FUZZY LOGIC IN CONTROL SYSTEMS:
FUZZY LOGIC CONTROLLER – II

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Fuzzy Logic in Control Systems: Fuzzy Logic Controller-II

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I. Decision Making Logic

As was pointed out already, an FLC may be regarded as a means of emulating a skilled human operator. More generally, the use of an FLC may be viewed as still another step in the direction of modeling human decision making within the conceptual framework of fuzzy logic and approximate reasoning. In this context, the forward data-driven inference (*generalized modus ponens*) plays an especially important role. In what follows, we shall investigate fuzzy implication functions, the sentence connectives *and* and *also*, compositional operators, inference mechanisms and other concepts which are closely related to the decision making logic of an FLC.

A. Fuzzy Implication Functions

In general, a fuzzy control rule is a fuzzy relation which is expressed as a fuzzy implication. In fuzzy logic, there are many ways in which a fuzzy implication may be defined. The definition of a fuzzy implication may be expressed as a fuzzy implication function. The choice of a fuzzy implication function reflects not only the intuitive criteria for implication but also the effect of connective *also.*

1. Basic Properties of a Fuzzy Implication Function

The choice of a fuzzy implication function involves a number of criteria, which are discussed in [3,24,2,71,18,52,19,116,85,72,96]. In particular, Baldwin and Pilsworth [3] considered the following basic characteristics of a fuzzy implication function: fundamental property, smoothness property, unrestricted inference, symmetry of *generalized modus ponens* and *generalized modus tollens*, and a measure of propagation of fuzziness. All of these properties are justified on purely intuitive grounds. We prefer to say that the inference (consequence) should be as close to the *input truth function value* as possible, rather than be equal to it. This gives us a more flexible criterion for choosing a fuzzy implication function. Furthermore, in a chain of implications, it is necessary to consider the *fuzzy syllogism* [147] associated with each fuzzy implication function before we can talk about the propagation of fuzziness.

Fukami, Mizumoto, and Tanaka [24] have proposed a set of intuitive criteria for choosing a fuzzy implication function which constrains the relations between the antecedents and consequents of a conditional proposition, with the latter playing the role of a premise in approximate reasoning. As is well-known, there are two important fuzzy implication inference rules in approximate reasoning. They are the *generalized modus ponens* (GMP) and the *generalized modus tollens* (GMT). Specifically,

$$\begin{array}{l} \textit{premise } 1: \ x \textit{ is } A' \\ \underline{\textit{premise } 2: \ \textit{if } x \textit{ is } A \textit{ then } y \textit{ is } B} \\ \textit{consequence}: y \textit{ is } B' \end{array} \qquad \text{(GMP)}$$

$$\begin{array}{l} \textit{premise } 1: \ y \textit{ is } B' \\ \underline{\textit{premise } 2: \ \textit{if } x \textit{ is } A \textit{ then } y \textit{ is } B} \\ \textit{consequence}: x \textit{ is } A' \end{array} \qquad \text{(GMT)}$$

in which A, A', B and B' are fuzzy predicates; the propositions above the line are the premises; and the proposition below the line is the consequence. The proposed criteria are summarized in Table I and Table II. We note that if a causal relation between x *is* A and y *is* B is not strong in a fuzzy implication, the satisfaction of criterion 2-2 and criterion 3-2 is allowed. Criterion 4-2 is interpreted as *if* x *is* A *then* y *is* B, *else* y *is not* B. Although this relation is not valid in formal logic, we often make such an interpretation in everyday reasoning. The same applies to criterion 8.

2. *Families of Fuzzy Implication Functions*

Following Zadeh's [146] introduction of the compositional rule of inference in approximate reasoning, a number of researchers have proposed various implication functions in which the antecedents and consequents contain fuzzy variables. Indeed, nearly forty distinct fuzzy implication functions have been described in the literature. In general, they can be classified into three main categories: the *fuzzy conjunction*, the *fuzzy disjunction*, and the *fuzzy implication*. The former two bear a close relation to a fuzzy Cartesian product. The latter is a generalization of implication in multiple-valued logic, and relates to the extension of material implication, implication in propositional calculus, *modus ponens*, and *modus tollens* [18]. In what follows, after a short review of triangular norms and triangular co-norms, we shall give the definitions of fuzzy conjunction, fuzzy disjunction, and fuzzy implication. Some fuzzy implication functions, which are often employed in an FLC and are commonly found in the literature, will be derived.

Definition 1: Triangular Norms. The triangular norm $*$ is a two-place function from $[0,1]\times[0,1]$ to $[0,1]$, i.e., $* : [0,1]\times[0,1] \rightarrow [0,1]$, which includes *intersection, algebraic product, bounded product*, and *drastic product.* The greatest triangular norm is the intersection and the least one is the drastic product. The operations associated with triangular norms are defined for all $x, y \in [0,1]$:

intersection $\quad x \wedge y = \min\{x, y\}$

algebraic product $\quad x \cdot y = xy$

bounded product $\quad x \odot y = \max\{0, x+y-1\}$

drastic product
$$x \cap y = \begin{cases} x & y=1 \\ y & x=1 \\ 0 & x, y < 1 \end{cases}$$

Definition 2: Triangular Co-Norms. The triangular co-norms $\dot{+}$ is a two-place function from $[0,1]\times[0,1]$ to $[0,1]$, i.e., $\dot{+} : [0,1]\times[0,1]$ to $[0,1]$, which includes *union, algebraic sum, bounded sum, drastic sum*, and *disjoint sum.* The operations associated with triangular co-norms are defined for all $x, y \in [0,1]$:

union $\quad x \vee y = \max\{x, y\}$

algebraic sum $\quad x \hat{+} y = x + y - xy$

bounded sum $\quad x \oplus y = \min\{1, x + y\}$

drastic sum
$$x \cup y = \begin{cases} x & y=0 \\ y & x=0 \\ 1 & x, y > 0 \end{cases}$$

disjoint sum $\quad x \Delta y = \max\{\min(x, 1-y), \min(1-x, y)\}$

The triangular norms are employed for defining conjunctions in approximate reasoning, while the triangular co-norms serve the same role for disjunctions. A fuzzy control rule, *if* x *is* A *then* y *is* B, is represented by a fuzzy implication function and is denoted by $A \rightarrow B$, where A and B are fuzzy sets in universes U and V with membership functions μ_A and μ_B, respectively.

Definition 3: Fuzzy Conjunction. The *fuzzy conjunction* is defined for all $u \in U$ and $v \in V$ by

$$A \rightarrow B = A \times B$$
$$= \int_{U \times V} \mu_A(u) * \mu_B(v) /(u,v)$$

where $*$ is an operator representing a triangular norm.

Definition 4: Fuzzy Disjunction. The *fuzzy disjunction* is defined for all $u \in U$ and $v \in V$ by

$$A \rightarrow B = A \times B$$
$$= \int_{U \times V} \mu_A(u) \dotplus \mu_B(v) /(u,v)$$

where $\dotplus$ is an operator representing a triangular co-norm.

Definition 5: Fuzzy Implication. The *fuzzy implication* is associated with five families of fuzzy implication functions in use. As before, $*$ denotes a triangular norm and $\dotplus$ is a triangular co-norm.

4.1) material implication

$$A \rightarrow B = (not\ A) \dotplus B$$

4.2) propositional calculus

$$A \rightarrow B = (not\ A) \dotplus (A * B)$$

4.3) extended propositional calculus

$$A \rightarrow B = (not\ A \times not\ B) \dotplus B$$

4.4) generalization of *modus ponens*

$$A \rightarrow B = sup\ \{\ c \in [0,1], A * c \leq B\ \}$$

4.5) generalization of *modus tollens*

$$A \rightarrow B = inf\ \{\ t \in [0,1], B \dotplus t \leq A\ \}$$

Based on these definitions, many fuzzy implication functions may be generated by employing the triangular norms and co-norms. For example, by using the definition of the fuzzy conjunction, Mamdani's mini fuzzy implication, R_c, is obtained if the intersection operator is used. Larsen's product fuzzy implication, R_p, is obtained if the algebraic product is used. Furthermore, R_{bp} and R_{dp} are obtained if the bounded product and the drastic product are used, respectively. The following fuzzy implications, which are often adopted in an FLC, will be discussed more at a later point.

Mini operation rule of fuzzy implication [Mamdani]

$$R_c = A \times B$$

$$= \int_{U \times V} \mu_A(u) \wedge \mu_B(v) \, /(u,v)$$

Product operation rule of fuzzy implication [Larsen]

$$R_p = A \times B$$

$$= \int_{U \times V} \mu_A(u)\mu_B(v) \, /(u,v)$$

Arithmetic rule of fuzzy implication [Zadeh]

$$R_a = (not\ A \times V) \oplus (U \times B)$$

$$= \int_{U \times V} 1 \wedge (1 - \mu_A(u) + \mu_B(v)) \, /(u,v)$$

Maxmin rule of fuzzy implication [Zadeh]

$$R_m = (A \times B) \cup (not\ A \times V)$$

$$= \int_{U \times V} (\mu_A(u) \wedge \mu_B(v)) \vee (1 - \mu_A(u)) \, /(u,v)$$

Standard sequence fuzzy implication

$$R_s = A \times V \rightarrow U \times B$$

$$= \int_{U \times V} (\mu_A(u) > \mu_B(v)) \, /(u,v)$$

$$\text{where } \mu_A(u) > \mu_B(v) = \begin{cases} 1 & \mu_A(u) \le \mu_B(v) \\ 0 & \mu_A(u) > \mu_B(v) \end{cases}$$

Boolean fuzzy implication

$$R_b = (not\ A \times V) \cup (U \times B)$$

$$= \int_{U \times V} (1 - \mu_A(u)) \vee (\mu_B(v)) \,/(u, v)$$

Goguen's fuzzy implication

$$R_\Delta = A \times V \rightarrow U \times B$$

$$= \int_{U \times V} (\mu_A(u) \gg \mu_B(v)) \,/(u, v)$$

$$\text{where } \mu_A(u) \gg \mu_B(v) = \begin{cases} 1 & \mu_A(u) \le \mu_B(v) \\ \dfrac{\mu_B(u)}{\mu_A(v)} & \mu_A(u) > \mu_B(v) \end{cases}$$

We note that Zadeh's arithmetic rule follows from Definition 5.1 by using the bounded sum operator; Zadeh's maxmin rule follows from Definition 5.2 by using the intersection and union operators; the standard sequence implication follows from Definition 5.4 by using the bounded product; Boolean fuzzy implication follows from Definition 5.1 by using the union; and Goguen's fuzzy implication follows from Definition 5.4 by using the algebraic product.

3. *Choice of a Fuzzy Implication Function*

First, we investigate the consequences resulting from applying the above forms of fuzzy implication in fuzzy inference and, in particular, the GMP and GMT. The inference is based on the sup-min compositional rule of inference. In the GMP, we examine the consequence of the following compositional equation:

$$B' = A' \circ R$$

where R: fuzzy implication (relation)

$\circ$: sup–min compositional operator

A': a fuzzy set which has the form

$$A = \int_U \mu_A(u)/u$$

$$very\ A = A^2 = \int_U \mu_A^2(u)/u$$

$$more\ or\ less\ A = A^{0.5} = \int_U \mu_A^{0.5}(u)/u$$

$$not\ A = \int_U 1 - \mu_{A(u)}/u$$

Similarly, in the GMT, we examine the consequence of the following equation:

$$A' = R \circ B'$$

where R: fuzzy implication (relation)

B': a fuzzy set which has the form

$$not\ B = \int_V 1 - \mu_B(u)/u$$

$$not\ very\ B = \int_V 1 - \mu_B^2(u)/u$$

$$not\ more\ or\ less\ B = \int_V 1 - \mu_B^{0.5}(u)/u$$

$$B = \int_V \mu_B(u)/u$$

The Case of R_p : Larsen's Product Rule

A method for computing the *generalized modus ponens* and the *generalized modus tollens* laws of inference is described in [3]. The graphs corresponding to Larsen's fuzzy implication R_p are given in Figure 1. The graph with parameter μ_A is used for the GMP and the graph with μ_B is used for the GMT.

Larsen's Product Rule in GMP:

Suppose that $A' = A^\alpha$ (α>0); then the consequence B_p' is inferred as follows:

$$B_p' = A^\alpha \circ R_p$$

$$= \int_U \mu_A^\alpha(u)/u \circ \int_{U \times V} \mu_A(u) \cdot \mu_B(v)/(u,v).$$

The membership function $\mu_{B_p'}$ of the fuzzy set B_p' is pointwise defined for all $v \in V$ by

$$\mu_{B_p'}(v) = \sup_{u \in U} \min \{\mu_A^\alpha(u), \mu_A(u)\mu_B(v)\}$$

$$= \sup_{u \in U} S_p(\mu_A^\alpha(u))$$

where

$$S_p(\mu_A^\alpha(u)) \triangleq \min \{\mu_A^\alpha(u), \mu_A(u)\mu_B(v)\}.$$

{A'=A}:

The values of $S_p(\mu_A(u))$ with a parameter $\mu_B(v)$, say $\mu_B(v)$=0.3 and 0.8, are indicated in Figure 2 by a broken line and dotted line, respectively. The membership function $\mu_{B_p'}$ is obtained by

$$\mu_{B_p'}(v) = \sup_{u \in U} \min \{\mu_A(u), \mu_A(u)\mu_B(v)\}$$

$$= \sup_{u \in U} \mu_A(u)\mu_B(v)$$

$$= \mu_B(v), \quad \mu_A(u)=1.$$

{A'=A^2}:

The values of $S_p(\mu_A^2(u))$ with a parameter $\mu_B(v)$, say $\mu_B(v)$=0.3 and 0.8, are indicated in Figure 3 by a broken line and dotted line, respectively. The membership function $\mu_{B_p'}$ may be expressed as

$$\mu_{B_p'}(v) = \sup_{u \in U} \min \{\mu_A^2(u), \mu_A(u)\mu_B(v)\}$$

$$= \mu_B(v).$$

{A'=$A^{0.5}$}:

The values of $S_p(\mu_A^{0.5}(u))$ with a parameter $\mu_B(v)$, say $\mu_B(v)$=0.3 and 0.8, are indicated in Figure 4 by a broken line and dotted line, respectively. The membership function $\mu_{B_p'}$ is given by

$$\mu_{B_p'}(v) = \sup_{u \in U} \min \{\mu_A^{0.5}(u), \mu_A(u)\mu_B(v)\}$$

$$= \mu_B(v).$$

{A'=not A}:

The values of $S_p(1-\mu_A(u))$ with a parameter $\mu_B(v)$, say $\mu_B(v)$=0.3 and 0.8, are indicated in Figure 5 by a broken line and dotted line, respectively. The membership function $\mu_{B_p'}$ is given by

$$\mu_{B_p'}(v) = \sup_{u \in U} \min \{1-\mu_A(u), \mu_A(u)\mu_B(v)\}$$

$$= \frac{\mu_B(v)}{1+\mu_B(v)}.$$

Larsen's Product Rule in GMT:

Suppose that B'=not B^α (α>0); then the consequence A_p' is inferred as follows:

$$A_t' = R_p \circ (not\ B^{\alpha})$$

$$= \int_{U\times V} \mu_A(u)\mu_B(v)/(u,v) \circ \int_V (1-\mu_B^{\alpha}(v))/v.$$

The membership function $\mu_{A_t'}$ of the fuzzy set A_t' is pointwise defined for all $u \in U$ by

$$\mu_{A_t'}(u) = \sup_{v\in V} \min\ \{\mu_A(u)\mu_B(v),\ 1-\mu_B^{\alpha}(v)\}$$

$$= \sup_{v\in V} S_t(\mu_B^{\alpha}(v))$$

where

$$S_t(1-\mu_B^{\alpha}(v)) \triangleq \min\ \{\mu_A(u)\mu_B(v),\ 1-\mu_B^{\alpha}(v)\}.$$

{B'=not B}:

The values of $S_t(1-\mu_B(v))$ with a parameter $\mu_A(u)$, say $\mu_A(u)$=0.3 and 0.8, are indicated in Figure 6 by a broken line and dotted line, respectively. The membership function $\mu_{A_t'}$ is given by

$$\mu_{A_t'}(u) = \sup_{v\in V} \min\ \{\mu_A(u)\mu_B(v),\ 1-\mu_B(v)\}$$

$$= \frac{\mu_A(u)}{1+\mu_A(u)}.$$

{B'=*not* B^2}:

The values of $S_t(1-\mu_B^2(v))$ with a parameter $\mu_A(u)$, say $\mu_A(u)$=0.3 and 0.8, are indicated in Figure 7 by a broken line and dotted line, respectively. The membership function $\mu_{A_t'}$ is given by

$$\mu_{A_t'}(u) = \sup_{v\in V} \min\ \{\mu_A(u)\mu_B(v),\ 1-\mu_B^2(v)\}$$

$$= \frac{\mu_A(u)\sqrt{\mu_A^2(u)+4} - \mu_A(u)}{2}.$$

{B'=*not* $B^{0.5}$}:

The values of $S_t(1-\mu_B^{0.5}(v))$ with a parameter $\mu_A(u)$, say $\mu_A(u)$=0.3 and 0.8, are indicated in Figure 8 by a broken line and dotted line, respectively. The membership function $\mu_{A_t'}$ is given by

$$\mu_{A_t'}(u) = \sup_{v\in V} \min\ \{\mu_A(u)\mu_B(v),\ 1-\mu_B^{0.5}(v)\}$$

$$= \frac{2\mu_A(u) + 1 - \sqrt{4\mu_A + 4}}{2\mu_A(u)}.$$

{B'=B}:

The values of $S_t(\mu_B(v))$ with a parameter $\mu_A(u)$, say $\mu_A(u)$=0.3 and 0.8, are indicated in Figure 9 by a broken line and dotted line, respectively. The membership function $\mu_{A_t'}$ is given by

$$\mu_{A_t'}(u) = \sup_{v \in V} \{\mu_A(u)\mu_B(v), \mu_B(v)\}$$

$$= \mu_A(u).$$

The remaining consequences [24] inferred by $R_a, R_c, R_m, R_s, R_b, R_\Delta$ can be obtained by the same method as described above. The results are summarized in Table III and Table IV.

By employing the intuitive criteria in Table I and Table II in Table III and Table IV, we can determine how well a fuzzy implication function satisfies them. This information is summarized in Table V.

In FLC applications, a control action is determined by the observed inputs and the control rules, without the consequent of one rule serving as the antecedent of another. In effect, the FLC functions as a one-level forward data-driven inference (GMP). Thus, the backward goal-driven inference (GMT), chaining inference mechanisms (syllogisms), and contraposition do not play a role in the FLC, since there is no neeed to infer a fuzzy control action through the use of these inference mechanisms.

Although R_c and R_p do not have a well-defined logical structure, the results tabulated in Table V indicate that they are well-suited for approximate reasoning, especially for the *generalized modus ponens*.

R_m has a logical structure which is similar to R_b. R_a is based on the implication rule in Lukasiewicz's logic L_{Aleph}. However, R_m and R_a are not well-suited for approximate reasoning since the inferred consequences do not always fit our intuition. Furthermore, for multiple-valued logical systems, R_b and R_Δ have significant shortcomings. Overall, R_s yields reasonable results and thus constitutes an appropriate choice for use in approximate reasoning.

B. Interpretation of Sentence Connectives and, also

In most of the existing FLCs, the sentence connective *and* is usually implemented as a fuzzy conjunction in a Cartesian product space in which the underlying variables take values in different universes of discourse. As an illustration, in *if* (A *and* B) *then* C, the antecedent is interpreted as a fuzzy set in the product space $U \times V$, with the membership function given by

$$\mu_{A \times B}(u,v) = \min \{\mu_A(u), \mu_B(v)\}$$

or

$$\mu_{A \times B}(u,v) = \mu_A(u) \cdot \mu_B(v)$$

where U and V are the universes of discourse associated with A and B, respectively.

When a fuzzy system is characterized by a set of fuzzy control rules, the ordering of the rules is immaterial. This necessitates that the sentence connective *also* should have the properties of commutativity and associativity (See A and C in Section III in Part I and D in this Section). In this connection, it should be noted that the operators in triangular norms and co-norms possess these properties and thus qualify as the candidates for the interpretation of the connective *also.* In general, we use the triangular co-norms in association with fuzzy conjunction and disjunction, and the triangular norms in association with fuzzy implication. The experimental results [52,53,54,96,73] and the theoretical studies [18,85,116,19] relate to this issue.

Kiszka et al. [52] described a preliminary investigation of the fuzzy implication functions and the sentence connective *also* in the context of the fuzzy model of a DC series motor. In later work, they presented additional results for fuzzy implication functions and the connective *also* in terms of the union and intersection operators [53,54].

Our investigation leads to some preliminary conclusions. First, the connective *also* has a substantial influence on the quality of a fuzzy model, as we might expect. Fuzzy implication functions such as R_s, R_Δ, and R_a with the connective *also* defined as the union operator, and R_c, R_p, R_{bp}, and R_{dp} defined as the intersection, yield satisfactory results. These fuzzy implication functions differ in the number of mathematical operations which are needed for computer implementation.

Recently, Stachowicz and Kochanska [96] studied the characteristics of thirty eight types of fuzzy implication along with nine different interpretations (in terms of triangular norms and co-norms) of the connective *also*, based on various forms of the operational curve of a series motor. Based on their results, we tabulate in Table VI a summary of the most appropriate pairs for the FLC of the fuzzy implication function and the connective *also.*

Additional results relating to the interpretation of the connective *also* as the union and the intersection are reported in [73]. The investigation in question is based on a plant model with first order delay. It is established that the fuzzy implication functions R_c, R_p, R_{bp}, R_{dp} with the connective *also* as the union operator yield the best control results. Furthermore, the fuzzy implications R_s and R_g are not well-suited for control applications even though they yield reasonably good results in approximate reasoning.

From a practical point of view, the computational aspects of an FLC require a simplification of the fuzzy control algorithm. In this perspective, Mamdani's R_c and Larsen's R_p with the connective *also* as the union operator appear to be better suited for constructing fuzzy models than the other methods in FLC applications. We will have more to say about these methods at a later point.

C. Compositional Operators

In a general form, a compositional operators may be expressed as the sup-star composition, where "star" denotes an operator, e.g., min, product, etc., which is chosen to fit a specific application. In the literature, four kinds of compositional operators can be used in the compositional rule of inference, namely:

sup-min operation [Zadeh,1973],

sup-product operation [Kaufmann,1975],

sup-bounded-product operation [Mizumoto,1981],

sup-drastic-product operation [Mizumoto,1981].

In FLC applications, the sup-min and sup-product compositional operators are the most frequently used. The reason is obvious when the computational aspects of an FLC are considered. However, interesting results can be obtained if we apply the sup-product, sup-bounded-product and sup-drastic-product operations with different fuzzy implication functions in approximate reasoning [70,72]. The inferred results employing these compositional operators are better than those employing the sup-min operator. Further investigation of these issues in the context of the accuracy of fuzzy models may provide interesting results.

D. Inference Mechanisms

The inference mechanisms employed in an FLC are generally much simpler than those used in a typical expert system since in an FLC the consequent of a rule is not applied to the antecedent of another. In other words, in FLC we do not employ the chaining inference mechanism since the control actions are based on an one-level forward data-driven inference (GMP).

The rule base of an FLC is usually derived from expert knowledge. Typically, the rule base has the form of a MIMO system[1]

$$R = \{R_{MIMO}^1, R_{MIMO}^2, \cdots, R_{MIMO}^n\}$$

where R_{MIMO}^i represents the rule: *if* (x *is* A_i *and* $\cdots$, *and* y *is* B_i) *then* (z_1 *is* $C_i, \cdots, z_q$ *is* D_i). The antecedent of R_{MIMO}^i forms a fuzzy set $A_i \times \cdots \times B_i$ in the product space $U \times \cdots \times V$. The consequent is the union of q independent control actions. Thus, the i^{th} rule R_{MIMO}^i may be represented as a fuzzy implication

$$R_{MIMO}^i : \ (A_i \times \cdots \times B_i) \rightarrow (z_1 + \cdots + z_q)$$

from which it follows that the rule base R may be represented as the union

$$\begin{aligned}
R &= \{ \bigcup_{i=1}^{n} R_{MIMO}^i \} \\
&= \{ \bigcup_{i=1}^{n} [\ (A_i \times \cdots \times B_i) \rightarrow (z_1 + \cdots + z_q)\ ]\ \} \\
&= \{ \bigcup_{i=1}^{n} [(A_i \times \cdots \times B_i) \rightarrow z_1)], \bigcup_{i=1}^{n} [(A_i \times \cdots \times B_i) \rightarrow z_2)], \cdots, \\
&\qquad \bigcup_{i=1}^{n} [(A_i \times \cdots \times B_i) \rightarrow z_q)]\ \} \\
&= \{ \bigcup_{k=1}^{q} \bigcup_{i=1}^{n} [(A_i \times \cdots \times B_i) \rightarrow z_k)]\ \} \\
&= \{RB_{MISO}^1, RB_{MISO}^2, \cdots, RB_{MISO}^q\}.
\end{aligned}$$

In effect, the rule base R of an FLC is composed of a set of sub-rule-bases RB_{MISO}^i, with each sub-rule-base RB_{MISO}^i consisting of n fuzzy control rules with multiple process state variables and a single control variable. The general rule structure of a MIMO fuzzy system can therefore be represented as a collection of MISO fuzzy systems:

$$R = \{RB_{MISO}^1, RB_{MISO}^2, \cdots, RB_{MISO}^q\}$$

where RB_{MISO}^k represents the rule: *if* (x *is* A_i *and* $\cdots$, *and* y *is* B_i) *then* (z_k *is* D_i), $i=1,2, \ldots ,n$.

Let us consider the following general form of MISO fuzzy control rules in the case of two-input-single-output fuzzy systems:

$$\textit{Input} : x \textit{ is } A' \textit{ and } y \textit{ is } B'$$

$$R_1 : \textit{if } x \textit{ is } A_1 \textit{ and } y \textit{ is } B_1 \textit{ then } z \textit{ is } C_1$$

1 See Section III of Part I in Fuzzy Conditional Statements and Fuzzy Control Rules.

$$also\ \ R_2 : if\ x\ is\ A_2\ and\ y\ is\ B_2\ \ then\ z\ is\ C_2$$

$$\cdots\ \cdots\ \cdots\ \cdots$$

$$\cdots\ \cdots\ \cdots\ \cdots$$

$$\underline{also\ \ R_n : if\ x\ is\ A_n\ \ and\ y\ is\ B_n\ \ then\ z\ is\ C_n}$$

$$z\ is\ C'$$

where x, y, and z are linguistic variables representing the process state variables and the control variable, respectively; A_i, B_i, and C_i are linguistic values of the linguistic variables x, y, and z in the universes of discourse U, V, and W, respectively, with $i=1,2, \cdots ,n$.

The fuzzy control rule, *if* (x *is* A_i *and* y *is* B_i) *then* (z *is* C_i) is implemented as a fuzzy implication (relation) R_i and is defined as:

$$\mu_{R_i} \triangleq \mu_{(A_i\ and\ B_i\ \rightarrow C_i)}(u,\ v,\ w) = [\mu_{A_i}(u)\ and\ \mu_{B_i}(v)] \rightarrow \mu_{C_i}(w)$$

where A_i *and* B_i is a fuzzy set $A_i{\times}B_i$ in $U{\times}V$, $R_i \triangleq (A_i\ and\ B_i) \rightarrow C_i$ is a fuzzy implication (relation) in $U{\times}V{\times}W$ and $\rightarrow$ denotes a fuzzy implication function.

The consequence C' is deduced from the sup-star compositional rule of inference employing the definitions of a fuzzy implication function and the connectives *and* and *also*.

In what follows, we shall consider some useful properties of the FLC inference mechanism. First, we would like to show that the sup-min operator denoted by $\circ$ and the connective *also* as the union operator are commutative. Thus, the fuzzy control action inferred from the complete set of fuzzy control rules is equivalent to the aggregated result derived from individual control rules. Furthermore, as will be shown later, the same properties are possessed by the sup-product operator. However, the conclusion in question does not apply when the fuzzy implication is used in its traditional logical sense [18,19]. More specifically, we have

Lemma 1: $(A',B') \circ \bigcup_{i=1}^{n} R_i = \bigcup_{i=1}^{n} (A',B') \circ R_i$.

Proof:

$$C' = (A',B') \circ \bigcup_{i=1}^{n} R_i$$

$$= (A',B') \circ \bigcup_{i=1}^{n} (A_i \text{ and } B_i \rightarrow C_i).$$

The membership function $\mu_{C'}$ of the fuzzy set C' is pointwise defined for all $w \in W$ by

$$\mu_{C'}(w) = (\mu_{A'}(u), \mu_{B'}(v)) \circ \max_{u,v,w} (\mu_{R_1}(u,v,w),\ \mu_{R_2}(u,v,w),\ \cdots,\ \mu_{R_n}(u,v,w))$$

$$= \sup_{u,v} \min\ \{\ (\mu_{A'}(u),\ \mu_{B'}(v)),\ \max_{u,v,w} (\mu_{R_1}(u,v,w),\ \mu_{R_2}(u,v,w),\ \cdots,\ \mu_{R_n}(u,v,w))\ \}$$

$$= \sup_{u,v} \max_{u,v,w} \{ \min [(\mu_{A'}(u), \mu_{B'}(v)), \mu_{R_1}(u,v,w)], \cdots, \min [(\mu_{A'}(u),\mu_{B'}(v)), \mu_{R_n}(u,v,w)] \}$$

$$= \max_{u,v,w} \{ [(\mu_{A'}(u),\mu_{B'}(v)) \circ \mu_{R_1}(u,v,w)], \cdots, [(\mu_{A'}(u),\mu_{B'}(v)) \circ \mu_{R_n}(u,v,w)] \}.$$

Therefore,

$$C' = [(A',B') \circ R_1] \cup [(A',B') \circ R_2] \cup \cdots \cup [(A',B') \circ R_n]$$

$$= \bigcup_{i=1}^{n} (A',B') \circ R_i$$

$$= \bigcup_{i=1}^{n} (A',B') \circ (A_i \text{ and } B_i \rightarrow C_i)$$

$$\triangleq \bigcup_{i=1}^{n} C_i'. \qquad \text{QED}$$

Lemma 2: For the fuzzy conjunctions R_c, R_p, R_{bp} and R_{dp}, we have:

$$(A',B')\circ(A_i \text{ and } B_i \rightarrow C_i) = [A'\circ(A_i \rightarrow C_i)] \cap [B'\circ(B_i \rightarrow C_i)] \quad \text{if } \mu_{A_i \times B_i} = \mu_{A_i} \wedge \mu_{B_i}$$

$$(A',B')\circ(A_i \text{ and } B_i \rightarrow C_i) = [A'\circ(A_i \rightarrow C_i)] \cdot [B'\circ(B_i \rightarrow C_i)] \quad \text{if } \mu_{A_i \times B_i} = \mu_{A_i} \cdot \mu_{B_i}$$

Proof:

$$C_i' = (A',B') \circ (A_i \text{ and } B_i \rightarrow C_i)$$

$$\mu_{C_i'} = (\mu_{A'},\mu_{B'}) \circ (\mu_{A_i \times B_i} \rightarrow \mu_{C_i})$$

$$= (\mu_{A'},\mu_{B'}) \circ (\min (\mu_{A_i},\mu_{B_i}) \rightarrow \mu_{C_i})$$

$$= (\mu_{A'},\mu_{B'}) \circ \min [(\mu_{A_i} \rightarrow \mu_{C_i}), (\mu_{B_i} \rightarrow \mu_{C_i})]$$

$$= \sup_{u,v} \min \{ [(\mu_{A'},\mu_{B'}), \min [(\mu_{A_i} \rightarrow \mu_{C_i}), (\mu_{B_i} \rightarrow \mu_{C_i})] \}$$

$$= \sup_{u,v} \min \{ \min [\mu_{A'}, (\mu_{A_i} \rightarrow \mu_{C_i})], \min [\mu_{B'}, (\mu_{B_i} \rightarrow \mu_{C_i})] \}$$

$$= \min \{ [\mu_{A'} \circ (\mu_{A_i} \rightarrow \mu_{C_i})], [\mu_{B'} \circ (\mu_{B_i} \rightarrow \mu_{C_i})] \}.$$

Hence, we obtain

$$C_i' = [A'\circ(A_i \rightarrow C_i)] \cap [B'\circ(B_i \rightarrow C_i)]. \qquad \text{QED}$$

Let us consider two special cases which follow from the above lemma, and which play an important role in FLC applications.

Lemma 3: If the inputs are fuzzy singletons, namely, $A'=u_0$, $B'=v_0$, then the consequences employing Mamdani's minimum operation rule R_c and Larsen's product operation rule R_p, respectively, may be expressed simply as

$$(i)\quad \begin{matrix} R_c: \alpha_i^{\wedge} \wedge \mu_{C_i}(w) \\ R_p: \alpha_i^{\wedge} \cdot \mu_{C_i}(w) \end{matrix} \qquad (ii)\quad \begin{matrix} R_c: \alpha_i^{\bullet} \wedge \mu_{C_i}(w) \\ R_p: \alpha_i^{\bullet} \cdot \mu_{C_i}(w) \end{matrix}$$

where $\alpha_i^{\wedge} = \mu_{A_i}(u_0) \wedge \mu_{B_i}(v_0)$ and $\alpha_i^{\bullet} = \mu_{A_i}(u_0) \cdot \mu_{B_i}(v_0)$.

Proof:

(i):

$$C_i' = [A' \circ (A_i \rightarrow C_i)] \cap [B' \circ (B_i \rightarrow C_i)]$$

$$\mu_{C_i'} = \min \{ [\mu_0 \circ (\mu_{A_i}(u) \rightarrow \mu_{C_i}(w))], [v_0 \circ (\mu_{B_i}(v) \rightarrow \mu_{C_i}(w))] \}$$

$$= \min \{ [\mu_{A_i}(u_0) \rightarrow \mu_{C_i}(w)], [\mu_{B_i}(v_0) \rightarrow \mu_{C_i}(w)] \}.$$

(ii):

$$C_i' = [A' \circ (A_i \rightarrow C_i)] \cdot [B' \circ (B_i \rightarrow C_i)]$$

$$\mu_{C_i'} = [\mu_0 \circ (\mu_{A_i}(u) \rightarrow \mu_{C_i}(w))] \cdot [v_0 \circ (\mu_{B_i}(v) \rightarrow \mu_{C_i}(w))]$$

$$= [\mu_{A_i}(u_0) \rightarrow \mu_{C_i}(w)] \cdot [\mu_{B_i}(v_0) \rightarrow \mu_{C_i}(w)].$$

QED

As will be seen in following section, the last lemma not only simplifies the process of computation, but also provides a graphic interpretation of the fuzzy inference mechanism in the FLC. Turning to the sup-product operator, which is denoted as •, we have

Lemma 1': $(A', B') \bullet \bigcup_{i=1}^{n} R_i = \bigcup_{i=1}^{n} (A', B') \bullet R_i$.

Lemma 2': For the fuzzy conjunctions R_c, R_p, R_{bp} and R_{dp}, we have:

$$(A', B') \bullet (A_i \text{ and } B_i \rightarrow C_i) = [A' \bullet (A_i \rightarrow C_i)] \cap [B' \bullet (B_i \rightarrow C_i)] \quad \text{if } \mu_{A_i \times B_i} = \mu_{A_i} \wedge \mu_{B_i}$$

$$(A', B') \bullet (A_i \text{ and } B_i \rightarrow C_i) = [A' \bullet (A_i \rightarrow C_i)] \cdot [B' \bullet (B_i \rightarrow C_i)] \quad \text{if } \mu_{A_i \times B_i} = \mu_{A_i} \cdot \mu_{B_i}$$

Lemma 3': If the inputs are fuzzy singletons, namely, $A' = u_0$, $B' = v_0$, then the consequences employing Mamdani's minimum operation rule R_c and Larsen's product operation rule R_p, respectively, may be expressed simply as

$$(i)\quad \begin{matrix} R_c: \alpha_i^{\wedge} \wedge \mu_{C_i}(w) \\ R_p: \alpha_i^{\wedge} \cdot \mu_{C_i}(w) \end{matrix} \qquad (ii)\quad \begin{matrix} R_c: \alpha_i^{\bullet} \wedge \mu_{C_i}(w) \\ R_p: \alpha_i^{\bullet} \cdot \mu_{C_i}(w) \end{matrix}$$

where $\alpha_i^{\wedge} = \mu_{A_i}(u_0) \wedge \mu_{B_i}(v_0)$ and $\alpha_i^{\bullet} = \mu_{A_i}(u_0) \cdot \mu_{B_i}(v_0)$.

Therefore, we can assert that

$$R_c: \ \mu_{C'} = \bigcup_{i=1}^{n} \alpha_i \wedge \mu_{C_i}$$

$$R_p: \ \mu_{C'} = \bigcup_{i=1}^{n} \alpha_i \cdot \mu_{C_i},$$

where the weighting factor (firing strength) α_i is a measure of the contribution of the i^{th} rule to the fuzzy control action. The weighting factor in question may be determined by two methods. The first uses the minimum operation in the Cartesian product, which is widely used in FLC applications. The second employs the algebraic product in the Cartesian product, thus preserving the contribution of each input variable rather than the dominant one only. In this respect, it appears to be a reasonable choice in many FLC applications.

For simplicity, assume that we have two fuzzy control rules, as follows:

$$R_1: \ \textit{if } x \textit{ is } A_1 \textit{ and } y \textit{ is } B_1 \textit{ then } z \textit{ is } C_1,$$

$$R_2: \ \textit{if } x \textit{ is } A_2 \textit{ and } y \textit{ is } B_2 \textit{ then } z \textit{ is } C_2.$$

Figure 10 illustrates a graphic interpretation of lemma 2 under R_c and $\alpha_i^\wedge$. Figure 11 shows a graphic interpretation of lemma 2 under R_p and $\alpha_i^\wedge$.

In on-line processes, the states of a control system play an essential role in control actions. The inputs are usually measured by sensors and are crisp. In some cases, it may be expedient to convert the input data into fuzzy sets[2]. In general, however, a crisp value may be treated as a fuzzy singleton. Then the firing strengths α_1 and α_2 of the first and second rules may be expressed as

$$\alpha_1 = \mu_{A_1}(x_0) \wedge \mu_{B_1}(y_0)$$

$$\alpha_2 = \mu_{A_2}(x_0) \wedge \mu_{B_2}(y_0),$$

where $\mu_{A_i}(x_0)$ and $\mu_{B_1}(y_0)$ play the role of the degrees of partial match between the user-supplied data and the data in the rule base. These relations play a central role in the four types of fuzzy reasoning which are currently employed in FLC applications and are described in the following.

1. Fuzzy Reasoning of the First Type: Mamdani's minimum operation rule as a fuzzy implication function

Fuzzy reasoning of the first type is associated with the use of Mamdani's minimum operation rule R_c as a fuzzy implication function. In this mode of reasoning, the i^{th} rule leads

2 See Section IV of Part I in Fuzzification Strategies.

to the control decision

$$\mu_{C_i'}(w) = \alpha_i \wedge \mu_{C_i}(w)$$

which implies that the membership function μ_C of the inferred consequence C is pointwise given by

$$\begin{aligned}\mu_C(w) &= \mu_{C_1'} \vee \mu_{C_2'} \\ &= [\alpha_1 \wedge \mu_{C_1}(w)] \vee [\alpha_2 \wedge \mu_{C_2}(w)].\end{aligned}$$

To obtain a deterministic control action, a defuzzification strategy is required, as will be discussed at a later point. The fuzzy reasoning process is illustrated in Figure 12, which shows a graphic interpretation of Lemma 3 in terms of Mamdani's method R_c.

2. Fuzzy Reasoning of the Second Type: Larsen's product operation rule as a fuzzy implication function

Fuzzy reasoning of the second type is based on the use Larsen's product operation rule R_p as a fuzzy implication function. In this case,, the i^{th} rule leads to the control decision

$$\mu_{C_i'}(w) = \alpha_i \cdot \mu_{C_i}(w).$$

Consequently, the membership function μ_C of the inferred consequence C is pointwise given by

$$\begin{aligned}\mu_C(w) &= \mu_{C_1'} \vee \mu_{C_2'} \\ &= [\alpha_1 \cdot \mu_{C_1}(w)] \vee [\alpha_2 \cdot \mu_{C_2}(w)].\end{aligned}$$

From C, a crisp control action can be deduced through the use of a defuzzification operator. The fuzzy reasoning process is illustrated in Figure 13, which shows a graphic interpretation of Lemma 3 in terms of Larsen's method R_p.

3. Fuzzy Reasoning of the Third Type: Tsukamoto's method with linguistic terms as monotonic membership functions

This method was proposed by Tsukamoto [117]. It is a simplified method based on the fuzzy reasoning of the first type in which the membership functions of fuzzy sets A_i, B_i and C_i are monotonic. However, in our derivation, A_i and B_i are not required to be monotonic but C_i is.

In Tsukamoto's method, the result inferred from the first rule is α_1 such that $\alpha_1=C_1(y_1)$. The result inferred from the second rule is α_2 such that $\alpha_2=C_2(y_2)$. Correspondingly, a crisp control action may be expressed as the weighted combination (Figure 14)

$$z_0 = \frac{\alpha_1 y_1 + \alpha_2 y_2}{\alpha_1 + \alpha_2}.$$

4. Fuzzy Reasoning of the Fourth Type: The consequence of a rule is a function of input linguistic variables

Fuzzy reasoning of the fourth type employs a modified version of states evaluation function[3]. In this mode of reasoning, the i^{th} fuzzy control rule is of the form

$$R_i : \text{ if } (x \text{ is } A_i, \cdots \text{ and } y \text{ is } B_i) \text{ then } z = f_i(x, \cdots y),$$

where $x, \cdots ,y$, and z are linguistic variables representing process state variables and the control variable, respectively; $A_i, \cdots, B_i$ are linguistic values of the linguistic variables $x, \cdots, y$ in the universes of discourse $U, \cdots, V$, respectively with $i=1,2, \cdots ,n$; and f_i is a function of the process state variables $x, \cdots ,y$ defined in the input subspaces.

For simplicity, assume that we have two fuzzy control rules as follows:

$$R_1 : \text{ if } x \text{ is } A_1 \text{ and } y \text{ is } B_1 \text{ then } z = f_1(x, y),$$

$$R_2 : \text{ if } x \text{ is } A_2 \text{ and } y \text{ is } B_2 \text{ then } z = f_2(x, y).$$

The inferred value of the control action from the first rule is $\alpha_1 f_1(x_0, y_0)$.

The inferred value of the control action from the second rule is $\alpha_2 f_2(x_0, y_0)$.

Correspondingly, a crisp control action is given by

$$z_0 = \frac{\alpha_1 f_1(x_0, y_0) + \alpha_2 f_2(x_0, y_0)}{\alpha_1 + \alpha_2}.$$

This method was proposed by Takagi and Sugeno [103] and has been applied to guide a model car smoothly along a crank-shaped track [98] and to park a car in a garage [97,99].

3 See Section V of Part I in Types of Fuzzy Control Rules.

II. Defuzzification Strategies

Basically, defuzzification is a mapping from a space of fuzzy control actions defined over an output universe of discourse into a space of nonfuzzy (crisp) control actions. It is employed because in many practical applications a crisp control action is required.

A defuzzification strategy is aimed at producing a nonfuzzy control action that best represents the possibility distribution of an inferred fuzzy control action. Unfortunately, there is no systematic procedure for choosing a defuzzification strategy. Zadeh [142] first pointed out this problem and made tentative suggestions for dealing with it. At present, the commonly used strategies may be described as the Max criterion, the Mean of Maximum, and the Center of Area.

A. The Max Criterion Method

The Max Criterion produces the point at which the possibility distribution of the control action reaches a maximum value.

B. The Mean of Maximum Method (MOM)

The MOM strategy generates a control action which represents the mean value of all local control actions whose membership functions reach the maximum. More specifically, in the case of a discrete universe, the control action may be expressed as

$$z_0 = \sum_{j=1}^{l} \frac{w_j}{l}$$

where w_j is the support value at which the membership function reaches the maximum value $\mu_z(w_j)$, and l is the number of such support values.

C. The Center of Area Method (COA)

The widely used COA strategy generates the center of gravity of the possibility distribution of a control action. In the case of a discrete universe, this method yields

$$z_0 = \frac{\sum_{j=1}^{n} \mu_z(w_j) \cdot w_j}{\sum_{j=1}^{n} \mu_z(w_j)}$$

where n is the number of quantization levels of the output.

Figure 15 shows a graphical interpretation of various defuzzification strategies. Braae and Rutherford [5] presented a detailed analysis of various defuzzification strategies (COA, MOM) and concluded that the COA strategy yields superior results (also see [58]). However, the MOM strategy yields a better transient performance while the COA strategy yields a better steady-state performance [94]. It should be noted that when the MOM strategy is used, the performance of an FLC is similar to that of a multi-level relay system [48], while the COA strategy yields results which are similar to those obtainable with a conventional PI controller [46]. An FLC based on the COA generally yields a lower mean square error than that based on the MOM [111]. Furthermore, the MOM strategy yields a better performance than the Max criterion strategy [52].

III. Applications and Recent Developments

A. Applications

During the past several years, fuzzy logic has found numerous applications in fields ranging from finance to earthquake engineering [62]. In particular, fuzzy control has emerged as one of the most active and fruitful areas for research in the application of fuzzy set theory. In many applications, the FLC based systems have proved to be superior in performance to conventional systems.

Notable applications of FLC include the heat exchange [80], warm water process [47], activated sludge process [113,35], traffic junction [82], cement kiln [59,118], aircraft flight control [58], turning process [92], robot control [119,94,106,8,34], model-car parking and turning [97,98,99], automobile speed control [74,75], water purification process [127], elevator control [23], automobile transmission control [40], power systems and nuclear reactor control [4,51], fuzzy memory devices [107,108,120,128,129,133], and fuzzy computer [132]. In this connection, it should be noted that the first successful industrial application of the FLC was the cement kiln control system developed by the Danish cement plant manufacturer F. L. Smidth in 1979. An ingenious application is Sugeno's fuzzy car which has the capability of learning from examples. More recently, predictive fuzzy control systems have been proposed and successfully applied to automatic train operation systems and automatic container crane operation systems [135,136,137,138,139]. In parallel with these developments, a great deal of progress has been made in the design of fuzzy hardware and its use in so-called fuzzy computers [132].

B. Recent Developments

1. Sugeno's Fuzzy Car

One of the most interesting applications of the FLC is the *fuzzy car* designed by Sugeno. Sugeno's car has successfully followed a crank-shaped track and parked itself in a garage [98,97,99].

The control policy incorporated in Sugeno's car is represented by a set of fuzzy control rules which have the form:

$$R_i : \text{ if } x \text{ } is \text{ } A_i \text{ }, \cdots \text{ and } y \text{ } is \text{ } B_i \text{ } then \text{ } z = a_0^i + a_1^i x + \cdots + a_n^i y$$

where $x, \cdots$, and y are linguistic variables representing the distances and orientation in relation to the boundaries of the track; $A_i, \cdots$, and B_i are linguistic values of $x, \cdots$, and y. z is the value of the control variable of the i^{th} control rule; and a_0^i, ..., and a_n^i are the parameters entering in the identification algorithm [103,99].

The inference mechanism of Sugeno's fuzzy car is based on fuzzy reasoning of the fourth type, with the parameters $a_0^i, \cdots$, and a_n^i identified by training. The training process involves a skilled operator who guides the fuzzy model car under different conditions. In this way, Sugeno's car has the capability of learning from examples.

2. FLC Hardware Systems

A high speed FLC hardware system employing fuzzy reasoning of the first type has been proposed by Yamakawa [130,131]. It is composed of 15 control rule boards and an action interface (i.e, a defuzzifier based on the COA). It can handle fuzzy linguistic rules labeled as NL, NM, NS, ZR, PS, PM, PL. The operational speed is approximately 10 mega fuzzy logical inferences per second (FLIPS).

The FLC hardware system has been tested by an application to the stabilization of inverted pendulum mounted on a vehicle. Two pendulums with different parameters were controlled by the same set of fuzzy control rules (Table VII). It is worthy of note that only seven fuzzy control rules achieve this result. Each control rule board and action interface has been integrated to a 40-pin chip.

3. Fuzzy Automatic Train Operation (ATO) Systems

Hitachi Ltd. has developed a Fuzzy Automatic Train Operation System (ATO) which has been in use in the Sendai-city subway system in Japan since July, 1987. In this system, an object evaluation fuzzy controller predicts the performance of each candidate control command

and selects the most likely control command based on skilled human operator's experience.

More specifically, fuzzy ATO comprises two rule bases which evaluate two major functions of a skilled operator based on the criteria of safety, riding comfort, stop gap accuracy, traceability of target velocity, energy consumption and running time. One is constant speed control (CSC) which starts a train and maintains a prescribed speed. The other is the train automatic stop control (TASC) which regulates a train speed in order to stop at the target position at a station. Each rule base consists of twelve object evaluation fuzzy control rules. The antecedent of every control rule performs the evaluation of train operation based on safety, riding comfort stop gap accuracy, etc.. The consequent determines the control action to be taken based on the degree of satisfaction of each criterion. The control action is the value of the train control notch which is evaluated every 100 milliseconds from the maximal evaluation of each candidate control action and it takes as a value a discrete number: positive value means "power notch"; negative value means "break notch".

The Sendai-city subsway system has been demonstrated to be superior in performance to the conventional PID ATO in riding comfort, stop gap accuracy, energy consumption, running time and robustness [135,136,139].

4. Fuzzy Automatic Container Crane Operation (ACO) Systems

In the application of FLC to the automatic operation of container-ship loading cranes, the principal performance criteria are safety, stop gap accuracy, container sway, and carrying time.

Fuzzy ACO involves two major operations: the trolley operation and the wire rope operation. Each operation comprises two function levels: a decision level and an activation level. Field tests of fuzzy ACO systems with real container cranes have been performed at the port of Kitakyusyu in Japan. The experimental results show that cargo handling ability of Fuzzy ACO by an unskilled operator is more than 30 containers per hour, which is comparable to the performance of a veteran operator. The tests have established that Fuzzy ACO controller has the capability of operating a crane as safely, accurately and skillfully as a highly experienced human operator [137,138,139].

5. Fuzzy Logic Chips and Fuzzy Computers

The first fuzzy logic chip was designed by Togai and Watanabe at AT&T Bell Laboratories in 1985 [107]. The fuzzy inference chip, which can process sixteen rules in parallel, consists of four major parts: a rule-set memory, an inference-processing unit, a controller, and an input-output circuitry. Recently, the rule-set memory has been implemented by a static random access memory (SRAM) to realize a capability for dynamic changes in the rule set. The inference-processing unit is based on the sup-min compositional rule of inference. Preliminary timing tests indicate that the chip can perform approximately 250,000 FLIPS at 16 mega HZ clock. A Fuzzy Logic Accelerator (FLA) based on this chip is currently under development [108,120]. Furthermore, in March 1989, the Microelectronics Center of North Carolina successfully completed the fabrication of the world's fastest fuzzy logic chip designed by Watanabe. The full-custom chip comprises 688,000 transistors and is capable of making 580,000 FLIPS.

In Japan, Yamakawa and Miki realized nine basic fuzzy logic functions by the standard CMOS process in current-mode circuit systems [128]. Later, a rudimentary concept of a fuzzy computer was proposed by Yamakawa and built by OMRON Tateishi Electric Co. Ltd [132]. The Yamakawa-OMRON computer comprises a fuzzy memory, a set of inference engines, a MAX block, a defuzzifier, and a control unit. The fuzzy memory stores linguistic fuzzy information in the form of membership functions. It has a binary RAM, a register, and a membership function generator [128]. A membership function generator (MFG) consists of a PROM, a pass transistor array, and a decoder. Every term in a term set is represented by a binary code and stored in a binary RAM. The corresponding membership functions are generated by the MFG via these binary codes. The inference engine employs MAX and MIN operations which are implemented by the Emitter Coupled Fuzzy Logic Gates (ECFL Gates) in voltage-mode circuit systems. The linguistic inputs which are represented by analog voltages distributed on data buses are fed into each inference engine in parallel. The results inferred from the rules are aggregated by a MAX block which implements the function of the connective *also* as a union operation, yielding a consequence which is a set of analog voltages distributed on output lines. In the FLC applications, a crisp control command necessitates an auxiliary defuzzifier. In this implementation, a fuzzy computer is capable of processing fuzzy information at the very high speed of approximately 10 mega FLIPS. It is indeed an important step not only in industrial applications but also in commonsense knowledge processing.

IV. Future Studies and Problems

In many of its applications, FLC is either designed by domain experts or in close collaboration with domain experts. Knowledge acquisition in FLC applications plays an important role in determining the level of performance of a fuzzy control system. However, domain experts and skilled operators do not structure their decision making in any formal way. As a result, the process of transferring expert knowledge into a usable knowledge base of an FLC is time-consuming and non-trivial. Although fuzzy logic provides an effective tool for linguistic knowledge representation and Zadeh's compositional rule of inference serves as a useful guideline, we are still in need of more efficient and more systematic methods for knowledge acquisition.

An FLC based on the fuzzy model of a process is needed when higher accuracy and reliability are required. However, the fuzzy modeling of a process is still not well understood due to difficulties in modeling the linguistic structure of a process and obtaining operating data in industrial process control [13,84,111,125,104,101].

Classical control theory has been well developed and provides an effective tool for mathematical system analysis and design when a precise model of a system is available. In a complementary way, FLC has found many practical applications as a means of replacing a skilled human operator. For further advances, what is needed at this juncture are well-founded procedures for system design. In response to this need, many researchers are engaged in the development of a theory of fuzzy dynamic systems which extends the fundamental notions of state [6], controllability [31], and stability [77,44,89,55].

Another direction that is beginning to be explored is that of the conception and design of fuzzy systems which have the capability to learn from experience. In this area, a combination of techniques drawn from both fuzzy logic and neural network theory may provide a powerful tool for the design of systems which can emulate the remarkable human ability to learn and adapt to changes in environment.

Acknowledgment

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References and Related Publications

[Please refer to Part I of this paper].

Figure Captions

Fig. 1. Diagrams for the calculation of membership functions. (a) μ_{R_p} *vs.* μ_A with the parameter μ_B. (b) μ_{R_p} *vs.* μ_B with the parameter μ_A.

Fig. 2-5 Approximate reasoning: Generalized Modus Ponens with Larsen's product operation rule.

Fig. 6-9 Approximate reasoning: Generalized Modus Tollens with Larsen's product operation rule.

Fig. 10. Graphical interpretation of lemma 2 under $\alpha^{\wedge}$ and R_c.

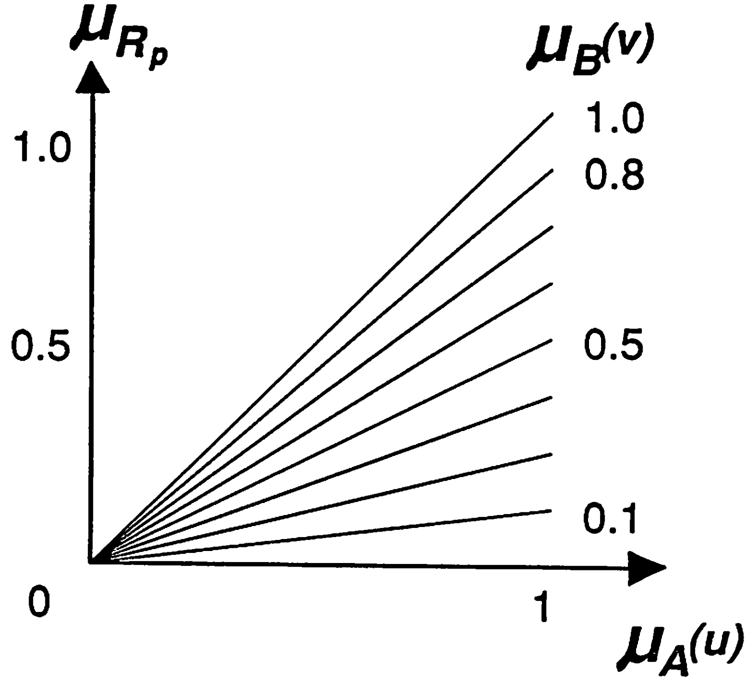
Fig. 11. Graphical interpretation of lemma 2 under $\alpha^{\bullet}$ and R_p.

Fig. 12. Diagrammatic representation of fuzzy reasoning 1.

Fig. 13. Diagrammatic representation of fuzzy reasoning 2.

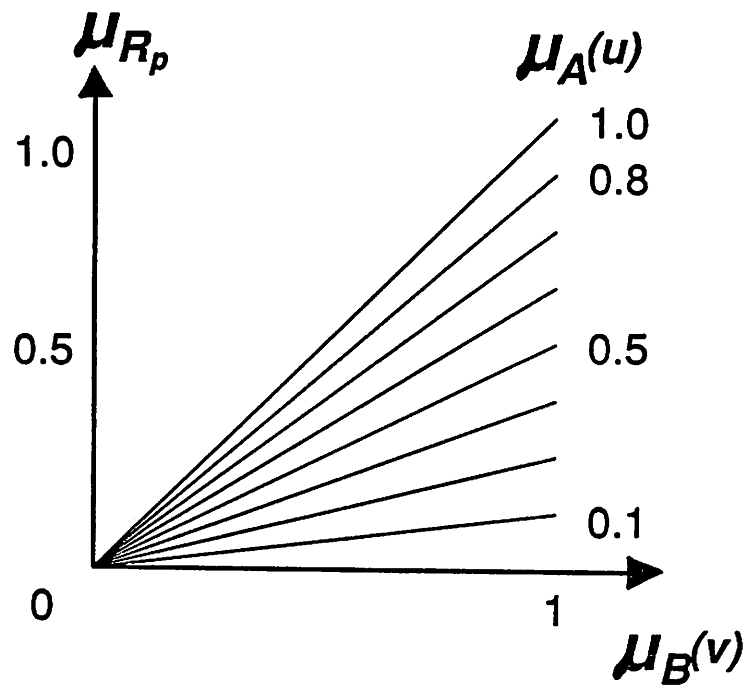
Fig. 14. Diagrammatic representation of fuzzy reasoning 3.

Fig. 15. Diagrammatic representation of various defuzzification strategies.



(a)

Fig. 1a



(b)

Fig. 1b

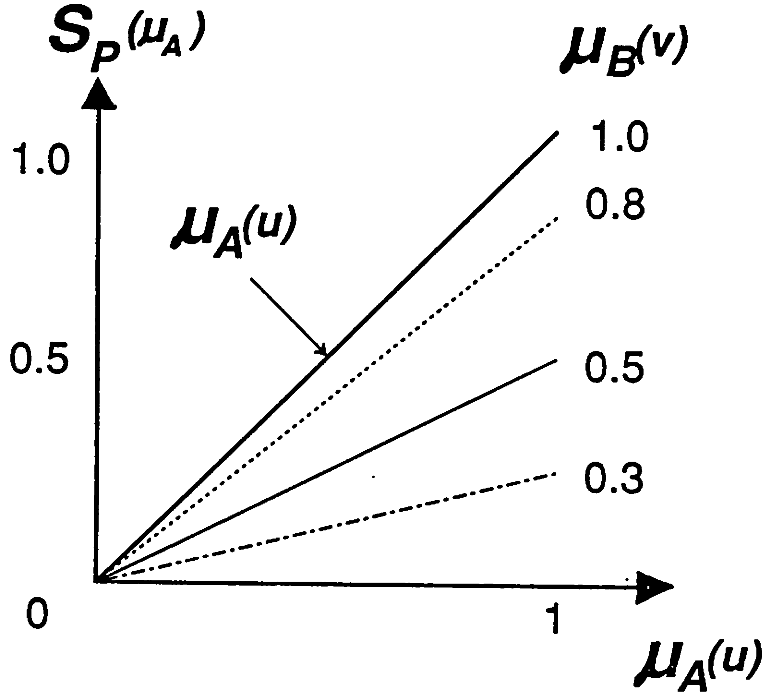


Fig. 2

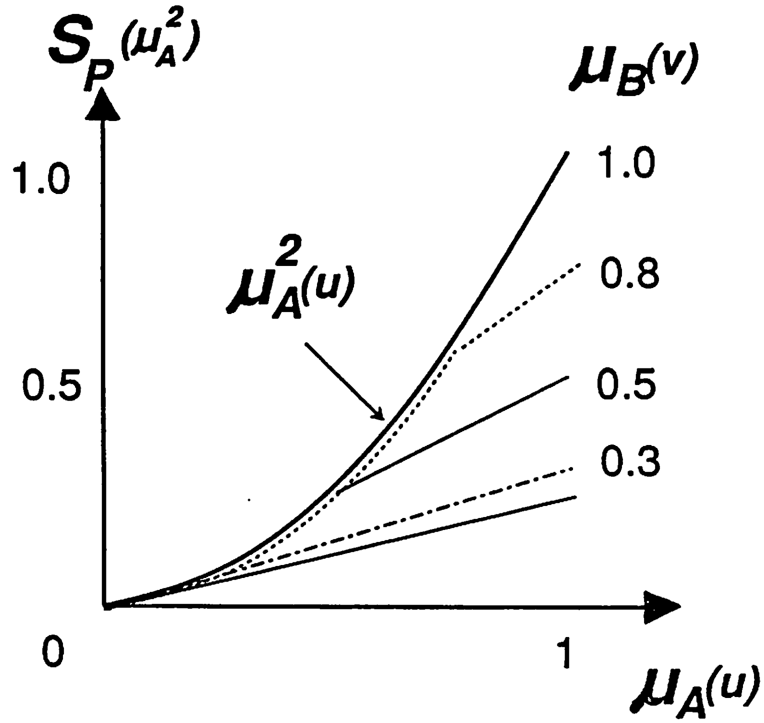


Fig. 3

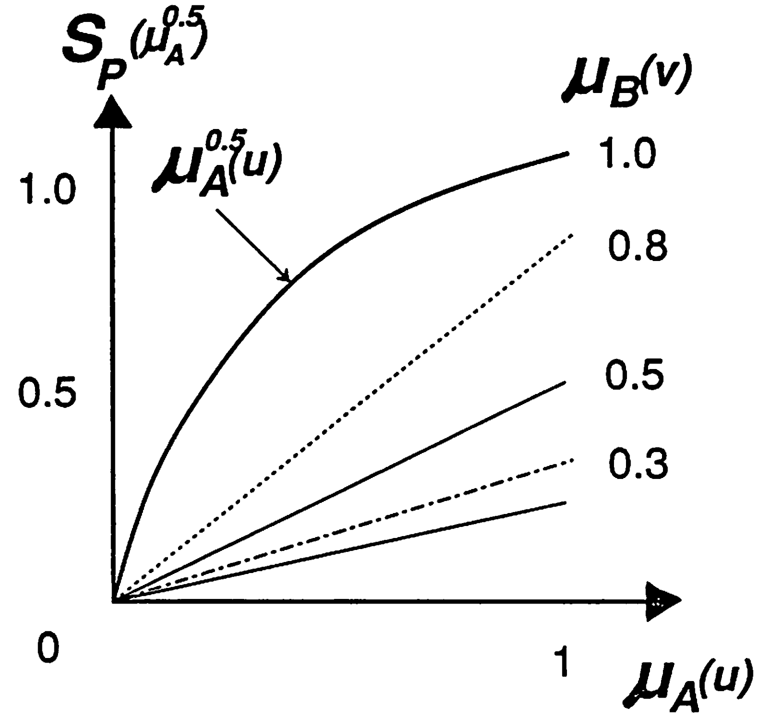


Fig. 4

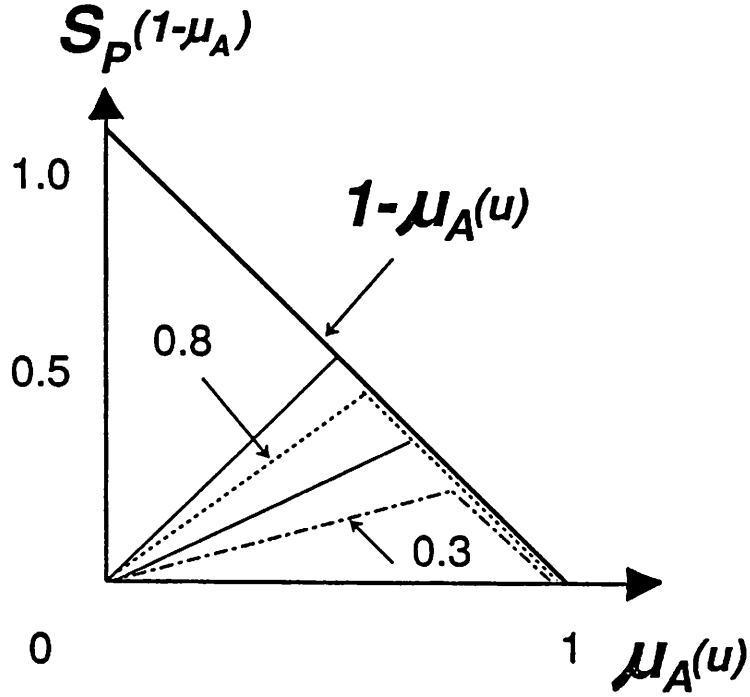


Fig. 5

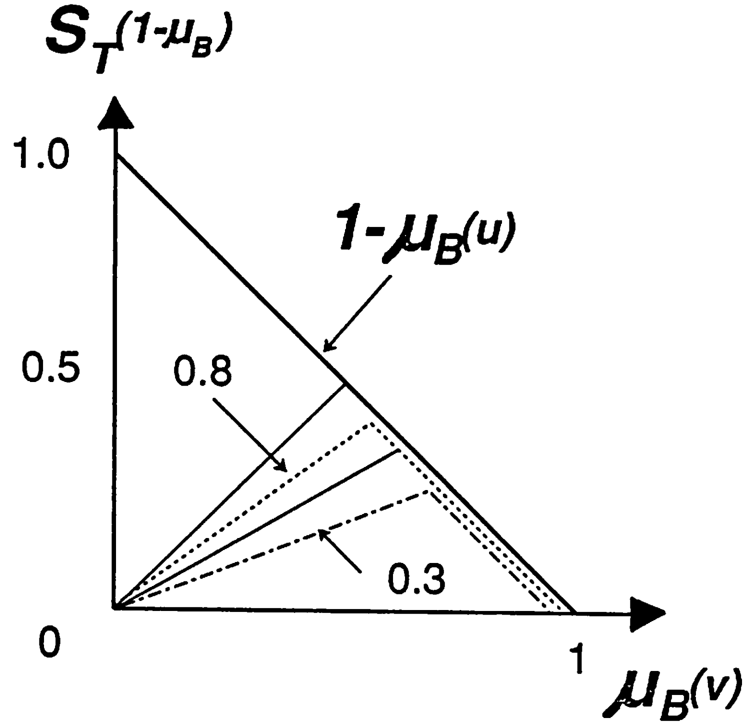


Fig. 6

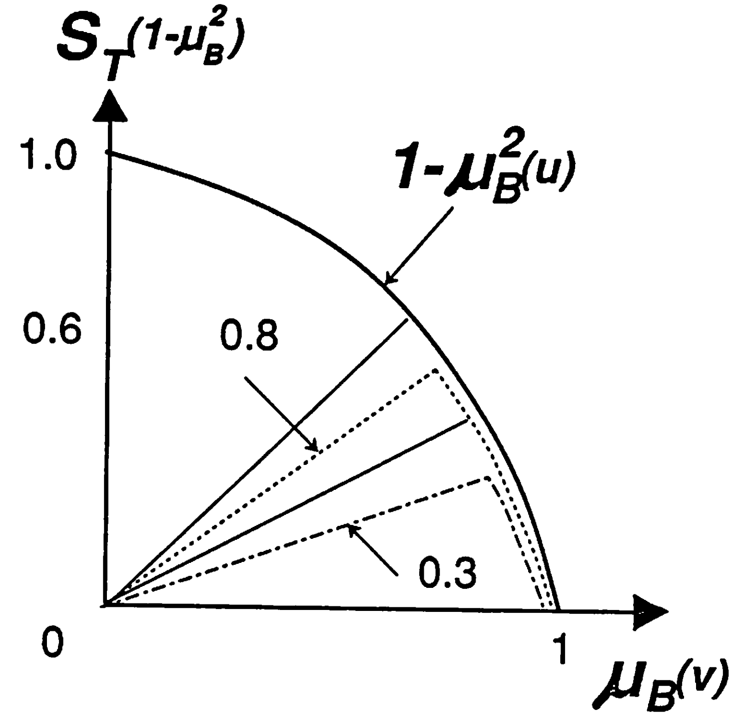


Fig. 7

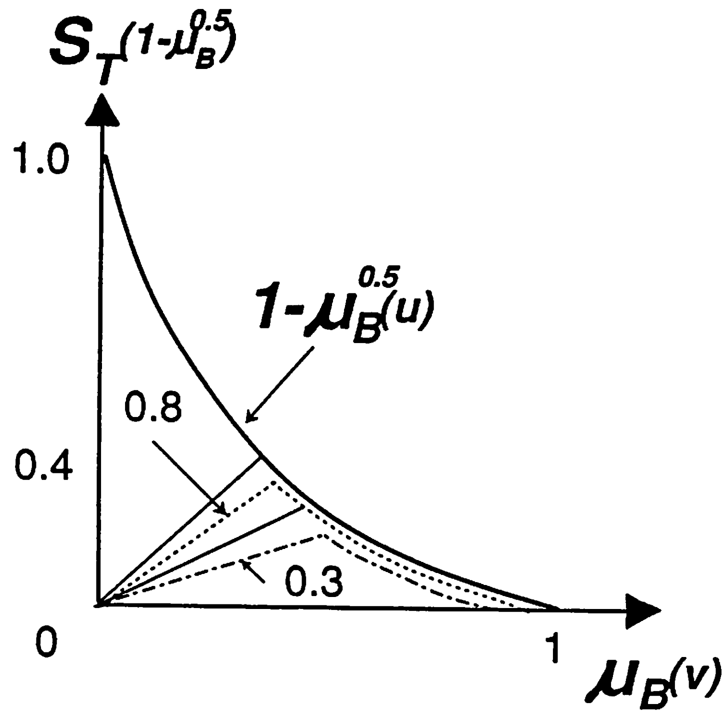


Fig. 8

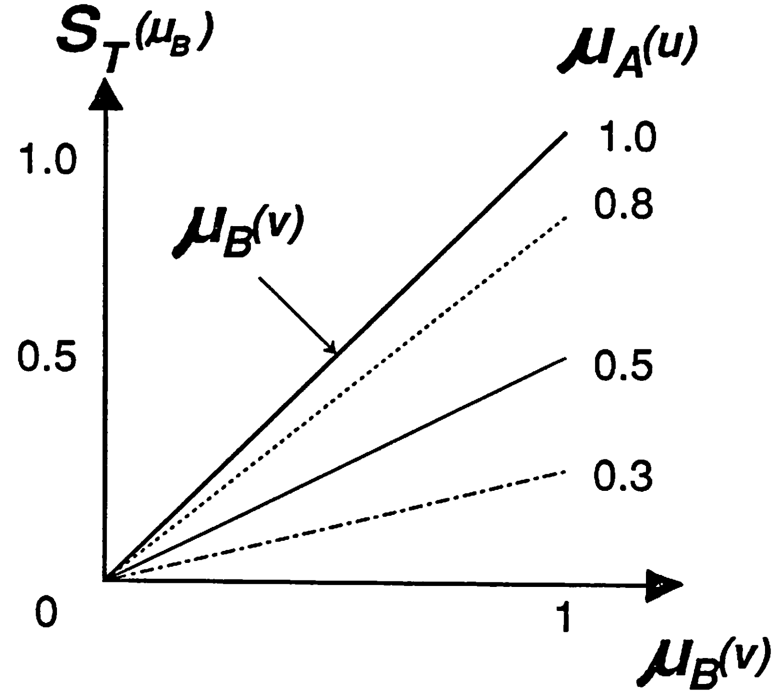
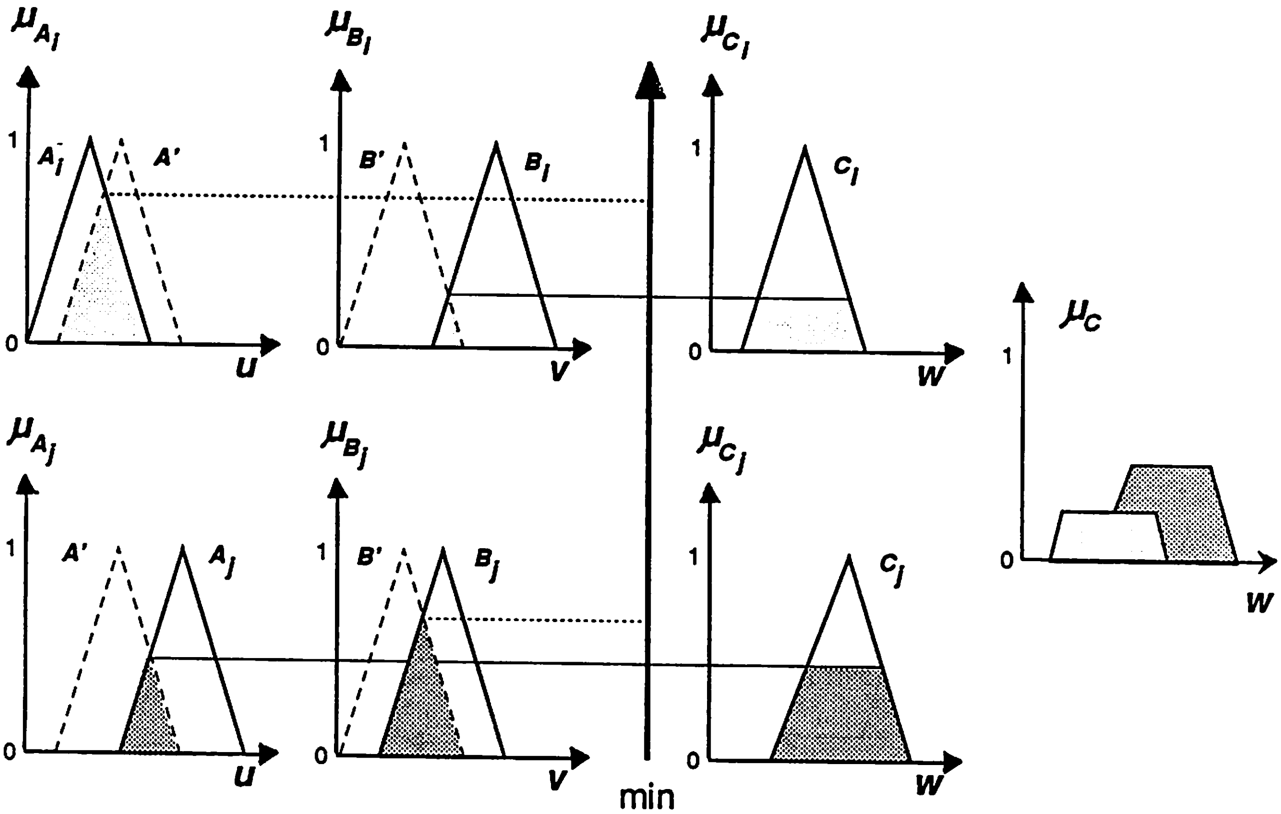
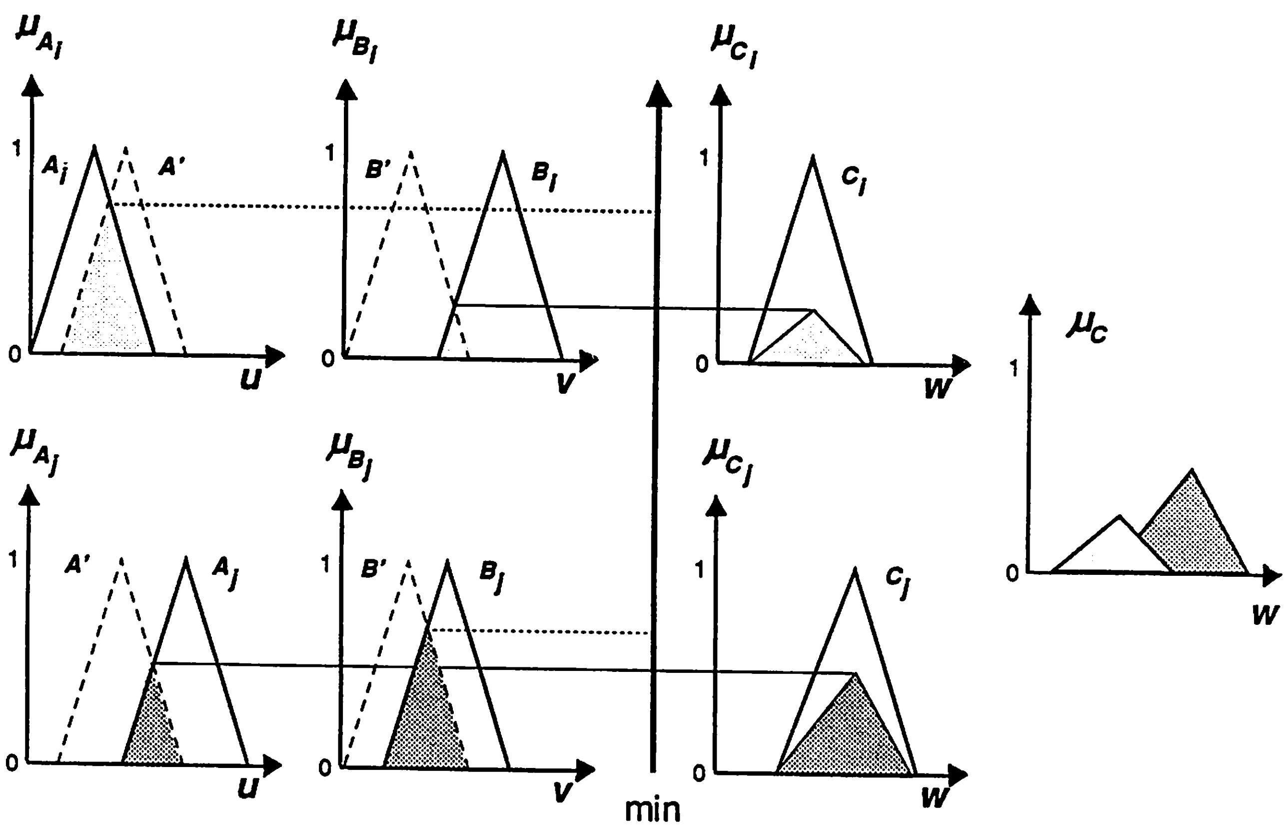


Fig. 9



μ_{A_i}
1
A_i
A'
0
u
μ_{B_i}
1
B'
B_i
0
v
μ_{C_i}
1
C_i
0
w
μ_C
1
0
w
μ_{A_j}
1
A'
A_j
0
u
μ_{B_j}
1
B'
B_j
0
v
min
μ_{C_j}
1
C_j
0
w

Fig. 10



μ_{A_i}
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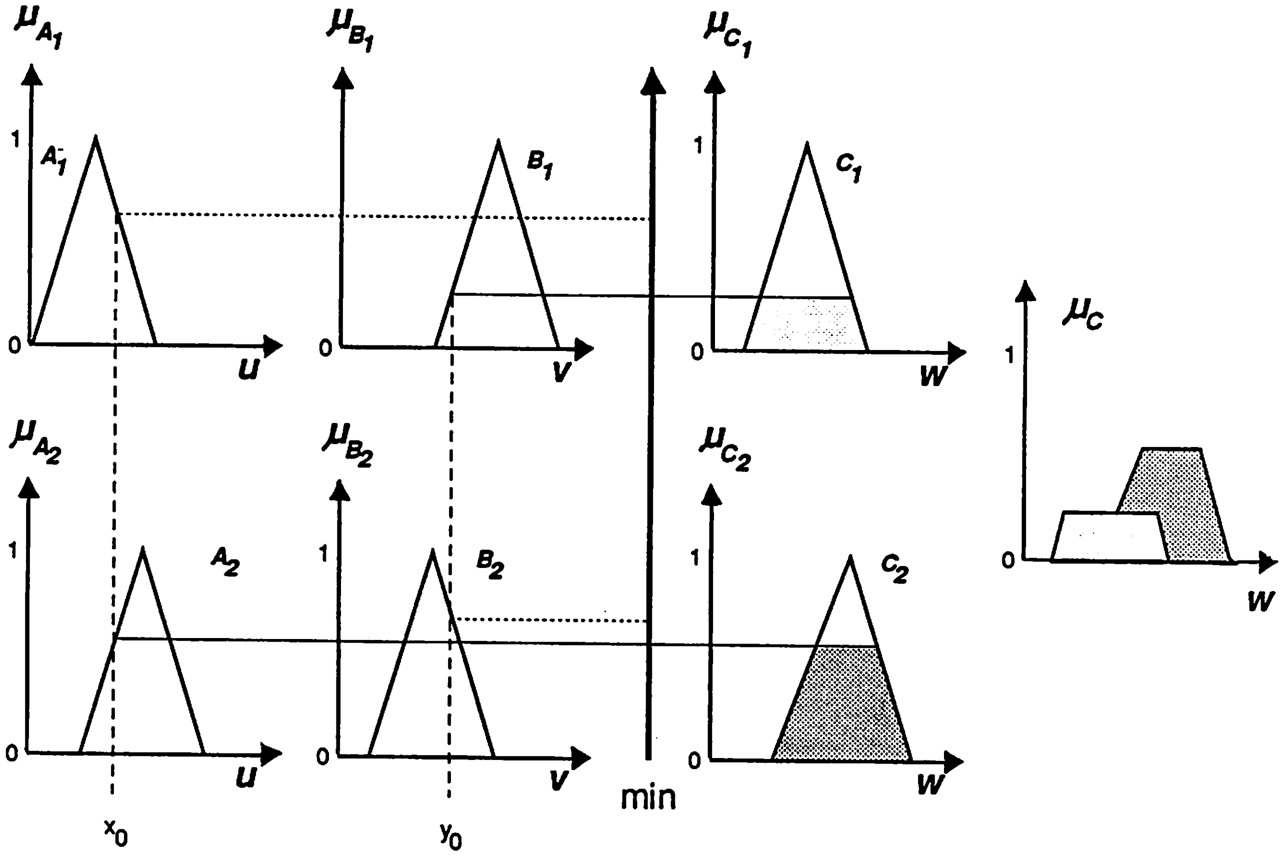
A_i
A'
0
u
μ_{B_i}
1
B'
B_i
0
v
μ_{C_i}
1
C_i
0
w
μ_C
1
0
w
μ_{A_j}
1
A'
A_j
0
u
μ_{B_j}
1
B'
B_j
0
v
min
μ_{C_j}
1
C_j
0
w

Fig. 11



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μ_{B_1}
μ_{C_1}
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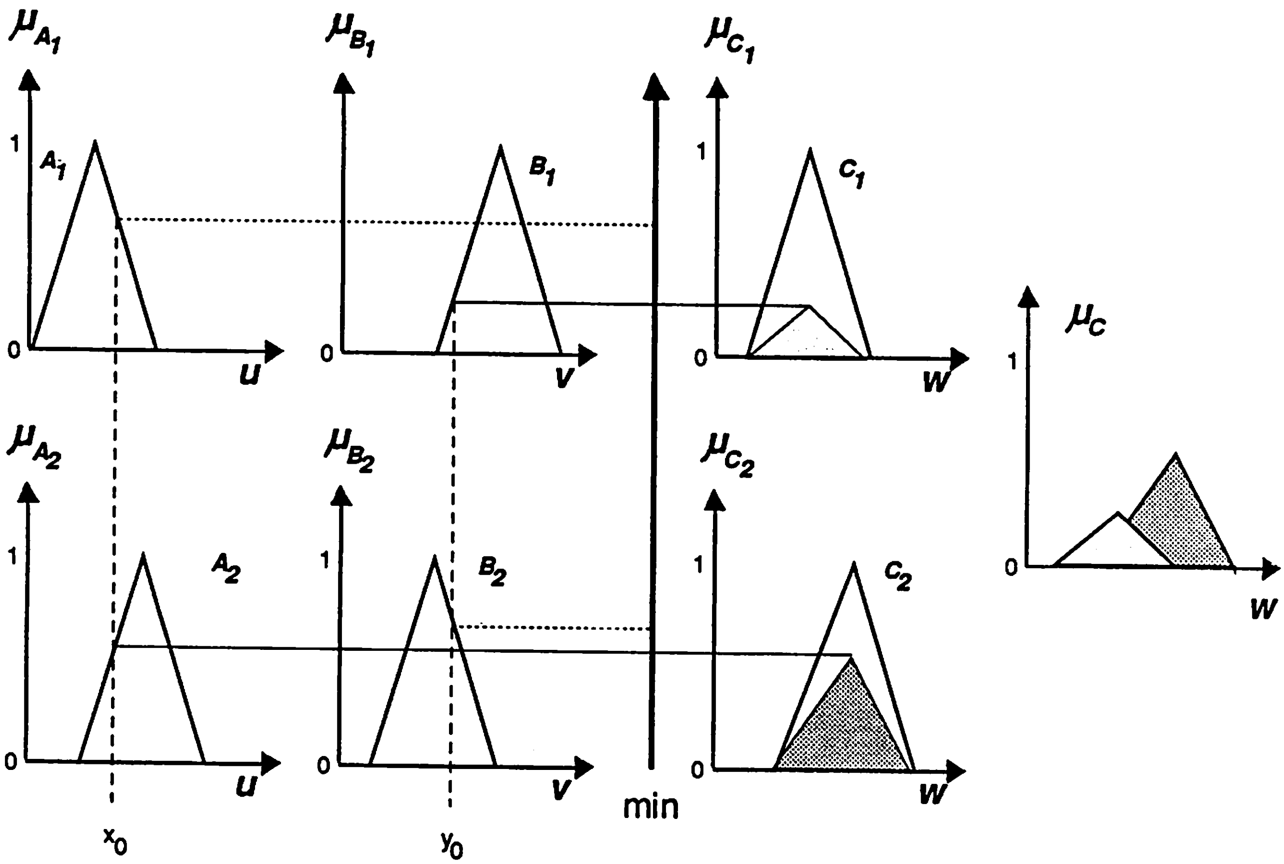
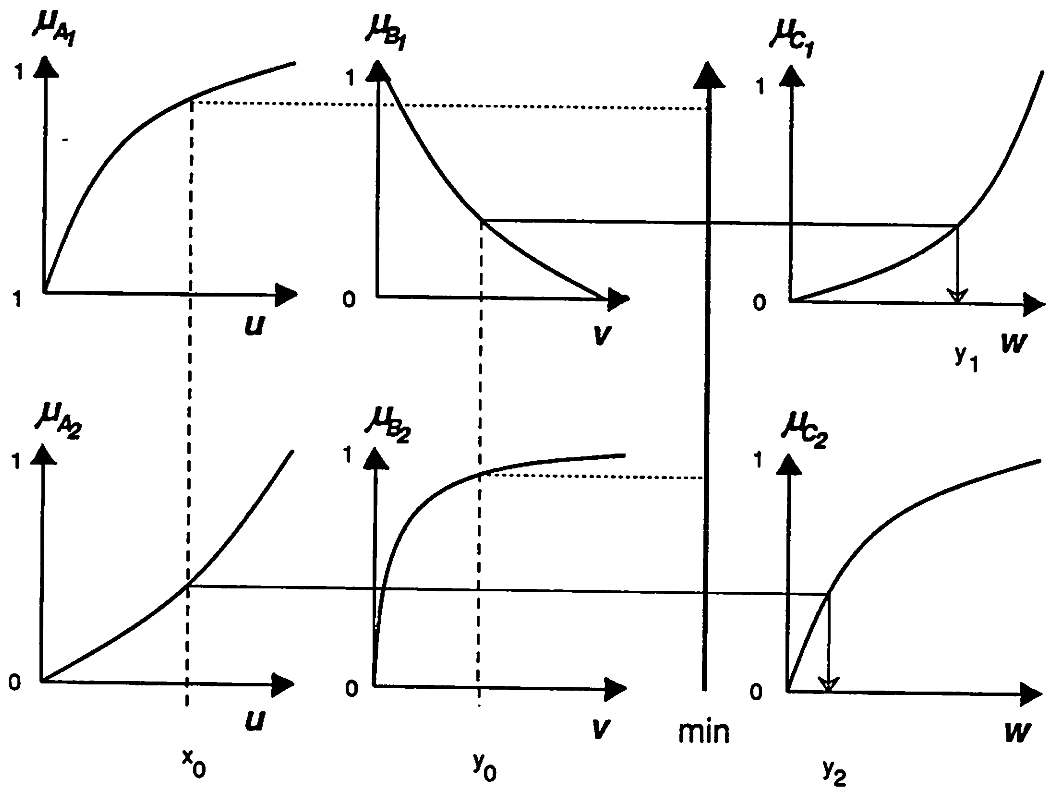
A_1
B_1
1
C_1
0
u
0
v
0
w
μ_C
1
μ_{A_2}
μ_{B_2}
μ_{C_2}
1
A_2
1
B_2
1
C_2
0
w
0
u
0
v
min
0
w
x_0
y_0

Fig. 12



μ_{A_1}
μ_{B_1}
μ_{C_1}
1
A_1
B_1
C_1
0
u
v
w
μ_C
μ_{A_2}
μ_{B_2}
μ_{C_2}
A_2
B_2
C_2
min
x_0
y_0

Fig. 13



μ_{A_1}
1
1
u
μ_{B_1}
1
0
v
μ_{C_1}
1
0
y_1
w
μ_{A_2}
1
0
u
x_0
μ_{B_2}
1
0
v
y_0
min
μ_{C_2}
1
0
w
y_2

Fig. 14

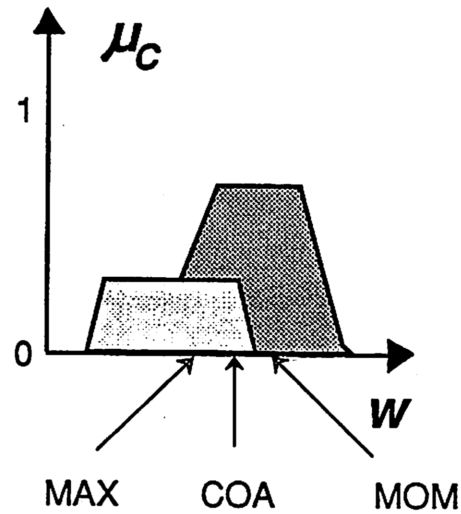


Fig. 15

Table Captions

TABLE I Intuitive Criteria relating Pre1 and Cons for Given Pre2 in GMP

TABLE II Intuitive Criteria relating Pre1 and Cons for Given Pre2 in GMT

TABLE III Summary of Inference Results for Generalized Modus Ponens

TABLE IV Summary of Inference Results for Generalized Modus Tollens

TABLE V Satisfaction of Various Fuzzy Implication Functions under Intuitive Criteria

TABLE VI Suitable Pairs of a Fuzzy Implication Function and Connective *also*

TABLE VII Fuzzy Control Rules for Inverted Pendulum Balancing

TABLE I

Intuitive Criteria relating Pre1 and Cons for Given Pre2 in GMP

	x is A'(Pre1)	y is B'(Cons)
criterion 1	x is A	y is B
criterion 2-1	x is very A	y is very B
criterion 2-2	x is very A	y is B
criterion 3-1	x is more or less A	y is more or less B
criterion 3-2	x is more or less A	y is B
criterion 4-1	x is not A	y is unknown
criterion 4-2	x is not A	y is not B

TABLE II

Intuitive Criteria relating Pre1 and Cons for Given Pre2 in GMT

	y is B'(Pre1)	x is A'(Cons)
criterion 5	y is not B	x is not A
criterion 6	y is not very B	x is not very A
criterion 7	y is not more or less B	x is not more or less A
criterion 8-1 criterion 8-2	y is B y is B	x is unknown x is A

TABLE III
Summary of Inference Results for Generalized Modus Ponens

	A	very A	more or less A	not A
R_c	μ_B	μ_B	μ_B	$0.5 \wedge \mu_B$
R_p	μ_B	μ_B	μ_B	$\dfrac{\mu_B}{1+\mu_B}$
R_a	$\dfrac{1+\mu_B}{2}$	$\dfrac{3+2\mu_B-\sqrt{5+4\mu_B}}{2}$	$\dfrac{\sqrt{5+4\mu_B}-1}{2}$	1
R_m	$0.5 \vee \mu_B$	$\dfrac{3-\sqrt{5}}{2} \vee \mu_B$	$\dfrac{\sqrt{5}-1}{2} \vee \mu_B$	1
R_b	$0.5 \vee \mu_B$	$\dfrac{3-\sqrt{5}}{2} \vee \mu_B$	$\dfrac{\sqrt{5}-1}{2} \vee \mu_B$	1
R_s	μ_B	μ_B^2	$\sqrt{\mu_B}$	1
R_Δ	$\sqrt{\mu_B}$	$\mu_B^{2/3}$	$\mu_B^{1/3}$	1

TABLE IV
Summary of Inference Results for Generalized Modus Tollens

	not B	not very B	not more or less B	B
R_c	$0.5 \wedge \mu_A$	$\frac{\sqrt{5}-1}{2} \wedge \mu_A$	$\frac{3-\sqrt{5}}{2} \wedge \mu_A$	μ_A
R_p	$\frac{\mu_A}{1+\mu_A}$	$\frac{\mu_A\sqrt{\mu_A^2+4}-\mu_A}{2}$	$\frac{2\mu_A+1-\sqrt{4\mu_A+1}}{2\mu_A}$	μ_A
R_a	$1-\frac{\mu_A}{2}$	$\frac{1-2\mu_A+\sqrt{1+4\mu_A}}{2}$	$\frac{3-\sqrt{1+\mu_A}}{2}$	1
R_m	$0.5 \vee (1-\mu_A)$	$(1-\mu_A) \vee (\frac{\sqrt{5}-1}{2} \wedge \mu_A)$	$\frac{3-\sqrt{5}}{2} \vee (1-\mu_A)$	$\mu_A \vee (1-\mu_A)$
R_b	$0.5 \vee (1-\mu_A)$	$\frac{\sqrt{5}-1}{2} \vee (1-\mu_A)$	$\frac{3-\sqrt{5}}{2} \vee (1-\mu_A)$	1
R_s	$1-\mu_A$	$1-\mu_A^2$	$1-\sqrt{\mu_A}$	1
μ_Δ	$\frac{1}{1+\mu_A}$	$\frac{\sqrt{1+4\mu_A^2}-1}{2\mu_A^2}$	$\frac{2+\mu_A-\sqrt{\mu_A^2+4\mu_A}}{2}$	1

TABLE V
Satisfaction of Various Fuzzy Implication Functions under Intuitive Criteria

	R_c	R_p	R_a	R_m	R_s	R_Δ	R_b
criteria 1	○	○	×	×	○	×	×
criteria 2-1	×	×	×	×	○	×	×
criteria 2-2	○	○	×	×	×	×	×
criteria 3-1	×	×	×	×	○	×	×
criteria 3-2	○	○	×	×	×	×	×
criteria 4-1	×	×	○	○	○	○	○
criteria 4-2	×	×	×	×	×	×	×
criteria 5	×	×	×	×	○	×	×
criteria 6	×	×	×	×	○	×	×
criteria 7	×	×	×	×	○	×	×
criteria 8-1	×	×	○	×	○	○	○
criteria 8-2	○	○	×	×	×	×	×

TABLE VI

Suitable Pairs of a Fuzzy Implication Function and Connective *also*

IMPLICATION RULE	CONNECTIVE ALSO
R_c R_p R_{bp} R_{dp}	$\cup$ $\hat{+}$ $\oplus$ $\dot{\cup}$ Δ
R_a	$\cap$ $\cdot$ $\odot$ $\dot{\cap}$
R_m	-
R_s R_Δ R_g	$(\cap$ $\cdot$ $\odot$ $\dot{\cap})$*
R_b	$\cdot$ $\odot$ $\dot{\cap}$

* it depends on the shape of reproduced curve which forms the set of fuzzy control rules.

TABLE VII
Fuzzy Control Rules for Inverted Pendulum Balancing

		Angle						
		NL	*NM*	*NS*	*ZR*	*PS*	*PM*	*PL*
	NL							
	NM							
Change	*NS*			*NS*		*ZR*		
of	*ZR*		*NM*		*ZR*		*PM*	
Angle	*PS*			*ZR*		*PS*		
	PM							
	PL							