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Multi-folding: A New Chaotic Attractor Formation Mechanism in a Driven *R-L-Diode* Circuit

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Abstract

The paper reports *multi-folding*, a new chaotic attractor formation mechanism in a driven *R-L-Diode* circuit. This mechanism is responsible for the *repeated appearance of period-*1 *attractors* in the bifurcation diagram observed when driven at a fairly low frequency and small amplitude. Extensive measurements are performed in order to simplify the circuit dynamics to a 1-dimensional map without loosing any essential qualitative features. The *multi-folding,* when couched in terms of this 1-dimentional map is characterized by its *multi-modality*.

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The purpose of this paper is to report a new mechanism, called ***multi-folding***, for chaotic attractor formation in a driven *R-L-Diode* circuit. Inspite of its simplicity, the driven *R-L-Diode* circuit exhibits a very rich variety of interesting phenomena [1-20], including period doubling, chaotic attractor, intermittency and crisis. One of the most interesting features of the bifurcation diagram associated with this is that the large periodic windows and the chaotic bands alternate while the period increases exactly by one [4,12-16]. It has been demonstrated that a "folding" mechanism is responsible for this phenomenon[18]. Several other works [10,19,20] have pointed out that qualitatively different bifurcations are taking place at the ***lower frequency*** and the ***smaller*** voltage source ***amplitude*** region of the bifurcation diagram. Figure 1 is the one-parameter bifurcation diagram of the circuit which motivated the present study. The horizontal axis is the ***amplitude*** of sinusoidal voltage source while the vertical axis is the current of the circuit sampled at a particular phase of the sinusoidal voltage source. The circuit parameters are;

R=75 Ω, L=2.5 mH, Diode: 3CC13,

DC bias voltage E_b=-1.0 V f = 30 kHz, $0 \leq E \leq 4.0$ V.

This bifurcation diagram is qualitatively different from those previously reported [4,12-16], in that rather than increasing the period of each successive periodic windows by one, ***period-1 windows*** and chaotic bands appear alternatively. Namely, something qualitatively different is happening at lower frequencies, smaller voltage source amplitudes and lower dc bias levels.

Our approach in the present work is to carefully observe the chaotic attractors and construct a simple ***one-dimensional discrete map*** model which captures the important qualitative features of the above observed bifurcation phenomena. The mechanism of our interest, in terms of the one-dimensional map model, turns out to be its ***multi-modality***. This, in turn, translates back to a "multi-folding" mechanism in the original dynamics.

First, we consider the dynamics of the *R-L-Diode* circuit. A fairly accurate equivalent circuit of a junction diode is given by a parallel connection of three nonlinear elements[21]:

(1) nonlinear resistor

$$I_d = I_s\,(\exp((q'v)/(kT))-1) \qquad (1)$$

(2) junction capacitor $C_j(v)$ due to the depletion region;

$$C_j(v) = C_{j0}/(1-v/V_{j0})^{1/2} \tag{2}$$

(3) diffusion capacitor $C_d\,(v)$ due to the rearrangement of the minority carrier density

$$C_d(v) = C_{d0}\exp((q'v)/(kT)) \tag{3}$$

where I_s, q', k, T, V_{j0}, C_{j0} and C_{d0} are the saturation current, electron charge, Boltzmann constant, the absolute temperature, the potential voltage of the pn junction, the junction capacitance at zero bias and the diffusion capacitance at zero bias, respectively.

Note that under reverse bias, the capacitor is dominated by the *junction* capacitor (2), whereas under forward bias, the capacitor is dominated by the ***diffusion*** capacitor (3). By measurements the capacitance is found to be 90 nF at 0.5 V (a positive bias) and 235 pF at -1.0 V (a negative bias). Note that the difference in the capacitance values is more than ***two orders of magnitude***. The diode exhibits also a rectification characteristic (1) : in the reverse bias region the resistance is almost infinite, whereas in the forward bias region the resistance is very small. For example at 0.5 V the resistance is 100 Ω. By carefully measuring the impedances of the capacitors and the resistor over a frequency range of more than 30 kHz, it was found that the impedances of the capacitors are much smaller than that of the resistor. Therefore the diode characteristic can be simplified and modeled by a 2-segment piecewise-linear capacitor[12] so that the dynamics of the *R-L-Diode* circuit can be accurately described by

$$\frac{dq}{dt} = i$$

$$L\frac{di}{dt} = -Ri - \begin{Bmatrix} \frac{1}{C_d}q & \text{if } q \geq 0 \\ \frac{1}{C_j}q & \text{if } q < 0 \end{Bmatrix} - E_d + E_b + E\sin(\omega t) \tag{4}$$

where C_d is the diffusion capacitance at 0.5 V bias, C_j is the junction capacitance at -1.0 V, E_d = 0.5 V is the break point voltage at which the capacitance value changes between the junction capacitance and the diffusion capacitance, i is the circuit current and q is the charge of the capacitor.

In order to uncover the attractor formation mechanism, various cross sections of the attractor are measured. Shown in Fig.2 are the cross sections measured at different phases of the input sinusoidal waveform. The horizontal axis is the diode voltage v_d and the vertical axis is the current i. The phases are increased in the order of (a), (b),---(f).

(a) The attractor is in the region which is dominated by the diffusion capacitor.

(b) A part of the attractor moves into the region which is dominated by the junction capacitor. When the attractor moves from the diffusion capacitor region into the junction capacitor region, it is ***stretched*** because of the difference of the vector fields in the diffusion capacitor region and the junction capacitor region .

(c) The attractor is ***curled up*** in the junction capacitor region and moves into the diffusion capacitor region again.

(d) The attractor is stretched again and curled up.

(e) The attractor moves into the diffusion capacitor region again. In this process the attractor is ***folded***.

(f) The attractor returns to the initial region.

Figure 3 gives the geometric structure corresponding to each figure in Fig.2, where the small triangle shows the reference orientation.

We now propose the one-dimensional map :

$$x_{n+1} = a(1 - \cos b(1 - x_n)) \tag{5}$$

as a model capable of reproducing the dynamics in Fig.3.

In order to show that (5) captures all the important features of the observed bifurcations, we will discuss the dependencies of a and b on E and f only roughly for our present purpose. Our analysis is based upon extensive laboratory measurements.

(i) *parameter a*

This parameter controls the extrema of the mapping. In other words a controls the size of the attractor. From the observations of Fig.2 the size of the attractor is proportional to E. Therefore a should be proportional to E, the amplitude of the voltage source. Moreover, a is inversely proportional to the dissipation which is given by $\exp(R/(2fL))$. Therefore

$$a \propto (E + a_1)\exp(-R/(2fL)) \tag{6}$$

would be an appropriate relationship, where a_1 is a parameter.

(ii) *parameter b*

This parameter controls the number of extrema of (5) which corresponds to the number of rotations in the junction capacitor region. The latter should be proportional to the imaginary part ω of the eigenvalue of the junction capacitor region, and the length t_4 of the time interval on which the attractor stays in the junction capacitor region; namely,

$$b \propto \omega \cdot t_4 + \theta_2 \tag{7}$$

where θ_2 represents the phase constant.

Based on these observations, we will now show that (5) captures essentially all the important bifurcation phenomena of the original circuit.

When E is increased, the attractor moves from the junction capacitor region to the diffusion capacitor region. Therefore t_4 is inversely proportional to the amplitude E. Note also that the imaginary part of the eigenvalue is given by

$$\frac{\sqrt{\frac{4L}{C_2} - R^2}}{2fL}$$

When E is increased it has been observed that the change of the number of rotations becomes more moderate. This factor is represented by θ_2, and the relationship

$$\theta_2 \propto b_1(1 - b_2/(E + b_2))$$

seems reasonable, where b_1 and b_2 are parameters. Therefore we will write (7) as

$$b \propto \frac{\sqrt{\frac{4L}{C_2} - R^2}}{2fL} \frac{b_3}{E + b_3} + b_1(1 - \frac{b_2}{E + b_2}) \tag{9}$$

where b_3 is a parameter. Figure 4 shows the bifurcation diagrams of (5) where the horizontal axis is E and the vertical axis is x_n. The parameter values are chosen as follow :

$R = 214\ \Omega$; $C_2 = 235$ pF; $L = 2.50$ mH; $a_1 = 0.15$;b_1=2.4; b_2=1.0 ;$b_3 = 0.2$

The frequency of the voltage source is fixed at f=35 kHz, while the amplitude of the voltage source is varied from 0 V to 1.5 V. Observe that the basic qualitative features of Fig.1 are clearly captured. The ***multi-modality*** of the sine function dominates the dynamics. As E increases, the number of extrema decreases. Figure 5 (a)-(c) show the orbits of (5) at the parameter values as indicated in (a)-(c), respectively in Fig.4.

It is clear that (5) undergoes a saddle-node bifurcation when it becomes *tangent* to the diagonal line. Since the extremum value of (5) is determined by the parameter a and since a is *monotonic with respect to E* (see (6)), the only possible reason for (5) to undergo repeated period one saddle-node bifurcations is its

$$\boxed{\textbf{\textit{multi-modality.}}}$$

Namely, the hills and valleys of (5) become tangent to the diagonal one by one. In terms of the original circuit dynamics, this means that an initial rectangle is mapped into a "multi-folded object" (see Fig.2,3), *i.e.*

$$\boxed{\textbf{\textit{multi-folding}}}$$

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REFERENCES

[1]P.S.Linsay, Phys. Rev. Lett. **47** (1981) 1349-1352.

[2]J.Testa, J.Perez and C.Jeffries, Phys. Rev. Lett. **48** (1982) 714-717.

[3]R.W.Rollins and E.R.Hunt, Phys. Rev. Lett. **49** (1982) 1295-1298.

[4]S.D.Bronson, D.Dewey and P.S.Linsay, Phys. Rev. A **28** (1983) 1201-1203.

[5]H.Ikezi, J.S.deGrassie and T.H.Jenson, Phys. Rev. A **28** (1983) 1207-1209.

[6]J.Cascais, R.Dilao and A.Noronha Da Costa, Phys. Lett. **93A** (1983) 213-216.

[7]D.Azzouz, R.Duhr and M.Hasler, IEEE Trans. CAS **30** (1983) 913-914.

[8]E.R.Hunt and R.W.Rollins, Phys. Rev. A **29** (1984) 1000-1002.

[9]T.Klinker, W.M.Ilse and W.Lauterborn, Phys. Lett. **101A** (1984) 371-375.

[10]M.F.Bocko, D.H.Douglass and H.Frutchy, Phys. Lett. **104A** (1984) 388-390.

[11]D.Azzouz, R.Duhr and M.Hasler, IEEE Trans. CAS **31** (1984) 1155-1157.

[12]T.Matsumoto, L.O.Chua and S.Tanaka, Phys. Rev. A **30** (1984) 1155-1157.

[13]T.H.Yoon, J.W.Song, S.Y.Shin and J.W.Ra, Phys. Rev. A **30** (1984) 3347-3350.

[14]C.D.Jeffries, Phys. Scri. **T9** (1985) 11-26.

[15]S.Tanaka, T.Matsumoto and L.O.Chua, Proc. IEEE Int. Symp. on Circuits and Systems (1985) 851-854.

[16]J.M.Perez, Phys. Rev. A **32** (1985) 2990-2992.

[17]J.Mevissen, R.Seal and L.Waters, Phys. Rev. A **32** (1985) 2990-2992.

[18]S.Tanaka, T.Matsumoto and L.O.Chua, Physica **28D** (1987) 317-344.

[19]J.Noguchi, S.Tanaka, T.Matsumoto and L.O.Chua, IEICE Tech. Rep. NLP88-24 (1988) 33-44.

[20]J.H.Baxter, M.F.Bocko and D.H.Douglass, Phys.Rev. A **41** (1990) 619-625.

[21]E.S.Yang, Fundamentals of Semiconductor Devices (McGraw-Hill, New York, 1987).

FIGURE CAPTIONS

Figure 1 One-parameter bifurcation diagram.

The horizontal axis is the amplitude of the voltage source E (0.5 V/div), the vertical axis is the inductor current i_L (2.0 mA/div), and the source frequency f is 30 kHz.

Figure 2 Observed cross sections of the *R-L-Diode* circuit at E=2.4 V, f=50 kHz.

The horizontal axis is the diode voltage v_d (5.0 V/div), and the vertical axis is the inductor current i_L (2.0 mA/div). Since the origin is not located at the center of each figure, the axes are indicated by arrows.

Figure 3 Geometric model of the attractor formation. Each figure corresponds to the one in same position in Fig.2.

Figure 4 One-parameter bifurcation diagram of the one-dimentional map.

$0\ \mathrm{V} \leq E \leq 1.5\ \mathrm{V},\ f = 35\ \mathrm{kHz}.$

Figure 5 Orbits of the one-dimensional map.

(a) E = 0.18 V. (b) E = 0.36 V. (c) E = 0.85 V.

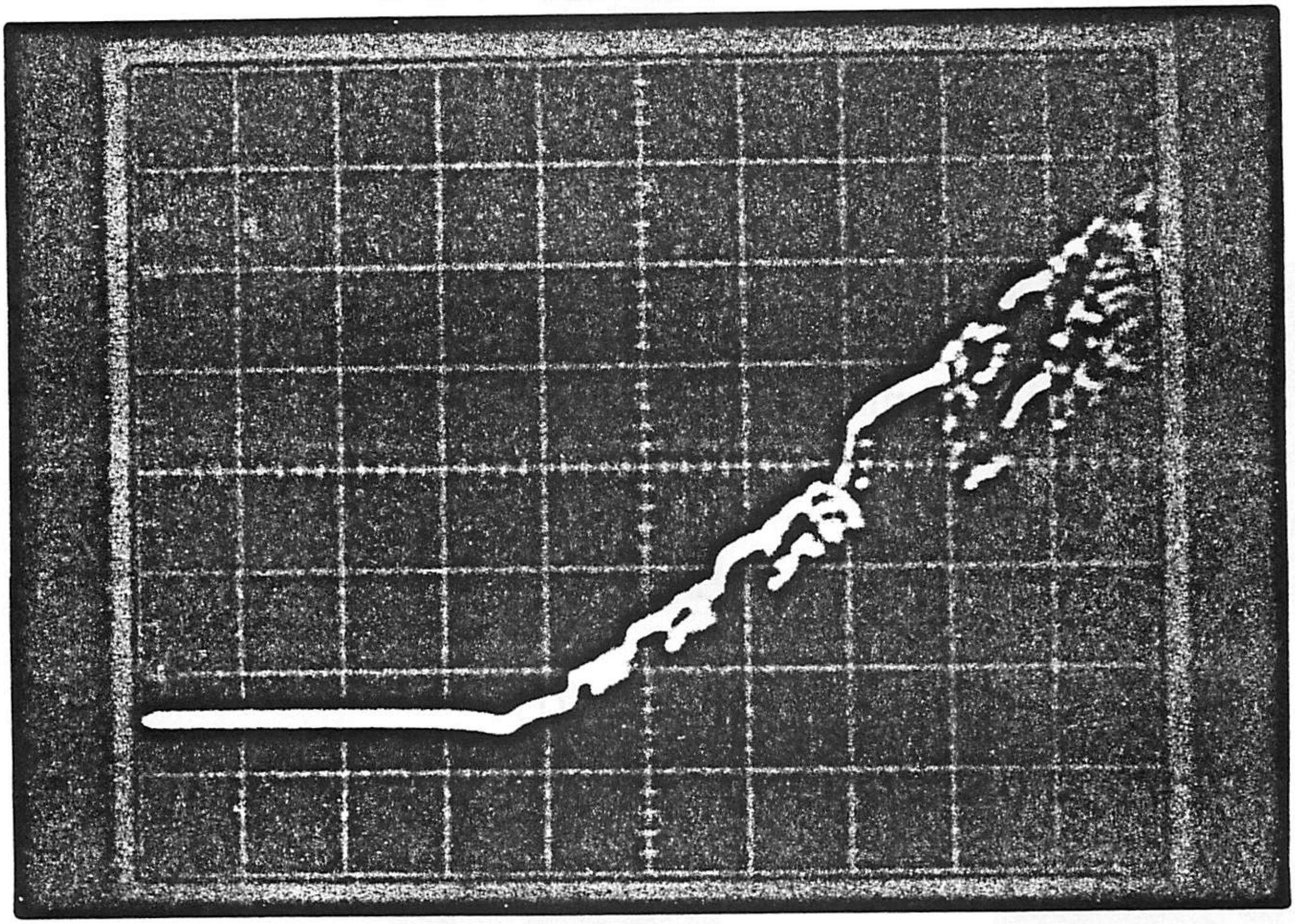

Figure 1

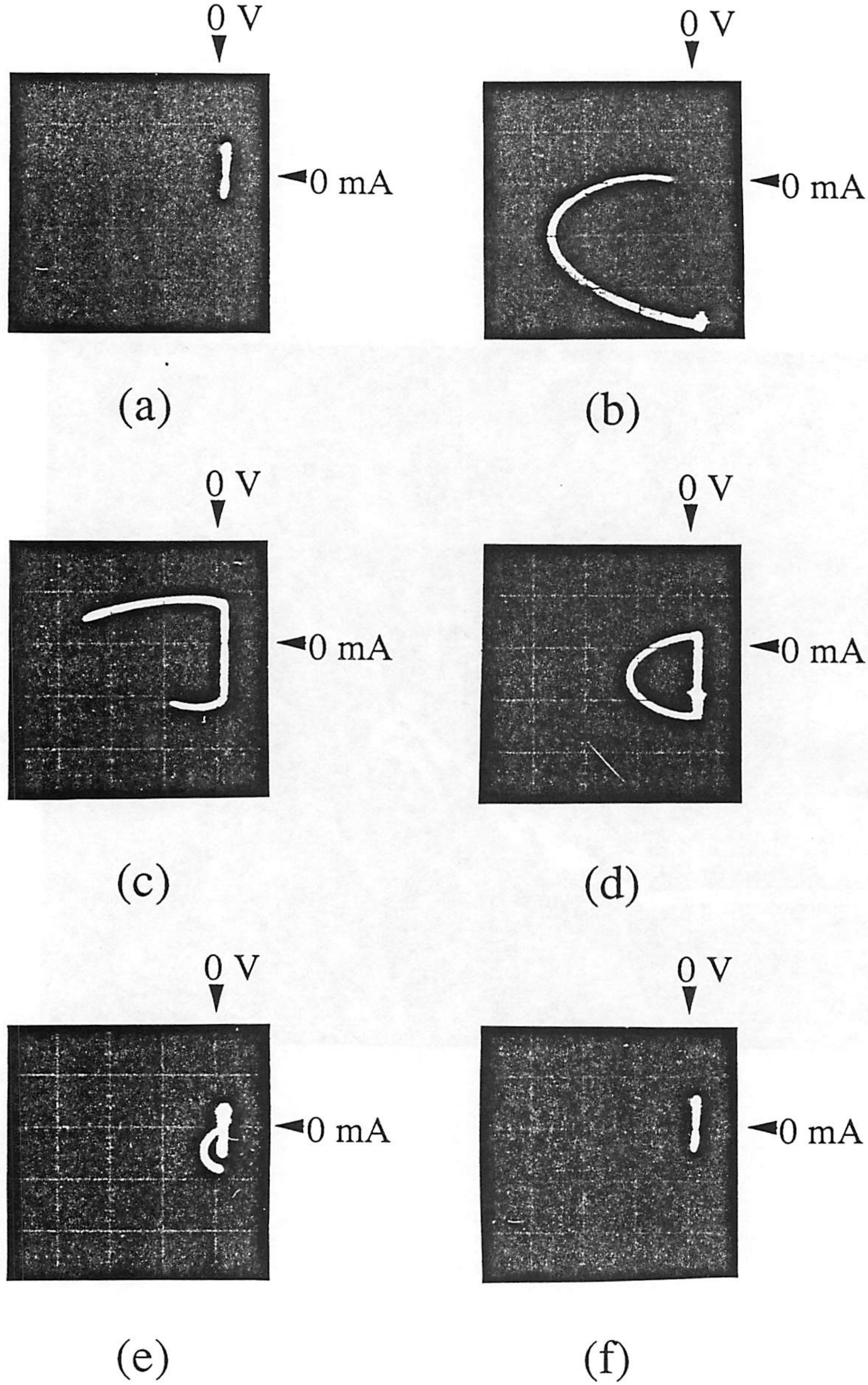

Figure 2

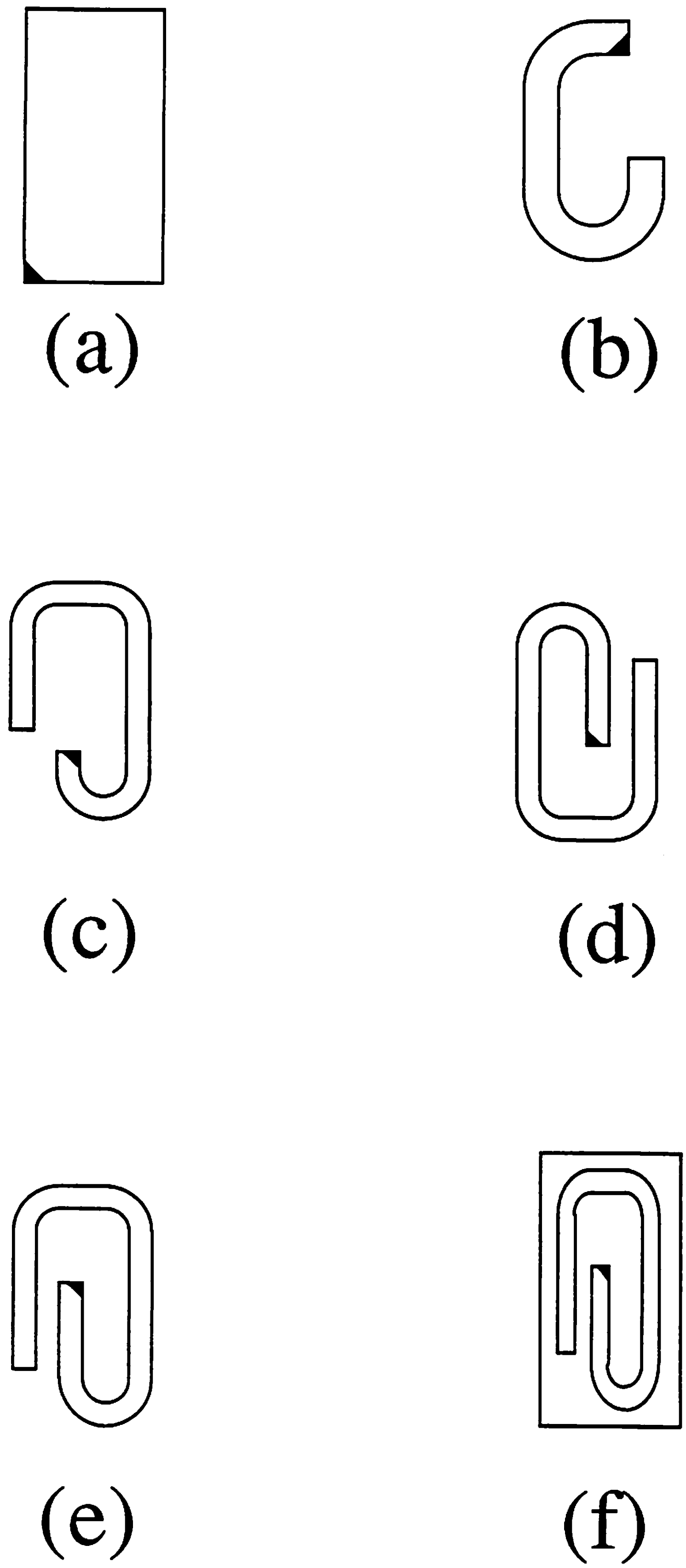

Figure 3

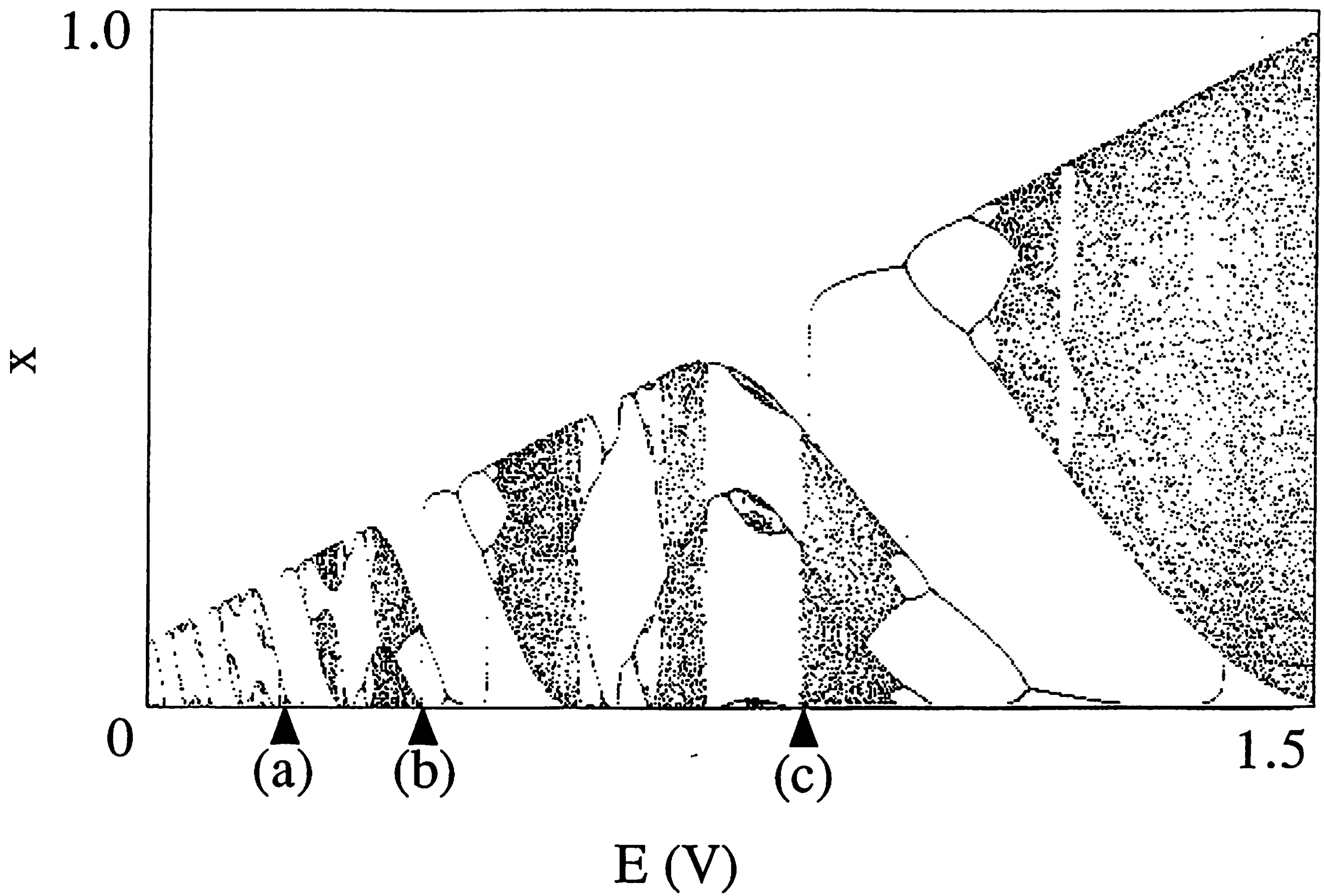

Figure 4

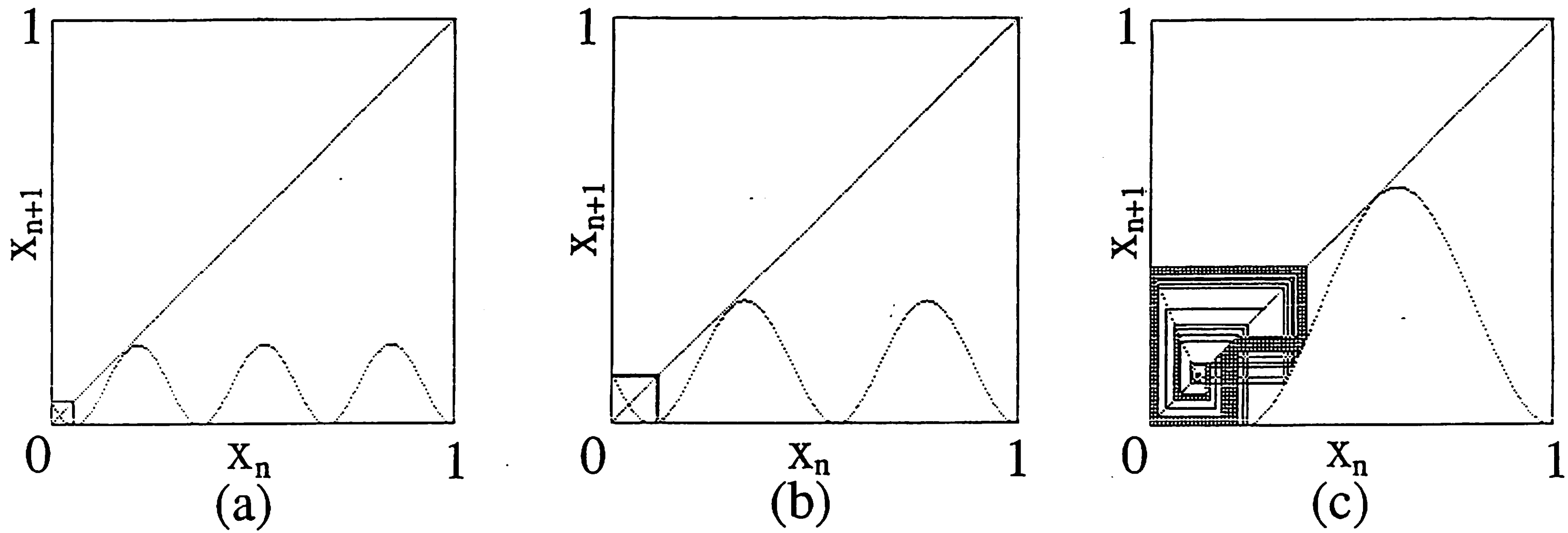

Figure 5