CNN CLONING TEMPLATE: HOLE-FILLER

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T. Matsumoto, L. O. Chua, and R. Furukawa

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CNN CLONING TEMPLATE †: HOLE-FILLER

T.Matsumoto, L.O.Chua and R.Furukawa ††

† This research is supported in part by the Japanese Ministry of Education, the Institute of Science and Engineering, Waseda University and by the U.S.Office of Naval Research under Contract N000 14-89-J1402.

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Abstract A CNN template ***for hole-filling*** is reported.

Consider the CNN (Cellular Neural Network) [1] defined by

$$C\frac{dv_{x_{ij}}}{dt} = -\frac{1}{R_x} v_{x_{ij}} + A * v_{y_{ij}} + B * v_{u_{ij}} + I \qquad (1)$$

$$v_{y_{ij}} = \frac{1}{2}\left(|v_{x_{ij}} + 1| - |v_{x_{ij}} - 1|\right) \qquad 1 \le i \le M,\ 1 \le j \le N$$

where * denotes the two dimensioned "convolution operator". Without loss of generality assume $R_x = 1$.

Suppose that a bipolar (±1) 2-d image $U=[u_{ij}]_{\substack{1\le i\le M\\ 1\le j\le N}}$ is given, e.g., Fig.1. The following template for(1) ***fills up any hole*** in **U** :

$$A = \begin{array}{|c|c|c|}\hline 0 & 1 & 0 \\ \hline 1 & 2 & 1 \\ \hline 0 & 1 & 0 \\ \hline \end{array}\ , \quad B = b = 4 \ \text{(a scalar)} \qquad (2)$$

$$I = -1$$

Specifically, if

(i) the given bipolar image u_{ij} ***is fed into*** $v_{u_{ij}}$ in(1) and held at that value for all $t \ge 0$, and if
(ii) ***all initial states*** $v_{x_{ij}}(0) = +1$, $1\le i\le M$, $1\le j\le N$.
then the final states $v_{x_{ij}}(\infty)$ are given by Fig.2. Figure 3(a)-(e) shows $v_{x_{ij}}(t)$, at t[μs]= 0, 0.5, 1, 1.5,∞ respectively. Of cource Fig.3(e) is identical to Fig.2. Let us explain why this phenomenon occurs. One can easily check that

(a) $\dfrac{dv_{x_{ij}}(t)}{dt} < 0$ if and only if

$$u_{ij} = -1 \ \textit{and} \quad y_{ij+1}(t)\,,\, y_{ij-1}(t)\,,\, y_{i-1j}(t)\,,\, y_{i+1j}(t) < +1 \qquad (3)$$

(b) $\dfrac{dv_{x_{ij}}(t)}{dt} \ge 0$ if and only if

$$u_{ij} = +1 \ \textit{or} \quad y_{ij+1}(t) = y_{ij-1}(t) = y_{i-1j}(t) = y_{i+1j}(t) = +1 \qquad (4)$$

First consider an *edge cell* C(i,j) where i=1 or M or j=1orN,and recall that $v_{xij}(0) = +1$. If $u_{ij}=-1$, i.e.,the cell value of the original image is negative, then (a) implies $dv_{xij}(0)/dt < 0$ because at least one of $y_{ij+1}(t)$, $y_{ij-1}(t)$, $y_{i-1j}(t)$ and $y_{i+1j}(t)$ is not present. (See Fig.4)

Since at all the edge cells $dv_{xij}(0)/dt < 0$, the cells next to the edges would behave in the same manner as time goes. A similar phenomenon occurs at those cells that are located two cells away from the edge, and so on. It is like *a wave* propagated from the edges into the inside of the plane. Therefore, *the propagation is stopped* if

(A) $u_{ij} = +1$, or
(B) there in a hole in U.

Namely, if (A) or (B) is satisfied , then condition (4) is satisfied. Since the stability of CNN is guaranteed [1], one obtains the desired result. (See Fig.5)

Of cource, the part of the original image U which does not form a loop, would remain as it is. Also the "thickness" of the hole does not have to be one.

In the previous argument, we considered the so called *"8-connected" neighborhood.* Namely, two neighboring cells *located diagonally* are also considered to be connected as well as those located horizontally and vertically. If one considers a "4-connected" neighborhood, instead, then the two neighboring cells located diagonally are not considered to be connected. In this case one considers Fig.6.(a). Then the following template fills the holes :

$$A = \begin{array}{|c|c|c|} \hline 1 & 1 & 1 \\ \hline 1 & 2 & 1 \\ \hline 1 & 1 & 1 \\ \hline \end{array} \quad , \; B = b = 8 \;\; \text{(a scalar)}$$

$$I = -1$$

Fig.6 (b)-(e) show the hole-filling process. The mechanism can be explained in a manner similar to the above.

References

[1] Leon O. Chua and Lin Yang, "Cellular Neural Network", IEEE Trans CAS vol.35, No 10, pp1257 - 1272 and 1273 - 1290, 1988

Figure Captions

Fig.1 A 2-d image with holes.

Fig.2 Holes are filled by CNN.

Fig.3 Hole-filling process. (a) $t = 0$, (b) $t = 0.5$, (c) $t = 1.0$, (d) $t = 1.5$, (e) $t = \infty$.

Fig.4 State variable of an edge cell decrease monotonically.

Fig.5 State variable is non-decreasing if $u_{ij} = +1$ or the cell $C(i,j)$ is in a hole.

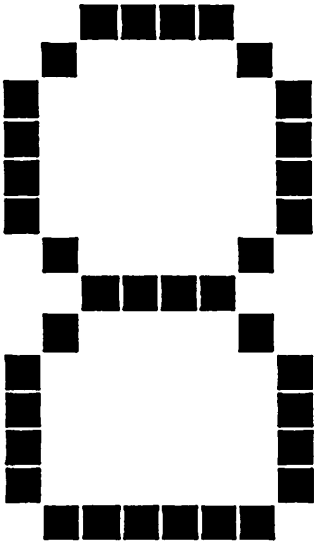


Fig.1

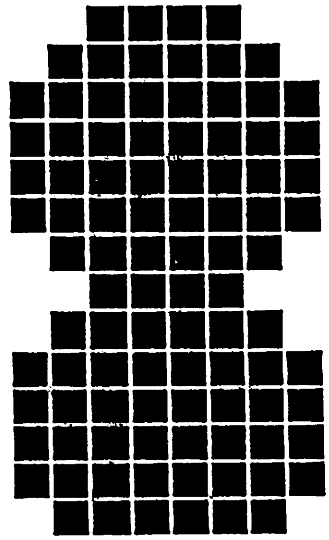


Fig.2

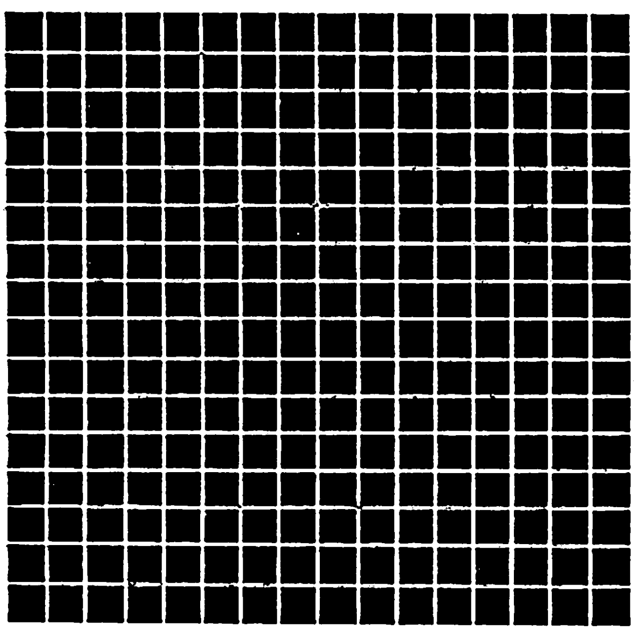
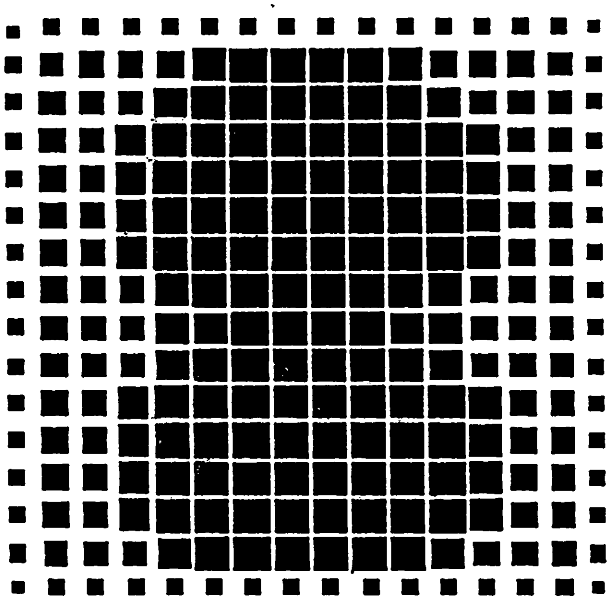
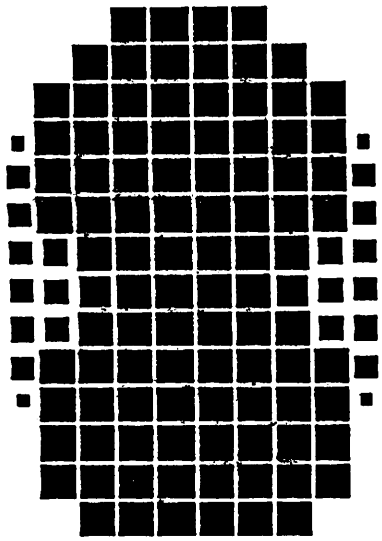


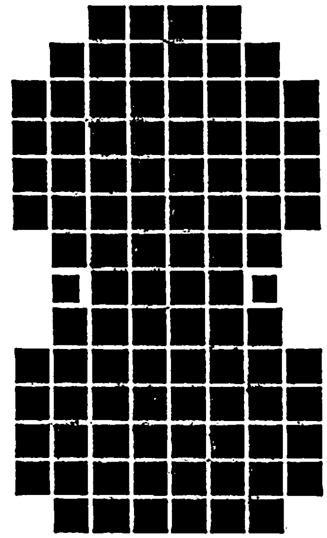
Fig.3(a)



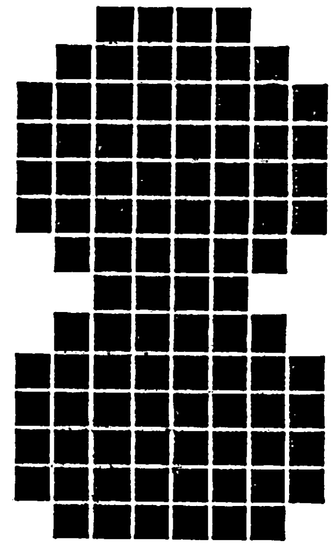
(b)



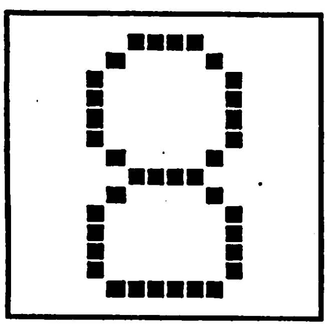
(c)

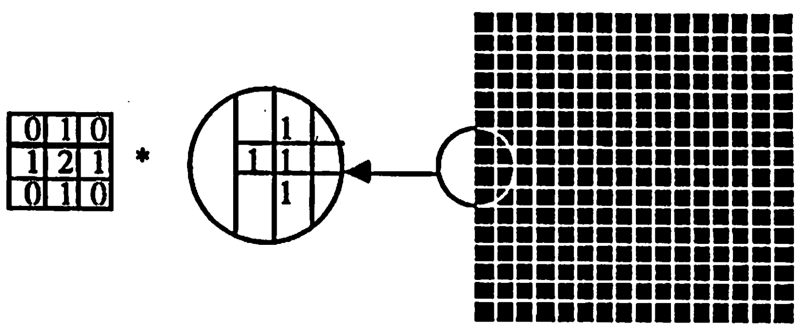


(d)



(e)

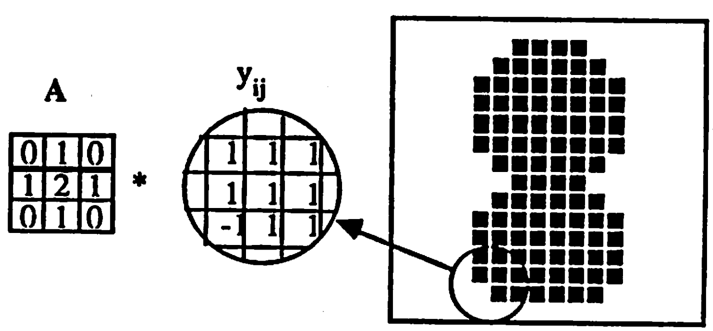




$$-x_{ij} + A*y_{ij} + bu_{ij} - I = -1 + 5 - 4 - 1 = -1$$

Fig.4(a)

(b)



$$-x_{ij} + A * y_{ij} + bu_{ij} - I = -1 + 6 - 4 - 1 = 0$$

Fig.5

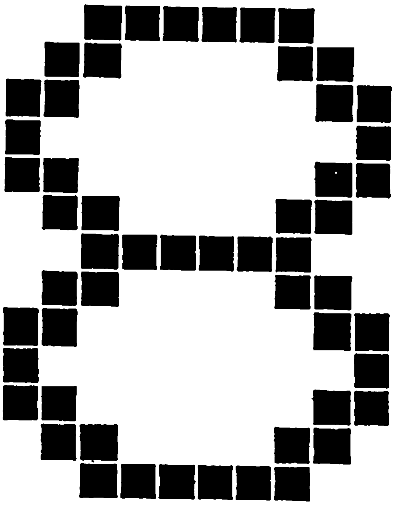
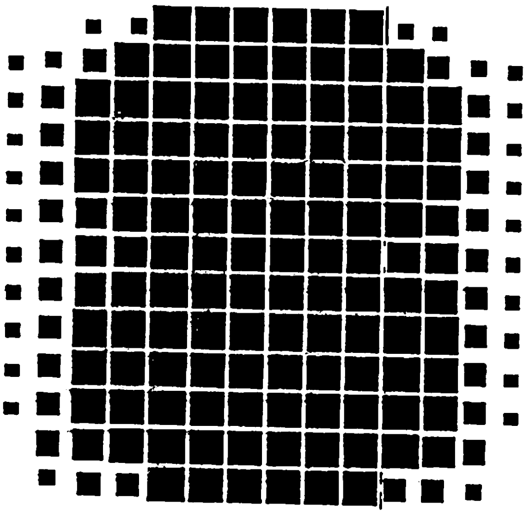
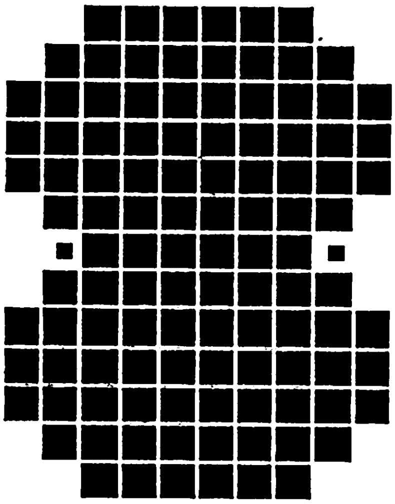


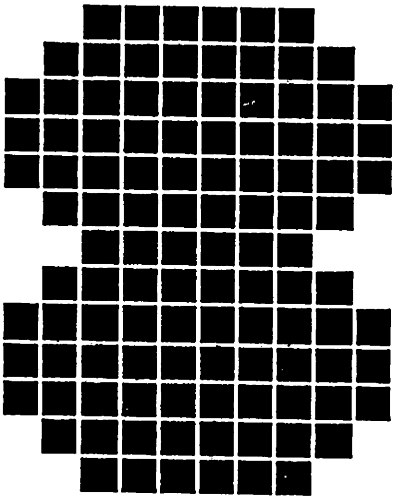
Fig.6 (a)



(b)



(c)



(d)