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Power Absorption Corresponding to Ion Losses in Parallel Plate RF Discharges

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ABSTRACT

A simple model of a symmetric parallel plate rf discharge is studied to illustrate how such discharges may absorb power from an rf power supply in order to sustain dc power losses corresponding to the steady acceleration of ions through the sheaths. The motions of the sheath boundaries over one period are derived assuming that the current density varies as $J_0 \sin\omega t$. One finds that the sheath thickness increases discontinuously at one sheath whenever the plasma contacts the opposing electrode. This implies that the external power supply delivers an electron pulse from the electrode at higher potential to the electrode at lower potential, so that some power is being absorbed in a pulsed fashion. The power absorbed by the discharge is also calculated for the portions of the rf cycle where the current varies sinusoidally. It is found that power is generated by the discharge in this phase of the rf cycle, with the energy coming from the deflating sheaths. It is further shown that the sum of the pulsed power absorption and smooth power generation, averaged over one rf period, is equal to the dc ion power losses arising from ions falling through the time averaged sheath potentials.

I. Introduction

RF parallel plate discharges are usually operated with a series blocking capacitor to prevent the circulation of a dc current through the discharge and the external circuit. The blocking capacitor prevents any dc power from entering the discharge. It is known, however, that some power loss in rf discharges occurs as the steady flow of ions accelerated across the discharge sheaths.[1,2] This is especially true in thick sheaths, where the ion motion is determined by the time-averaged fields. In discharges with large sheath voltages, the dc power expended in ion acceleration is the major power loss mechanism that the rf power supply must support. In fact, ion acceleration is one of the primary purposes of discharges used for sputtering.[3] The question arises as to how the discharge manages to convert the rf power from the generator into the dc power given to ion acceleration. This paper discusses a simple model of this process. The time-averaged power absorbed by the discharge is derived from the current density and voltage wave forms. The question of the absorption of power corresponding to the work done in accelerating ions is also of interest to those constructing models for the discharge impedance; however this issue is not investigated in the present paper.

II. Theory

A. Motion of the Sheath Boundary

The simple model of the discharge sheath used here is depicted in Figure 1. The ion density is constant at value n_0 throughout the discharge. The electrode has a surface charge density σ, and the sheath thickness is denoted by s. Both σ and s vary in time. The electron density is modeled as a step function with the discontinuity at a distance s from the electrode, marking the position of the sheath boundary. In the sheath the electron density is zero, and beyond the sheath boundary the electron density is also n_0. As a further simplification, the electric field is taken to be zero in the plasma.

The motion of the sheath boundary may be derived by considering the various charge densities in the system. Given that the electric field in the plasma is zero, an application of Gauss' law across the sheath implies that the ion charge density in the sheath must balance the surface charge density on the electrode:

$$\sigma(t) + en_0 s(t) = 0 \quad . \tag{1}$$

Here e is the electronic charge, and the time dependence of σ and s is explicitly noted. As depicted in Figure 1, the charge density σ is changed by the external current density J_{ext} and the ion current density, which may be written in terms of the Bohm speed $u_B=(kT_e/m_i)^{1/2}$. Thus, we have

$$\frac{d\sigma}{dt} = en_0 u_B + J_{ext} \quad . \tag{2}$$

Combining equations (1) and (2) yields

$$en_0 u_B + J_{ext} + en_0 \frac{ds}{dt} = 0 \quad , \tag{3}$$

which can then be integrated for s(t):

$$s(t) = -u_B t - \frac{1}{en_0}\int_0^t dt J_{ext} + s(0) \quad . \tag{4}$$

Equation (4) can be evaluated for the case where

$$J_{ext}(t) = J_0 \sin\omega t \tag{5}$$

and with the initial conditions

$$\begin{aligned} s(\tau) &= 0 \\ \frac{ds}{dt}(\tau) &= 0 \end{aligned} \tag{6}$$

at some time τ when the plasma touches the electrode. The solution is

$$s(t) = u_B(\tau - t) + \frac{J_0}{en_0\omega}(\cos\omega t - \cos\omega\tau) \tag{7}$$

where τ is given by

$$\sin\omega\tau = -en_0 u_B / J_0 \tag{8}$$

The right side of equation (8) is approximately the ratio of the Bohm speed to the characteristic speed of the sheath, ωs_0, where s_0 is the sheath oscillation amplitude. For a frequency of 13.6MHz, s_0=0.5cm, kT_e=3eV, and an atomic weight of 40amu, this ratio is approximately 1/156. Thus equation (8) implies that τ is slightly larger than half a period. Denoting the rf period by T, τ may be written

$$\tau = T/2 + \delta \ , \quad \delta < T \quad . \tag{9}$$

A graph of s(t) is given in Figure 2. One notable feature is that the value of s at t=0 is not equal to the value of s at t=T. This can be understood by considering both electrodes of a symmetric parallel plate discharge. Such a system and the relevant quantities are shown in Figure 3. The symmetry of the discharge requires that

$$s_B(t) = s_A(t \pm \frac{T}{2}) \quad . \tag{10}$$

Solutions for s_A and s_B that are consistent with equation (10) appear in Figure 4. Equation (7) is used to describe s_A over the time interval $\delta<t<T+\delta$. The solution for s_B is obtained by translating parts of s_A according to equation (10). These solutions were chosen so that the sheath at one electrode increases discontinuously whenever the thickness of the opposing sheath vanishes. If ϵ represents an infinitesimal time element, then by equation (7)

$$\begin{aligned} s_A(\delta+\epsilon) &= u_B T/2 + (2J_0\cos\omega\delta)/(en_0\omega) \\ s_A(T+\delta-\epsilon) = s_A(\delta-\epsilon) &= -u_B T/2 + (2J_0\cos\omega\delta)/(en_0\omega) \quad . \end{aligned} \tag{11}$$

Thus the sheath at electrode A increases in thickness by $u_B T$ around $t=\delta$ when the plasma contacts electrode B. Equation (1), governing the charge densities, implies that the negative surface charge density on electrode A increases in some infinitesimal time interval around $t=\delta$. This can be interpreted as meaning that electrons are absorbed at electrode B and are transferred through the external power supply to electrode A. Because electrode A is at a lower potential at this moment, this process requires energy expenditure by the power supply.

B. Power Calculation

The expression for the time-averaged power P absorbed by the discharge involves the integral of J_{ext} and the voltage between the electrodes V_{AB}, but because of the current pulses occurring every half cycle and the discontinuities occurring in the sheath thicknesses, it is useful to write this integral as a sum of four parts:

$$P = \frac{1}{T}\left\{\int_{\delta-\epsilon}^{\delta+\epsilon} dt\, V_{AB} J_{ext} + \int_{\delta+\epsilon}^{\tau-\epsilon} dt\, V_{AB} J_{ext} + \int_{\tau-\epsilon}^{\tau+\epsilon} dt\, V_{AB} J_{ext} + \int_{\tau+\epsilon}^{T+\delta-\epsilon} dt\, V_{AB} J_{ext}\right\} \quad (12)$$

Here, ϵ is an infinitesimal time element, and the quantity P is actually a power density with units of watts/m^2. The first and third integrals represent the work done on the current pulses by the source as the plasma contacts electrode B and A, respectively. The second and fourth integrals represent the work done during the portions of the rf cycle in which the current and voltage vary smoothly. A symmetric parallel plate discharge possesses the symmetry that the state of the discharge at some time t is related to the state of the discharge at time t-T/2 by a spatial inversion, or equivalently,

$$\begin{aligned} V_{AB}(t) &= -V_{AB}(t-\frac{T}{2}) \\ J_{ext}(t) &= -J_{ext}(t-\frac{T}{2}) \end{aligned} \quad (13)$$

Consequently, the first and third integrals are equal, the second and fourth integrals are equal.

First, the discharge voltage V_{AB} will be found. As illustrated in Figure 3, the discharge voltage may be expressed in terms of the sheath voltages V_{pA} and V_{pB}:

$$V_{AB} = V_{pB} - V_{pA} \quad . \quad (14)$$

Expressions for the sheath voltages follow from Poisson's equation and the boundary condition that the electric field vanishes at the sheath boundary. Given that the charge density in the sheath has a uniform value of en_0, one finds

$$V_{pi} = \frac{1}{2}\frac{en_0}{\epsilon_0}s_i^2 \quad , i = A \text{ or } B \quad , \tag{15}$$

where ϵ_0 is the permittivity of free space. Thus, V_{AB} is given by

$$V_{AB} = \frac{1}{2}\frac{en_0}{\epsilon_0}(s_B{}^2 - s_A{}^2) \quad . \tag{16}$$

The first integral in equation (12) can be rewritten as an integral over the sheath thickness s_A as follows: In the infinitesimal time interval around $t=\delta$, J_{ext} takes the form of a current pulse and the ion flux appearing equation (2) contributes nothing. The charge continuity expressed by equation (2) then becomes in differential form

$$J_{ext}dt = d\sigma_A \tag{17}$$

where σ_A is the charge density on electrode A. Equation (1) provides the differential relation between the charge density and sheath thickness:

$$d\sigma_A = -en_0 ds_A \quad . \tag{18}$$

In addition, the sheath voltage V_{pB} is zero in the infinitesimal time interval around $t=\delta$, and hence the voltage across the discharge is $-V_{pA}$. Hence, the form for the first integral in equation (12) becomes

$$\int_{\delta-e}^{\delta+e} dt\, V_{AB} J_{ext} = \int_{s_A(\delta-e)}^{s_A(\delta+e)} dt \left(-\frac{1}{2}\right)\frac{en_0}{\epsilon_0} s_A{}^2(-en_0)ds_A \quad . \tag{19}$$

Equation (11) gives s_A at the two limits of equation (19). Evaluation of equation (19) then yields the work done on the current pulse at $t=\delta$:

$$\int_{\delta-\epsilon}^{\delta+\epsilon} dt\, V_{AB} J_{ext} = J_0^2 \frac{u_B T}{\epsilon_0 \omega^2}\left[2 + (\frac{\pi^2}{6} - 2)\left(\frac{e n_0 u_B}{J_0}\right)^2\right] . \tag{20}$$

The second and fourth integrals in equation (12) represent the work done by the source during the part of the rf cycle when the current density and discharge voltage are smoothly varying. The discharge voltage follows from equation (16). Only equation (7) for s_A on $\delta<t<T+\delta$ is required since one may show with equations (5) and (10) that

$$\left[\int_{\delta+\epsilon}^{\tau-\epsilon} dt + \int_{\tau+\epsilon}^{T+\delta-\epsilon} dt\right](s_B^2 J_{ext}) = -\left[\int_{\delta+\epsilon}^{\tau-\epsilon} dt + \int_{\tau+\epsilon}^{T+\delta-\epsilon} dt\right](s_A^2 J_{ext}) . \tag{21}$$

The evaluation of the second and fourth integrals in equation (12) yields

$$\left[\int_{\delta+\epsilon}^{\tau-\epsilon} dt + \int_{\tau+\epsilon}^{T+\delta-\epsilon} dt\right] V_{AB} J_{ext} = J_0^2 \frac{u_B T}{\epsilon_0 \omega^2}\left[-\frac{5}{2} + \left(\frac{e n_0 u_B}{J_0}\right)^2\right] . \tag{22}$$

The remarkable feature of this term is that it is negative. In the smoothly varying parts of the rf cycle, the discharge is actually generating power on average. This contribution to the total work is quite significant; neglecting the terms that are second order in $e n_0 u_B / J_0$, this amounts to 62.5% of the work associated with the current pulses (given by twice the value in equation (20)).

Equations (20) and (22) can be inserted into (12) to find the total power absorbed by the discharge. Twice the value of equation (20) must be used to account for the first and third integrals in equation (12). The result is

$$P = J_0^2 \frac{u_B}{\epsilon_0 \omega^2}\left[\frac{3}{2} + (\frac{\pi^2}{3} - 3)\left(\frac{e n_0 u_B}{J_0}\right)^2\right] . \tag{23}$$

Equation (23) represents the rf power expended by the external power supply in driving the current density J_{ext} through the discharge. This result can be compared to the power P_{ion} consumed in accelerating ions through the time-averaged fields in the sheaths:

$$P_{ion} = 2\left(en_0 u_B \langle V_{pA}\rangle\right) \tag{24}$$

where $\langle V_{pA}\rangle$ denotes the time-averaged value of the V_{pA}, and the factor of two accounts for both sheaths since $\langle V_{pA}\rangle$ is equal to $\langle V_{pB}\rangle$. The time-averaged sheath voltage can be evaluated from equations (15) and (7) over the time interval $\delta<t<T+\delta$. The result is

$$\begin{aligned} P_{ion} &= \frac{(en_0)^2 u_B}{\epsilon_0}\langle s_A{}^2\rangle \\ &= J_0^2 \frac{u_B}{\epsilon_0 \omega^2}\left[\frac{3}{2} + (\frac{\pi^2}{3} - 3)\left(\frac{en_0 u_B}{J_0}\right)^2\right] \end{aligned} \tag{25}$$

which is the same as the power absorbed by the discharge, equation (23). Thus equations (23) and (25) confirm an energy balance between two processes, the first in which the discharge absorbs power in a time modulated fashion, and the second in which the power leaves the discharge in a steady manner.

III. Discussion

The results demonstrate two features of the symmetric parallel plate discharge. The first is a mechanism enabling the external power supply to replenish the energy the discharge loses in accelerating ions to the electrodes. The second is that energy is supplied by the discharge when the sheaths suffer a decrease in thickness as a result of ion losses from the plasma, resulting in an accumulation of the remaining electrons.

The mechanism for power absorption indicated by the foregoing analysis is that of a transfer of electrons from the electrode at higher potential to the electrode at lower potential when the plasma is in contact with the electrode at higher potential. In an actual experimental arrangement this manifests itself as an increase in the conduction current to the electrode, which can load the power supply and distort the rf wave form. One can imagine that this mechanism is equivalent to moving ions back across the sheath potential drop: an ion moves back across the sheath not as a charged particle, but in a neutralized state as an atom. However, for an ion residing on the electrode at lower potential, the only path by which it can acquire an electron is through the external circuit, which first acquires electrons from the electrode at higher potential. Of course, this process of electron transfer requires work to be done by the power supply. These ideas show that one can think of the power supply and the discharge as an engine that continually cycles ions back and forth across the sheath potential drop.

The analysis of section II demonstrates that the discharge generates power during part of the rf cycle. The importance of this result is not that it predicts negative power absorption, but rather that it identifies an available source of energy. Indeed, the analysis has omitted the dissipative processes of sheath heating and ohmic heating that would contribute to the power absorbed.[4,5,6] The power generation depends on ion losses from the plasma. As ion are lost from the plasma, the number of plasma electrons increases, as does the volume they occupy. This means that the sheaths become thinner, so that the stored electrostatic energy is less than would be the case if no ion losses were allowed. Thus, as the sheaths deflate under the action of ion losses, they serve as a source of energy.

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Figure Captions

Figure 1. The uniform sheath model. The ion density is uniform and the electron density behaves as a step function. The ion current density J_{ion}, the current density from the external circuit J_{ext}, the surface charge density on the electrode σ, and the sheath thickness s(t) are indicated.

Figure 2. The solution for the sheath thickness s(t) from equation (7). The ratio of u_B to $J_0/(en_0\omega)$ was chosen to be 0.1 to emphasize the discontinuity in s and the displacement of τ from T/2.

Figure 3. The symmetric parallel plate rf discharge system. The positive directions for J_{ext}, V_{AB}, V_{pA}, and V_{pB} are indicated by the arrows.

Figure 4. Sheath widths s_A and s_B, from equations (7) and (10). The ratio of u_B to $J_0/(en_0\omega)$ was chosen to be 0.1, and the times δ and τ, when the width of one sheath is zero, are indicated.

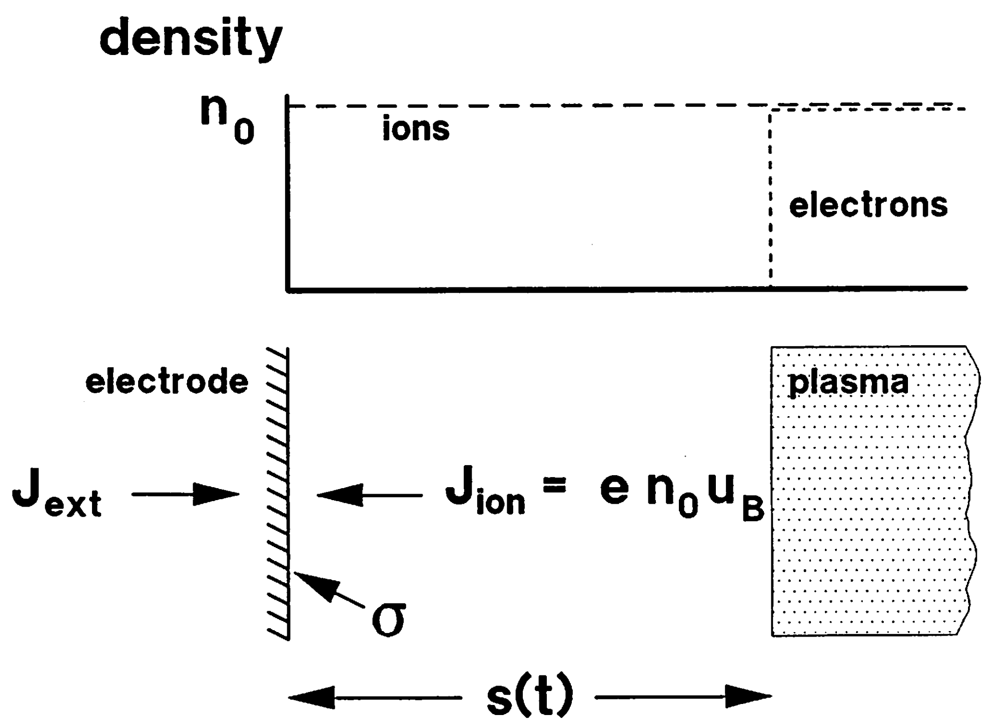
density
n_0
ions
electrons
electrode
plasma
J_{ext}
$J_{ion} = e\, n_0 u_B$
σ
s(t)

Figure 1

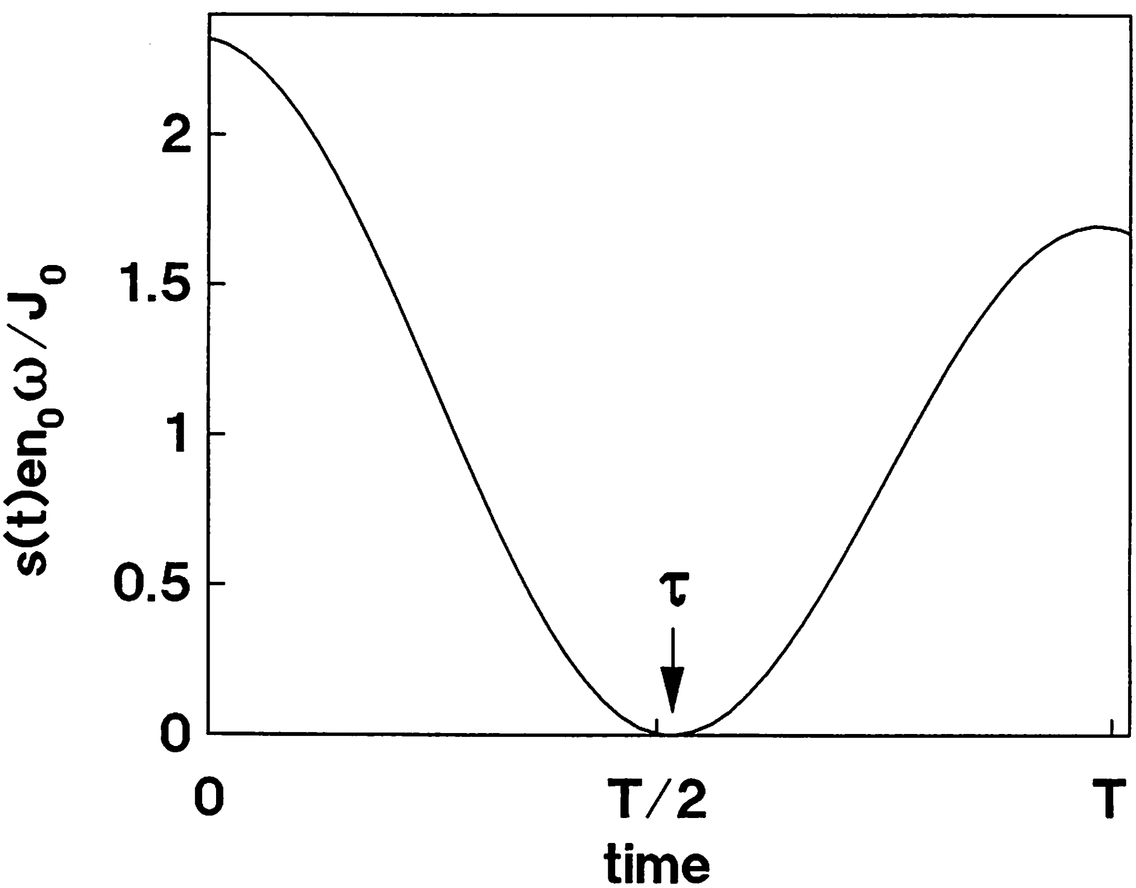

Figure 2

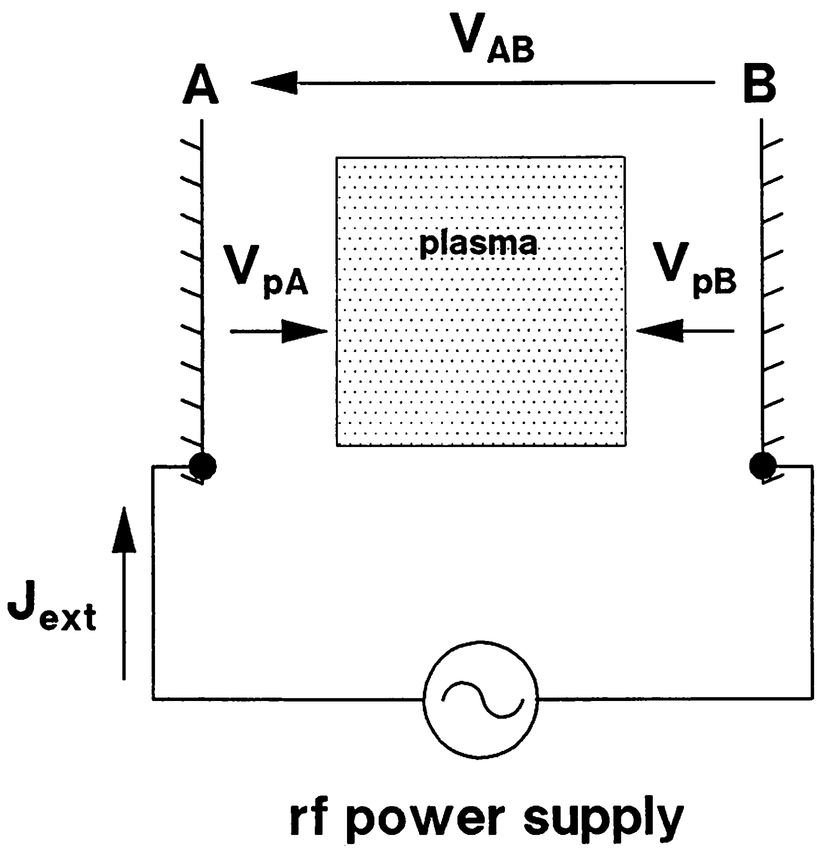

Figure 3

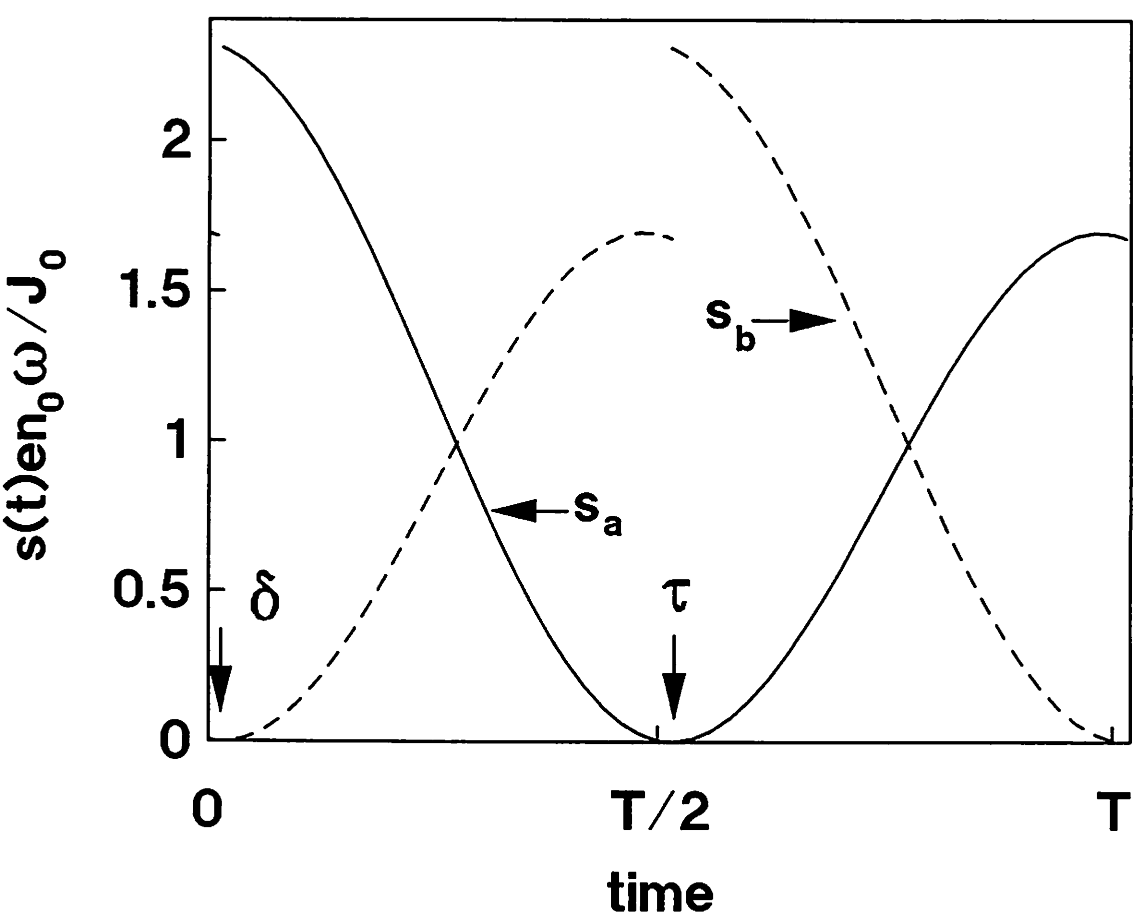

Figure 4