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NEURAL NETWORK**

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Memorandum No. UCB/ERL M89/86

19 July 1989

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# IMAGE THINNING WITH A CELLULAR NEURAL NETWORK †

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***Abstract*** Image thinning can be achieved in real time using a cellular neural network with 8 planes, each one defined by a set of "peeling templates", and a set of "stopping templates".

This note reports *thinning* with CNN ( Cellular Neural Network )<sup>[1],[2]</sup>. In the parlance of image processing, *thinning* refers to any system which transforms images into *one-pixel-thick* binary (0,1) or bipolar ( $\pm 1$ ) patterns while preserving the *connectivity* of the images.

Fig.1(a), for example, is a handwritten character "6" in a  $45 \times 45$  pixel plane, while Fig.1(b) is its thinned image obtained with a CNN. The *thinning* problem is much more difficult than it looks. Basically, two tasks must be implemented:

- (i) peeling the thick pixels off, and
- (ii) stopping the peeling process when the pixel size reduces to exactly one.

The first part can be achieved with relative ease. The main difficulty lies in (ii) because the "stopping decision" must be done *automatically*. If (i) keeps going, the patterns would disappear all together! In fact there is a vast variety of works on the subject of thinning. See e.g. [3]-[9]. All of them are *digital* and basically *sequential*. Our CNN thinning processor takes an entirely different view point: it is *analog* and *parallel*.

Let  $v_{x_p} \in \mathfrak{R}^{M \times N}$ ,  $p=1, \dots, 8$ , be the state vectors of the following "eight-plane" CNN:

$$C \frac{dv_{x_p}}{dt} = -\frac{1}{R} v_{x_p} + A_{pp} * v_{y_p} + \hat{I} + A_{pq} * v_{y_q} + k \sum_{r \neq (p+4), r \neq p} (v_{y_r} + \hat{I}_e) \quad (1)$$

where  $q = (p+4) \bmod 8$ ,  $\hat{I} = (I, \dots, I)$ ,  $\hat{I}_e = (I_e, \dots, I_e) \in \mathfrak{R}^{M \times N}$ ,  $I, I_e, C, R \in \mathfrak{R}$ ,  
 $v_y = (v_{y_{q11}}, \dots, v_{y_{qMN}})$ ,

$$v_{y_{pij}} = \frac{1}{2} ( |v_{x_{pij}} + 1| - |v_{x_{pij}} - 1| ), \quad 1 \leq i \leq M, \quad 1 \leq j \leq N,$$

and "\*" denotes a 2-dimensional "convolution" operator.

Using the choice of the particular templates given in Table 1 and Table 2, and particular parameter values  $I = I_e = -1.9\text{mA}$ ,  $R = 1\text{k}\Omega$ ,  $C = 1\text{nF}$ , we will demonstrate that 2-dimensional bipolar ( $\pm 1$ ) images can be thinned, where the unit used in Table 1 and Table 2 is  $10^{-3}\Omega^{-1}$ .

Fig.2 shows a schematic representation of (1). Observe the self similar structure of this CNN in that the  $p$ th plane and the  $q$ th plane are connected through  $A_{pq}$ , and then this pair is connected with the  $(p+2, (p+2+4)\text{mod } 8)$ - pair and so forth.

The templates  $A_{pp}$ 's defined in Table 1 are the "peeling" templates, while  $A_{pq}$ 's defined in Table 2 are the "stopping" templates. Note that although the  $A_{pp}$ 's are symmetric, the  $A_{pq}$ 's are not. Moreover,  $A_{62}$  and  $A_{84}$  have an extra -1 in addition to the usual  $3 \times 3$  window. Even though theoretical justifications have not been done yet, this CNN thinning processor works perfectly so far.

To demonstrate our thinning processor, another example is given in Fig.3.

Complete details including performance comparison with the conventional digital methods will appear elsewhere.

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### Table Captions

Table 1 The peeling templates.

Table 2 The stopping templates.

### Figure Captions

Fig.1 Thinning with CNN.

(a) Handwritten character "6" digitized in a  $45 \times 45$  pixel plane.

(b) Thinned image of (a).

Fig.2 Structure of the "thinning" CNN.

Fig.3 Another example of CNN thinning.

(a) Handwritten character "2".

(b) Thinned image of (a).



Table 1 The peeling templates

1.0	1.0	0.0
1.0	2.0	-1.0
0.0	-1.0	0.0

Plane No.1  
A<sub>11</sub>

1.0	1.0	1.0
0.0	3.0	0.0
-1.0	-1.0	-1.0

Plane No.2  
A<sub>22</sub>

0.0	1.0	1.0
-1.0	2.0	1.0
0.0	-1.0	0.0

Plane No.3  
A<sub>33</sub>

1.0	0.0	-1.0
1.0	3.0	-1.0
1.0	0.0	-1.0

Plane No.8  
A<sub>88</sub>

-1.0	0.0	1.0
-1.0	3.0	1.0
-1.0	0.0	1.0

Plane No.4  
A<sub>44</sub>

0.0	-1.0	0.0
1.0	2.0	-1.0
1.0	1.0	0.0

Plane No.7  
A<sub>77</sub>

-1.0	-1.0	-1.0
0.0	3.0	0.0
1.0	1.0	1.0

Plane No.6  
A<sub>66</sub>

0.0	-1.0	0.0
-1.0	2.0	1.0
0.0	1.0	1.0

Plane No.5  
A<sub>55</sub>

Table 2 The stopping templates

0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0
0.0	0.0	8.0	-1.0	0.0
0.0	0.0	-1.0	-2.0	0.0
0.0	0.0	0.0	0.0	0.0

$A_{15}(i,j;k,l)$

0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0
0.0	0.0	8.0	0.0	0.0
0.0	-1.0	-2.0	-1.0	0.0
0.0	0.0	0.0	0.0	0.0

$A_{26}(i,j;k,l)$

0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0
0.0	-1.0	8.0	0.0	0.0
0.0	-2.0	-1.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0

$A_{37}(i,j;k,l)$

0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	-1.0	0.0
0.0	0.0	8.0	-1.0	-1.0
0.0	0.0	0.0	-1.0	0.0
0.0	0.0	0.0	0.0	0.0

$A_{84}(i,j;k,l)$

0.0	0.0	0.0	0.0	0.0
0.0	-1.0	0.0	0.0	0.0
0.0	-2.0	8.0	0.0	0.0
0.0	-1.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0

$A_{48}(i,j;k,l)$

0.0	0.0	0.0	0.0	0.0
0.0	0.0	-2.0	0.0	0.0
0.0	0.0	8.0	-2.0	0.0
0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0

$A_{73}(i,j;k,l)$

0.0	0.0	-1.0	0.0	0.0
0.0	-1.0	-1.0	-1.0	0.0
0.0	0.0	8.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0

$A_{62}(i,j;k,l)$

0.0	0.0	0.0	0.0	0.0
0.0	0.0	-2.0	0.0	0.0
0.0	-2.0	8.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0

$A_{51}(i,j;k,l)$

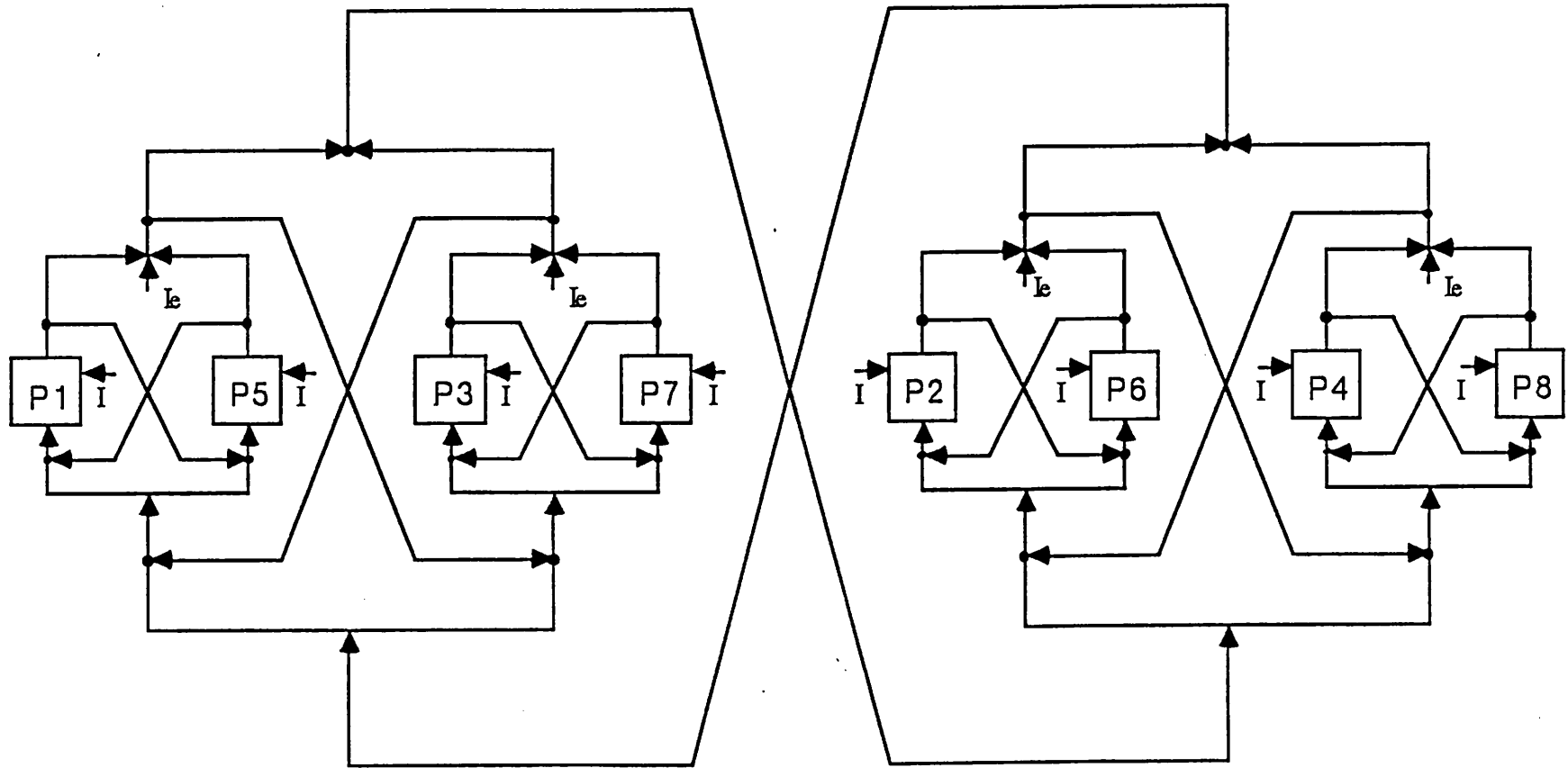
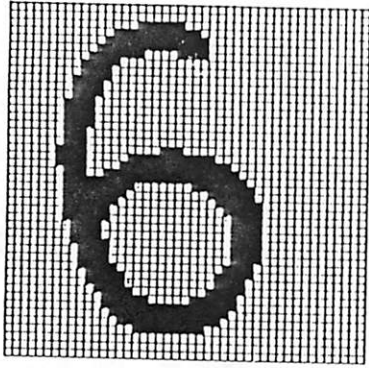
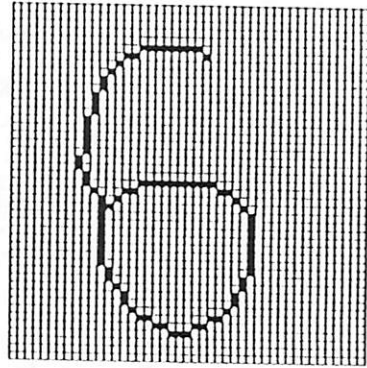


Fig.2 Structure of the "thinning" CNN

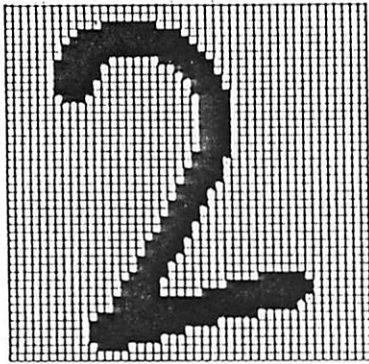


(a)

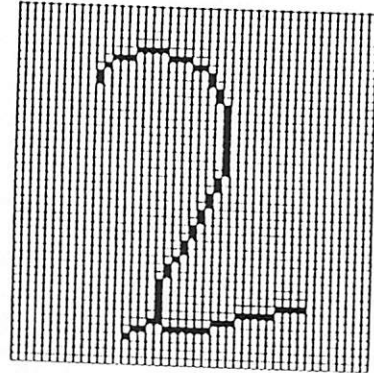


(b)

Fig.1



(a)



(b)

Fig.3