} \end{bmatrix} \begin{bmatrix} \dot{v}_{b,p} \\ \dot{w}_{b,p} \end{bmatrix} + \begin{bmatrix} w_{b,p} \times \hat{m} v_{b,p} \\ w_{b,p} \times \mathcal{I} w_{b,p} \end{bmatrix} \right\} + x_o. \tag{35}$$

The dynamics of the ith finger manipulator is given by

$$M_i(\theta_i)\ddot{\theta}_i + N_i(\theta_i, \dot{\theta}_i) = \tau_i - J_i^t(\theta_i) B_i x_i. \tag{36}$$

Here, as is common in the literature, $M_i(\theta_i) \in \Re^{m_i \times m_i}$ is the moment of inertia matrix of the ith finger manipulator, $N_i(\theta_i, \dot{\theta}_i) \in \Re^{m_i}$ is the centrifugal, Coriolis and gravitational force terms, τ_i is the vector of joint torque inputs and $B_i x_i \in \Re^6$ the vector of applied finger wrenches. Define

$$M(\theta) = \begin{bmatrix} M_1(\theta_1) & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & M_k(\theta_k) \end{bmatrix}, N(\theta, \dot{\theta}) = \begin{bmatrix} N_1(\theta_1, \dot{\theta}_1) \\ \vdots \\ N_k(\theta_k, \dot{\theta}_k) \end{bmatrix} \text{ and } \tau = \begin{bmatrix} \tau_1 \\ \vdots \\ \tau_k \end{bmatrix}. \tag{37}$$

Then, the finger dynamics can be grouped to yield

$$M(\theta)\ddot{\theta} + N(\theta, \dot{\theta}) = \tau - J_h^t(\theta) x. \tag{38}$$

The control objectives are to specify a set of joint torque inputs τ so that both the desired body trajectory (25) and the desired internal grasping force (28) can be realized.

The first main theorem of the paper is the following control algorithm for coordinated manipulation by robot hands with *non-redundant fingers* .

Theorem 2 *Assume that (A1) holds and the fingers are non-redundant, i.e.,* $m_i = n_i$, *for* $i = 1, \ldots k$. *Denoting the position trajectory tracking error by* $e_p \in \Re^6$, *and the internal grasping force error by* $e_f \in \Re^6$,

$$e_p(t) = \begin{bmatrix} r_{b,p}(t) \\ \phi_{b,p}(t) \end{bmatrix} - \begin{bmatrix} r^d_{b,p}(t) \\ \phi^d_{b,p}(t) \end{bmatrix} \text{ and } e_f(t) = x_o(t) - x^d_o(t). \tag{39}$$

Then, the control law specified by (40) realizes, with fixed points of contact, not only the desired trajectory of the object but also the desired internal grasping force.

$$\begin{aligned} \tau &= N(\theta, \dot\theta) + J_h^t G^+ \begin{bmatrix} w_{b,p} \times \hat m v_{b,p} \\ w_{b,p} \times \mathcal{I} w_{b,p} \end{bmatrix} - M(\theta) J_h^{-1} \dot J_h \dot\theta + M_h \dot U \begin{bmatrix} \dot r_{b,p} \\ \dot\phi_{b,p} \end{bmatrix} \\ &+ J_h^t (x^d_o - K_I \int e_f) + M_h U \left\{ \begin{bmatrix} \ddot r^d_{b,p} \\ \ddot\phi^d_{b,p} \end{bmatrix} - K_v \dot e_p - K_p e_p \right\}, \end{aligned} \tag{40}$$

where

$$M_h = M(\theta) J_h^{-1} G^t + J_h^t G^+ \begin{bmatrix} \hat m & 0 \\ 0 & \mathcal{I} \end{bmatrix} \tag{41}$$

and K_I *is a matrix such that the null space of* G *is* K_I*-invariant.*

Note that

$$G J_h^{-t} M_h = G J_h^{-t} M(\theta) J_h^{-1} G^t + \begin{bmatrix} \hat m & 0 \\ 0 & \mathcal{I} \end{bmatrix}$$

is called the generalized inertia matrix of the hand system.

<u>Remark:</u> *The first four components in (40) are used for cancellation of Coriolis, gravitational and centrifugal forces. These terms behave exactly like the nonlinearity cancellation terms in the computed torque control for a single manipulator; the term* $J_h^t(x^d_o - K_I \int e_f)$ *is the compensation for the internal grasping force loop, and the last term is the compensation for the position loop. We will see in the proof that the dynamics of the internal grasping force loop and that of the position loop are mutually decoupled. Consequently, we can design the force error integral gain* K_I *independently from the position feedback gains* K_v *and* K_p.

Proof.

The proof is very procedural and straightforward. First, for nonredundant fingers with fixed points of contact, internal motion of the fingers reflected by $\ddot\theta_o$ in (31) disappear and G is constant. Thus, the acceleration constraint equation (31) simplifies to

$$\ddot\theta = J_h^{-1} G^t \begin{bmatrix} \dot v_{b,p} \\ \dot w_{b,p} \end{bmatrix} - J_h^{-1} \dot J_h \dot\theta. \tag{42}$$

Substitute (42) and (35) into (38) we have

$$M\left\{J_h^{-1}G^t\begin{bmatrix}\dot{v}_{b,p}\\ \dot{w}_{b,p}\end{bmatrix}-J_h^{-1}\dot{J}_h\dot{\theta}\right\}+N=\tau-J_h^t\left\{G^+\begin{bmatrix}\hat{m} & 0\\ 0 & \mathcal{I}\end{bmatrix}\begin{bmatrix}\dot{v}_{b,p}\\ \dot{w}_{b,p}\end{bmatrix}+G^+\begin{bmatrix}w_{b,p}\times\hat{m}v_{b,p}\\ w_{b,p}\times\mathcal{I}w_{b,p}\end{bmatrix}\right\}-J_h^t x_o. \tag{43}$$

Linearizing (43) with the following control

$$\tau=N(\theta,\dot{\theta})+J_h^tG^+\begin{bmatrix}w_{b,p}\times\hat{m}v_{b,p}\\ w_{b,p}\times\mathcal{I}w_{b,p}\end{bmatrix}-M(\theta)J_h^{-1}\dot{J}_h\dot{\theta}+\tau_1 \tag{44}$$

where τ_1 is to be determined, we have that

$$\left\{M(\theta)J_h^{-1}G^t+J_h^tG^+\begin{bmatrix}\hat{m} & 0\\ 0 & \mathcal{I}\end{bmatrix}\right\}\begin{bmatrix}\dot{v}_{b,p}\\ \dot{w}_{b,p}\end{bmatrix}=\tau_1-J_h^tx_o, \tag{45}$$

or

$$M_h\begin{bmatrix}\dot{v}_{b,p}\\ \dot{w}_{b,p}\end{bmatrix}=\tau_1-J_h^tx_o.$$

Substitute (27) into the above equation, we have

$$M_h\left\{U\begin{bmatrix}\ddot{r}_{b,p}\\ \ddot{\phi}_{b,p}\end{bmatrix}+\dot{U}\begin{bmatrix}\dot{r}_{b,p}\\ \dot{\phi}_{b,p}\end{bmatrix}\right\}=\tau_1-J_h^tx_o. \tag{46}$$

Further, let the control input τ_1 be

$$\tau_1=M_hU\left\{\begin{bmatrix}\ddot{r}^d_{b,p}\\ \ddot{\phi}^d_{b,p}\end{bmatrix}-K_v\dot{e}_p-K_pe_p\right\}+M_h\dot{U}\begin{bmatrix}\dot{r}_{b,p}\\ \dot{\phi}_{b,p}\end{bmatrix}+J_h^t\left(x_o^d-K_I\int e_f\right) \tag{47}$$

and apply it to (46) to yield:

$$M_hU\{\ddot{e}_p+K_v\dot{e}_p+K_pe_p\}=-J_h^t(e_f+K_I\int e_f). \tag{48}$$

Multiply (48) by GJ_h^{-t}, we obtain the following equation.

$$GJ_h^{-t}M_hU\{\ddot{e}_p+K_v\dot{e}_p+K_pe_p\}=-G(e_f+K_I\int e_f)=0 \tag{49}$$

where we have used the facts that $\eta(G)$ is constant and the internal grasping forces lie in the null space of G, i.e.,

$$G(e_f+K_I\int e_f)=0. \tag{50}$$

Since $GJ_h^{-t}M_h=GJ_h^{-t}M(\theta)J_h^{-1}G^t+\begin{bmatrix}\hat{m} & 0\\ 0 & \mathcal{I}\end{bmatrix}$ is positive definite and U is non-singular, (49) implies that

$$\ddot{e}_p+K_v\dot{e}_p+K_p\dot{e}_p=0. \tag{51}$$

Thus, we have shown that the position trajectory tracking error e_p can be driven to zero with proper choice of the feedback gain matrices K_v and K_p.

The last step is to show that e_f also goes to zero. If we substitute (51) into (48) and notice that J_h is nonsingular, we have the following equation.

$$e_f + K_I \int e_f = 0. \tag{52}$$

With proper choice of K_I, the above equation implies that the internal grasping force error e_f converges to zero.

Q.E.D.

Theorem 2 provides a control law for coordinated manipulation by robot hands with non-redundant fingers. This implies that, for a point contact with friction, each finger of the hand has exactly three joints. But, in many industrial applications, several robots which often have more than three degrees of freedom are integrated to maneuver a massive load, or to perform a sophisticated task. Under the point contact model assumption the system has redundant degrees of freedom. It is desirable to have a control law that works for a robot hand with redundant degrees of freedom.

The control law of Theorem 2 can be modified for this purpose.

Corollary 4 *For a robot hand with redundant degrees of freedom, i.e., $m_i \geq n_i, i = 1, ...k$ assume that assumption A1 holds. Then, the control law given by (53) will realize both the desired object trajectory and the desired internal grasp force, with fixed points of contact.*

$$\tau = N(\theta, \dot{\theta}) + J_h^t G^+ \begin{bmatrix} w_{b,p} \times \hat{m} v_{b,p} \\ w_{b,p} \times \mathcal{I} w_{b,p} \end{bmatrix} - M J_h^+ \dot{J}_h \dot{\theta} + M J_h^+ (J_h M^{-1} J_h^t) M_h \dot{U} \begin{bmatrix} \dot{r}_{b,p} \\ \dot{\phi}_{b,p} \end{bmatrix} +$$
$$M J_h^+ (J_h M^{-1} J_h^t)(x_o^d - K_I \int e_f) + M J_h^+ (J_h M^{-1} J_h^t) \hat{M}_h \left\{ \begin{bmatrix} \ddot{r}_{b,p}^d \\ \ddot{\phi}_{b,p}^d \end{bmatrix} - K_v \dot{e}_p - K_p e_p \right\}, \tag{53}$$

where

$$\hat{M}_h = (J_h M^{-1} J_h^t)^{-1} G^t + G^+ \begin{bmatrix} \hat{m} & 0 \\ 0 & \mathcal{I} \end{bmatrix}. \tag{54}$$

and $J_h^+ = J_h^t (J_h J_h^t)^{-1}$.

4.2 A Control Algorithm for Rolling Motion

Controls for dexterous manipulation with rolling constraints have been studied in (Cole, Hauser and Sastry [2]), and (Kerr [6]). Here, we modify Theorem 2 to give a control law for rolling motion. The main difference of rolling motion from coordinated manipulation is that the grip Jacobian G, which depends on the contact coordinates given by (9), is time

varying. Consequently, the null space of $G(t)$, denoted by $V(t) \subset \Re^n$, is also time varying. In general $e_f(t) = x_o(t) - x_o^d(t) \in V(t)$ does not imply, unless $V(t)$ is time independent, that $\int_0^t e_f(\tau)d\tau \in V(t)$, nor $\dot{e}_f(t) \in V(t)$. Thus, we can not introduce dynamic feedback in the force loop, as we did in Theorem 2, to create linear force error equation. But,

$$G(t)e_f(t) = 0 \text{ implies that } G(t)\dot{e}_f(t) + \dot{G}(t)e_f(t) = 0. \tag{55}$$

Lemma 1 *Consider the following differential equation*

$$\dot{x}(t) = A(t)x(t). \tag{56}$$

Let $\mu(A(t)) = \lambda_{max}(A^*(t) + A(t))/2$ be the matrix measure of $A(t)$, where λ_{max} stands for the maximum eigenvalue value. Then,

$$\|x(t)\| \leq \|x(t_0)\| \exp \int_{t_0}^{t} \mu(A(\tau))d\tau.$$

In other words, if $\mu(A(t)) < 0, \forall t$, and $A(t)$ is sufficiently slow time-varying, then, the system (56) is exponentially stable.

Proposition 3 *Assume that Assumption (A1) holds for a robot hand with non-redundant degrees of freedom. Then, the following control law, along with the kinematic equations of contact given by (9), realizes both the desired position trajectory and the desired internal grasp force, for rolling contacts.*

$$\begin{aligned} \tau &= N(\theta,\dot{\theta}) + J_h^t G^+ \begin{bmatrix} w_{b,p} \times \hat{m} v_{b,p} \\ w_{b,p} \times \mathcal{I} w_{b,p} \end{bmatrix} - M(\theta) J_h^{-1} \dot{J}_h \dot{\theta} + M_h \dot{U} \begin{bmatrix} \dot{r}_{b,p} \\ \dot{\phi}_{b,p} \end{bmatrix} + \underline{M(\theta) J_h^{-1} \dot{G}^t \begin{bmatrix} v_{b,p} \\ w_{b,p} \end{bmatrix}} \\ &+ J_h^t \Big(x_o^d - \underline{\dot{e}_f/\delta - G^+ \dot{G} e_f/\delta} \Big) + M_h U \left\{ \begin{bmatrix} \ddot{r}_{b,p}^d \\ \ddot{\phi}_{b,p}^d \end{bmatrix} - K_v \dot{e}_p - K_p e_p \right\}, \end{aligned} \tag{57}$$

where

$$M_h = M(\theta) J_h^{-1} G^t + J_h^t G^+ \begin{bmatrix} \hat{m} & 0 \\ 0 & \mathcal{I} \end{bmatrix} \tag{58}$$

and δ is a sufficiently large number so that the force error equation can be made to be exponentially stable.

Note that in order to compare the control law with that of Theorem 2, we have underlined the terms which are different here.

Proof.

The proof is very similar to that of Theorem 2 and an outline of it is given here. Under rolling constraints, (31) becomes

$$\ddot{\theta} = J_h^{-1} G^t \begin{bmatrix} \dot{v}_{b,p} \\ \dot{w}_{b,p} \end{bmatrix} + J_h^{-1} \dot{G}^t \begin{bmatrix} v_{b,p} \\ w_{b,p} \end{bmatrix} - J_h^{-1} \dot{J}_h \dot{\theta}. \tag{59}$$

Substitute (59) and (35) into the system dynamic equation (38) and linearize the resulting equation with the appropriate terms in the control inputs (57), we get

$$M_h \begin{bmatrix} \dot{v}_{b,p} \\ \dot{w}_{b,p} \end{bmatrix} = \tau_1 - J_h^t x_o. \tag{60}$$

Substituting (27) into the above equation and applying the rest of the control inputs yield

$$M_h\{\ddot{e}_p + K_v \dot{e}_p + K_p e_p\} = -J_h^t \Big\{ e_f + \dot{e}_f/\delta + G^+ \dot{G} e_f/\delta \Big\}. \tag{61}$$

Multiply (61) by GJ_h^{-t} and notice that because

$$Ge_f = 0, \text{ and } (G\dot{e}_f + \dot{G}e_f)/\delta = 0.$$

we have

$$G(e_f + \dot{e}_f/\delta + G^+ \dot{G} e_f/\delta) = 0$$

which implies that

$$\ddot{e}_p + K_v \dot{e}_p + K_p e_p = 0. \tag{62}$$

This shows that the position error goes to zero. On the other hand, substituting (62) into (61), and using the fact that J_h is of full rank, we conclude that

$$(\delta I + G^+ \dot{G}) e_f + \dot{e}_f = 0. \tag{63}$$

Let $A(t) = -(\delta I + G^+ \dot{G})$. It is easy to see that by choosing δ sufficiently large, $\mu(A(t))$ is negative for all $t \in [t_0, t_f]$. Consequently, by Lemma 1 force error e_f also goes to zero.

Q.E.D.

Combining Proposition 3 with Corollary 4 produces a control law for rolling motion by a robot hand with redundant degrees of freedom.

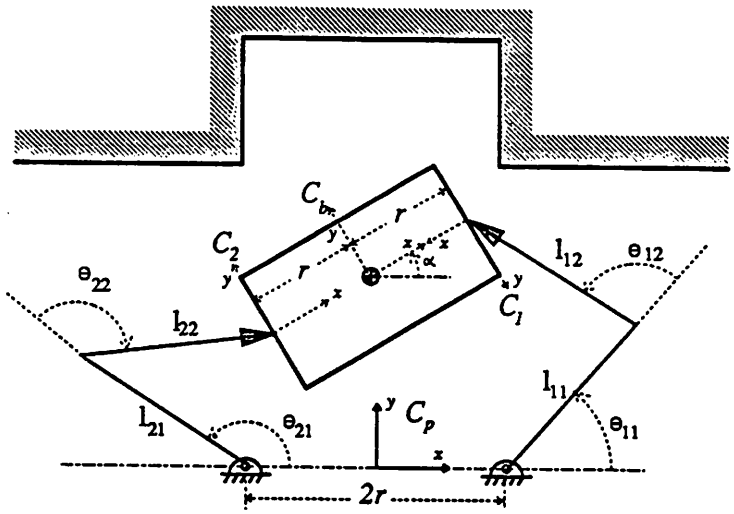

Figure 5: A two-fingered planar manipulation system.

4.3 Simulation

Consider the two-fingered planar manipulation system shown in Figure 5, where the two fingers are assumed to be identical. We model the contact to be a point contact with friction. Let the object width and the finger spacing be 2 units. The grip Jacobian and the hand Jacobian are

$$G = \begin{bmatrix} -1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 \\ 0 & -1 & 0 & -1 \end{bmatrix}$$

and

$$J_h = \begin{bmatrix} J_1 & 0 \\ 0 & J_2 \end{bmatrix}$$

where

$$J_1 = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} \begin{bmatrix} -\sin\theta_{11} - sin(\theta_{11}+\theta_{12}) & -\sin(\theta_{11}+\theta_{12}) \\ \cos\theta_{11} + \cos(\theta_{11}+\theta_{12}) & \cos(\theta_{11}+\theta_{12}) \end{bmatrix}$$

and

$$J_2 = \begin{bmatrix} -\cos\alpha & \sin\alpha \\ -\sin\alpha & -\cos\alpha \end{bmatrix} \begin{bmatrix} -\sin\theta_{21} - sin(\theta_{21}-\theta_{22}) & \sin(\theta_{21}-\theta_{22}) \\ \cos\theta_{21} + \cos(\theta_{21}-\theta_{22}) & -\cos(\theta_{22}+\theta_{22}) \end{bmatrix}.$$

The grasp will be stable and manipulable for the object along the following trajectory

$$x(t) = c_1 \sin(t),\ y(t) = c_2 + c_1 \cos(t), \alpha(t) = c_3 \sin(t).$$

With the control law of Theorem 2, we have simulated the system using a program designed to integrate differential equations with algebraic constraints. Figure 6 shows that the initial position error diminishes exponentially as predicted by (51).

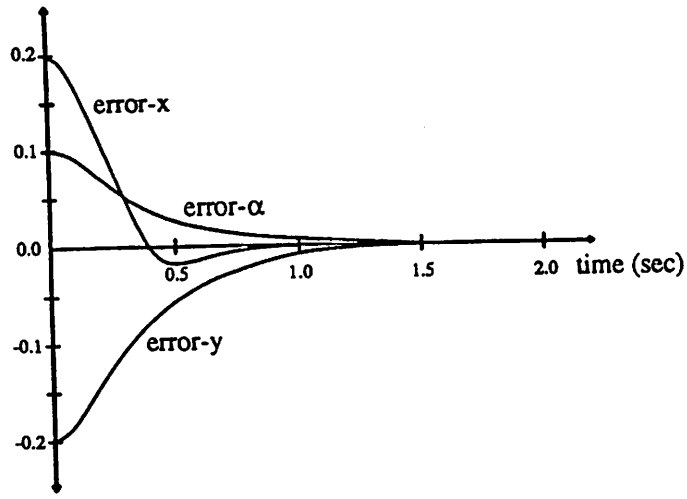

Figure 6: Position error from simulation.

5. Conclusions

In this paper, using the basic building blocks developed by previous investigators, we have formulated the kinematics of a multifingered robot hand system. For finger contact with an object, we have classified three types of useful contacts and developed control laws for dexterous manipulation by a robot hand under these contact constraints. We have shown that the control laws realize not only the desired position trajectory of the object, but also the desired internal grasp force.

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